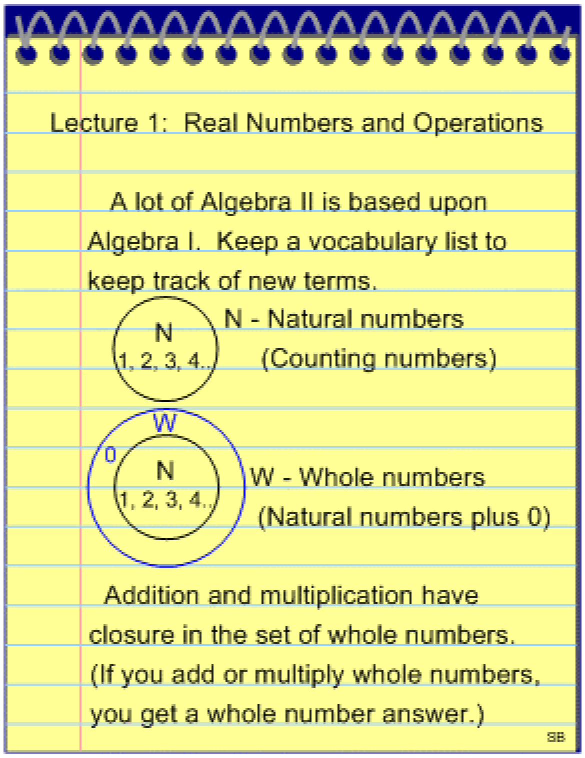
ALGEBRA II NOTES

Lecture 1 Notes

ALG2001-01



Lecture 1: Real Numbers and Operations

A lot of Algebra II is based upon Algebra I. Keep a vocabulary list to keep track of new terms.

N - Natural numbers (Counting numbers)

W - Whole numbers (Natural numbers plus 0)

Addition and multiplication have closure in the set of whole numbers. (If you add or multiply whole numbers, you get a whole number answer.)

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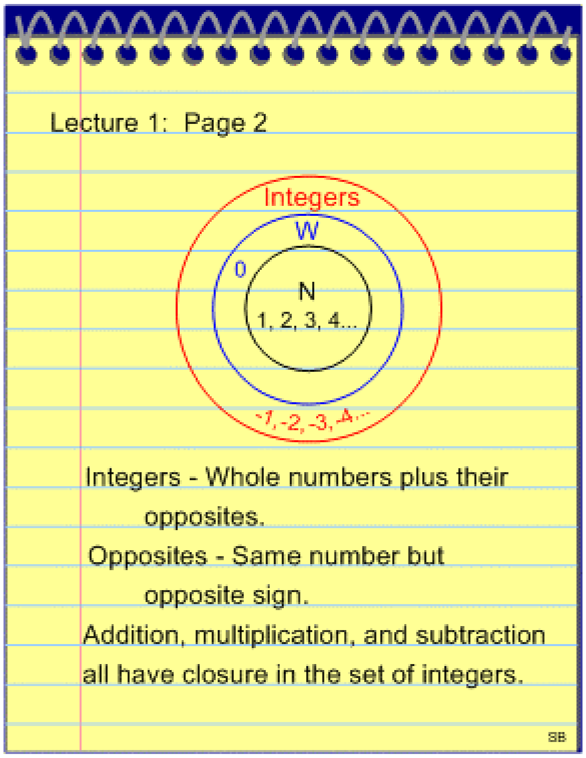

Lecture 1: Page 2

Integers - Whole numbers plus their opposites.

Opposites - Same number but opposite sign.

Addition, multiplication, and subtraction all have closure in the set of integers.

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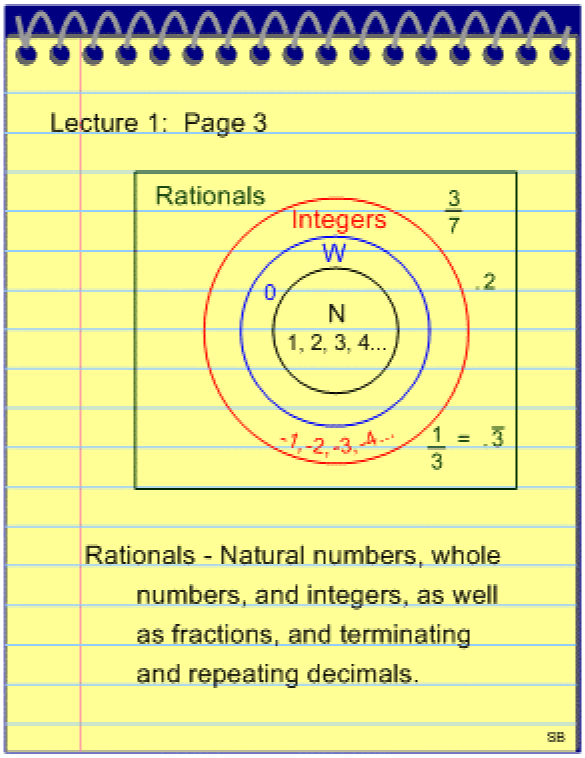

Lecture 1: Page 3

Rationals - Natural numbers, whole numbers, and integers, as well as fractions, and terminating and repeating decimals.

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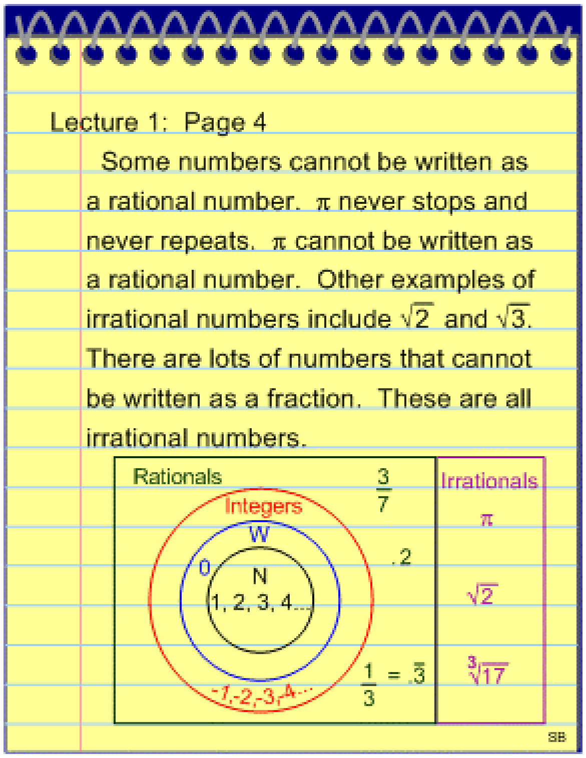

Lecture 1: Page 4

Some numbers cannot be written as a rational number. π never stops and never repeats. π cannot be written as a rational number. Other examples of irrational numbers include $\sqrt{2}$ and $\sqrt{3}$. There are lots of numbers that cannot be written as a fraction. These are all irrational numbers.

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Lecture 1 Notes, Continued

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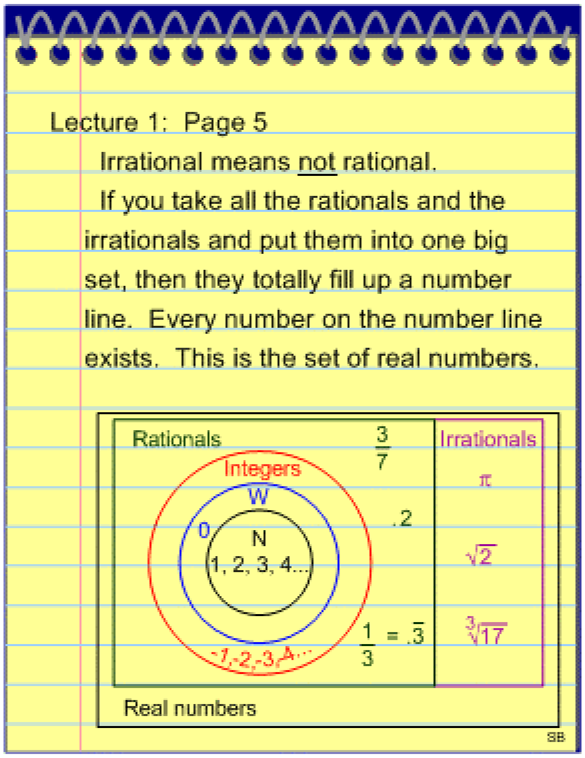

Lecture 1: Page 5

Irrational means not rational.

If you take all the rationals and the irrationals and put them into one big set, then they totally fill up a number line. Every number on the number line exists. This is the set of real numbers.

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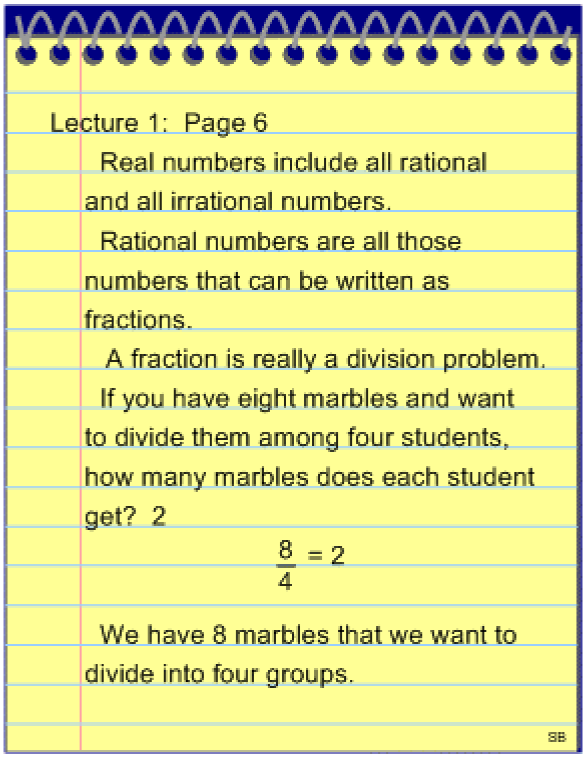

Lecture 1: Page 6

Real numbers include all rational and all irrational numbers.

Rational numbers are all those numbers that can be written as fractions.

A fraction is really a division problem.

If you have eight marbles and want to divide them among four students, how many marbles does each student get? 2

$$\frac{8}{4} = 2$$

We have 8 marbles that we want to divide into four groups.

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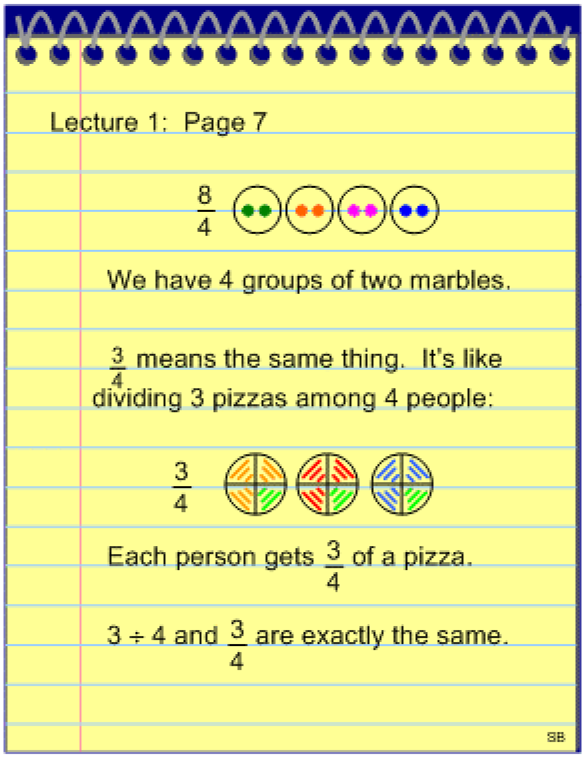

Lecture 1: Page 7

$\frac{8}{4}$

We have 4 groups of two marbles.

$\frac{3}{4}$ means the same thing. It's like dividing 3 pizzas among 4 people:

$\frac{3}{4}$

Each person gets $\frac{3}{4}$ of a pizza.

$3 \div 4$ and $\frac{3}{4}$ are exactly the same.

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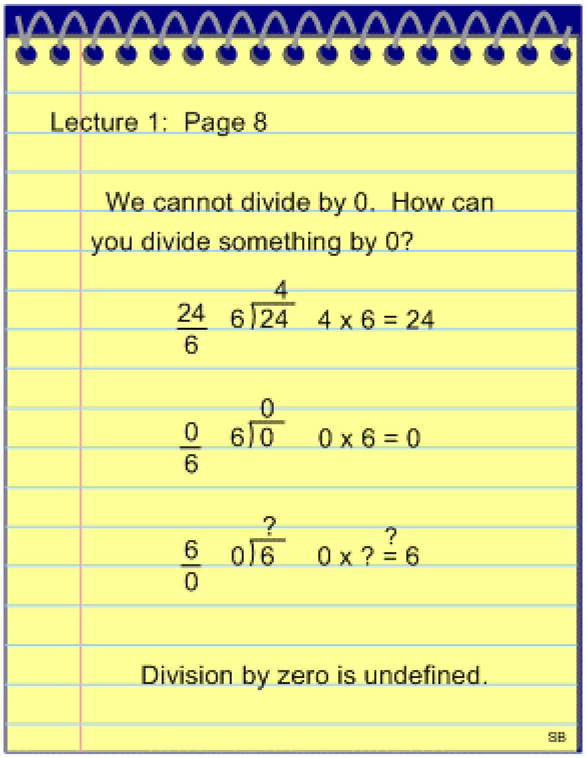


Lecture 1: Page 8

We cannot divide by 0. How can you divide something by 0?

$$\frac{24}{6} \quad 6\overset{4}{\overline{)24}} \quad 4 \times 6 = 24$$

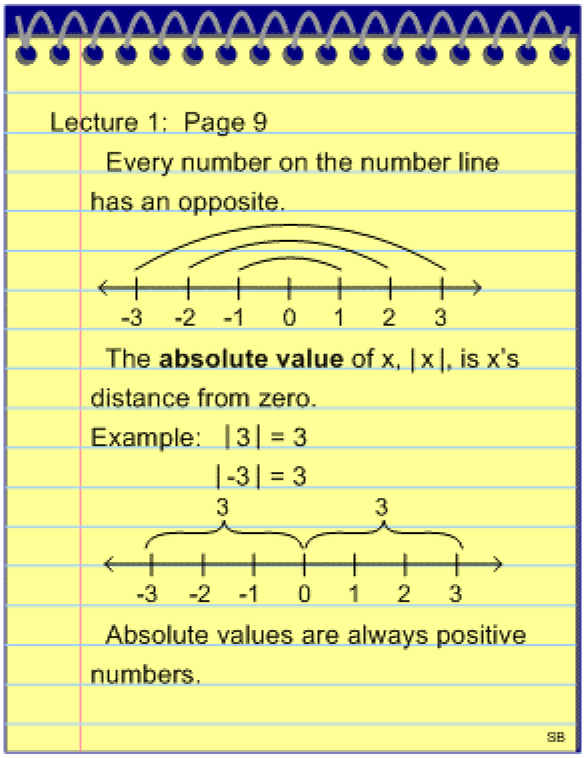
$$\frac{0}{6} \quad 6\overset{0}{\overline{)0}} \quad 0 \times 6 = 0$$

$$\frac{6}{0} \quad 0\overset{?}{\overline{)6}} \quad 0 \times ? \overset{?}{=} 6$$

Division by zero is undefined.

Lecture 1 Notes, Continued

ALG2001-09



Lecture 1: Page 9

Every number on the number line has an opposite.

The **absolute value** of x, |x|, is x's distance from zero.

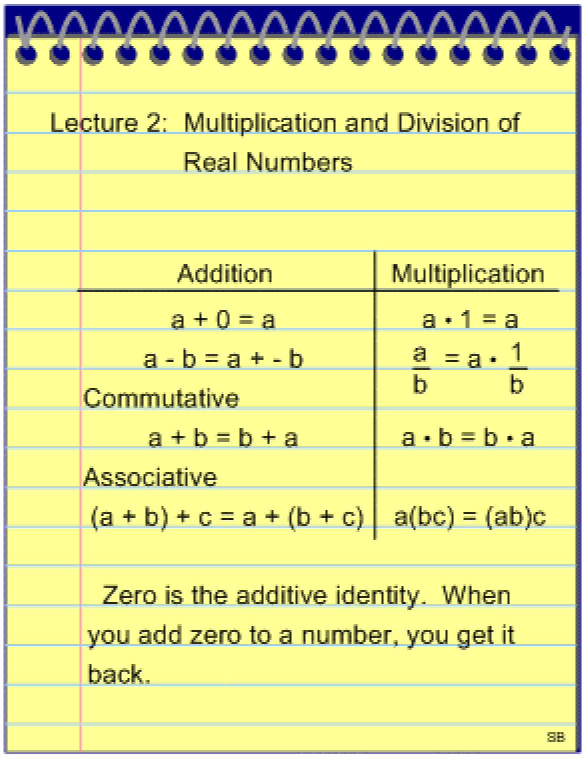
Example: |3| = 3

|-3| = 3

Absolute values are always positive numbers.

Lecture 2 Notes

ALG2002-01



Lecture 2: Multiplication and Division of Real Numbers

Addition	Multiplication
$a + 0 = a$	$a \cdot 1 = a$
$a - b = a + - b$	$\frac{a}{b} = a \cdot \frac{1}{b}$
Commutative	
$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	
$(a + b) + c = a + (b + c)$	$a(bc) = (ab)c$

Zero is the additive identity. When you add zero to a number, you get it back.

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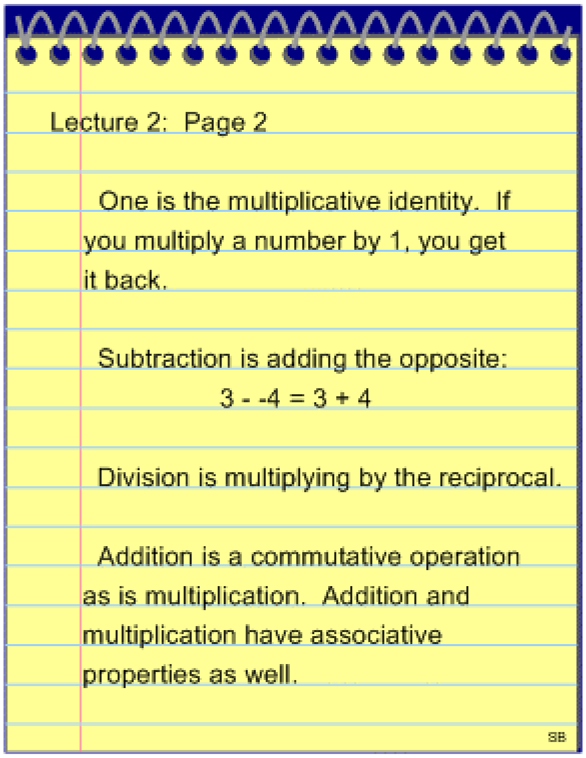

Lecture 2: Page 2

One is the multiplicative identity. If you multiply a number by 1, you get it back.

Subtraction is adding the opposite:

$3 - -4 = 3 + 4$

Division is multiplying by the reciprocal.

Addition is a commutative operation as is multiplication. Addition and multiplication have associative properties as well.

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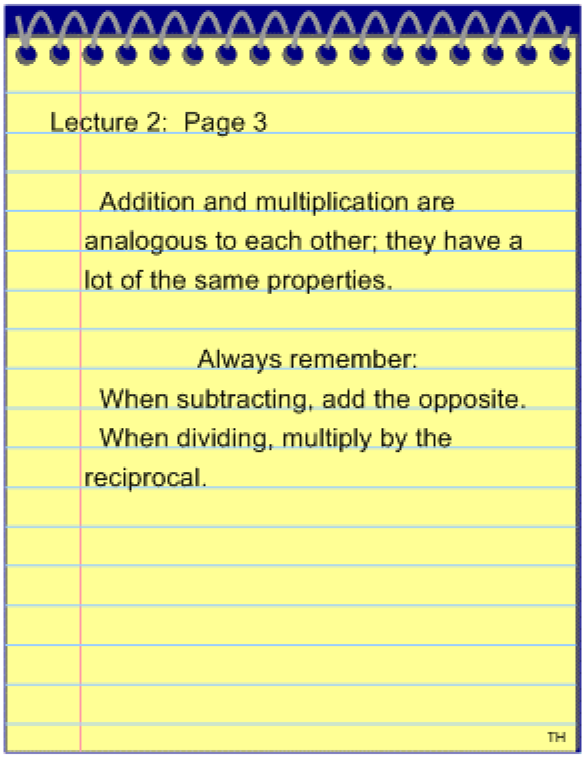

Lecture 2: Page 3

Addition and multiplication are analogous to each other; they have a lot of the same properties.

Always remember:

When subtracting, add the opposite.

When dividing, multiply by the reciprocal.

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Lecture 3 Notes

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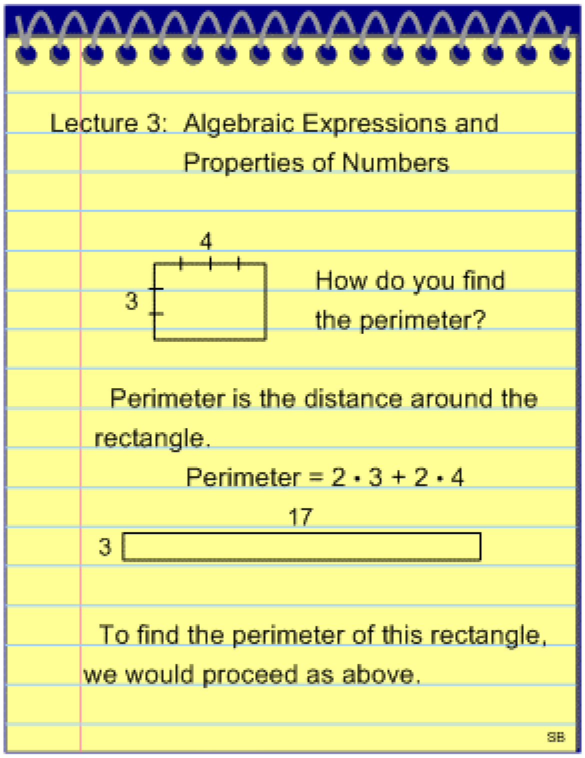


Lecture 3: Algebraic Expressions and Properties of Numbers

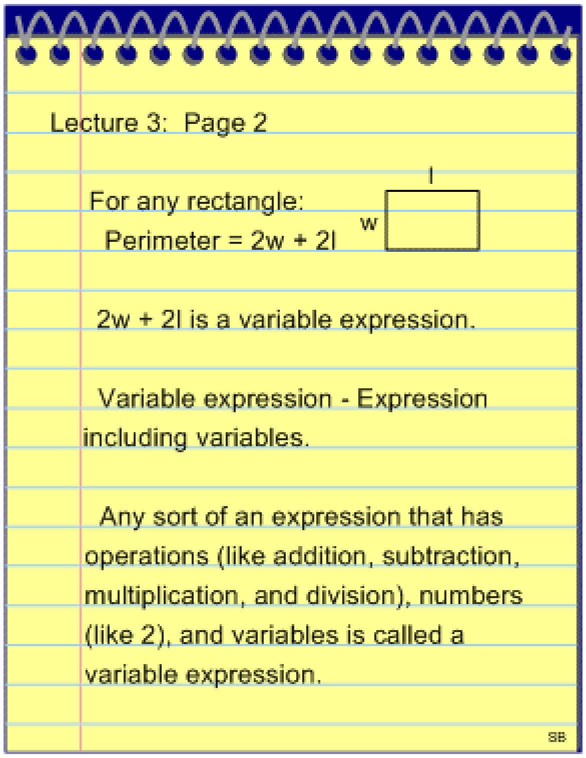
How do you find the perimeter?

Perimeter is the distance around the rectangle.

Perimeter = 2 • 3 + 2 • 4

To find the perimeter of this rectangle, we would proceed as above.

ALG2003-02



Lecture 3: Page 2

For any rectangle:
Perimeter = 2w + 2l

2w + 2l is a variable expression.

Variable expression - Expression including variables.

Any sort of an expression that has operations (like addition, subtraction, multiplication, and division), numbers (like 2), and variables is called a variable expression.

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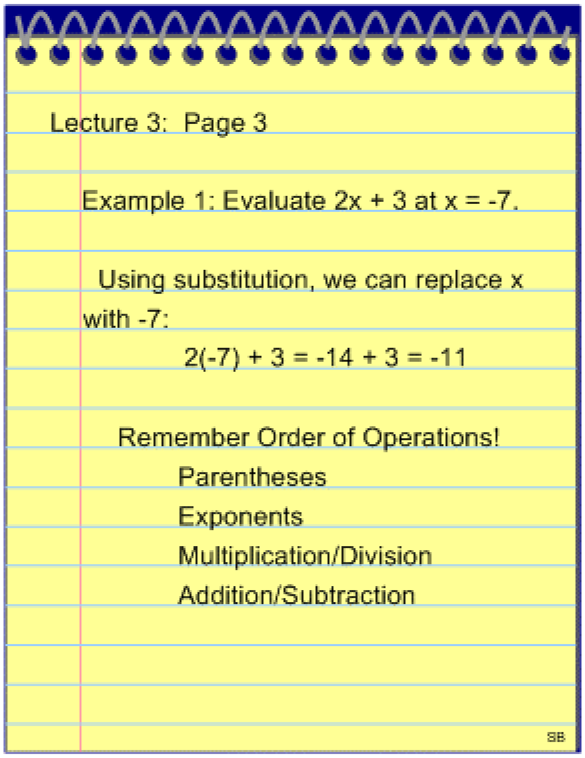


Lecture 3: Page 3

Example 1: Evaluate 2x + 3 at x = -7.

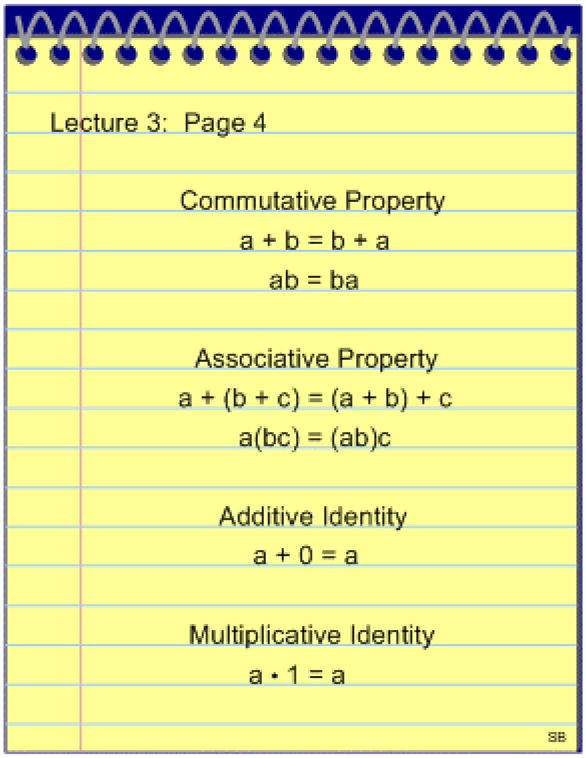
Using substitution, we can replace x with -7:

2(-7) + 3 = -14 + 3 = -11

Remember Order of Operations!

Parentheses
Exponents
Multiplication/Division
Addition/Subtraction

ALG2003-04



Lecture 3: Page 4

Commutative Property
a + b = b + a
ab = ba

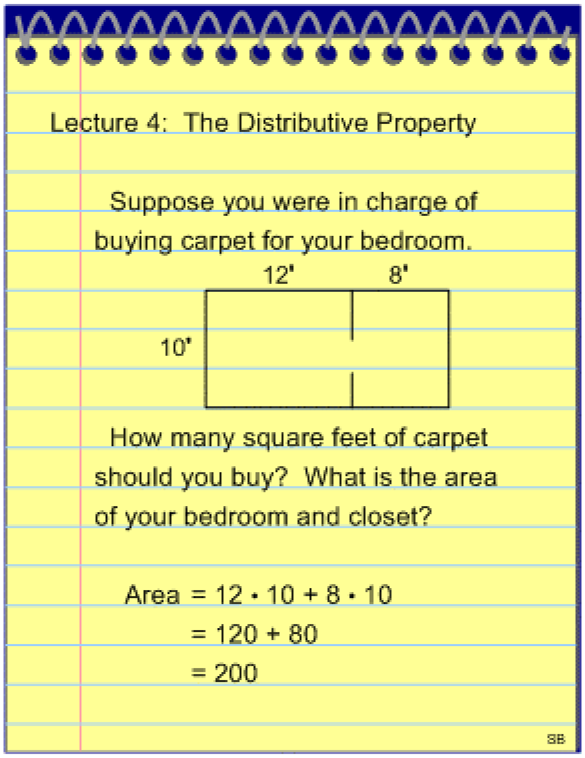
Associative Property
a + (b + c) = (a + b) + c
a(bc) = (ab)c

Additive Identity
a + 0 = a

Multiplicative Identity
a • 1 = a

Lecture 4 Notes

ALG2004-01



Lecture 4: The Distributive Property

Suppose you were in charge of buying carpet for your bedroom.

How many square feet of carpet should you buy? What is the area of your bedroom and closet?

Area = 12 • 10 + 8 • 10
= 120 + 80
= 200

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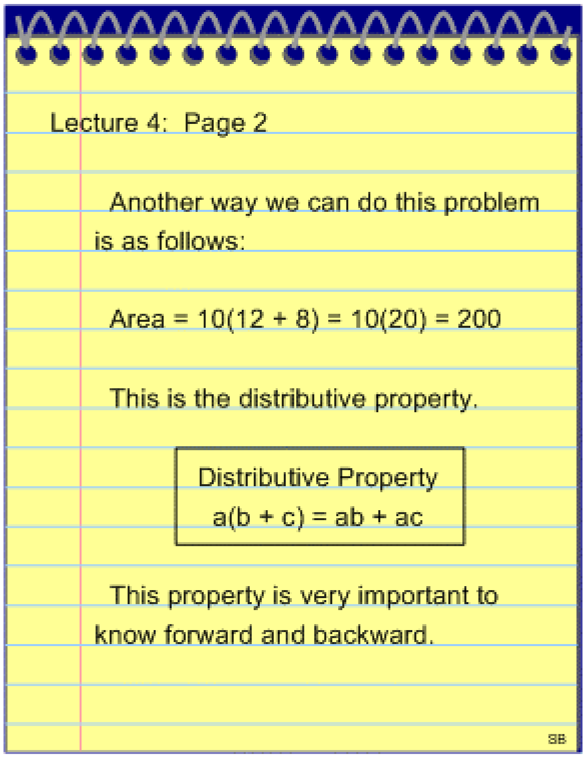

Lecture 4: Page 2

Another way we can do this problem is as follows:

Area = 10(12 + 8) = 10(20) = 200

This is the distributive property.

Distributive Property
a(b + c) = ab + ac

This property is very important to know forward and backward.

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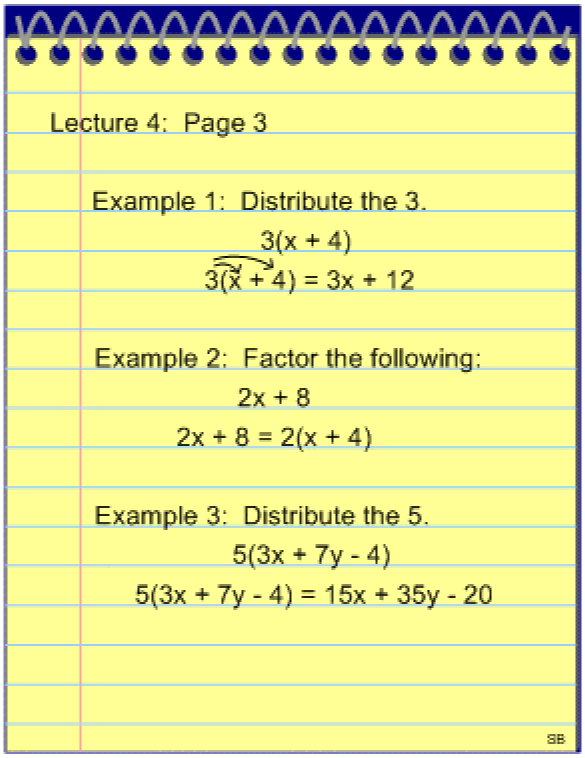

Lecture 4: Page 3

Example 1: Distribute the 3.

3(x + 4)

3(x + 4) = 3x + 12

Example 2: Factor the following:

2x + 8

2x + 8 = 2(x + 4)

Example 3: Distribute the 5.

5(3x + 7y - 4)

5(3x + 7y - 4) = 15x + 35y - 20

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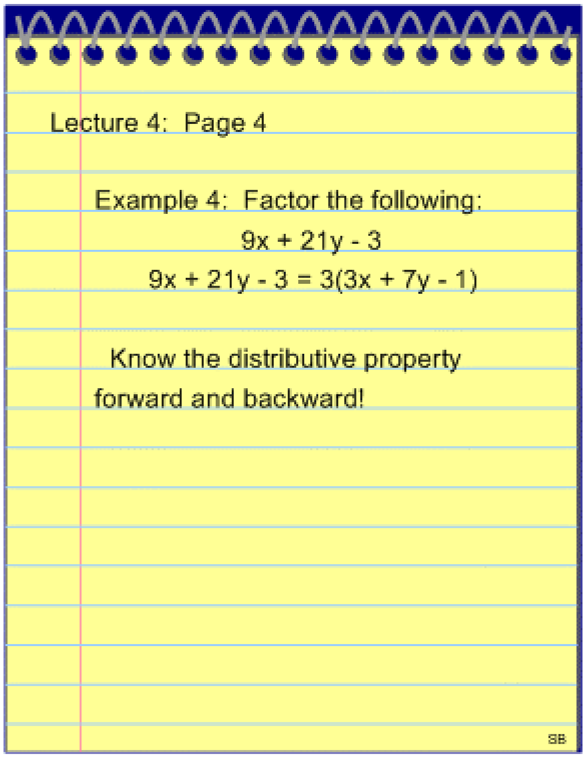

Lecture 4: Page 4

Example 4: Factor the following:

9x + 21y - 3

9x + 21y - 3 = 3(3x + 7y - 1)

Know the distributive property forward and backward!

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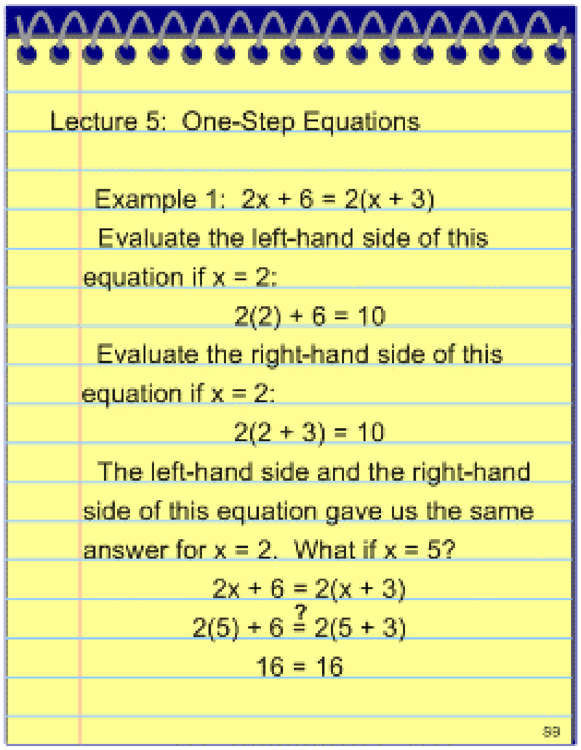
Lecture 5 Notes

ALG2005-01



Lecture 5: One-Step Equations

Example 1: $2x + 6 = 2(x + 3)$

Evaluate the left-hand side of this equation if $x = 2$:

$$2(2) + 6 = 10$$

Evaluate the right-hand side of this equation if $x = 2$:

$$2(2 + 3) = 10$$

The left-hand side and the right-hand side of this equation gave us the same answer for $x = 2$. What if $x = 5$?

$$2x + 6 = 2(x + 3)$$

$$2(5) + 6 \overset{?}{=} 2(5 + 3)$$

$$16 = 16$$

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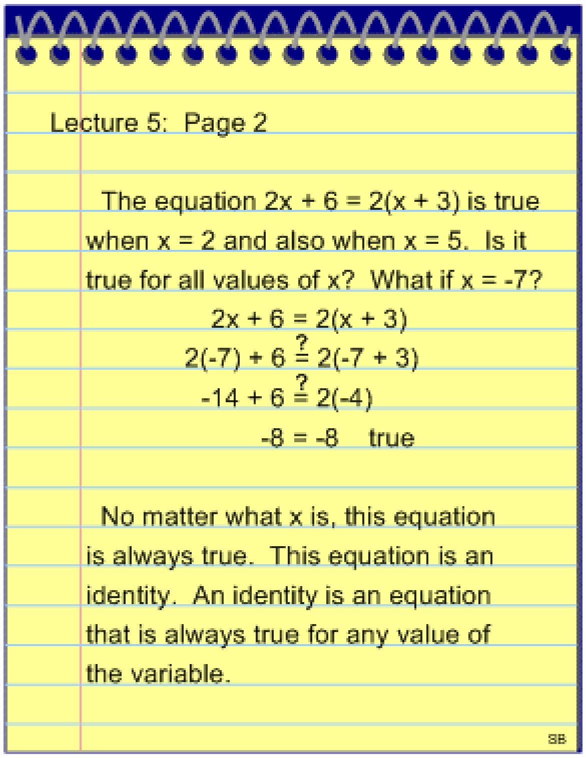

Lecture 5: Page 2

The equation $2x + 6 = 2(x + 3)$ is true when $x = 2$ and also when $x = 5$. Is it true for all values of x? What if $x = -7$?

$$2x + 6 = 2(x + 3)$$

$$2(-7) + 6 \overset{?}{=} 2(-7 + 3)$$

$$-14 + 6 \overset{?}{=} 2(-4)$$

$$-8 = -8 \quad \text{true}$$

No matter what x is, this equation is always true. This equation is an identity. An identity is an equation that is always true for any value of the variable.

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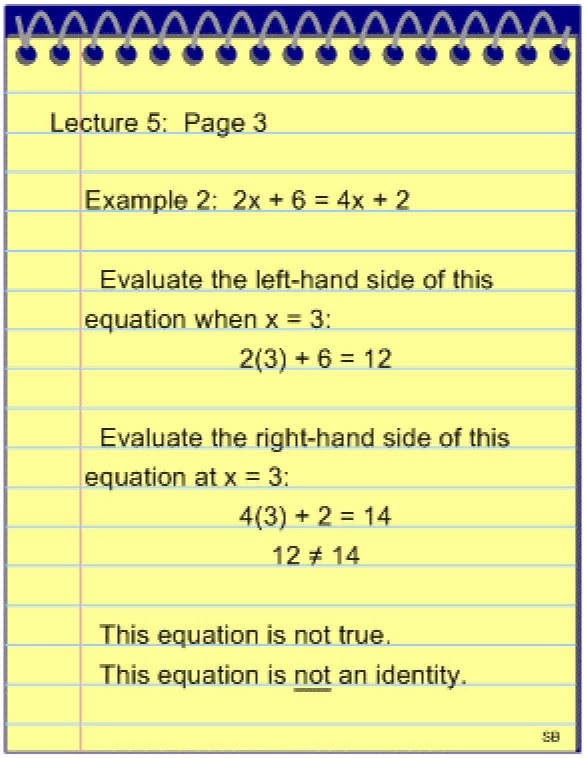

Lecture 5: Page 3

Example 2: $2x + 6 = 4x + 2$

Evaluate the left-hand side of this equation when $x = 3$:

$$2(3) + 6 = 12$$

Evaluate the right-hand side of this equation at $x = 3$:

$$4(3) + 2 = 14$$

$$12 \neq 14$$

This equation is not true.

This equation is <u>not</u> an identity.

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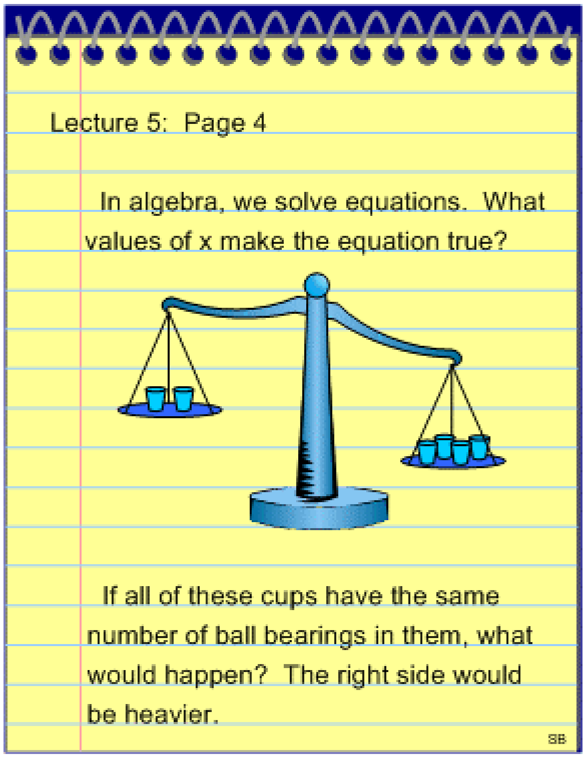

Lecture 5: Page 4

In algebra, we solve equations. What values of x make the equation true?

If all of these cups have the same number of ball bearings in them, what would happen? The right side would be heavier.

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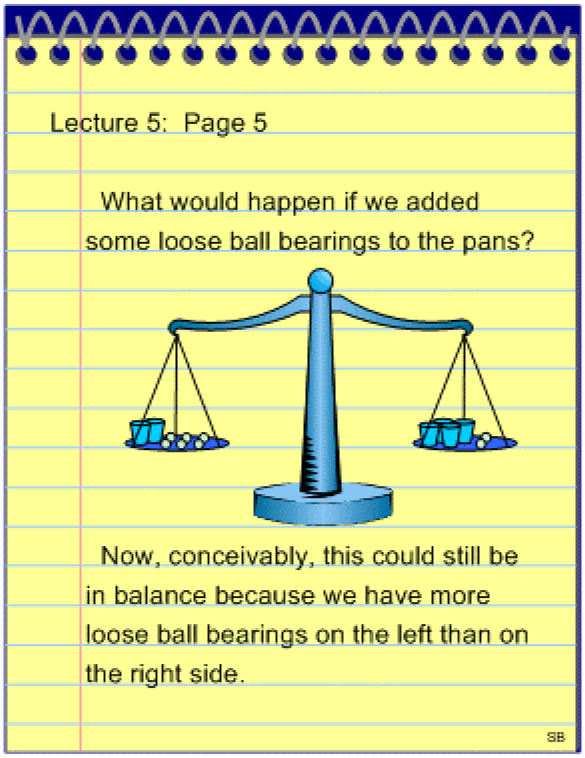
Lecture 5 Notes, Continued

ALG2005-05



Lecture 5: Page 5

What would happen if we added some loose ball bearings to the pans?

Now, conceivably, this could still be in balance because we have more loose ball bearings on the left than on the right side.

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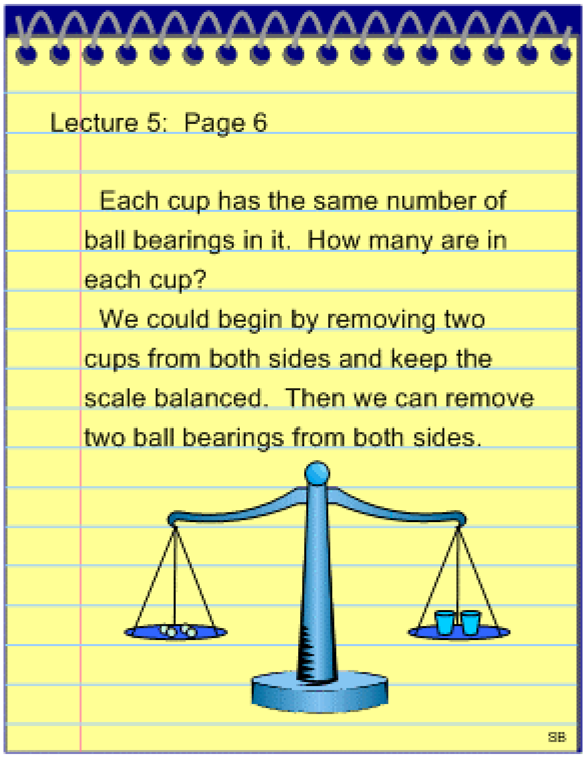

Lecture 5: Page 6

Each cup has the same number of ball bearings in it. How many are in each cup?

We could begin by removing two cups from both sides and keep the scale balanced. Then we can remove two ball bearings from both sides.

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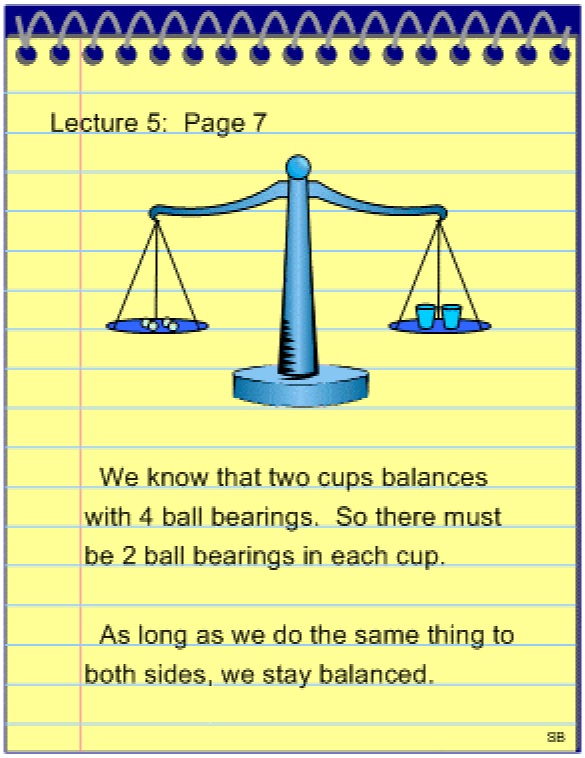

Lecture 5: Page 7

We know that two cups balances with 4 ball bearings. So there must be 2 ball bearings in each cup.

As long as we do the same thing to both sides, we stay balanced.

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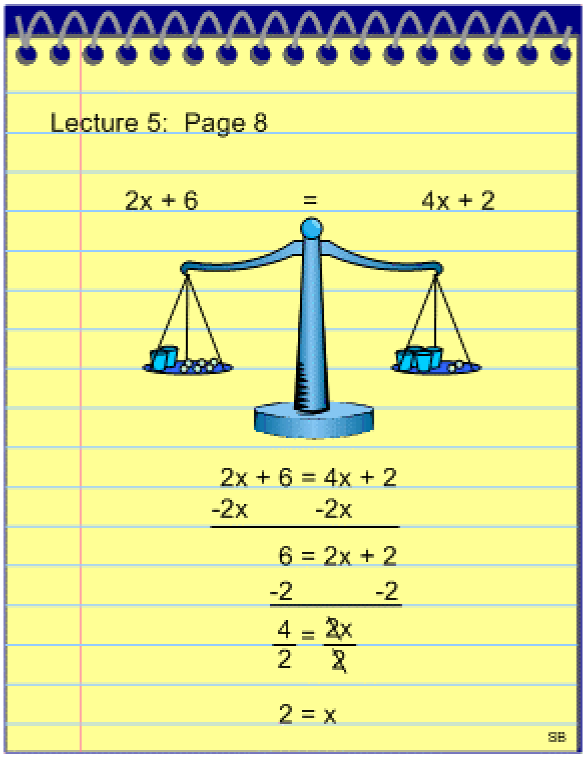

Lecture 5: Page 8

$2x + 6 \quad = \quad 4x + 2$

$$\begin{array}{rcl} 2x + 6 & = & 4x + 2 \\ -2x & & -2x \\ \hline 6 & = & 2x + 2 \\ -2 & & -2 \\ \hline \end{array}$$

$$\frac{4}{2} = \frac{\cancel{2}x}{\cancel{2}}$$

$$2 = x$$

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Lecture 5 Notes, Continued.

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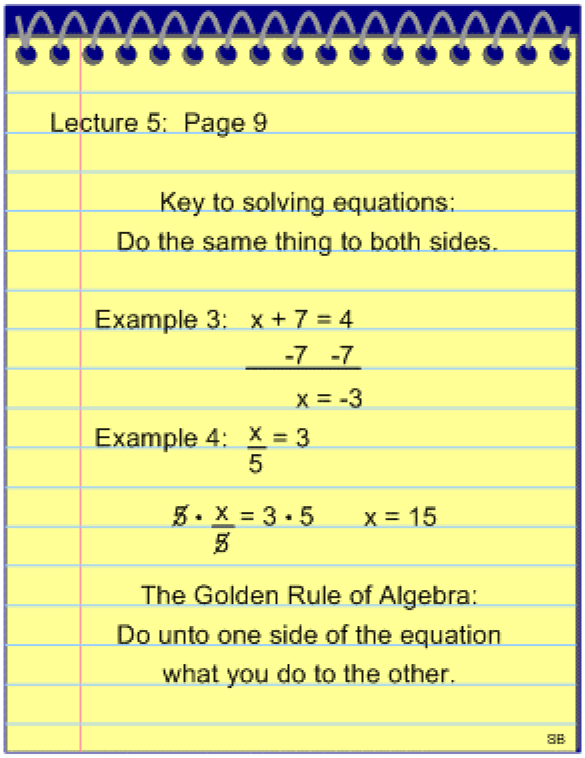

Lecture 5: Page 9

Key to solving equations:
Do the same thing to both sides.

Example 3: $x + 7 = 4$

$$\begin{array}{r} x + 7 = 4 \\ \underline{\;-7 \quad -7\;} \\ x = -3 \end{array}$$

Example 4: $\frac{x}{5} = 3$

$$\cancel{5} \cdot \frac{x}{\cancel{5}} = 3 \cdot 5 \qquad x = 15$$

The Golden Rule of Algebra:
Do unto one side of the equation
what you do to the other.

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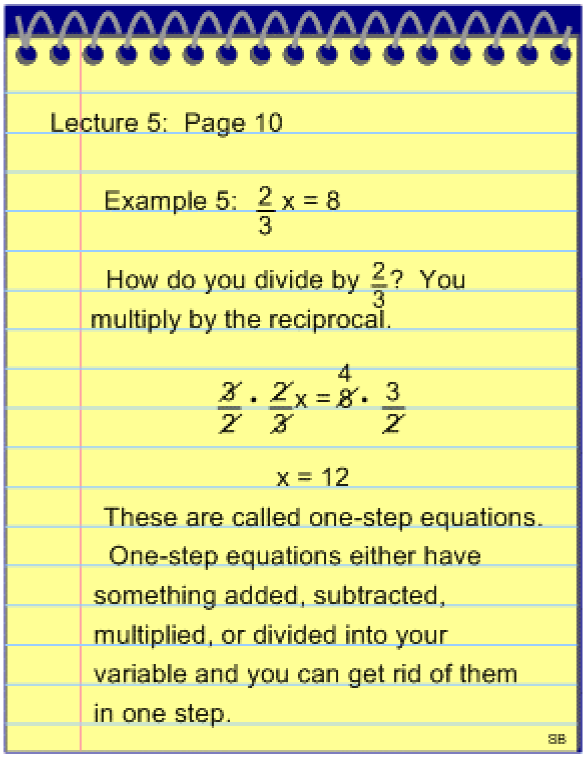

Lecture 5: Page 10

Example 5: $\frac{2}{3}x = 8$

How do you divide by $\frac{2}{3}$? You multiply by the reciprocal.

$$\frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}}x = \overset{4}{\cancel{8}} \cdot \frac{3}{\cancel{2}}$$

$$x = 12$$

These are called one-step equations. One-step equations either have something added, subtracted, multiplied, or divided into your variable and you can get rid of them in one step.

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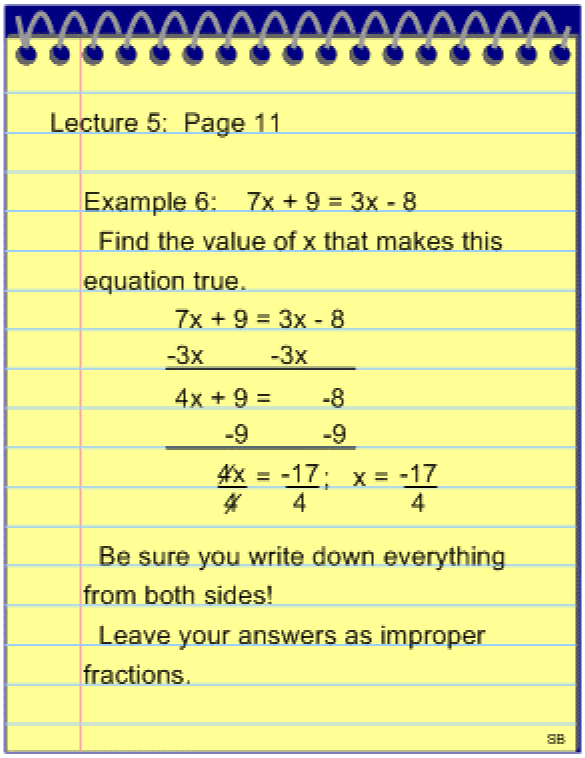

Lecture 5: Page 11

Example 6: $7x + 9 = 3x - 8$

Find the value of x that makes this equation true.

$$\begin{array}{rcl} 7x + 9 & = & 3x - 8 \\ \underline{-3x \quad\;\;} & & \underline{-3x \quad\;\;} \\ 4x + 9 & = & -8 \\ \underline{\quad -9} & & \underline{-9} \end{array}$$

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{-17}{4}; \quad x = \frac{-17}{4}$$

Be sure you write down everything from both sides!

Leave your answers as improper fractions.

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Lecture 6 Notes

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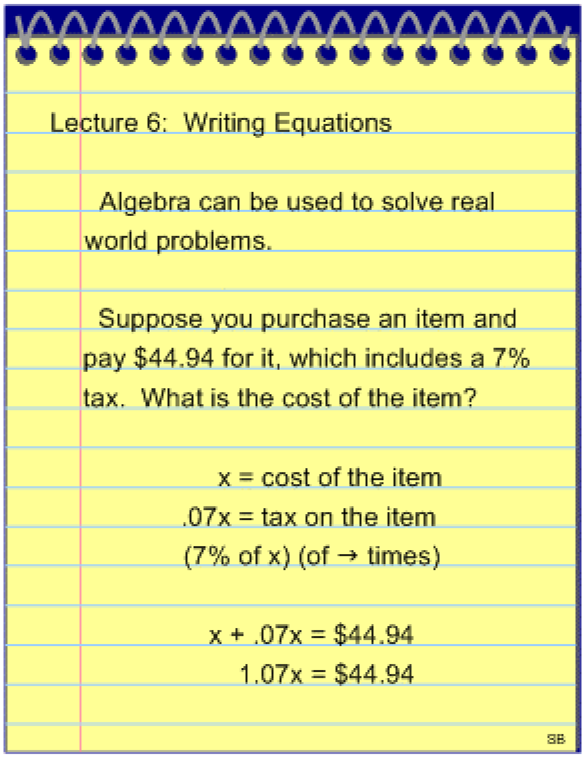

Lecture 6: Writing Equations

Algebra can be used to solve real world problems.

Suppose you purchase an item and pay $44.94 for it, which includes a 7% tax. What is the cost of the item?

x = cost of the item

.07x = tax on the item

(7% of x) (of → times)

$x + .07x = \$44.94$

$1.07x = \$44.94$

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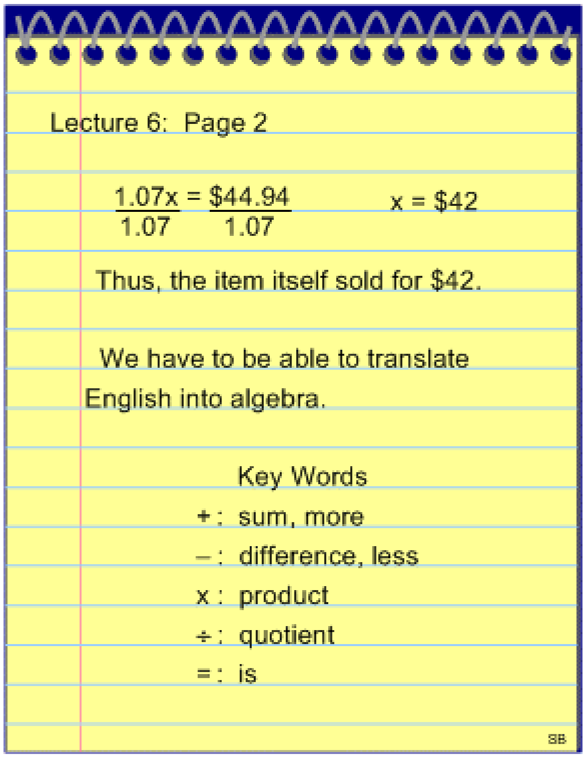


Lecture 6: Page 2

$\frac{1.07x}{1.07} = \frac{\$44.94}{1.07}$ $x = \$42$

Thus, the item itself sold for $42.

We have to be able to translate English into algebra.

Key Words

+ : sum, more

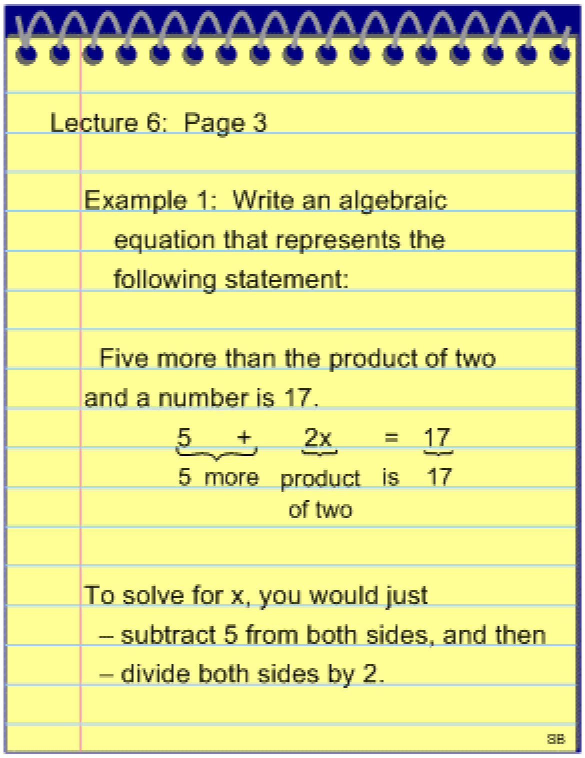
– : difference, less

x : product

÷ : quotient

= : is

ALG2006-03



Lecture 6: Page 3

Example 1: Write an algebraic equation that represents the following statement:

Five more than the product of two and a number is 17.

$$\underbrace{5 \quad +}_{\text{5 more}} \quad \underbrace{2x}_{\text{product of two}} \quad \underset{\text{is}}{=} \quad \underbrace{17}_{17}$$

To solve for x, you would just
- subtract 5 from both sides, and then
- divide both sides by 2.

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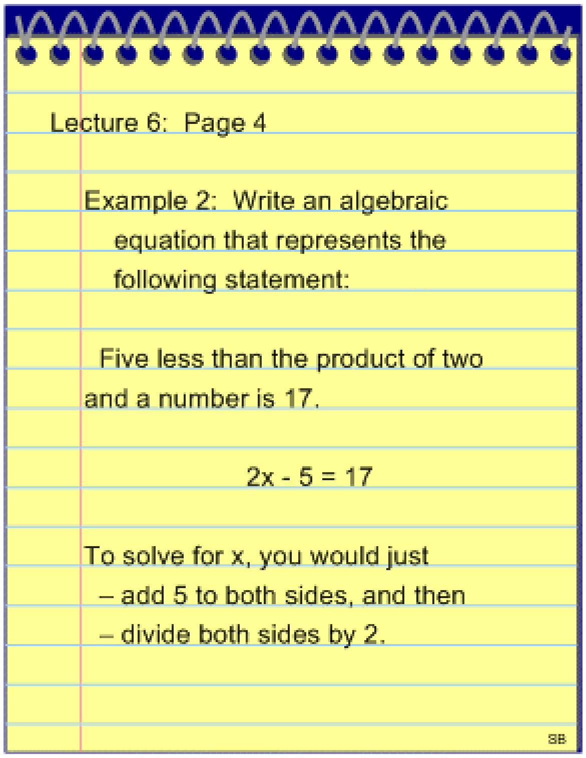


Lecture 6: Page 4

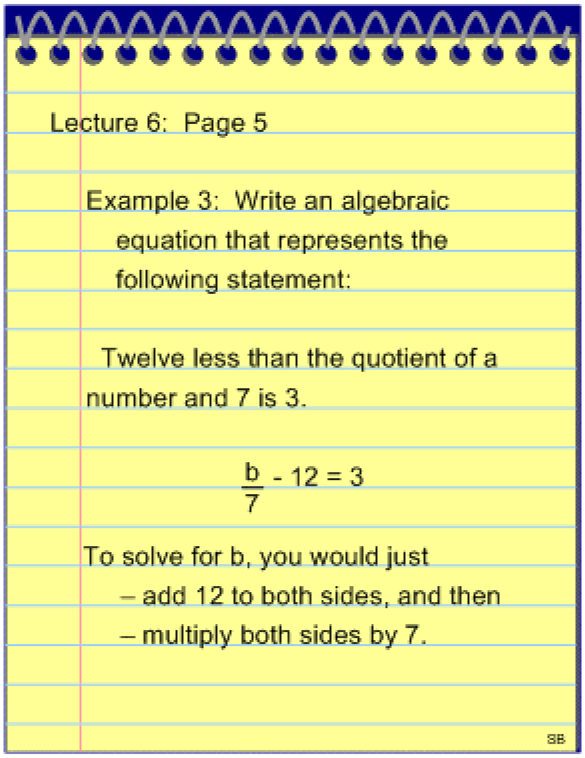
Example 2: Write an algebraic equation that represents the following statement:

Five less than the product of two and a number is 17.

$2x - 5 = 17$

To solve for x, you would just
- add 5 to both sides, and then
- divide both sides by 2.

ALG2006-05



Lecture 6: Page 5

Example 3: Write an algebraic equation that represents the following statement:

Twelve less than the quotient of a number and 7 is 3.

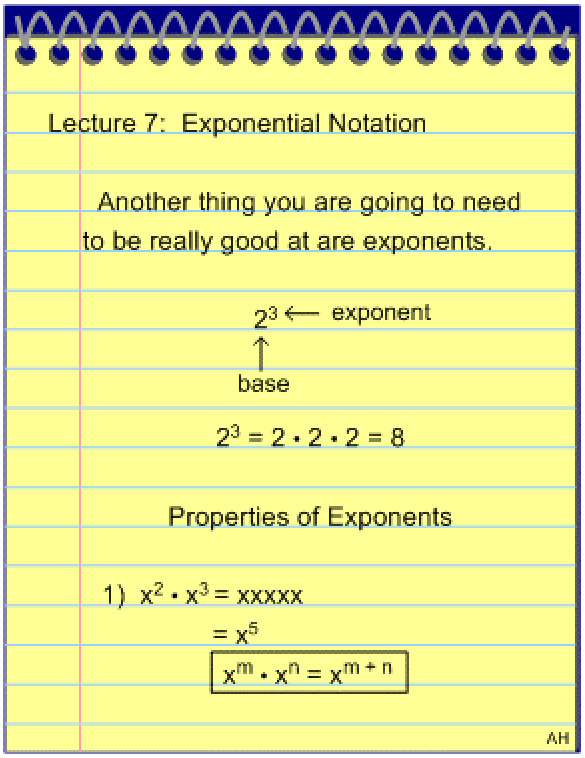
$$\frac{b}{7} - 12 = 3$$

To solve for b, you would just
- add 12 to both sides, and then
- multiply both sides by 7.

SB

Lecture 7 Notes

ALG2007-01



Lecture 7: Exponential Notation

Another thing you are going to need to be really good at are exponents.

$2^3 \leftarrow$ exponent

$\uparrow$

base

$2^3 = 2 \cdot 2 \cdot 2 = 8$

Properties of Exponents

1) $x^2 \cdot x^3 = xxxxx$

$= x^5$

$$\boxed{x^m \cdot x^n = x^{m+n}}$$

AH

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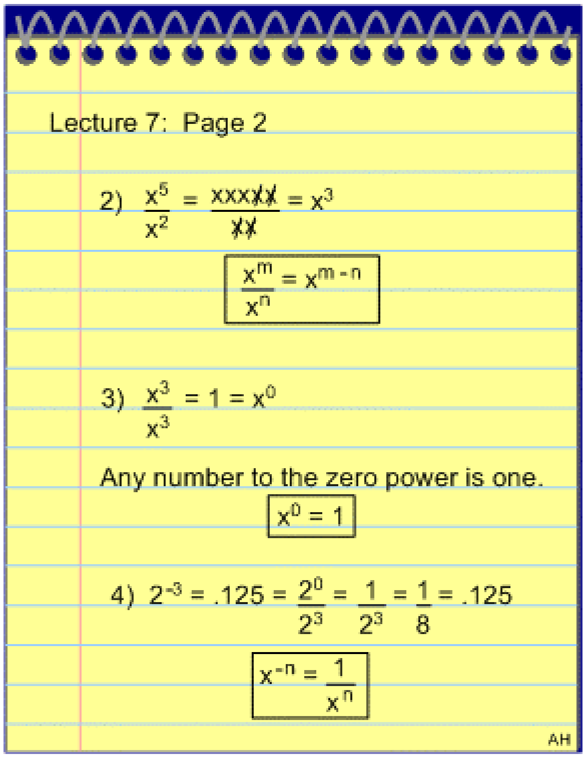


Lecture 7: Page 2

2) $\frac{x^5}{x^2} = \frac{xxx\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}} = x^3$

$$\boxed{\frac{x^m}{x^n} = x^{m-n}}$$

3) $\frac{x^3}{x^3} = 1 = x^0$

Any number to the zero power is one.

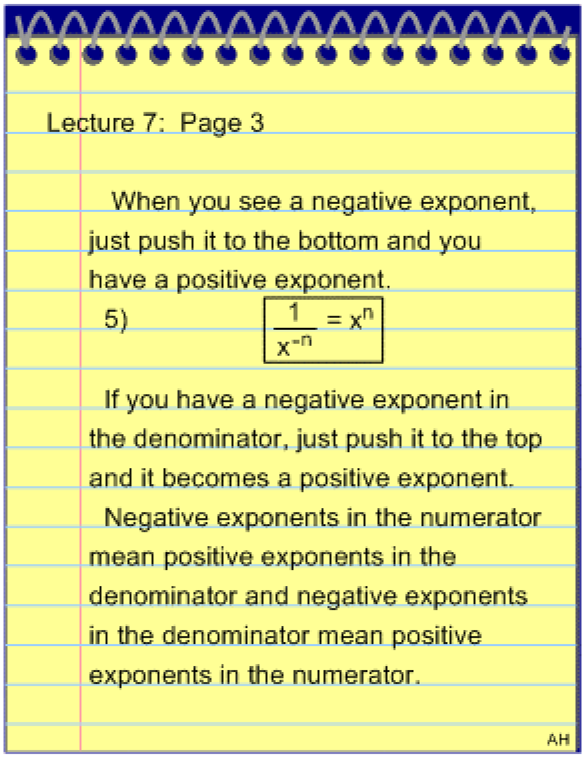
$$\boxed{x^0 = 1}$$

4) $2^{-3} = .125 = \frac{2^0}{2^3} = \frac{1}{2^3} = \frac{1}{8} = .125$

$$\boxed{x^{-n} = \frac{1}{x^n}}$$

AH

ALG2007-03



Lecture 7: Page 3

When you see a negative exponent, just push it to the bottom and you have a positive exponent.

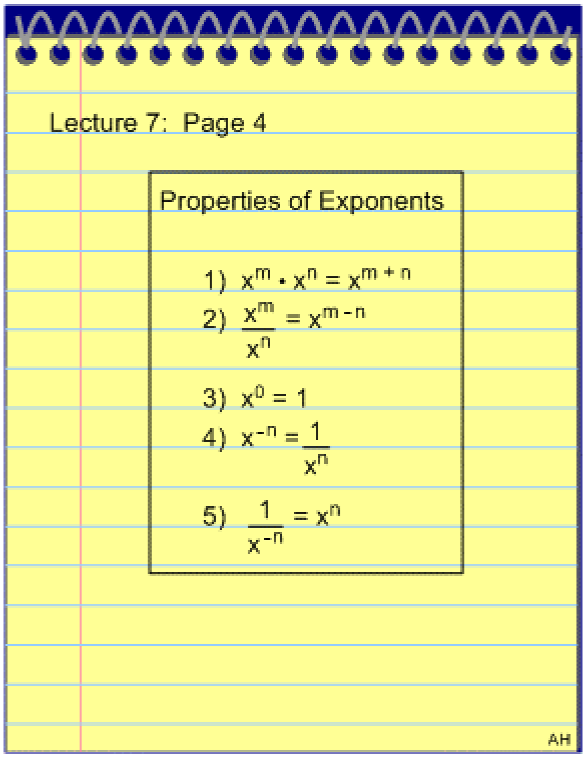
5) $\boxed{\frac{1}{x^{-n}} = x^n}$

If you have a negative exponent in the denominator, just push it to the top and it becomes a positive exponent.

Negative exponents in the numerator mean positive exponents in the denominator and negative exponents in the denominator mean positive exponents in the numerator.

AH

ALG2007-04



Lecture 7: Page 4

Properties of Exponents

1) $x^m \cdot x^n = x^{m+n}$

2) $\frac{x^m}{x^n} = x^{m-n}$

3) $x^0 = 1$

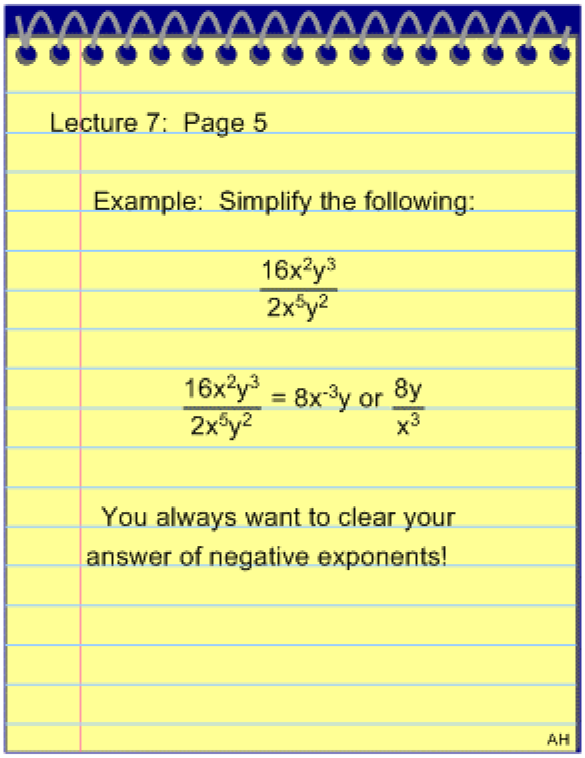
4) $x^{-n} = \frac{1}{x^n}$

5) $\frac{1}{x^{-n}} = x^n$

AH

Lecture 7 Notes, Continued

ALG2007-05



Lecture 7: Page 5

Example: Simplify the following:

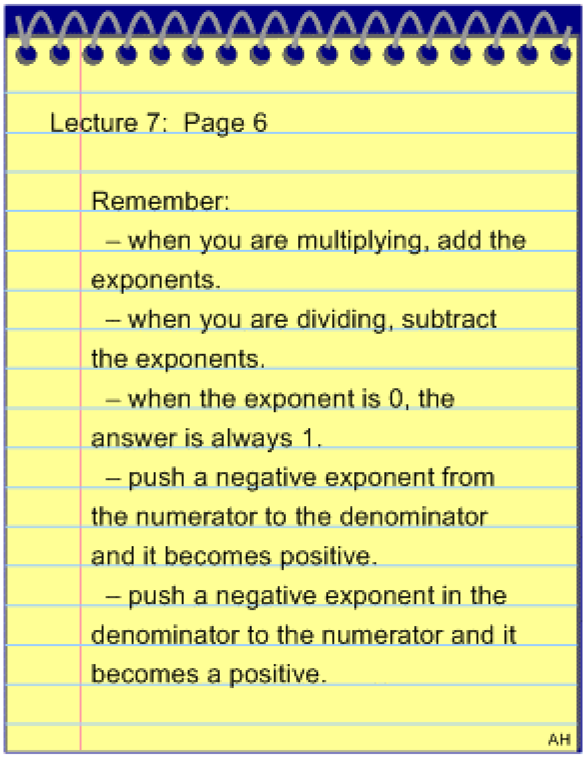
$$\frac{16x^2y^3}{2x^5y^2}$$

$$\frac{16x^2y^3}{2x^5y^2} = 8x^{-3}y \text{ or } \frac{8y}{x^3}$$

You always want to clear your answer of negative exponents!

AH

ALG2007-06



Lecture 7: Page 6

Remember:

– when you are multiplying, add the exponents.

– when you are dividing, subtract the exponents.

– when the exponent is 0, the answer is always 1.

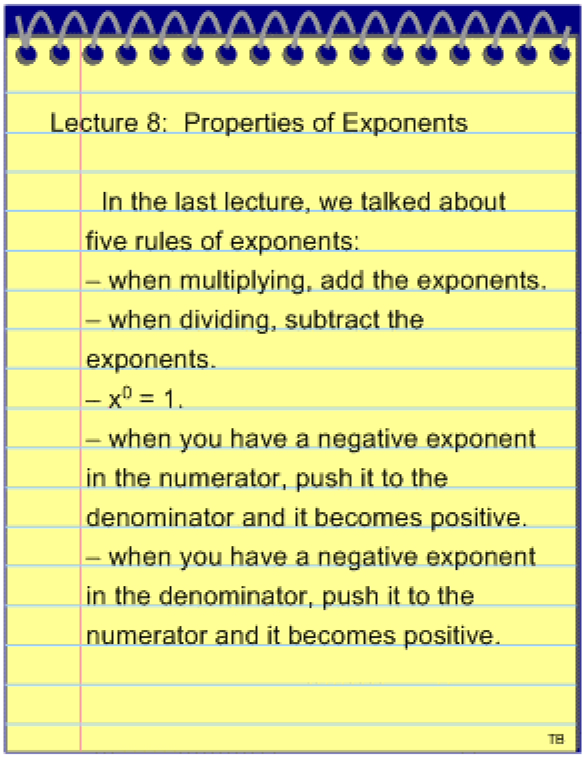
– push a negative exponent from the numerator to the denominator and it becomes positive.

– push a negative exponent in the denominator to the numerator and it becomes a positive.

AH

Lecture 8 Notes

ALG2008-01



Lecture 8: Properties of Exponents

In the last lecture, we talked about five rules of exponents:

– when multiplying, add the exponents.

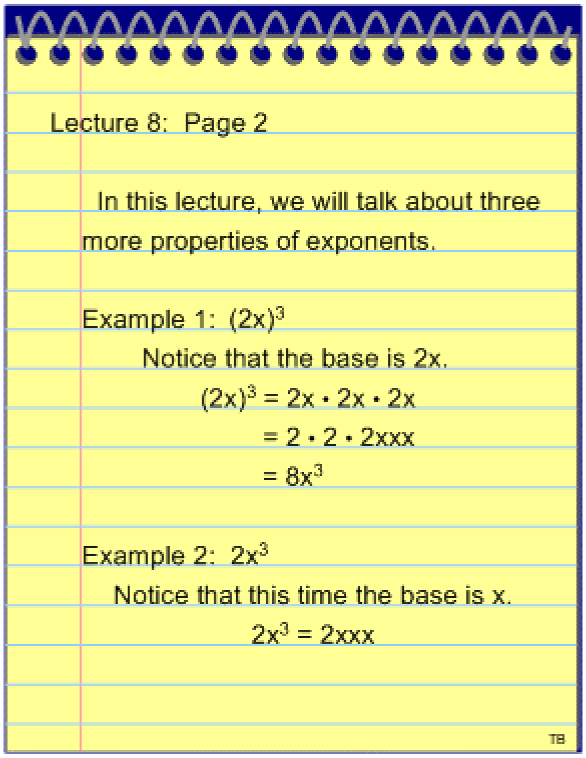
– when dividing, subtract the exponents.

– $x^0 = 1$.

– when you have a negative exponent in the numerator, push it to the denominator and it becomes positive.

– when you have a negative exponent in the denominator, push it to the numerator and it becomes positive.

ALG2008-02



Lecture 8: Page 2

In this lecture, we will talk about three more properties of exponents.

Example 1: $(2x)^3$

Notice that the base is 2x.

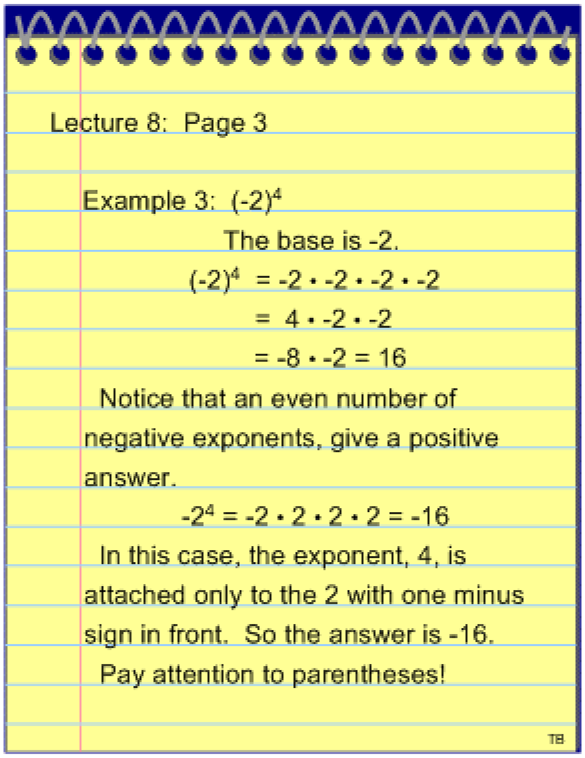
$$(2x)^3 = 2x \cdot 2x \cdot 2x$$
$$= 2 \cdot 2 \cdot 2xxx$$
$$= 8x^3$$

Example 2: $2x^3$

Notice that this time the base is x.

$$2x^3 = 2xxx$$

ALG2008-03



Lecture 8: Page 3

Example 3: $(-2)^4$

The base is -2.

$$(-2)^4 = -2 \cdot -2 \cdot -2 \cdot -2$$
$$= 4 \cdot -2 \cdot -2$$
$$= -8 \cdot -2 = 16$$

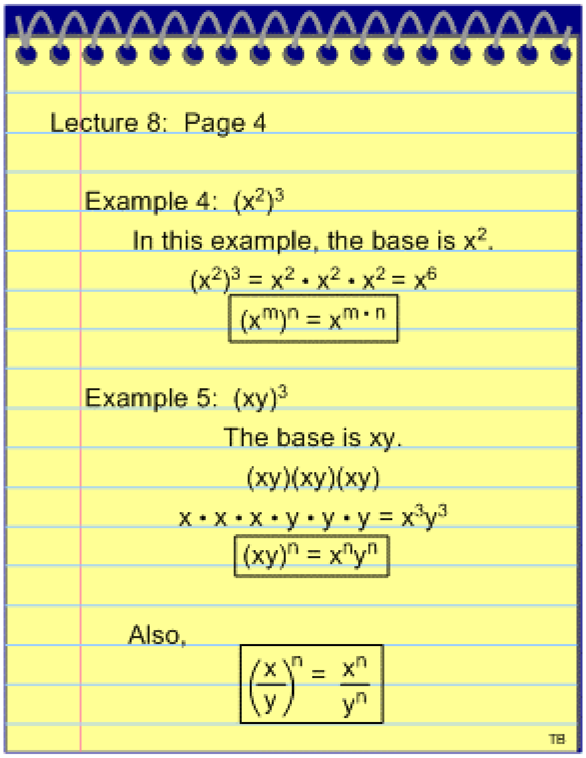
Notice that an even number of negative exponents, give a positive answer.

$$-2^4 = -2 \cdot 2 \cdot 2 \cdot 2 = -16$$

In this case, the exponent, 4, is attached only to the 2 with one minus sign in front. So the answer is -16.

Pay attention to parentheses!

ALG2008-04



Lecture 8: Page 4

Example 4: $(x^2)^3$

In this example, the base is x^2.

$$(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x^6$$
$$\boxed{(x^m)^n = x^{m \cdot n}}$$

Example 5: $(xy)^3$

The base is xy.

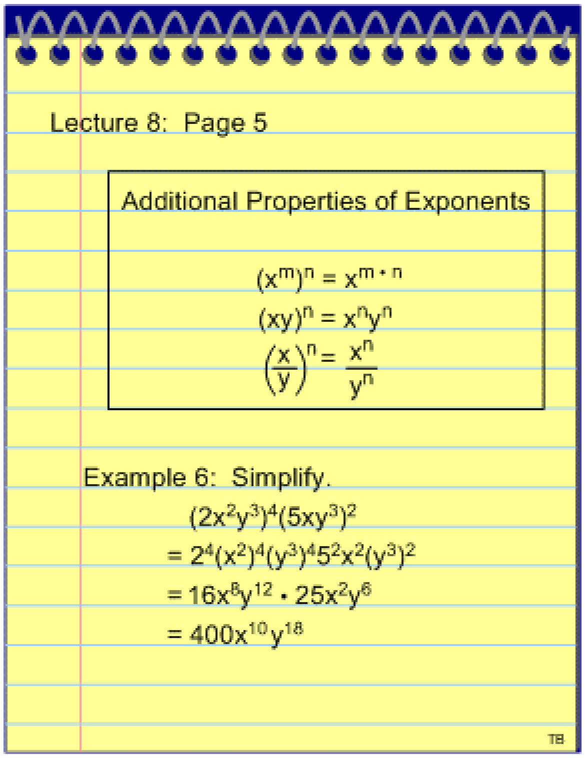
$$(xy)(xy)(xy)$$
$$x \cdot x \cdot x \cdot y \cdot y \cdot y = x^3y^3$$
$$\boxed{(xy)^n = x^ny^n}$$

Also,

$$\boxed{\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}}$$

Lecture 8 Notes, Continued

ALG2008-05



Lecture 8: Page 5

Additional Properties of Exponents

$(x^m)^n = x^{m \cdot n}$

$(xy)^n = x^n y^n$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

Example 6: Simplify.

$(2x^2y^3)^4(5xy^3)^2$

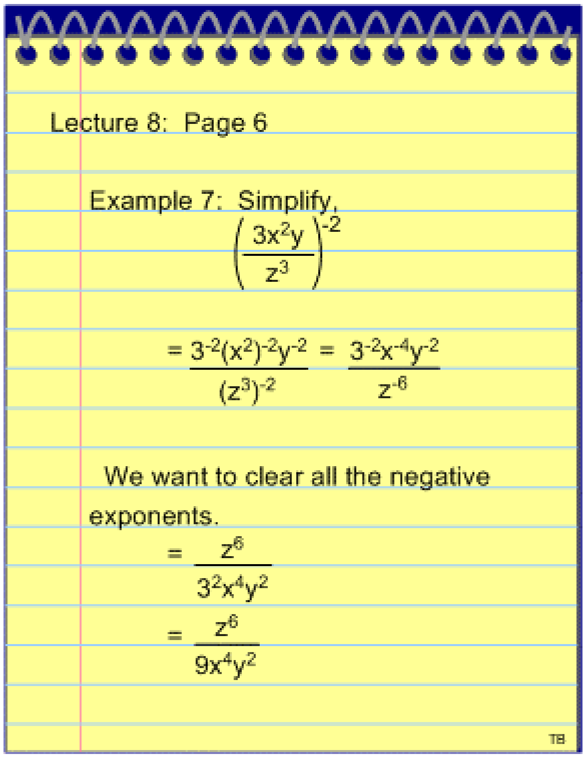
$= 2^4(x^2)^4(y^3)^4 5^2x^2(y^3)^2$

$= 16x^8y^{12} \cdot 25x^2y^6$

$= 400x^{10}y^{18}$

TB

ALG2008-06



Lecture 8: Page 6

Example 7: Simplify,

$\left(\frac{3x^2y}{z^3}\right)^{-2}$

$= \frac{3^{-2}(x^2)^{-2}y^{-2}}{(z^3)^{-2}} = \frac{3^{-2}x^{-4}y^{-2}}{z^{-6}}$

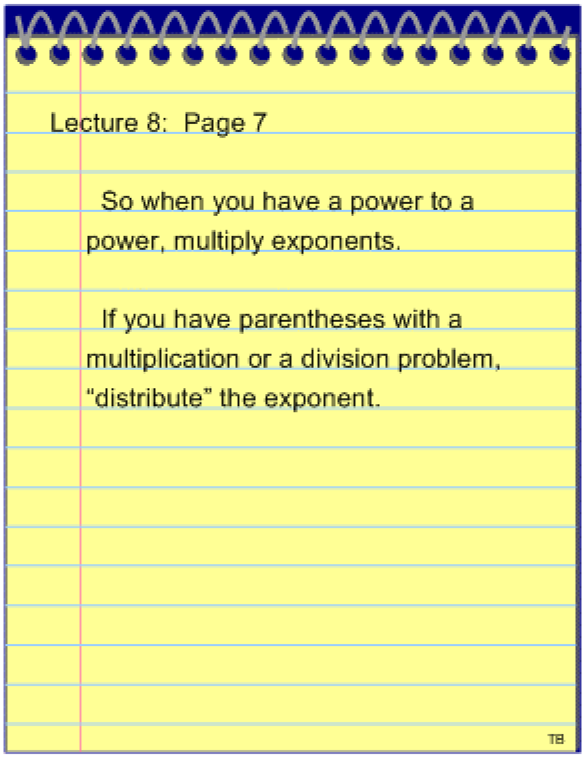
We want to clear all the negative exponents.

$= \frac{z^6}{3^2x^4y^2}$

$= \frac{z^6}{9x^4y^2}$

TB

ALG2008-07



Lecture 8: Page 7

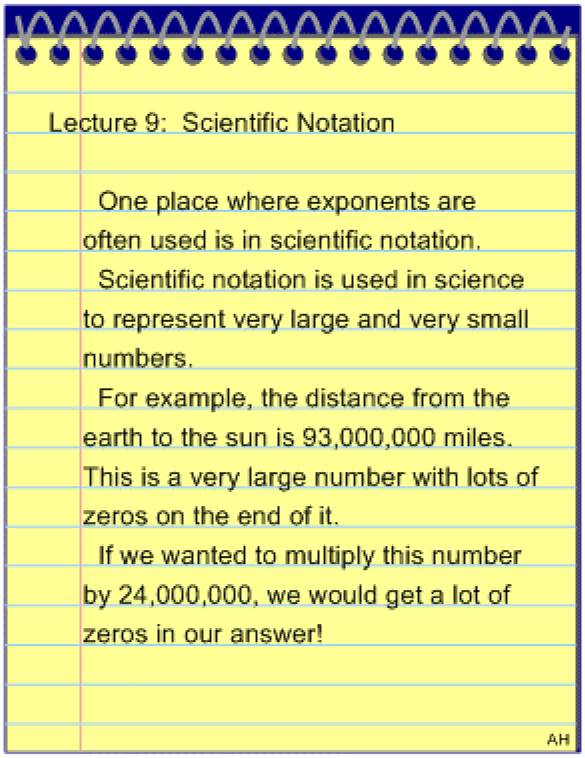
So when you have a power to a power, multiply exponents.

If you have parentheses with a multiplication or a division problem, “distribute” the exponent.

TB

Lecture 9 Notes

ALG2009-01



Lecture 9: Scientific Notation

One place where exponents are often used is in scientific notation.

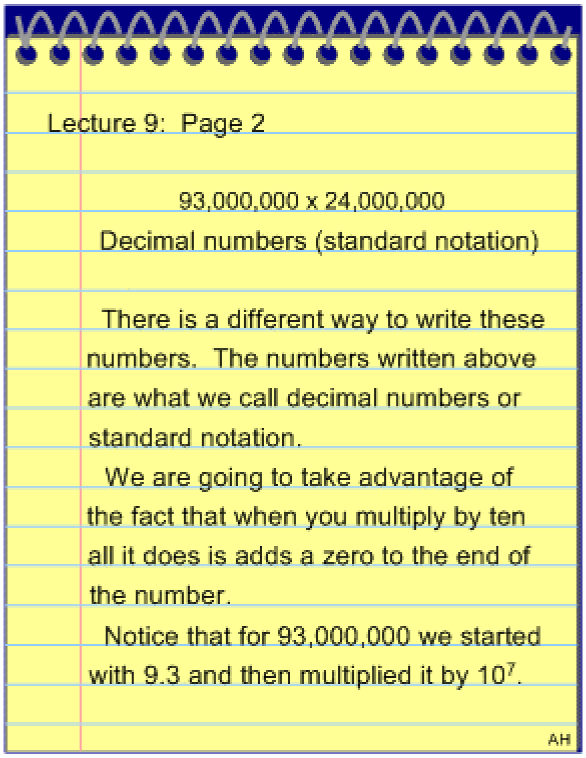
Scientific notation is used in science to represent very large and very small numbers.

For example, the distance from the earth to the sun is 93,000,000 miles. This is a very large number with lots of zeros on the end of it.

If we wanted to multiply this number by 24,000,000, we would get a lot of zeros in our answer!

AH

ALG2009-02



Lecture 9: Page 2

93,000,000 x 24,000,000

Decimal numbers (standard notation)

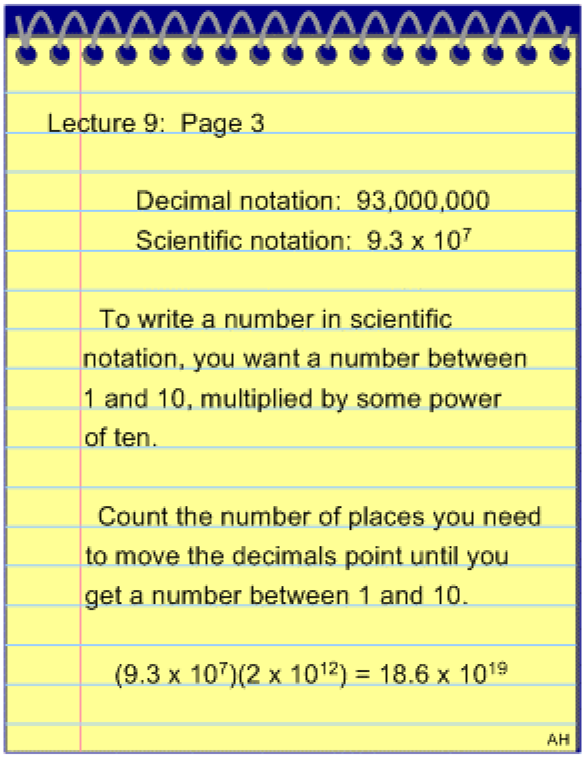
There is a different way to write these numbers. The numbers written above are what we call decimal numbers or standard notation.

We are going to take advantage of the fact that when you multiply by ten all it does is adds a zero to the end of the number.

Notice that for 93,000,000 we started with 9.3 and then multiplied it by 10^7.

AH

ALG2009-03



Lecture 9: Page 3

Decimal notation: 93,000,000

Scientific notation: 9.3×10^7

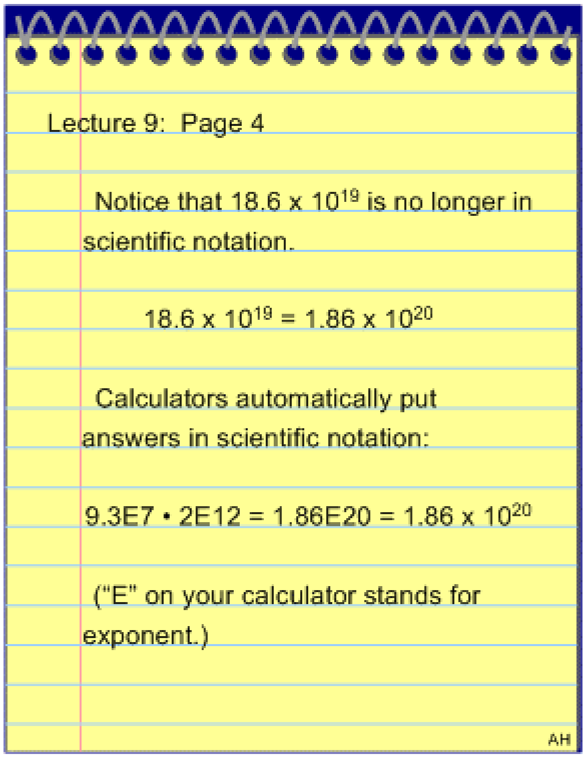
To write a number in scientific notation, you want a number between 1 and 10, multiplied by some power of ten.

Count the number of places you need to move the decimals point until you get a number between 1 and 10.

$(9.3 \times 10^7)(2 \times 10^{12}) = 18.6 \times 10^{19}$

AH

ALG2009-04



Lecture 9: Page 4

Notice that 18.6×10^{19} is no longer in scientific notation.

$18.6 \times 10^{19} = 1.86 \times 10^{20}$

Calculators automatically put answers in scientific notation:

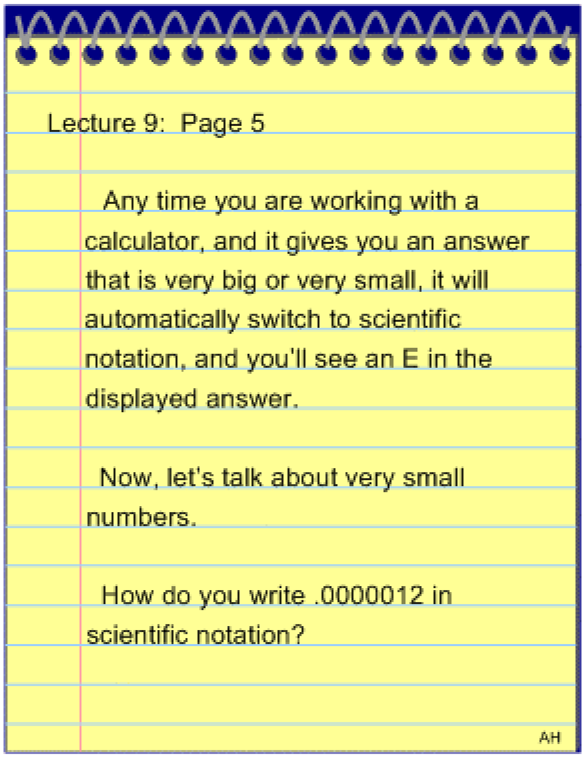
$9.3E7 \cdot 2E12 = 1.86E20 = 1.86 \times 10^{20}$

("E" on your calculator stands for exponent.)

AH

Lecture 9 Notes, Continued

ALG2009-05



Lecture 9: Page 5

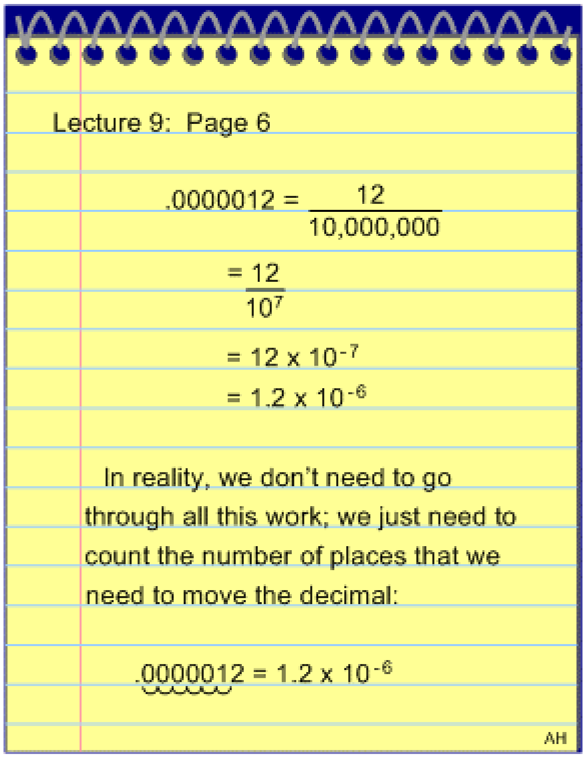
Any time you are working with a calculator, and it gives you an answer that is very big or very small, it will automatically switch to scientific notation, and you'll see an E in the displayed answer.

Now, let's talk about very small numbers.

How do you write .0000012 in scientific notation?

AH

ALG2009-06



Lecture 9: Page 6

$$.0000012 = \frac{12}{10{,}000{,}000}$$

$$= \frac{12}{10^7}$$

$$= 12 \times 10^{-7}$$

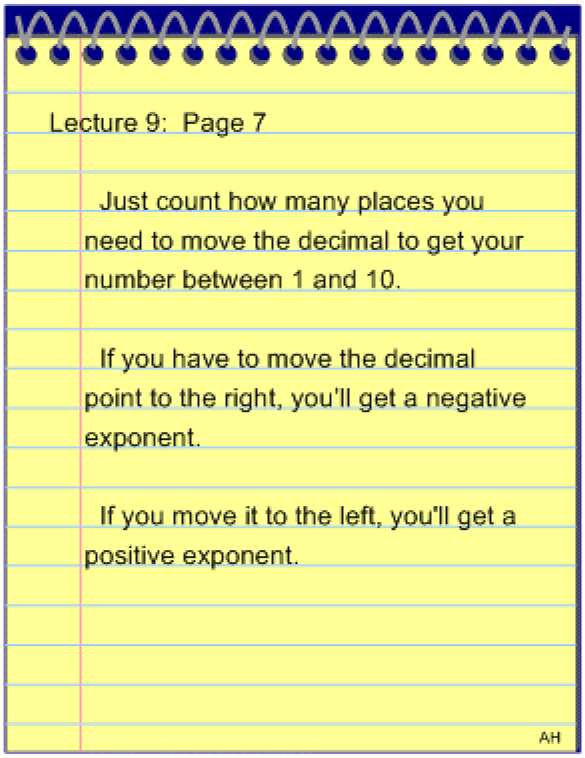
$$= 1.2 \times 10^{-6}$$

In reality, we don't need to go through all this work; we just need to count the number of places that we need to move the decimal:

$$.0000012 = 1.2 \times 10^{-6}$$

AH

ALG2009-07



Lecture 9: Page 7

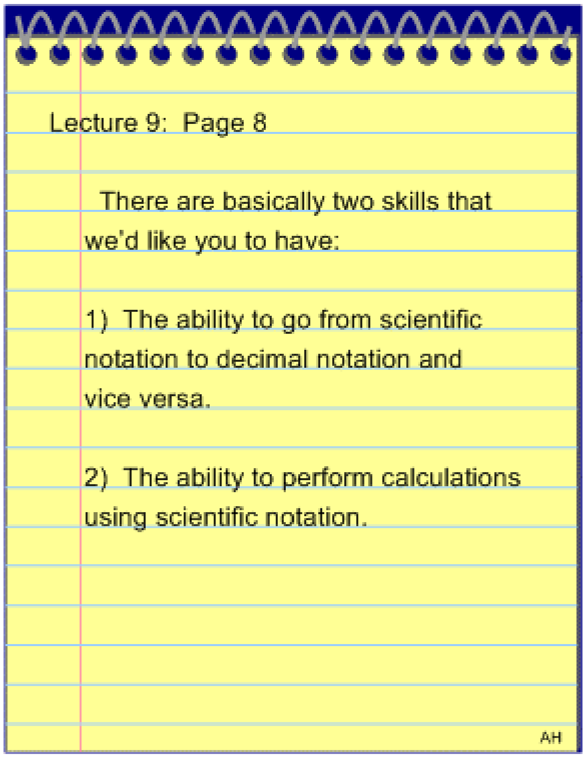
Just count how many places you need to move the decimal to get your number between 1 and 10.

If you have to move the decimal point to the right, you'll get a negative exponent.

If you move it to the left, you'll get a positive exponent.

AH

ALG2009-08



Lecture 9: Page 8

There are basically two skills that we'd like you to have:

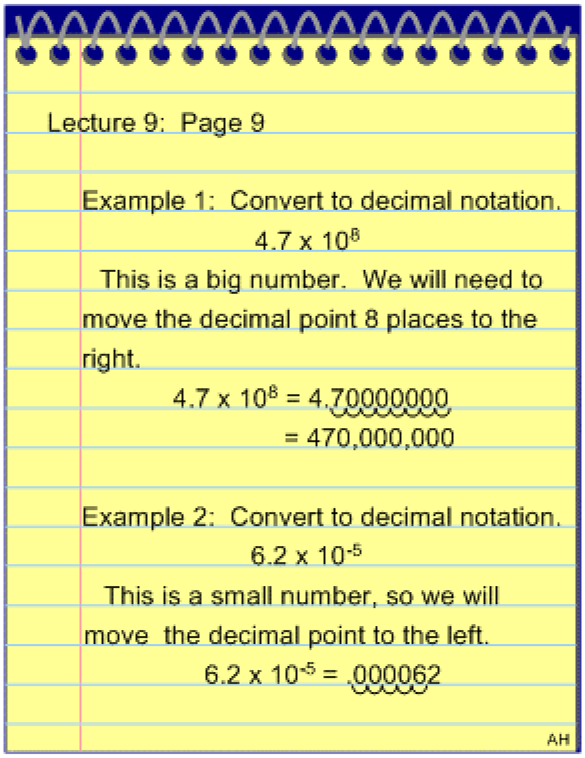
1) The ability to go from scientific notation to decimal notation and vice versa.

2) The ability to perform calculations using scientific notation.

AH

Lecture 9 Notes, Continued

ALG2009-09



Lecture 9: Page 9

Example 1: Convert to decimal notation.

$4.7 \text{ x } 10^8$

This is a big number. We will need to move the decimal point 8 places to the right.

$4.7 \text{ x } 10^8 = 4.70000000$

$= 470{,}000{,}000$

Example 2: Convert to decimal notation.

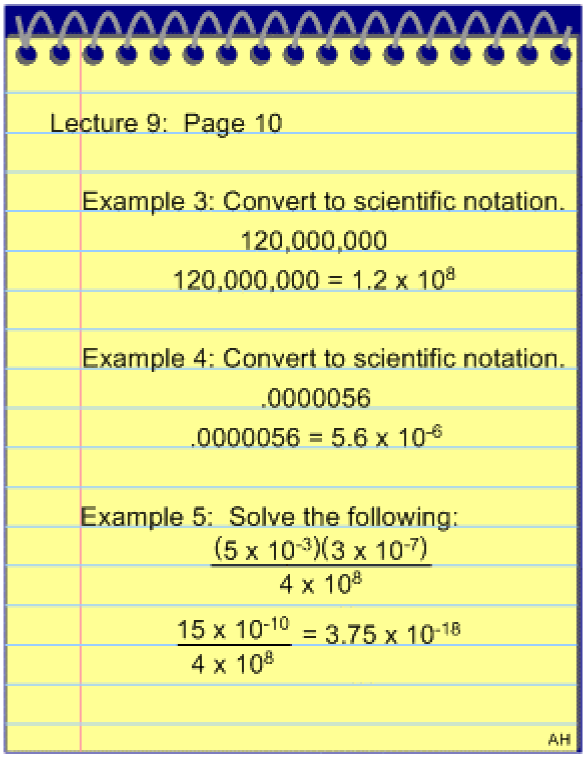
$6.2 \text{ x } 10^{-5}$

This is a small number, so we will move the decimal point to the left.

$6.2 \text{ x } 10^{-5} = .000062$

AH

ALG2009-10



Lecture 9: Page 10

Example 3: Convert to scientific notation.

120,000,000

$120{,}000{,}000 = 1.2 \text{ x } 10^8$

Example 4: Convert to scientific notation.

.0000056

$.0000056 = 5.6 \text{ x } 10^{-6}$

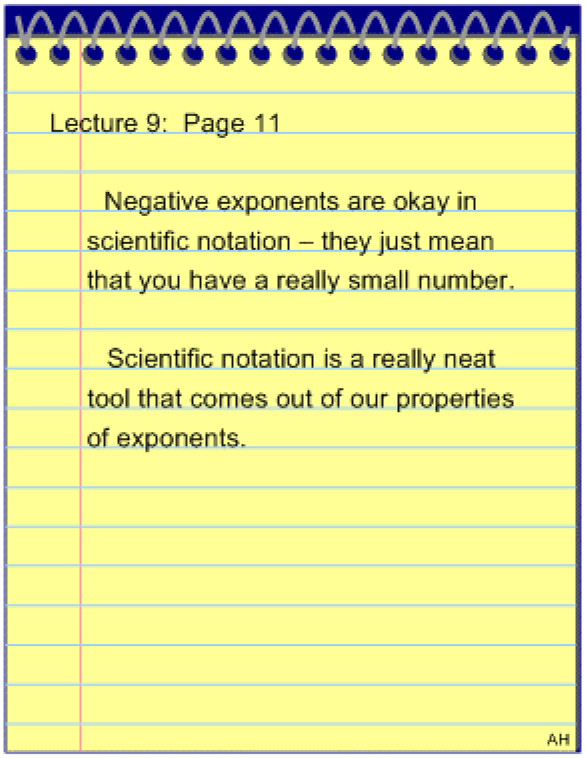
Example 5: Solve the following:

$$\frac{(5 \text{ x } 10^{-3})(3 \text{ x } 10^{-7})}{4 \text{ x } 10^8}$$

$$\frac{15 \text{ x } 10^{-10}}{4 \text{ x } 10^8} = 3.75 \text{ x } 10^{-18}$$

AH

ALG2009-11



Lecture 9: Page 11

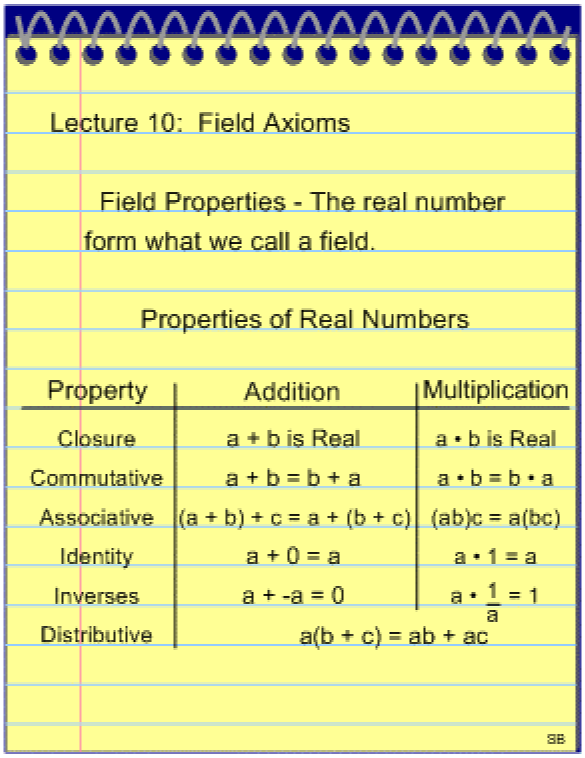
Negative exponents are okay in scientific notation – they just mean that you have a really small number.

Scientific notation is a really neat tool that comes out of our properties of exponents.

AH

Lecture 10 Notes

ALG2010-01



Lecture 10: Field Axioms

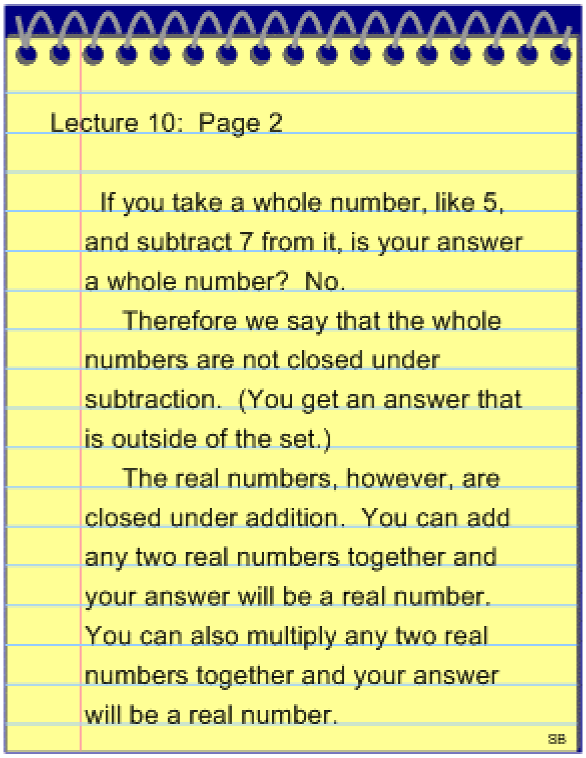
Field Properties - The real number form what we call a field.

Properties of Real Numbers

Property	Addition	Multiplication
Closure	a + b is Real	a • b is Real
Commutative	a + b = b + a	a • b = b • a
Associative	(a + b) + c = a + (b + c)	(ab)c = a(bc)
Identity	a + 0 = a	a • 1 = a
Inverses	a + -a = 0	$a \cdot \frac{1}{a} = 1$
Distributive	a(b + c) = ab + ac	

SB

ALG2010-02



Lecture 10: Page 2

If you take a whole number, like 5, and subtract 7 from it, is your answer a whole number? No.

Therefore we say that the whole numbers are not closed under subtraction. (You get an answer that is outside of the set.)

The real numbers, however, are closed under addition. You can add any two real numbers together and your answer will be a real number. You can also multiply any two real numbers together and your answer will be a real number.

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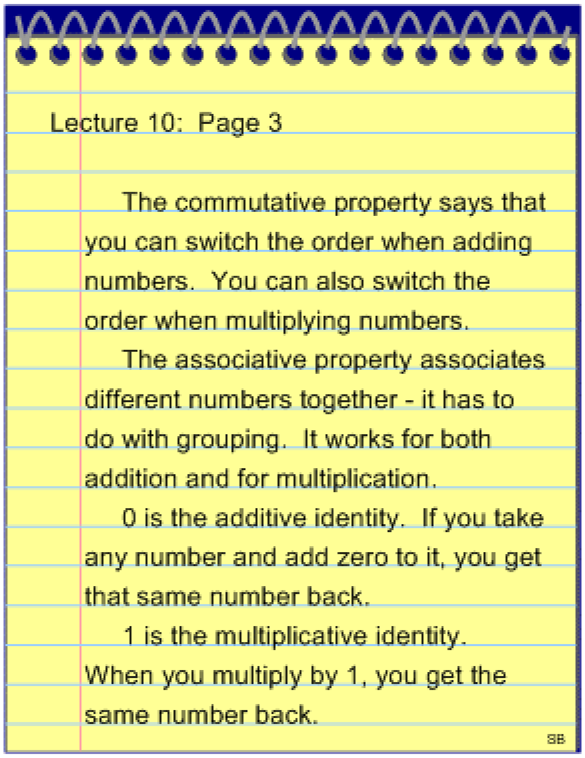
ALG2010-03



Lecture 10: Page 3

The commutative property says that you can switch the order when adding numbers. You can also switch the order when multiplying numbers.

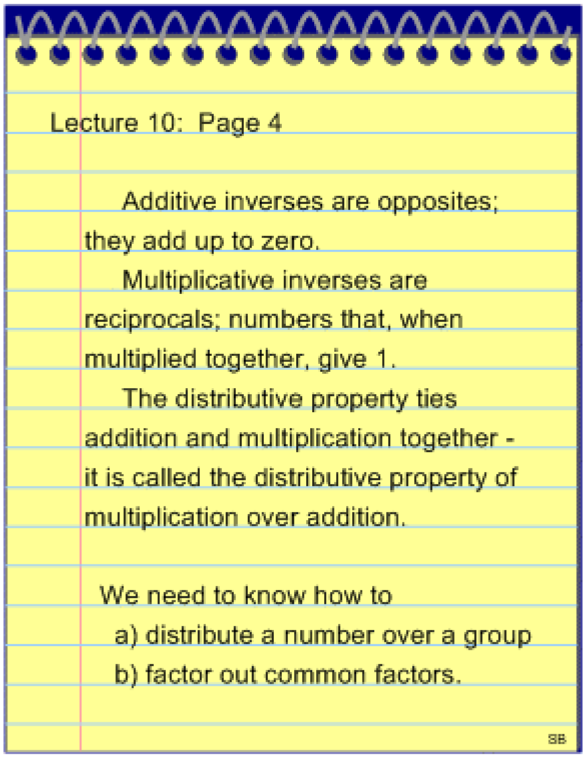
The associative property associates different numbers together - it has to do with grouping. It works for both addition and for multiplication.

0 is the additive identity. If you take any number and add zero to it, you get that same number back.

1 is the multiplicative identity. When you multiply by 1, you get the same number back.

SB

ALG2010-04



Lecture 10: Page 4

Additive inverses are opposites; they add up to zero.

Multiplicative inverses are reciprocals; numbers that, when multiplied together, give 1.

The distributive property ties addition and multiplication together - it is called the distributive property of multiplication over addition.

We need to know how to

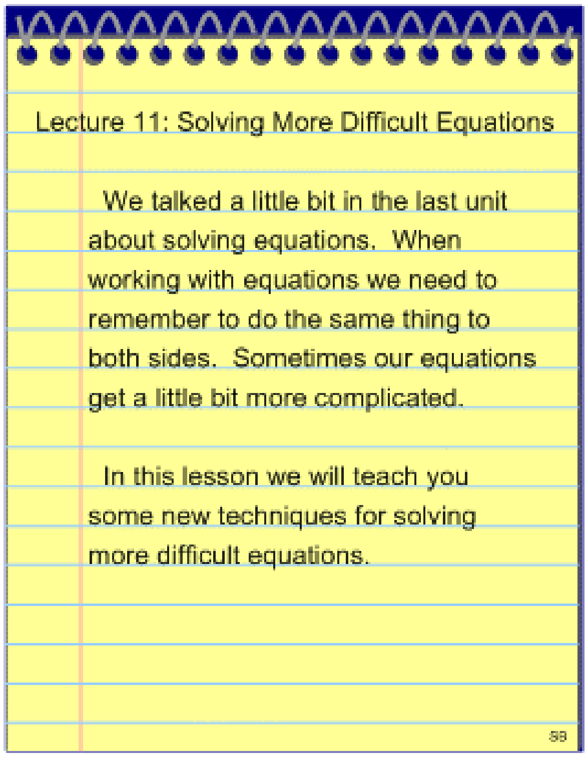
a) distribute a number over a group

b) factor out common factors.

SB

Lecture 11 Notes

ALG2011-01



Lecture 11: Solving More Difficult Equations

We talked a little bit in the last unit about solving equations. When working with equations we need to remember to do the same thing to both sides. Sometimes our equations get a little bit more complicated.

In this lesson we will teach you some new techniques for solving more difficult equations.

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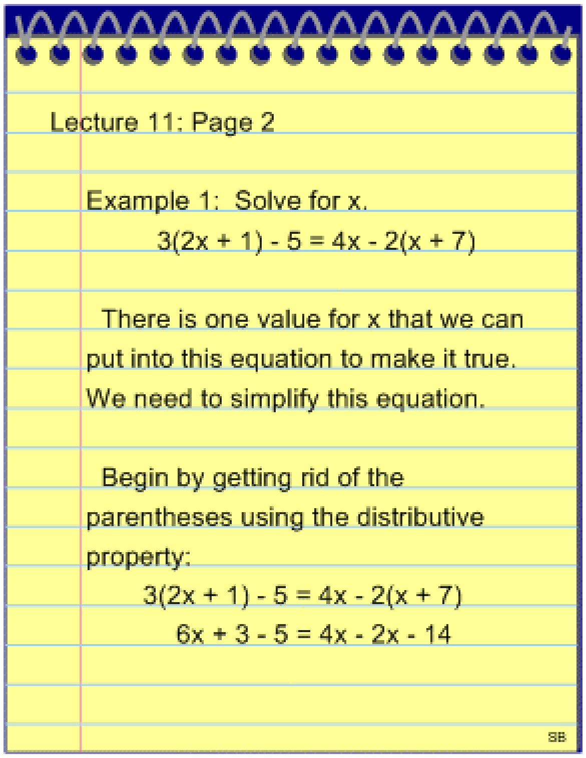
ALG2011-02



Lecture 11: Page 2

Example 1: Solve for x.

$3(2x + 1) - 5 = 4x - 2(x + 7)$

There is one value for x that we can put into this equation to make it true. We need to simplify this equation.

Begin by getting rid of the parentheses using the distributive property:

$3(2x + 1) - 5 = 4x - 2(x + 7)$

$6x + 3 - 5 = 4x - 2x - 14$

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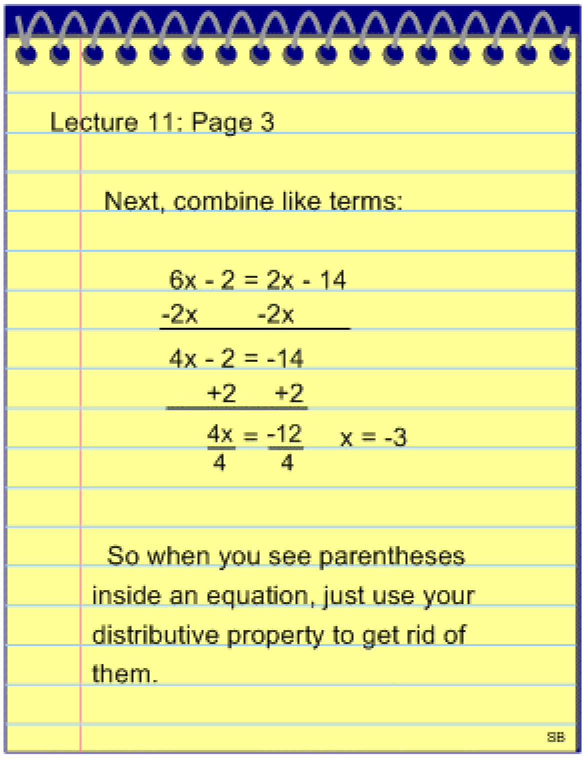
ALG2011-03



Lecture 11: Page 3

Next, combine like terms:

$6x - 2 = 2x - 14$

$-2x \quad\quad -2x$

$4x - 2 = -14$

$+2 \quad +2$

$\frac{4x}{4} = \frac{-12}{4} \quad x = -3$

So when you see parentheses inside an equation, just use your distributive property to get rid of them.

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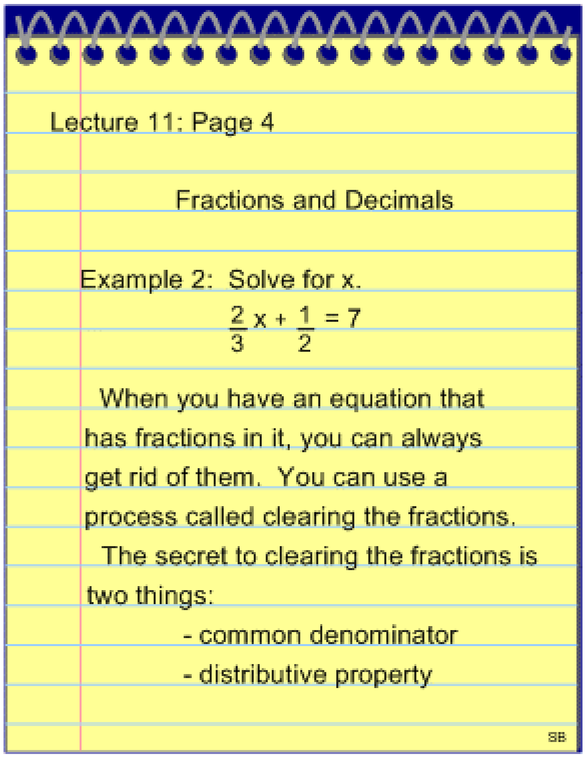
ALG2011-04



Lecture 11: Page 4

Fractions and Decimals

Example 2: Solve for x.

$\frac{2}{3}x + \frac{1}{2} = 7$

When you have an equation that has fractions in it, you can always get rid of them. You can use a process called clearing the fractions.

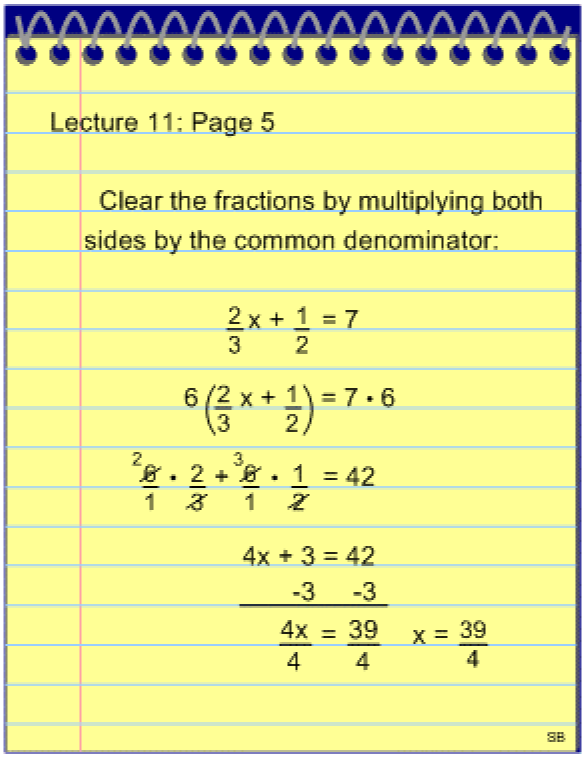
The secret to clearing the fractions is two things:

- common denominator
- distributive property

SB

Lecture 11 Notes, Continued

ALG2011-05



Lecture 11: Page 5

Clear the fractions by multiplying both sides by the common denominator:

$$\frac{2}{3}x + \frac{1}{2} = 7$$

$$6\left(\frac{2}{3}x + \frac{1}{2}\right) = 7 \cdot 6$$

$$\frac{\overset{2}{\cancel{6}}}{1} \cdot \frac{2}{\cancel{3}} + \frac{\overset{3}{\cancel{6}}}{1} \cdot \frac{1}{\cancel{2}} = 42$$

$$4x + 3 = 42$$

$$\underline{\quad -3 \quad -3}$$

$$\frac{4x}{4} = \frac{39}{4} \qquad x = \frac{39}{4}$$

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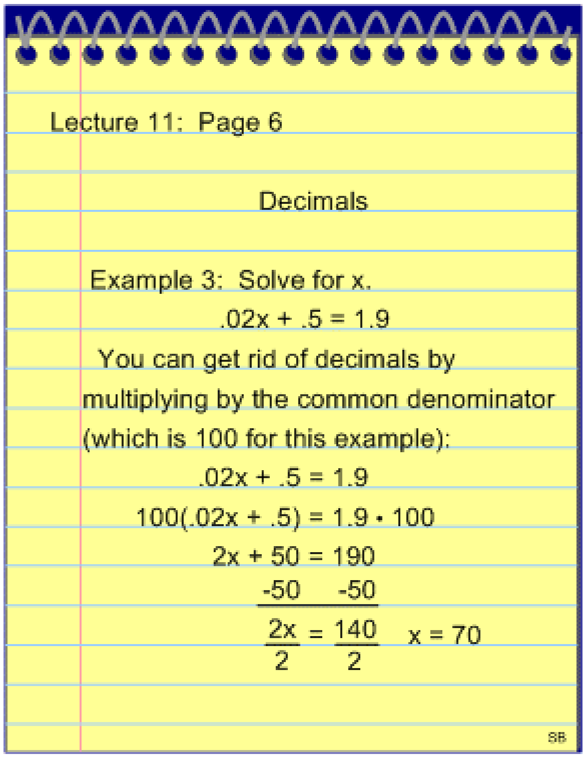
ALG2011-06



Lecture 11: Page 6

Decimals

Example 3: Solve for x.

$$.02x + .5 = 1.9$$

You can get rid of decimals by multiplying by the common denominator (which is 100 for this example):

$$.02x + .5 = 1.9$$

$$100(.02x + .5) = 1.9 \cdot 100$$

$$2x + 50 = 190$$

$$\underline{-50 \quad -50}$$

$$\frac{2x}{2} = \frac{140}{2} \qquad x = 70$$

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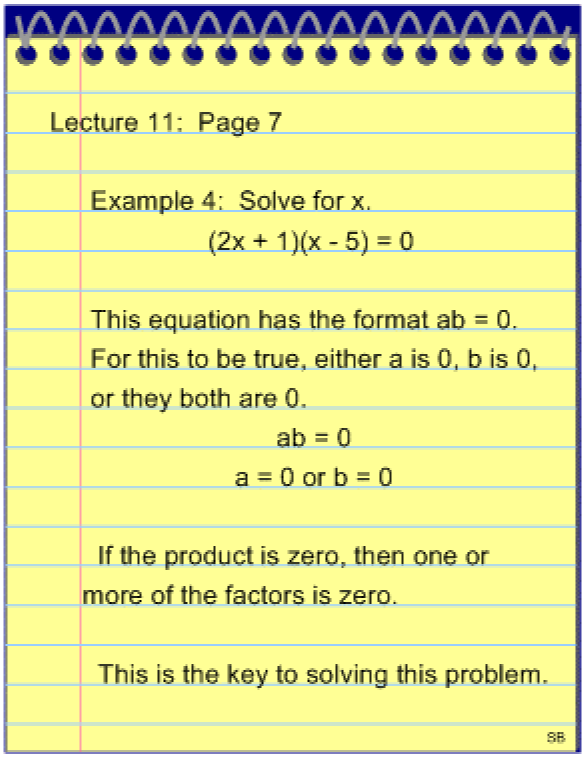
ALG2011-07



Lecture 11: Page 7

Example 4: Solve for x.

$$(2x + 1)(x - 5) = 0$$

This equation has the format ab = 0. For this to be true, either a is 0, b is 0, or they both are 0.

$$ab = 0$$

$$a = 0 \text{ or } b = 0$$

If the product is zero, then one or more of the factors is zero.

This is the key to solving this problem.

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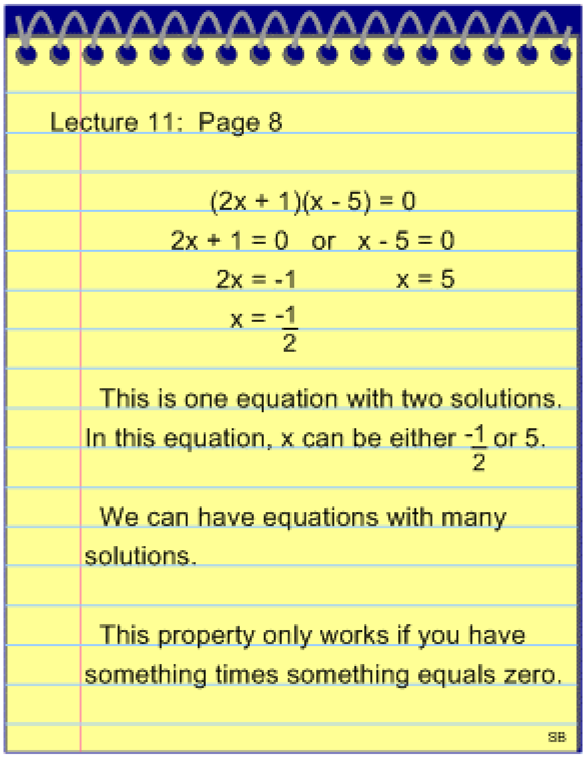
ALG2011-08



Lecture 11: Page 8

$$(2x + 1)(x - 5) = 0$$

$$2x + 1 = 0 \quad \text{or} \quad x - 5 = 0$$

$$2x = -1 \qquad\qquad x = 5$$

$$x = \frac{-1}{2}$$

This is one equation with two solutions. In this equation, x can be either $\frac{-1}{2}$ or 5.

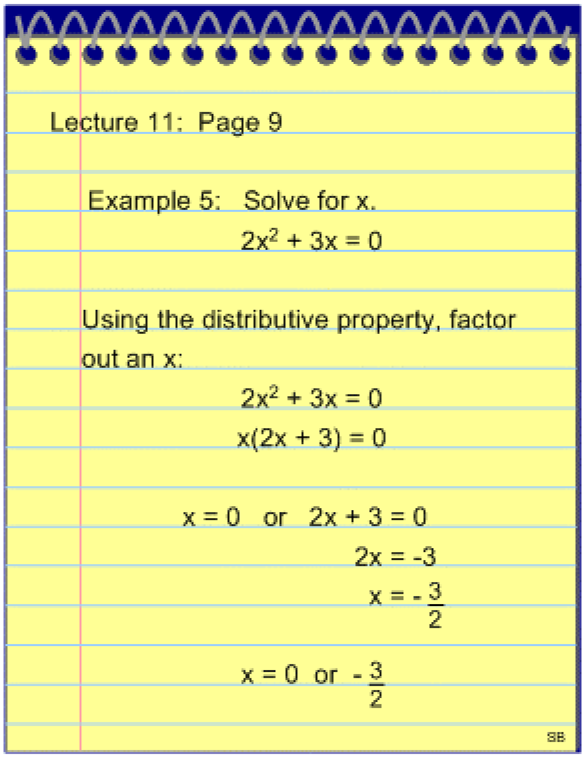
We can have equations with many solutions.

This property only works if you have something times something equals zero.

SB

Lecture 11 Notes, Continued

ALG2011-09



Lecture 11: Page 9

Example 5: Solve for x.

$$2x^2 + 3x = 0$$

Using the distributive property, factor out an x:

$$2x^2 + 3x = 0$$

$$x(2x + 3) = 0$$

$$x = 0 \quad \text{or} \quad 2x + 3 = 0$$

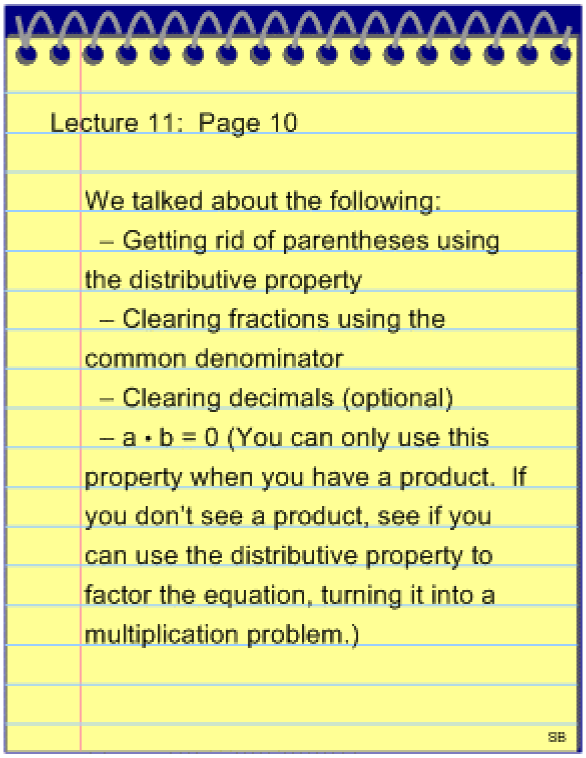
$$2x = -3$$

$$x = -\frac{3}{2}$$

$$x = 0 \ \text{ or } \ -\frac{3}{2}$$

SB

ALG2011-10



Lecture 11: Page 10

We talked about the following:

– Getting rid of parentheses using the distributive property

– Clearing fractions using the common denominator

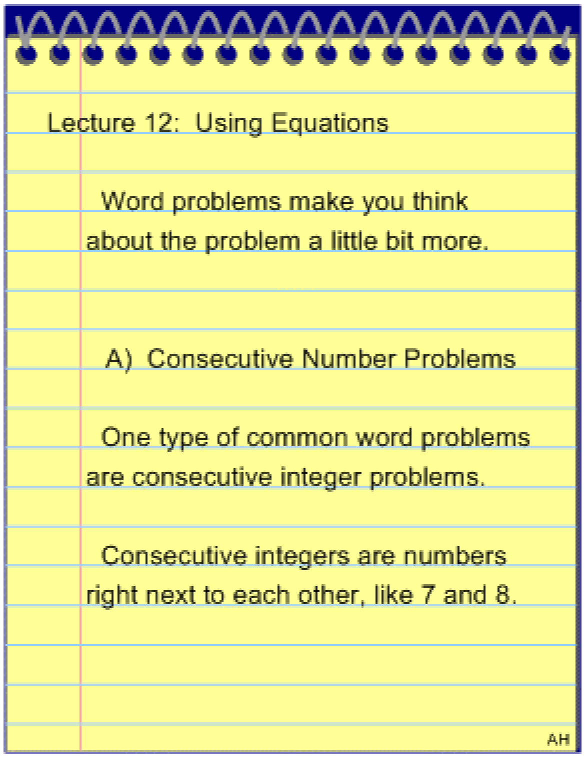
– Clearing decimals (optional)

– $a \cdot b = 0$ (You can only use this property when you have a product. If you don't see a product, see if you can use the distributive property to factor the equation, turning it into a multiplication problem.)

SB

Lecture 12 Notes

ALG20012-01



Lecture 12: Using Equations

Word problems make you think about the problem a little bit more.

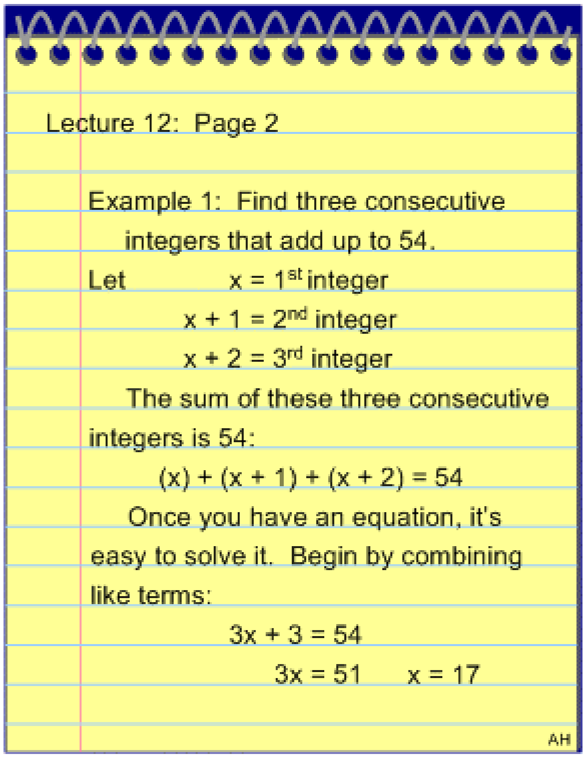
A) Consecutive Number Problems

One type of common word problems are consecutive integer problems.

Consecutive integers are numbers right next to each other, like 7 and 8.

AH

ALG2012-02



Lecture 12: Page 2

Example 1: Find three consecutive integers that add up to 54.

Let x = 1st integer
$x + 1$ = 2nd integer
$x + 2$ = 3rd integer

The sum of these three consecutive integers is 54:

$$(x) + (x + 1) + (x + 2) = 54$$

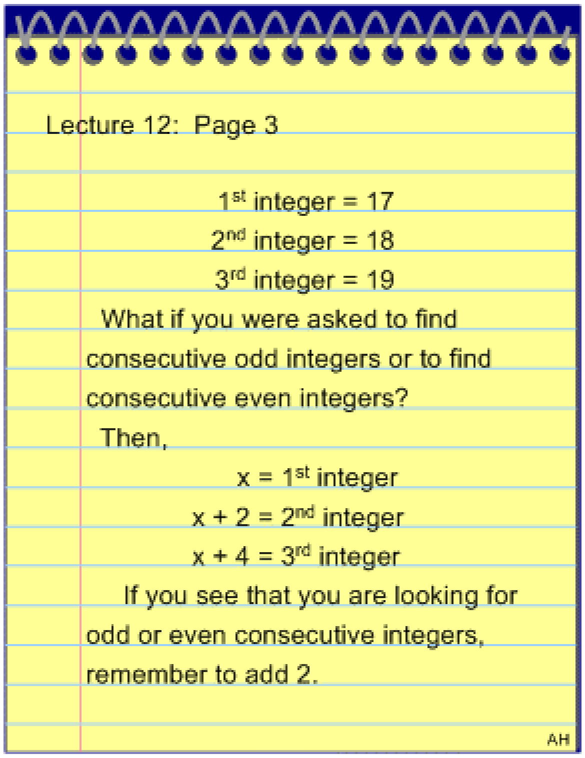
Once you have an equation, it's easy to solve it. Begin by combining like terms:

$$3x + 3 = 54$$

$$3x = 51 \qquad x = 17$$

AH

ALG2012-03



Lecture 12: Page 3

1st integer = 17
2nd integer = 18
3rd integer = 19

What if you were asked to find consecutive odd integers or to find consecutive even integers?

Then,

x = 1st integer
$x + 2$ = 2nd integer
$x + 4$ = 3rd integer

If you see that you are looking for odd or even consecutive integers, remember to add 2.

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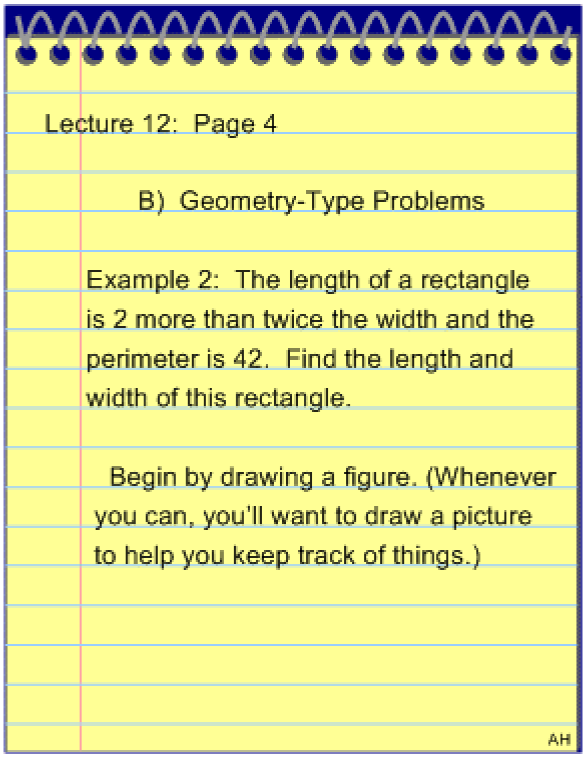
ALG2012-04



Lecture 12: Page 4

B) Geometry-Type Problems

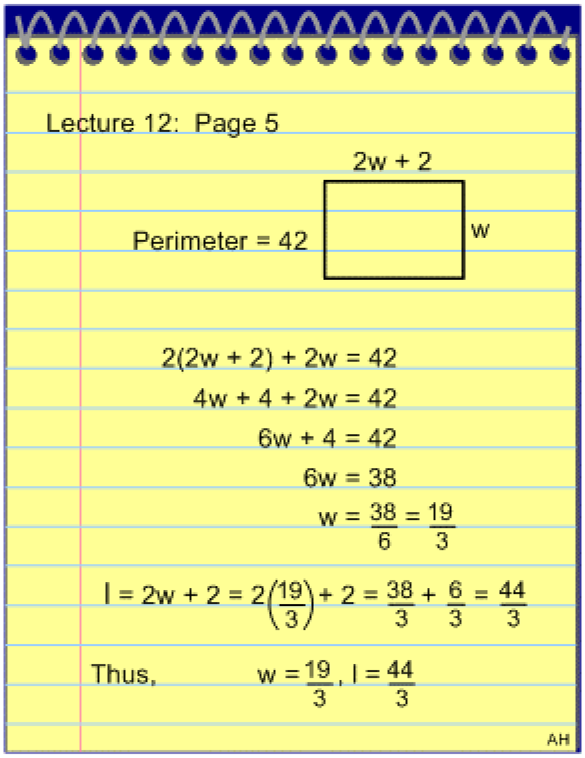
Example 2: The length of a rectangle is 2 more than twice the width and the perimeter is 42. Find the length and width of this rectangle.

Begin by drawing a figure. (Whenever you can, you'll want to draw a picture to help you keep track of things.)

AH

Lecture 12 Notes, Continued

ALG2012-05



Lecture 12: Page 5

Perimeter = 42

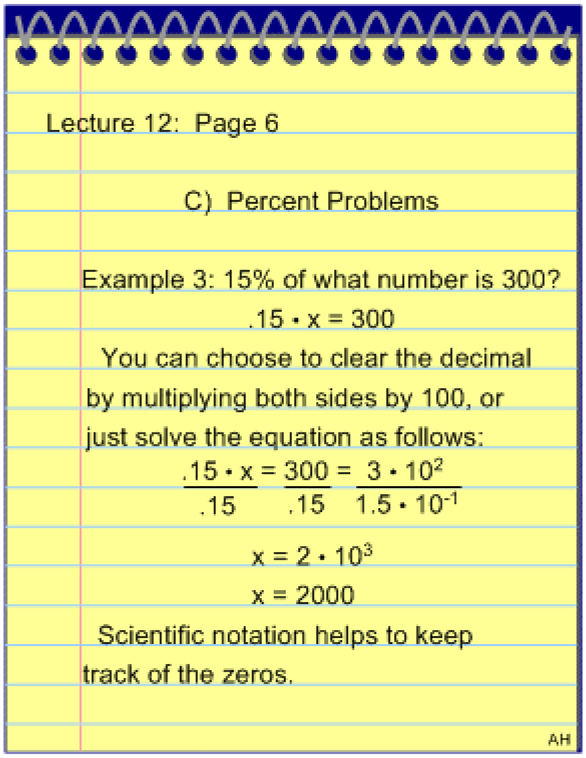
$$2(2w + 2) + 2w = 42$$
$$4w + 4 + 2w = 42$$
$$6w + 4 = 42$$
$$6w = 38$$
$$w = \frac{38}{6} = \frac{19}{3}$$

$$l = 2w + 2 = 2\left(\frac{19}{3}\right) + 2 = \frac{38}{3} + \frac{6}{3} = \frac{44}{3}$$

Thus, $w = \frac{19}{3}, \ l = \frac{44}{3}$

AH

ALG2012-06



Lecture 12: Page 6

C) Percent Problems

Example 3: 15% of what number is 300?

$$.15 \cdot x = 300$$

You can choose to clear the decimal by multiplying both sides by 100, or just solve the equation as follows:

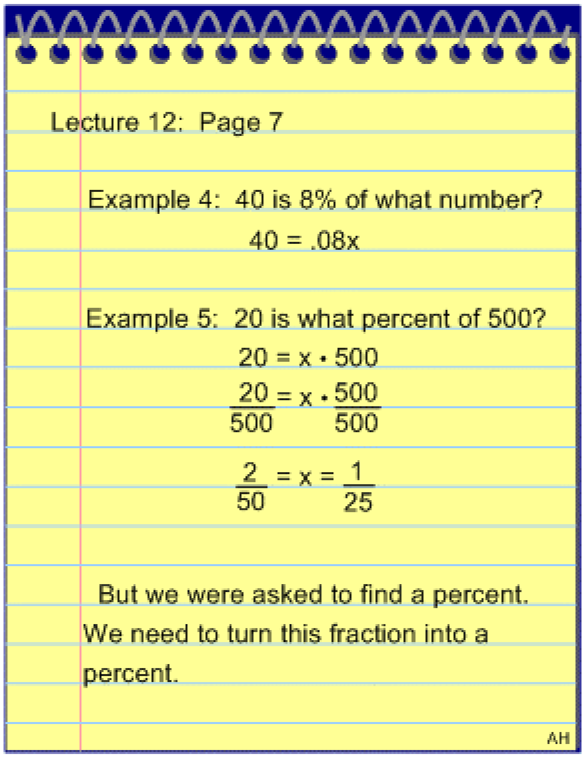
$$\frac{.15 \cdot x}{.15} = \frac{300}{.15} = \frac{3 \cdot 10^2}{1.5 \cdot 10^{-1}}$$

$$x = 2 \cdot 10^3$$
$$x = 2000$$

Scientific notation helps to keep track of the zeros.

AH

ALG2012-07



Lecture 12: Page 7

Example 4: 40 is 8% of what number?

$$40 = .08x$$

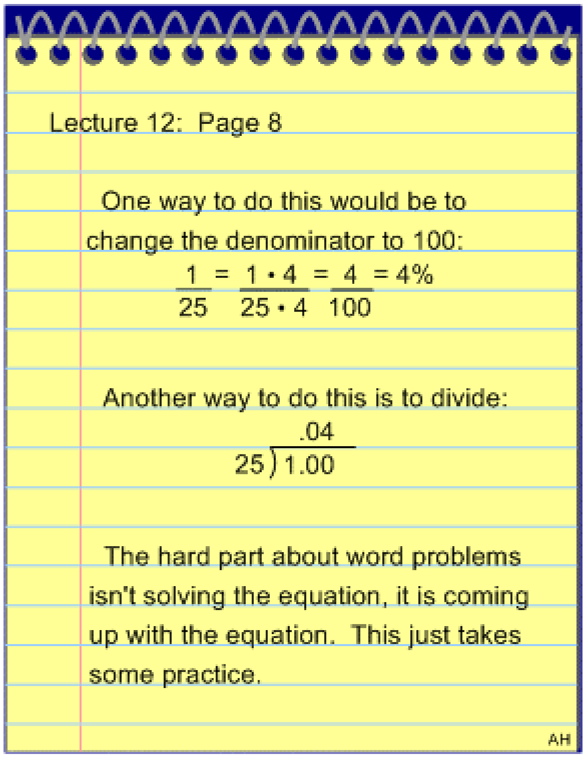
Example 5: 20 is what percent of 500?

$$20 = x \cdot 500$$
$$\frac{20}{500} = \frac{x \cdot 500}{500}$$
$$\frac{2}{50} = x = \frac{1}{25}$$

But we were asked to find a percent. We need to turn this fraction into a percent.

AH

ALG2012-08



Lecture 12: Page 8

One way to do this would be to change the denominator to 100:

$$\frac{1}{25} = \frac{1 \cdot 4}{25 \cdot 4} = \frac{4}{100} = 4\%$$

Another way to do this is to divide:

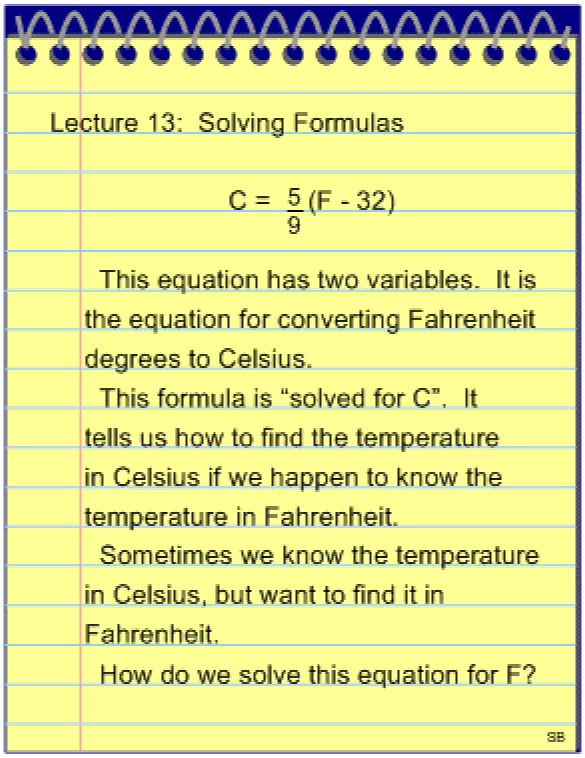
$$\begin{array}{r} .04 \\ 25\overline{)1.00} \end{array}$$

The hard part about word problems isn't solving the equation, it is coming up with the equation. This just takes some practice.

AH

Lecture 13 Notes

ALG2013-01



Lecture 13: Solving Formulas

$$C = \frac{5}{9}(F - 32)$$

This equation has two variables. It is the equation for converting Fahrenheit degrees to Celsius.

This formula is "solved for C". It tells us how to find the temperature in Celsius if we happen to know the temperature in Fahrenheit.

Sometimes we know the temperature in Celsius, but want to find it in Fahrenheit.

How do we solve this equation for F?

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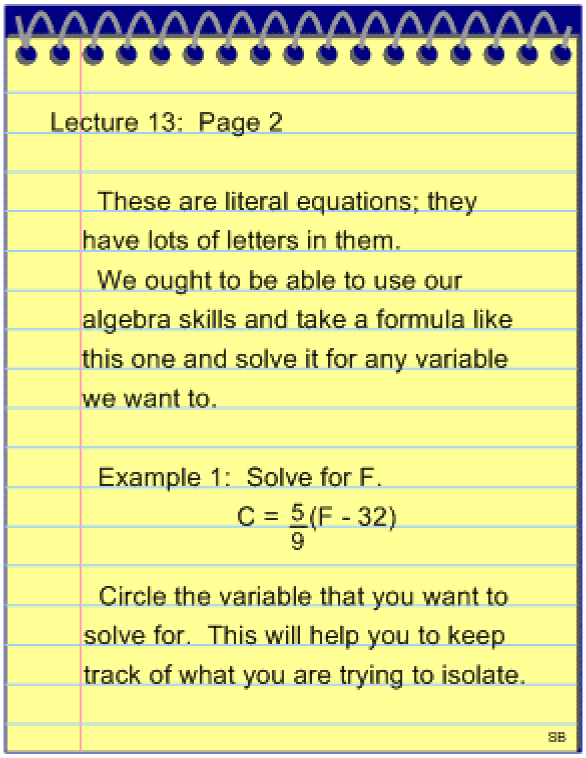
ALG2013-02



Lecture 13: Page 2

These are literal equations; they have lots of letters in them.

We ought to be able to use our algebra skills and take a formula like this one and solve it for any variable we want to.

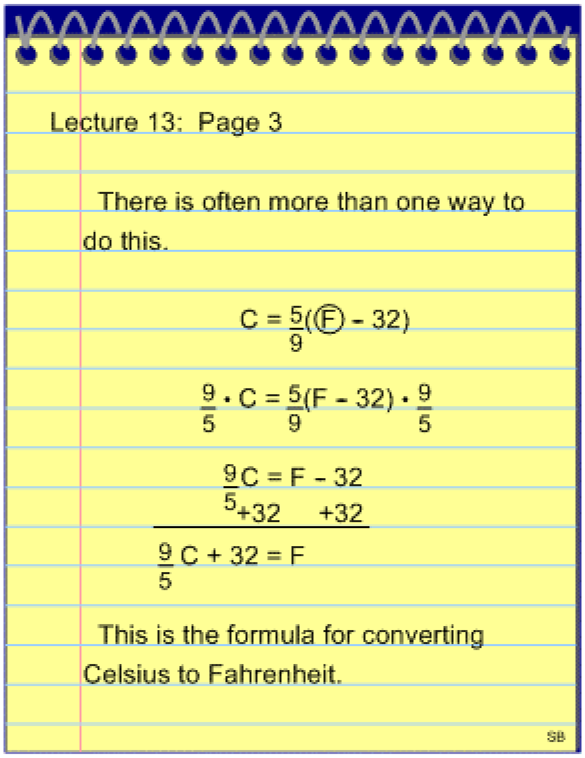
Example 1: Solve for F.

$$C = \frac{5}{9}(F - 32)$$

Circle the variable that you want to solve for. This will help you to keep track of what you are trying to isolate.

SB

ALG2013-03



Lecture 13: Page 3

There is often more than one way to do this.

$$C = \frac{5}{9}(\textcircled{F} - 32)$$

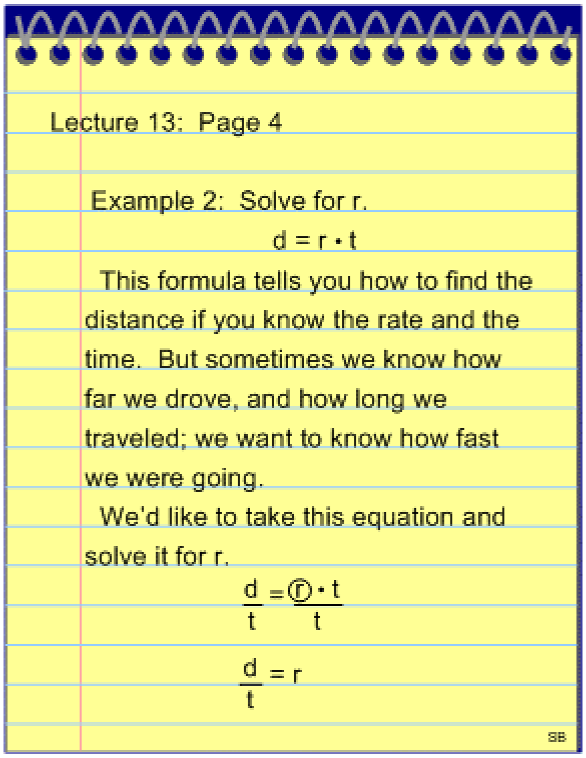
$$\frac{9}{5} \cdot C = \frac{5}{9}(F - 32) \cdot \frac{9}{5}$$

$$\begin{array}{l} \frac{9}{5}C = F - 32 \\ \underline{+32 \quad\quad +32} \\ \frac{9}{5}C + 32 = F \end{array}$$

This is the formula for converting Celsius to Fahrenheit.

SB

ALG2013-04



Lecture 13: Page 4

Example 2: Solve for r.

$$d = r \cdot t$$

This formula tells you how to find the distance if you know the rate and the time. But sometimes we know how far we drove, and how long we traveled; we want to know how fast we were going.

We'd like to take this equation and solve it for r.

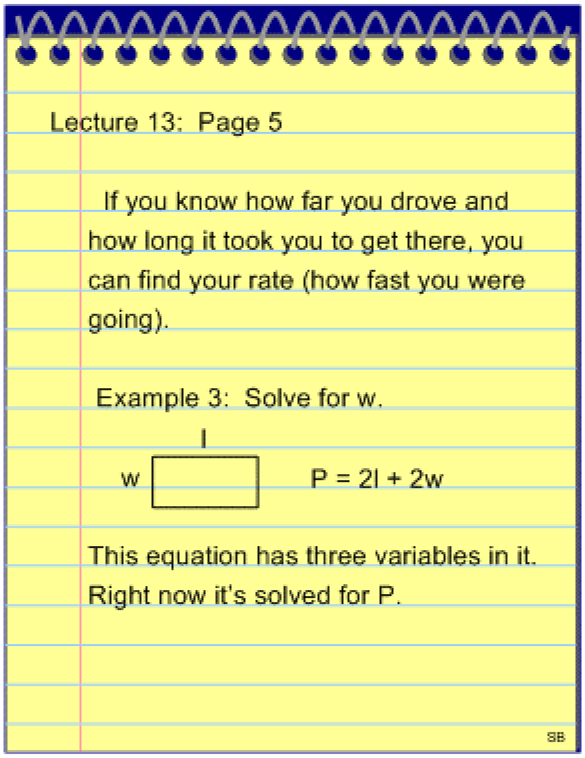
$$\frac{d}{t} = \frac{\textcircled{r} \cdot t}{t}$$

$$\frac{d}{t} = r$$

SB

Lecture 13 Notes, Continued

ALG2013-05



Lecture 13: Page 5

If you know how far you drove and how long it took you to get there, you can find your rate (how fast you were going).

Example 3: Solve for w.

l

w $\qquad$ $P = 2l + 2w$

This equation has three variables in it. Right now it's solved for P.

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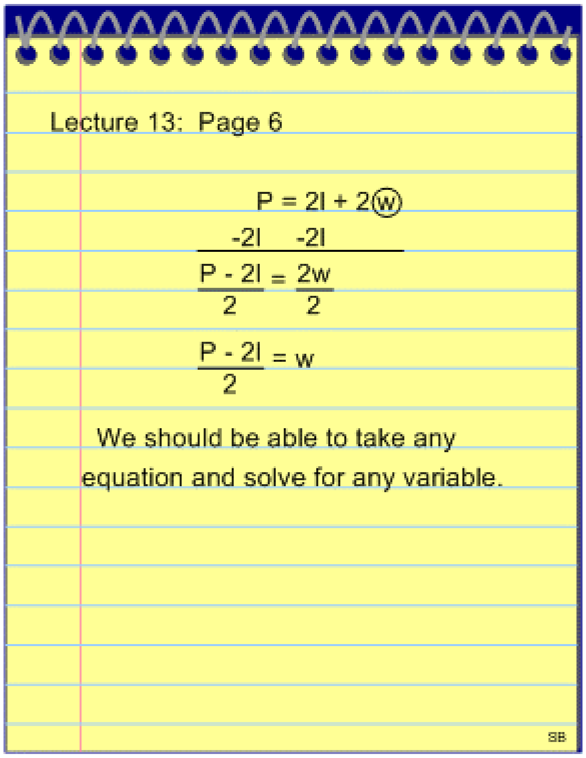
ALG2013-06



Lecture 13: Page 6

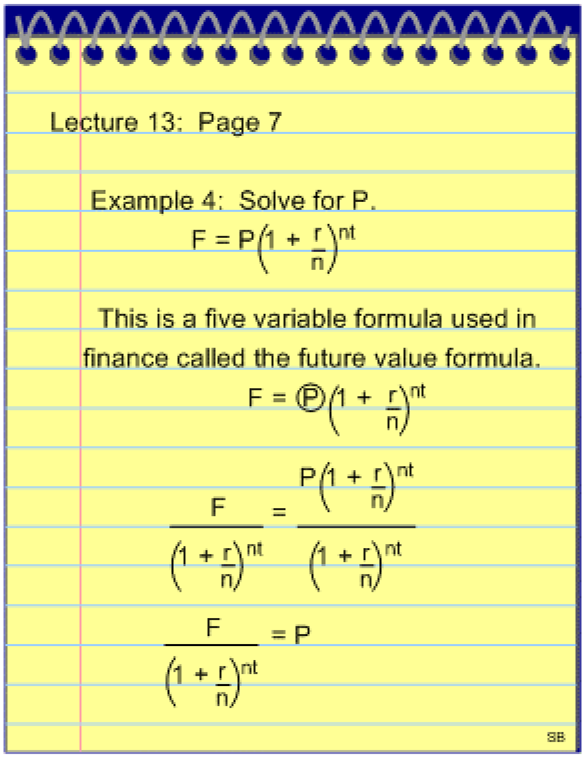
$$\begin{array}{rcl} P &=& 2l + 2\textcircled{w} \\ -2l && -2l \\ \hline \frac{P - 2l}{2} &=& \frac{2w}{2} \end{array}$$

$$\frac{P - 2l}{2} = w$$

We should be able to take any equation and solve for any variable.

SB

ALG2013-07



Lecture 13: Page 7

Example 4: Solve for P.

$$F = P\left(1 + \frac{r}{n}\right)^{nt}$$

This is a five variable formula used in finance called the future value formula.

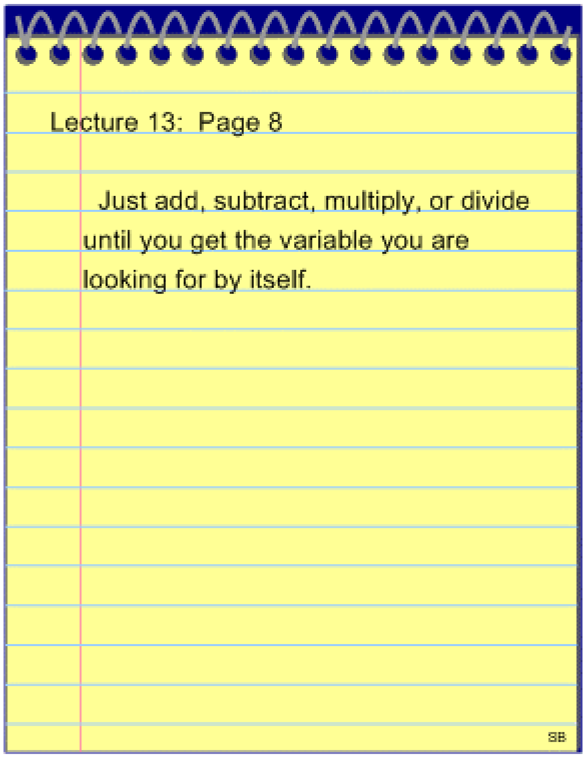
$$F = \textcircled{P}\left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = \frac{P\left(1 + \frac{r}{n}\right)^{nt}}{\left(1 + \frac{r}{n}\right)^{nt}}$$

$$\frac{F}{\left(1 + \frac{r}{n}\right)^{nt}} = P$$

SB

ALG20013-08



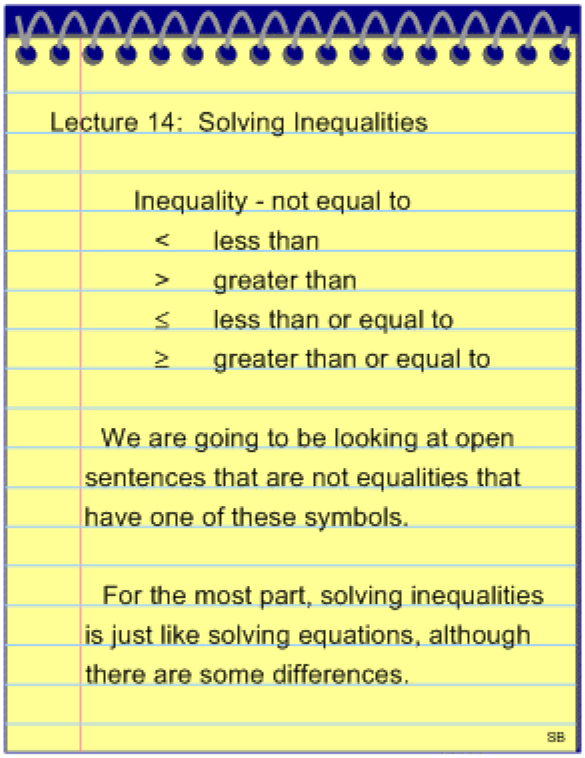
Lecture 13: Page 8

Just add, subtract, multiply, or divide until you get the variable you are looking for by itself.

SB

Lecture 14 Notes

ALG2014-01



Lecture 14: Solving Inequalities

Inequality - not equal to

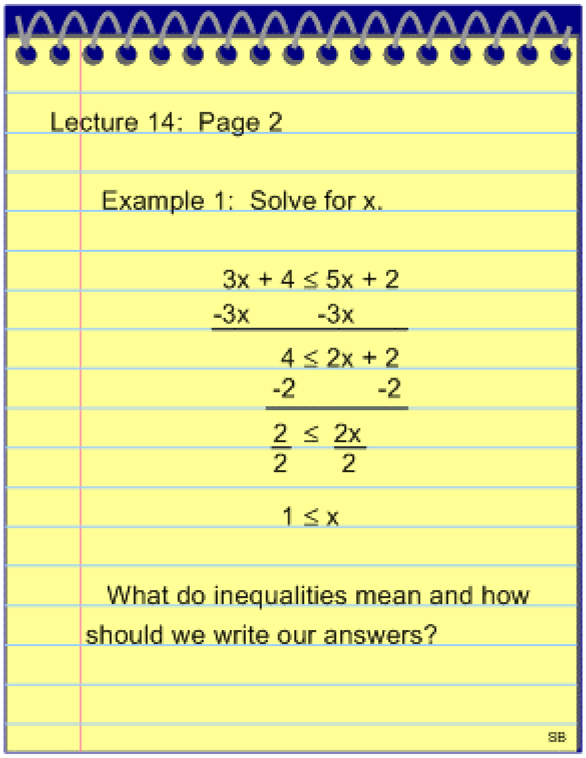
$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to

We are going to be looking at open sentences that are not equalities that have one of these symbols.

For the most part, solving inequalities is just like solving equations, although there are some differences.

SB

ALG2014-02



Lecture 14: Page 2

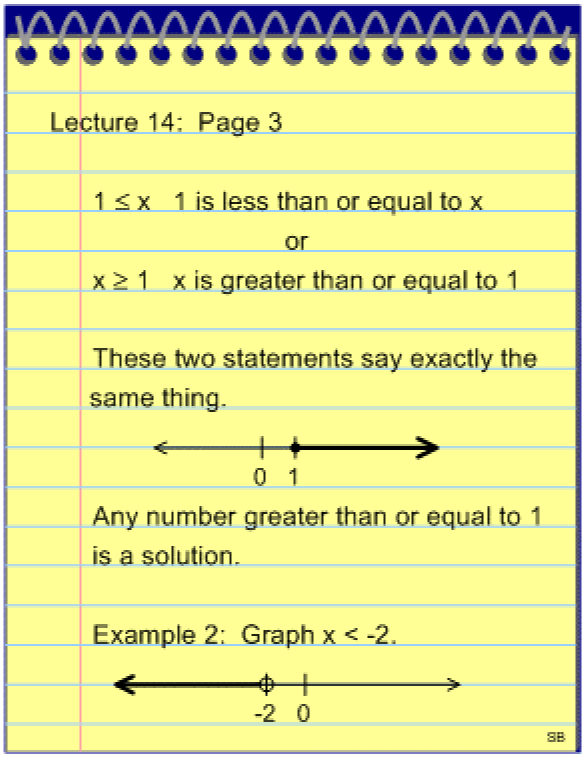
Example 1: Solve for x.

$$3x + 4 \leq 5x + 2$$
$$\underline{-3x \qquad -3x}$$
$$4 \leq 2x + 2$$
$$\underline{-2 \qquad\quad -2}$$
$$\frac{2}{2} \leq \frac{2x}{2}$$
$$1 \leq x$$

What do inequalities mean and how should we write our answers?

SB

ALG2014-03



Lecture 14: Page 3

$1 \leq x$ 1 is less than or equal to x

or

$x \geq 1$ x is greater than or equal to 1

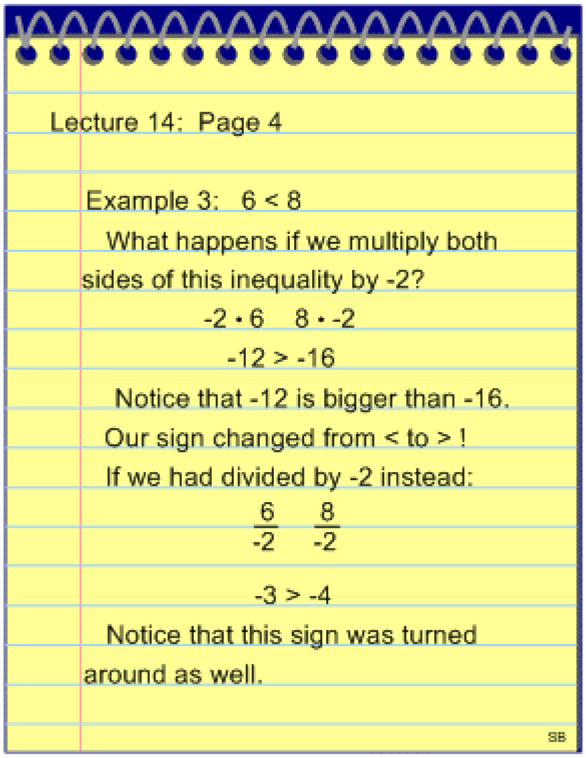
These two statements say exactly the same thing.

Any number greater than or equal to 1 is a solution.

Example 2: Graph $x < -2$.

SB

ALG2014-04



Lecture 14: Page 4

Example 3: $6 < 8$

What happens if we multiply both sides of this inequality by -2?

$$-2 \cdot 6 \quad 8 \cdot -2$$
$$-12 > -16$$

Notice that -12 is bigger than -16.

Our sign changed from < to > !

If we had divided by -2 instead:

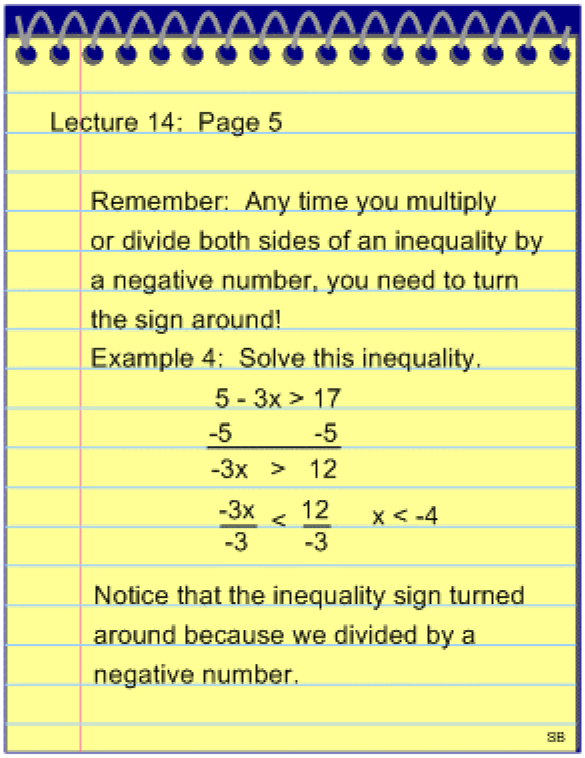
$$\frac{6}{-2} \quad \frac{8}{-2}$$
$$-3 > -4$$

Notice that this sign was turned around as well.

SB

Lecture 14 Notes, Continued

ALG2014-05



Lecture 14: Page 5

Remember: Any time you multiply or divide both sides of an inequality by a negative number, you need to turn the sign around!

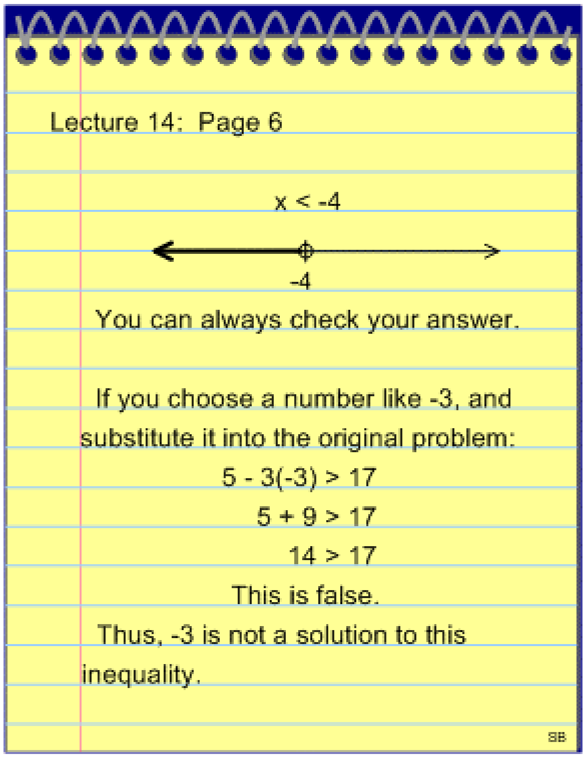
Example 4: Solve this inequality.

$$5 - 3x > 17$$
$$\underline{-5 \qquad -5}$$
$$-3x > 12$$
$$\frac{-3x}{-3} < \frac{12}{-3} \qquad x < -4$$

Notice that the inequality sign turned around because we divided by a negative number.

SB

ALG2014-06



Lecture 14: Page 6

$x < -4$

-4

You can always check your answer.

If you choose a number like -3, and substitute it into the original problem:

$$5 - 3(-3) > 17$$
$$5 + 9 > 17$$
$$14 > 17$$

This is false.

Thus, -3 is not a solution to this inequality.

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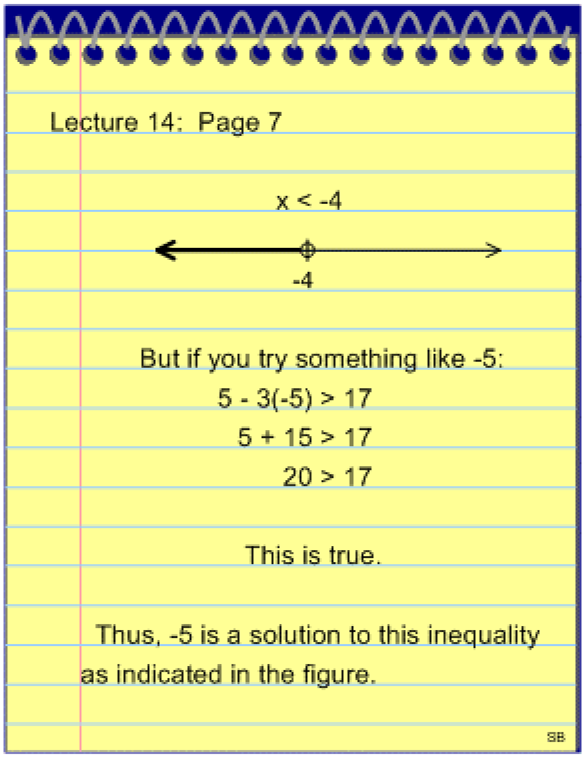
ALG2014-07



Lecture 14: Page 7

$x < -4$

-4

But if you try something like -5:

$$5 - 3(-5) > 17$$
$$5 + 15 > 17$$
$$20 > 17$$

This is true.

Thus, -5 is a solution to this inequality as indicated in the figure.

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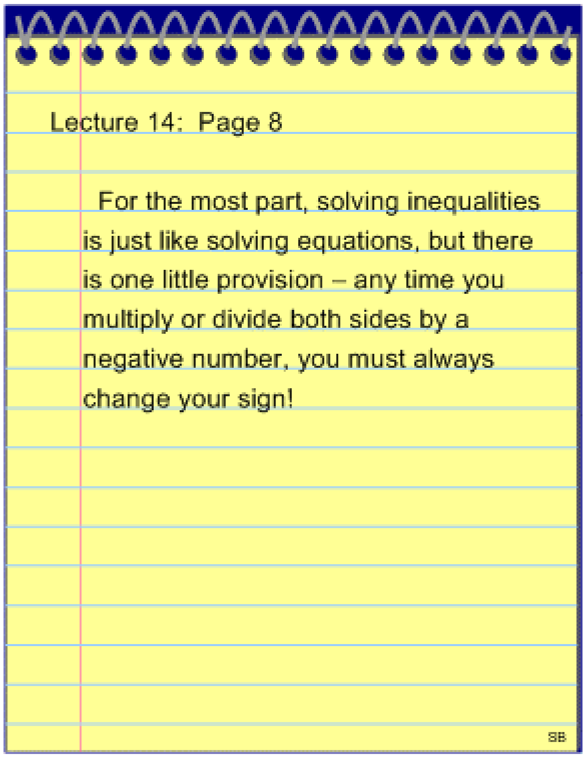
ALG2014-08



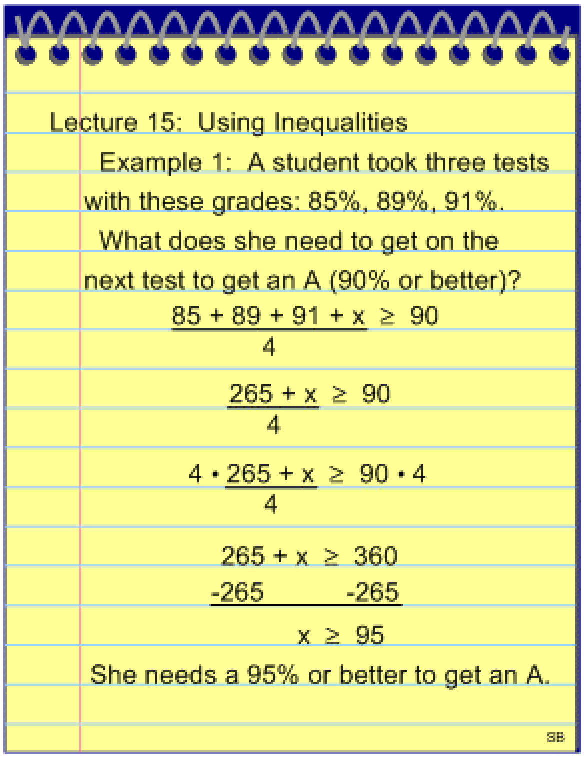
Lecture 14: Page 8

For the most part, solving inequalities is just like solving equations, but there is one little provision – any time you multiply or divide both sides by a negative number, you must always change your sign!

SB

Lecture 15 Notes

ALG2015-01



Lecture 15: Using Inequalities

Example 1: A student took three tests with these grades: 85%, 89%, 91%. What does she need to get on the next test to get an A (90% or better)?

$$\frac{85 + 89 + 91 + x}{4} \geq 90$$

$$\frac{265 + x}{4} \geq 90$$

$$4 \cdot \frac{265 + x}{4} \geq 90 \cdot 4$$

$$\begin{array}{rcl} 265 + x & \geq & 360 \\ \underline{-265} & & \underline{-265} \\ x & \geq & 95 \end{array}$$

She needs a 95% or better to get an A.

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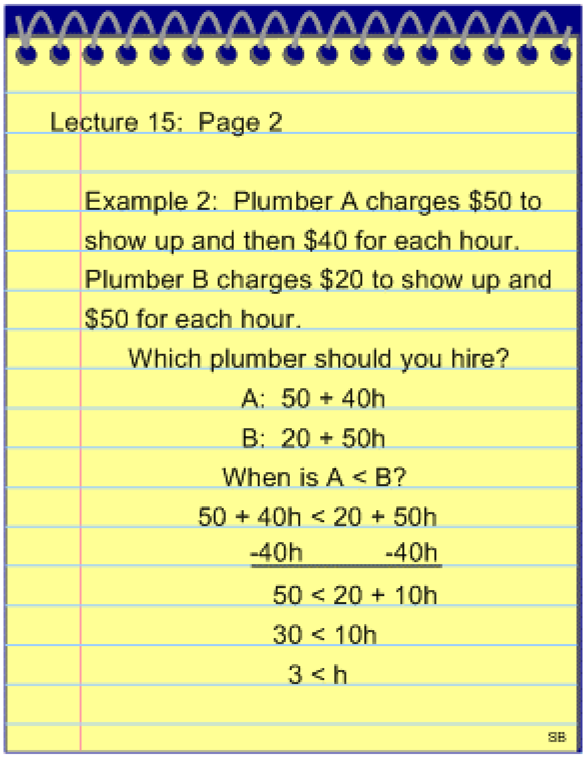
ALG2015-02



Lecture 15: Page 2

Example 2: Plumber A charges $50 to show up and then $40 for each hour. Plumber B charges $20 to show up and $50 for each hour.

Which plumber should you hire?

A: $50 + 40h$

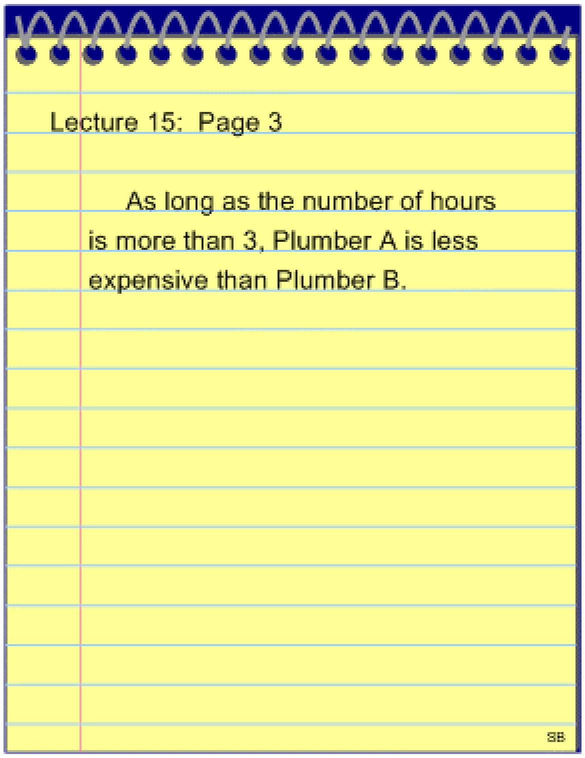
B: $20 + 50h$

When is A < B?

$$\begin{array}{rcl} 50 + 40h & < & 20 + 50h \\ \underline{-40h} & & \underline{-40h} \\ 50 & < & 20 + 10h \\ 30 & < & 10h \\ 3 & < & h \end{array}$$

SB

ALG2015-03



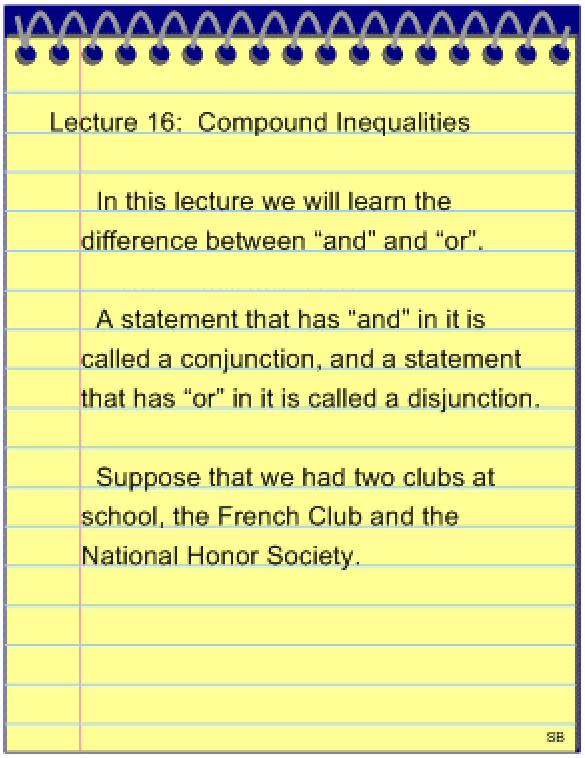
Lecture 15: Page 3

As long as the number of hours is more than 3, Plumber A is less expensive than Plumber B.

SB

Lecture 16 Notes

ALG2016-01



Lecture 16: Compound Inequalities

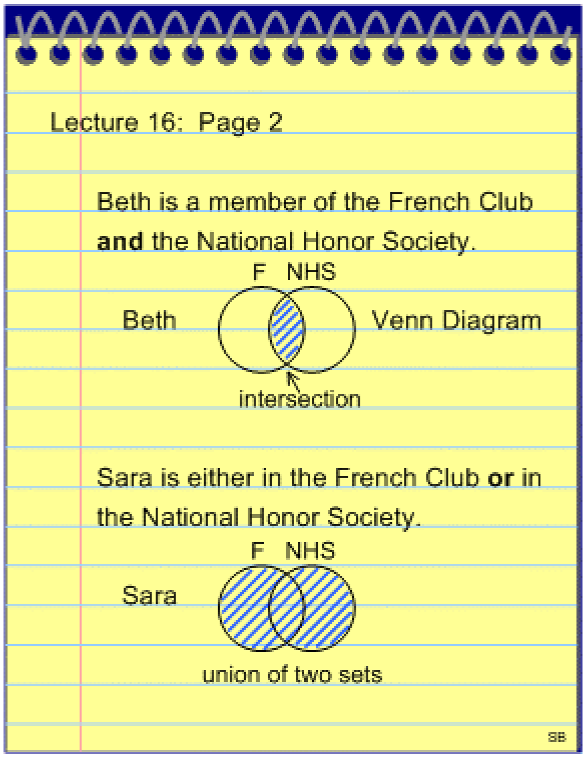
In this lecture we will learn the difference between "and" and "or".

A statement that has "and" in it is called a conjunction, and a statement that has "or" in it is called a disjunction.

Suppose that we had two clubs at school, the French Club and the National Honor Society.

SB

ALG2016-02



Lecture 16: Page 2

Beth is a member of the French Club **and** the National Honor Society.

Sara is either in the French Club **or** in the National Honor Society.

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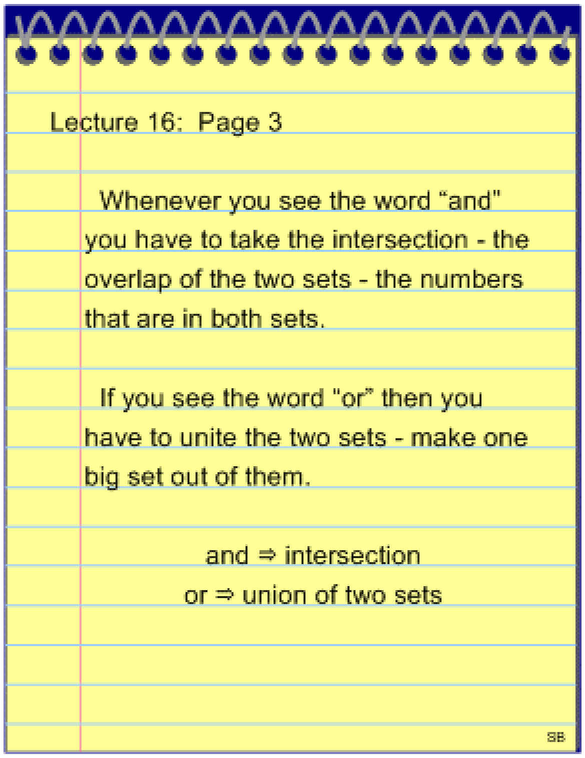
ALG2016-03



Lecture 16: Page 3

Whenever you see the word "and" you have to take the intersection - the overlap of the two sets - the numbers that are in both sets.

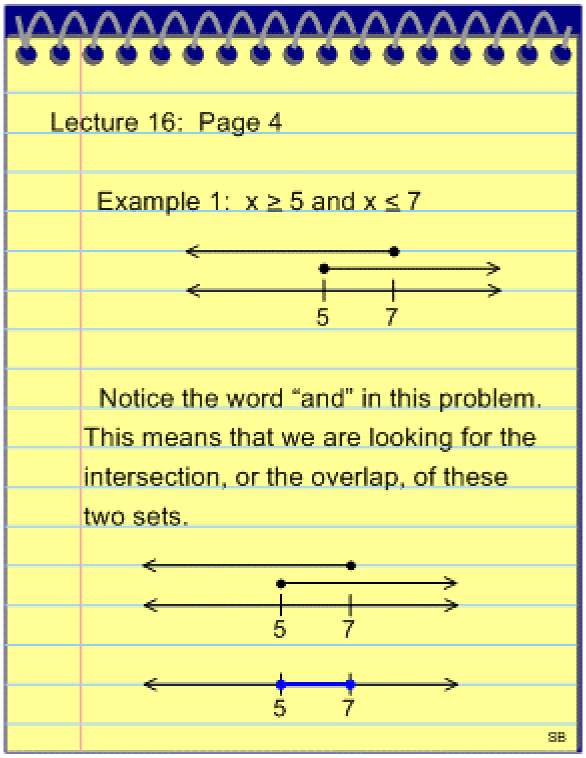
If you see the word "or" then you have to unite the two sets - make one big set out of them.

and ⇒ intersection

or ⇒ union of two sets

SB

ALG2016-04



Lecture 16: Page 4

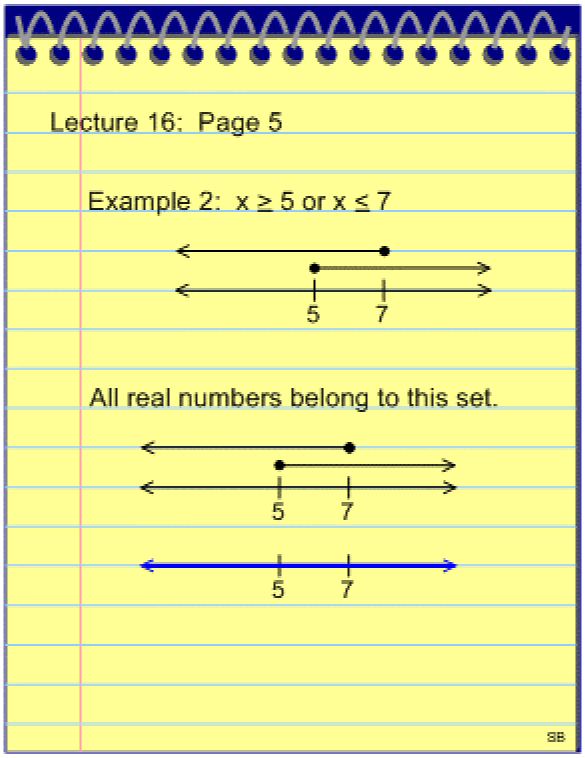
Example 1: $x \geq 5$ and $x \leq 7$

Notice the word "and" in this problem. This means that we are looking for the intersection, or the overlap, of these two sets.

SB

Lecture 16 Notes, Continued

ALG2016-05

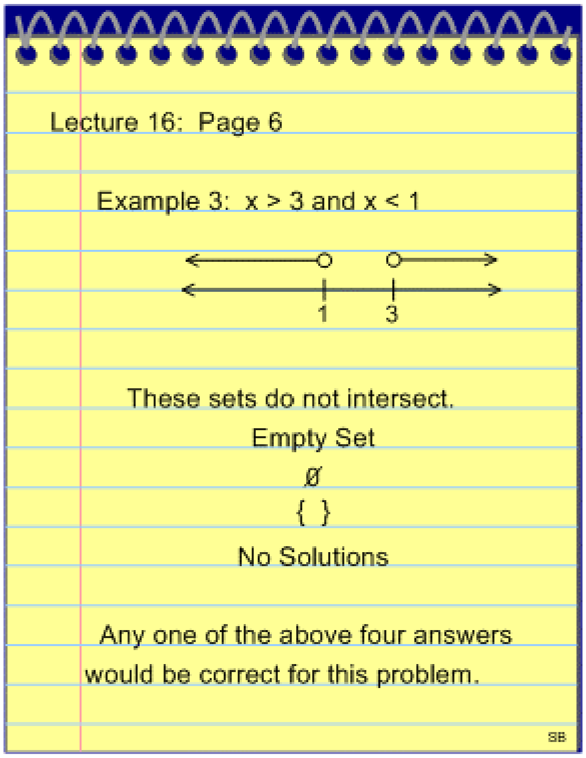


Lecture 16: Page 5

Example 2: $x \geq 5$ or $x \leq 7$

All real numbers belong to this set.

ALG2016-06



Lecture 16: Page 6

Example 3: $x > 3$ and $x < 1$

These sets do not intersect.

Empty Set

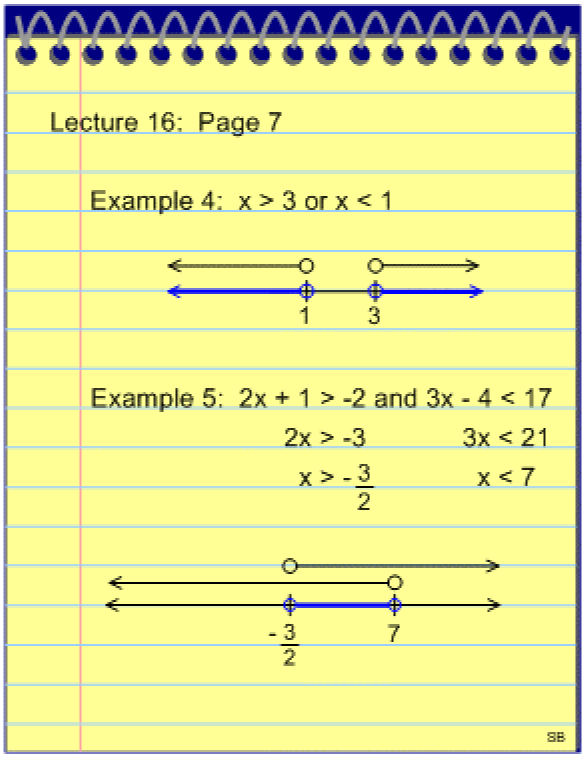
$\emptyset$

{ }

No Solutions

Any one of the above four answers would be correct for this problem.

ALG2016-07



Lecture 16: Page 7

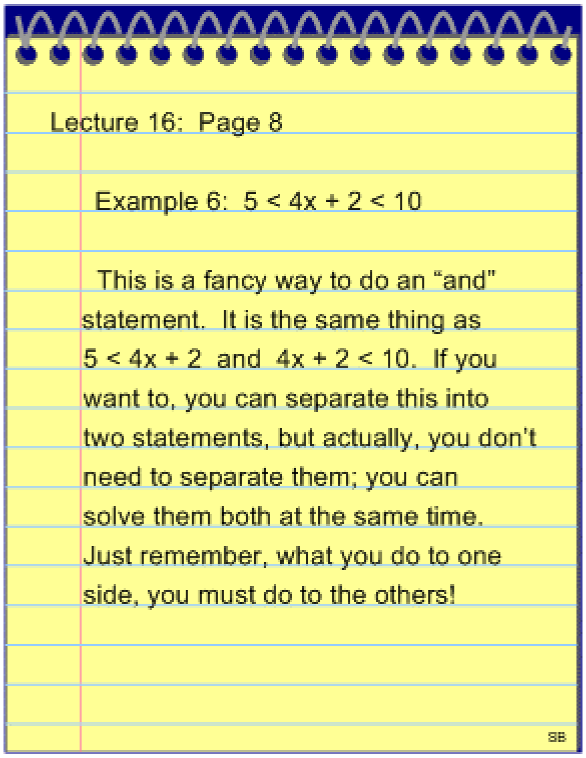
Example 4: $x > 3$ or $x < 1$

Example 5: $2x + 1 > -2$ and $3x - 4 < 17$

$2x > -3$ $3x < 21$

$x > -\frac{3}{2}$ $x < 7$

ALG2016-08



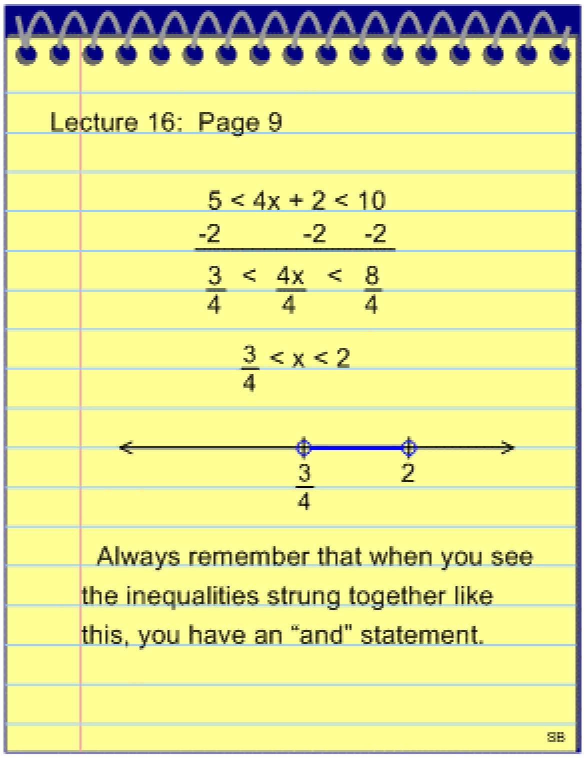
Lecture 16: Page 8

Example 6: $5 < 4x + 2 < 10$

This is a fancy way to do an “and” statement. It is the same thing as $5 < 4x + 2$ and $4x + 2 < 10$. If you want to, you can separate this into two statements, but actually, you don't need to separate them; you can solve them both at the same time. Just remember, what you do to one side, you must do to the others!

Lecture 16 Notes, Continued

ALG2016-09



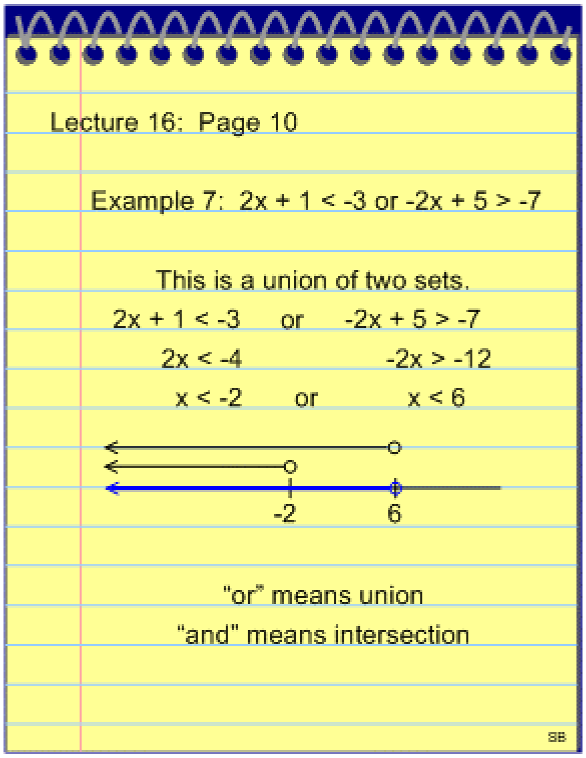
Lecture 16: Page 9

$$5 < 4x + 2 < 10$$
$$\underline{-2 \qquad -2 \quad -2}$$
$$\frac{3}{4} < \frac{4x}{4} < \frac{8}{4}$$
$$\frac{3}{4} < x < 2$$

Always remember that when you see the inequalities strung together like this, you have an "and" statement.

SB

ALG2016-10



Lecture 16: Page 10

Example 7: $2x + 1 < -3$ or $-2x + 5 > -7$

This is a union of two sets.

$2x + 1 < -3$ or $-2x + 5 > -7$

$2x < -4$ $\qquad$ $-2x > -12$

$x < -2$ or $x < 6$

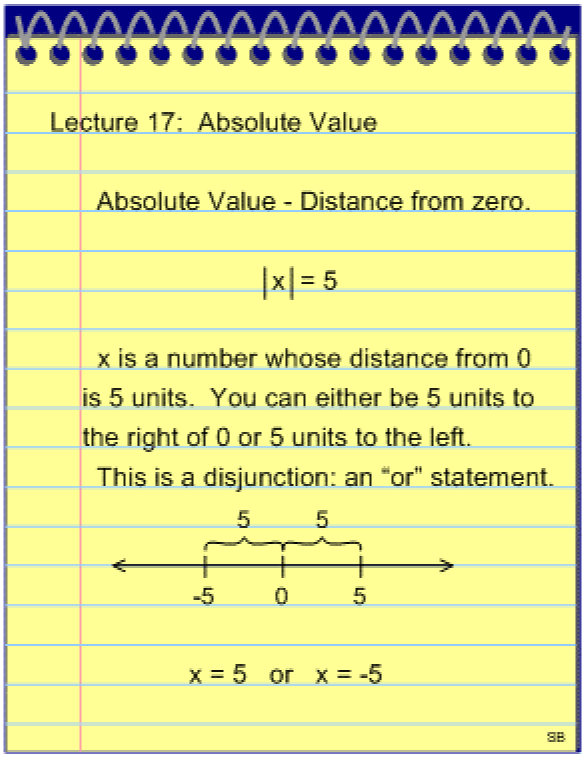
"or" means union

"and" means intersection

SB

Lecture 17 Notes

ALG2017-01



Lecture 17: Absolute Value

Absolute Value - Distance from zero.

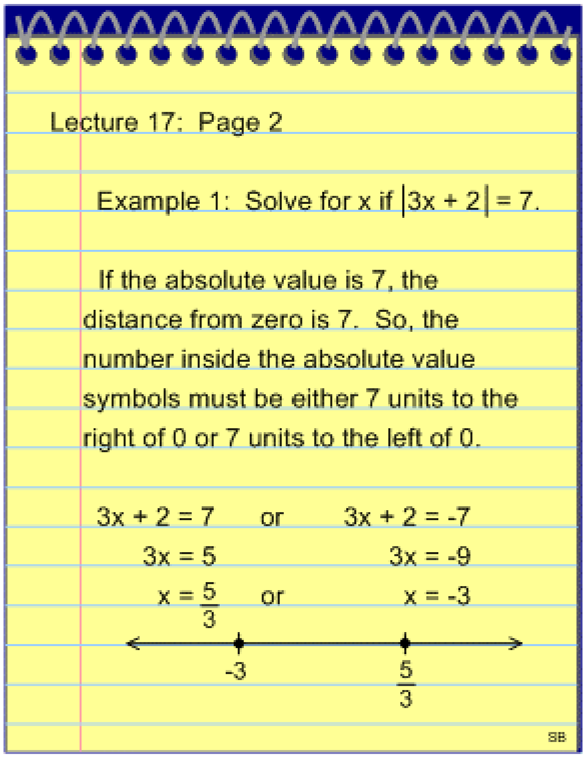
$|x| = 5$

x is a number whose distance from 0 is 5 units. You can either be 5 units to the right of 0 or 5 units to the left.

This is a disjunction: an "or" statement.

$x = 5$ or $x = -5$

ALG2017-02



Lecture 17: Page 2

Example 1: Solve for x if $|3x + 2| = 7$.

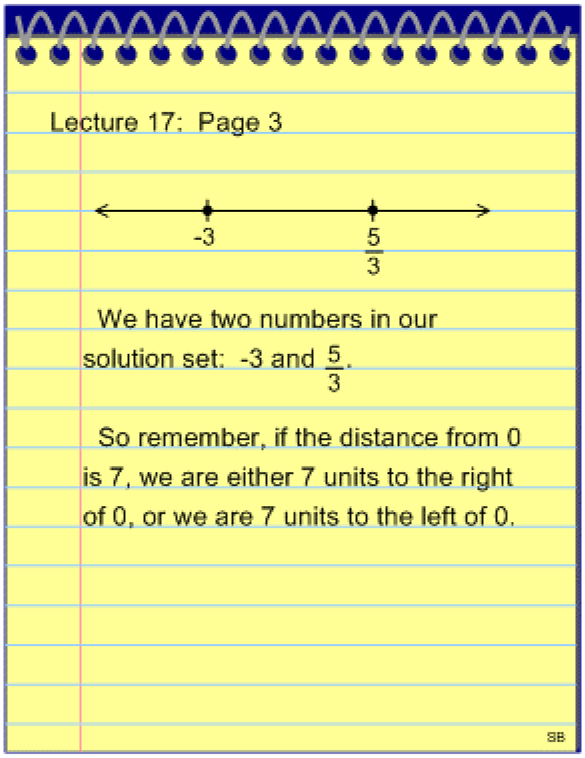
If the absolute value is 7, the distance from zero is 7. So, the number inside the absolute value symbols must be either 7 units to the right of 0 or 7 units to the left of 0.

$3x + 2 = 7$ or $3x + 2 = -7$

$3x = 5$ $\quad 3x = -9$

$x = \frac{5}{3}$ or $x = -3$

ALG2017-03

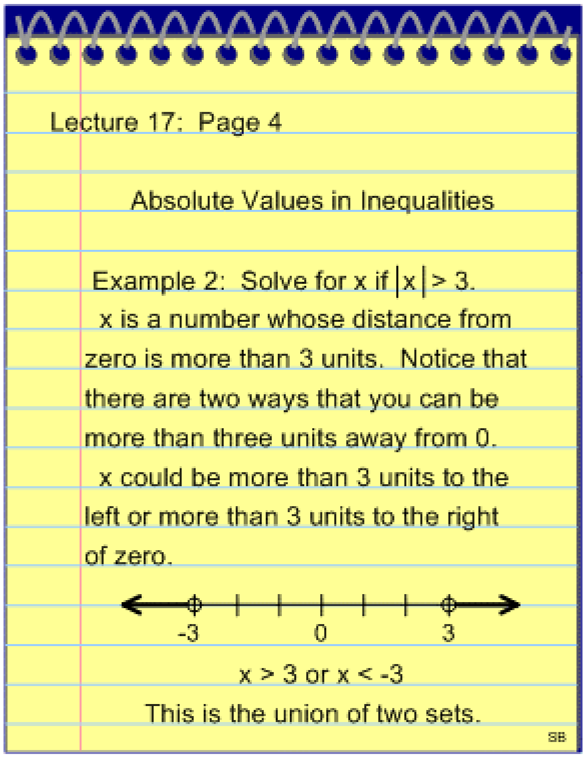


Lecture 17: Page 3

We have two numbers in our solution set: -3 and $\frac{5}{3}$.

So remember, if the distance from 0 is 7, we are either 7 units to the right of 0, or we are 7 units to the left of 0.

ALG2017-04



Lecture 17: Page 4

Absolute Values in Inequalities

Example 2: Solve for x if $|x| > 3$.

x is a number whose distance from zero is more than 3 units. Notice that there are two ways that you can be more than three units away from 0.

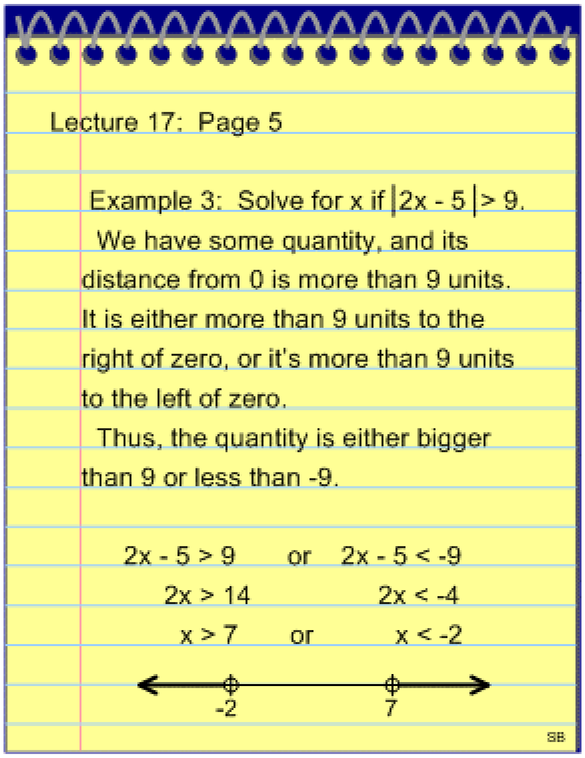
x could be more than 3 units to the left or more than 3 units to the right of zero.

$x > 3$ or $x < -3$

This is the union of two sets.

Lecture 17 Notes, Continued

ALG2017-05



Lecture 17: Page 5

Example 3: Solve for x if $|2x - 5| > 9$.

We have some quantity, and its distance from 0 is more than 9 units. It is either more than 9 units to the right of zero, or it's more than 9 units to the left of zero.

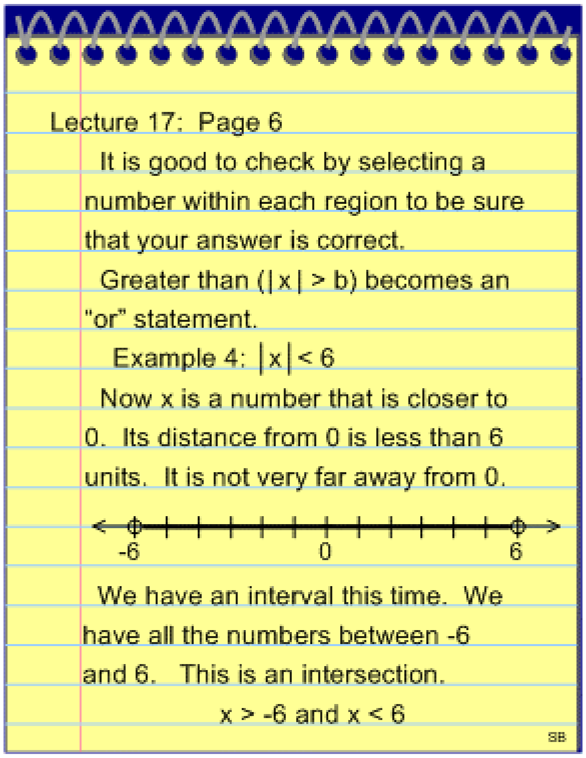
Thus, the quantity is either bigger than 9 or less than -9.

$2x - 5 > 9$ or $2x - 5 < -9$

$2x > 14$ $2x < -4$

$x > 7$ or $x < -2$

ALG2017-06



Lecture 17: Page 6

It is good to check by selecting a number within each region to be sure that your answer is correct.

Greater than ($|x| > b$) becomes an "or" statement.

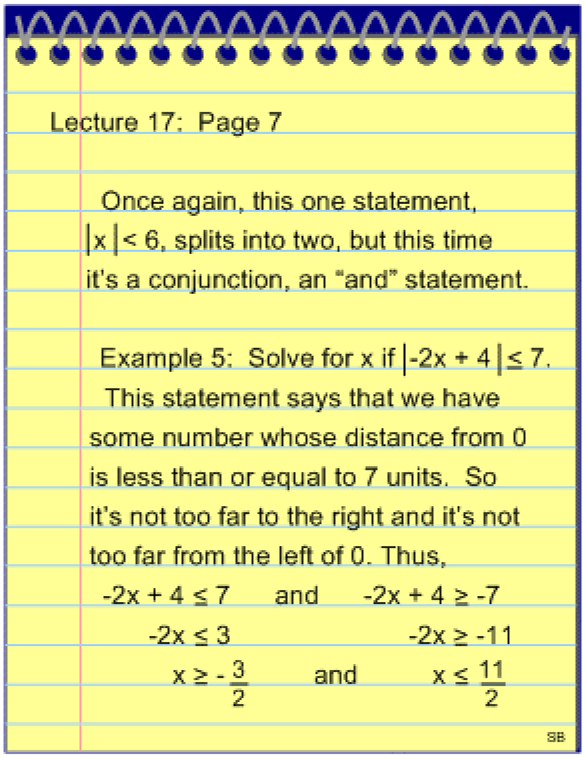
Example 4: $|x| < 6$

Now x is a number that is closer to 0. Its distance from 0 is less than 6 units. It is not very far away from 0.

We have an interval this time. We have all the numbers between -6 and 6. This is an intersection.

$x > -6$ and $x < 6$

ALG2017-07



Lecture 17: Page 7

Once again, this one statement, $|x| < 6$, splits into two, but this time it's a conjunction, an "and" statement.

Example 5: Solve for x if $|-2x + 4| \le 7$.

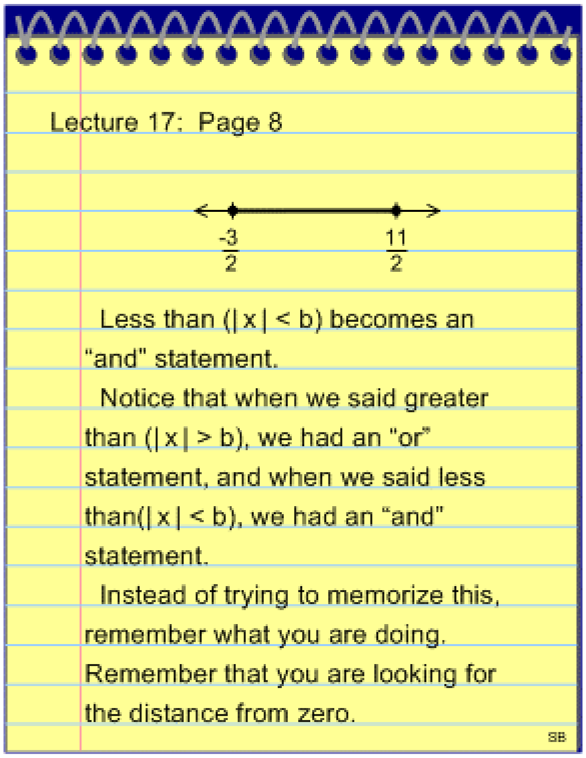
This statement says that we have some number whose distance from 0 is less than or equal to 7 units. So it's not too far to the right and it's not too far from the left of 0. Thus,

$-2x + 4 \le 7$ and $-2x + 4 \ge -7$

$-2x \le 3$ $-2x \ge -11$

$x \ge -\frac{3}{2}$ and $x \le \frac{11}{2}$

ALG2017-08



Lecture 17: Page 8

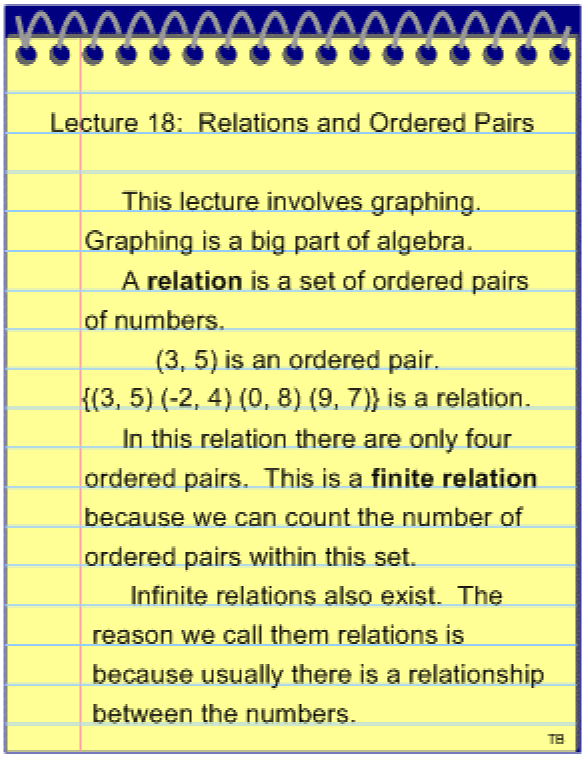
Less than ($|x| < b$) becomes an "and" statement.

Notice that when we said greater than ($|x| > b$), we had an "or" statement, and when we said less than($|x| < b$), we had an "and" statement.

Instead of trying to memorize this, remember what you are doing. Remember that you are looking for the distance from zero.

Lecture 18 Notes

ALG2018-01



Lecture 18: Relations and Ordered Pairs

This lecture involves graphing. Graphing is a big part of algebra.

A **relation** is a set of ordered pairs of numbers.

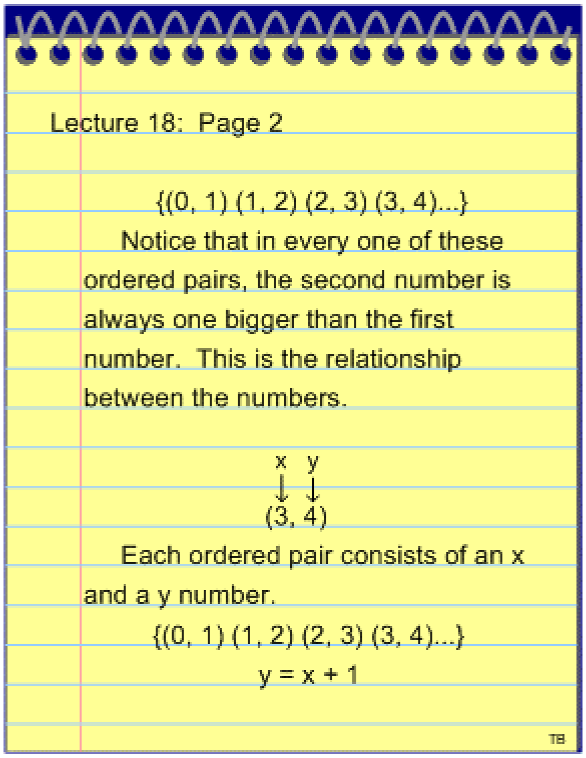
(3, 5) is an ordered pair.

{(3, 5) (-2, 4) (0, 8) (9, 7)} is a relation.

In this relation there are only four ordered pairs. This is a **finite relation** because we can count the number of ordered pairs within this set.

Infinite relations also exist. The reason we call them relations is because usually there is a relationship between the numbers.

ALG2018-02



Lecture 18: Page 2

{(0, 1) (1, 2) (2, 3) (3, 4)...}

Notice that in every one of these ordered pairs, the second number is always one bigger than the first number. This is the relationship between the numbers.

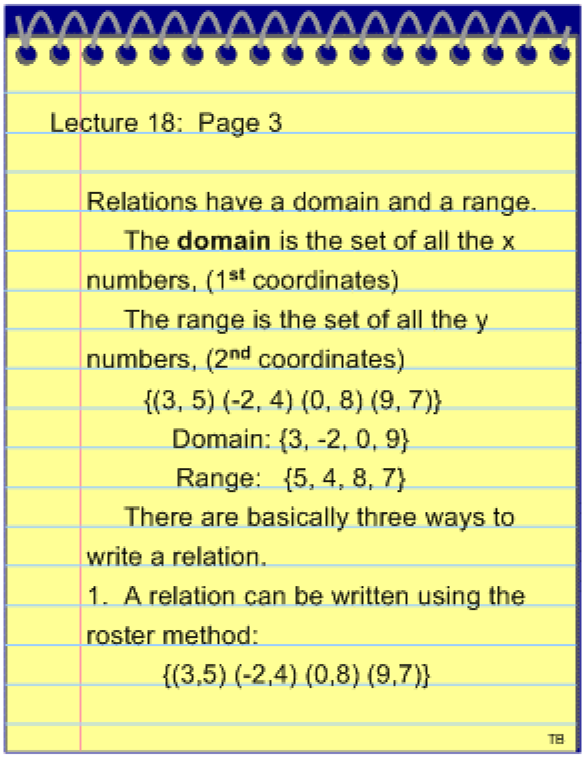
x y
↓ ↓
(3, 4)

Each ordered pair consists of an x and a y number.

{(0, 1) (1, 2) (2, 3) (3, 4)...}

$y = x + 1$

ALG2018-03



Lecture 18: Page 3

Relations have a domain and a range.

The **domain** is the set of all the x numbers, (1st coordinates)

The range is the set of all the y numbers, (2nd coordinates)

{(3, 5) (-2, 4) (0, 8) (9, 7)}

Domain: {3, -2, 0, 9}

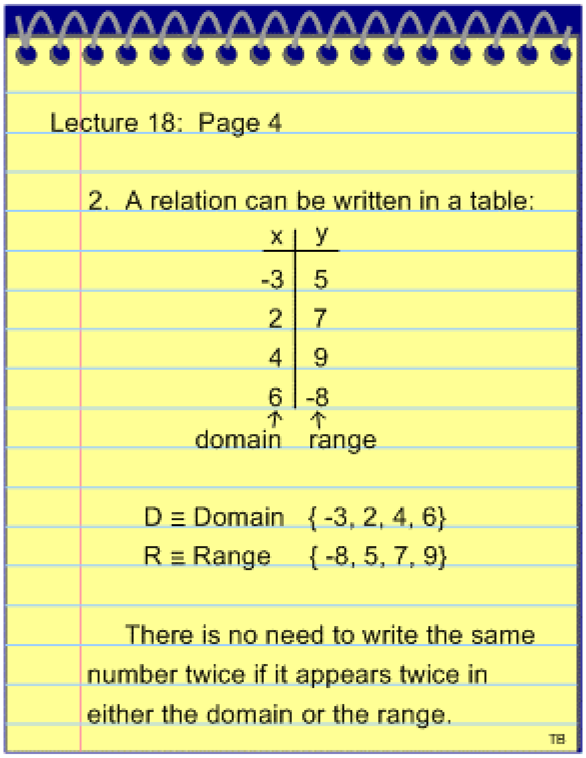
Range: {5, 4, 8, 7}

There are basically three ways to write a relation.

1. A relation can be written using the roster method:

{(3,5) (-2,4) (0,8) (9,7)}

ALG2018-04



Lecture 18: Page 4

2. A relation can be written in a table:

x	y
-3	5
2	7
4	9
6	-8

↑ domain ↑ range

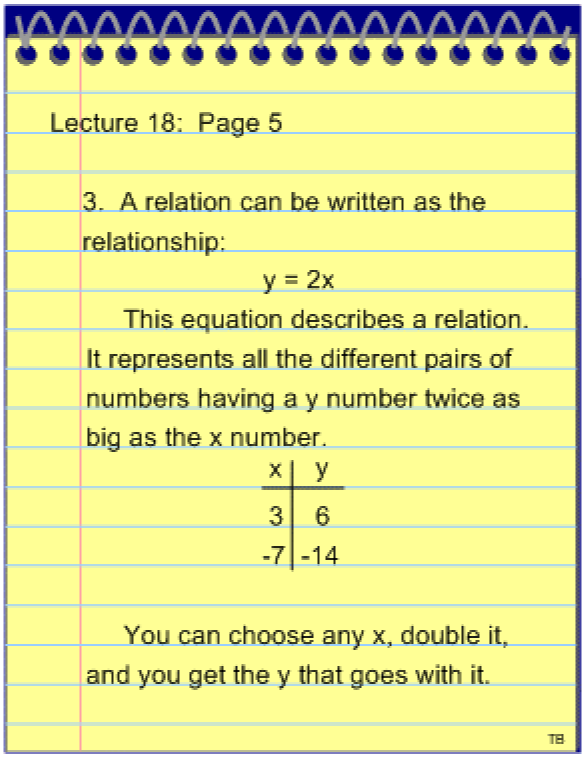
D ≡ Domain { -3, 2, 4, 6}

R ≡ Range { -8, 5, 7, 9}

There is no need to write the same number twice if it appears twice in either the domain or the range.

Lecture 18 Notes, Continued

ALG2018-05



Lecture 18: Page 5

3. A relation can be written as the relationship:

$$y = 2x$$

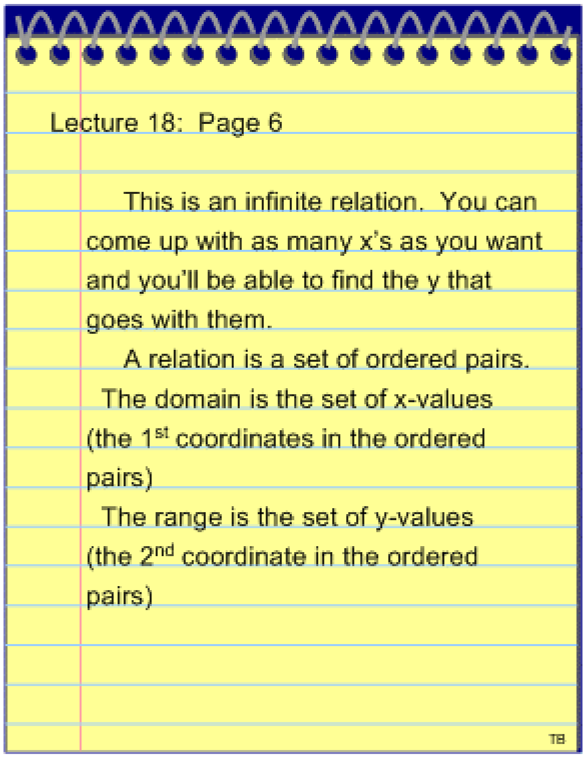
This equation describes a relation. It represents all the different pairs of numbers having a y number twice as big as the x number.

x	y
3	6
-7	-14

You can choose any x, double it, and you get the y that goes with it.

TB

ALG2018-06



Lecture 18: Page 6

This is an infinite relation. You can come up with as many x's as you want and you'll be able to find the y that goes with them.

A relation is a set of ordered pairs.

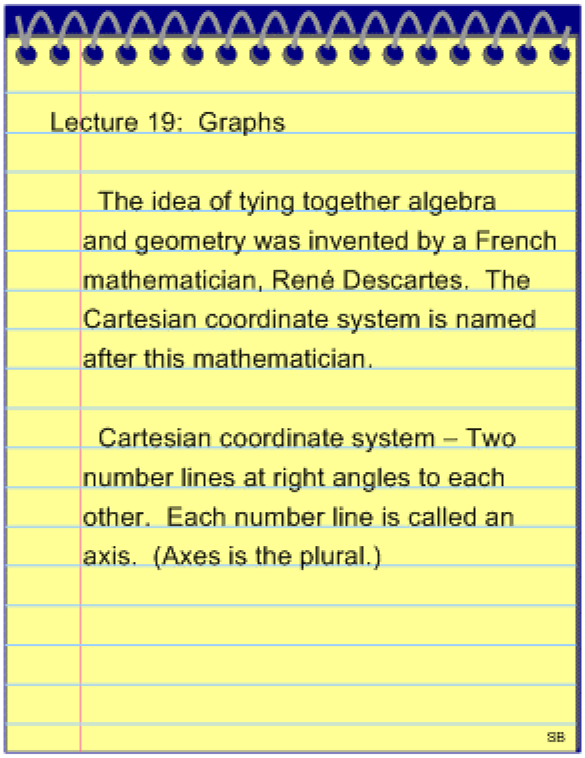
The domain is the set of x-values (the 1st coordinates in the ordered pairs)

The range is the set of y-values (the 2nd coordinate in the ordered pairs)

TB

Lecture 19 Notes

ALG2019-01

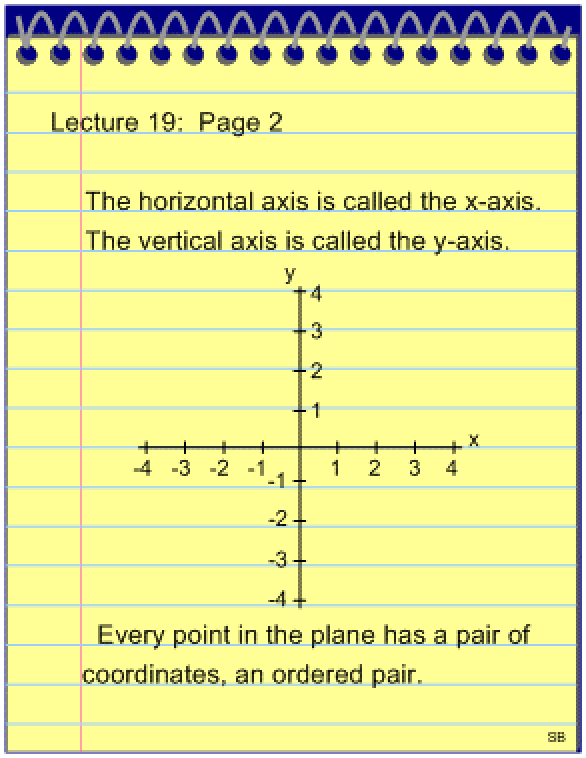


Lecture 19: Graphs

The idea of tying together algebra and geometry was invented by a French mathematician, René Descartes. The Cartesian coordinate system is named after this mathematician.

Cartesian coordinate system – Two number lines at right angles to each other. Each number line is called an axis. (Axes is the plural.)

ALG2019-02

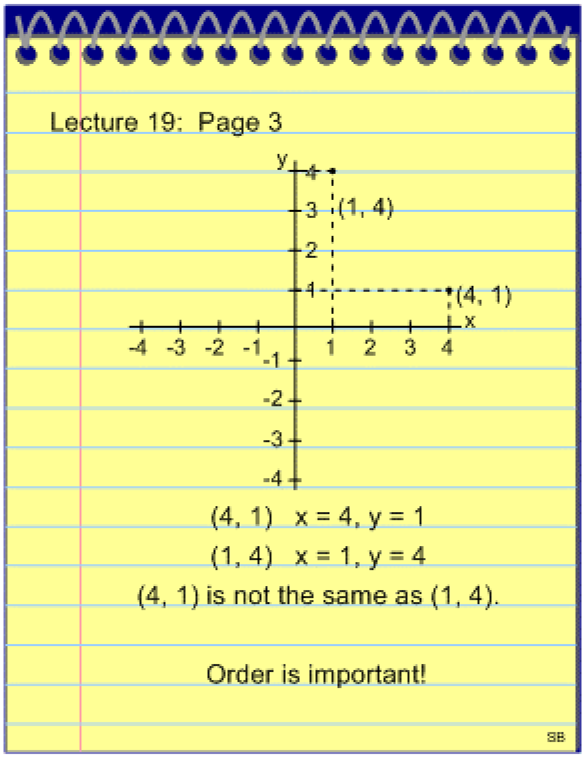


Lecture 19: Page 2

The horizontal axis is called the x-axis. The vertical axis is called the y-axis.

Every point in the plane has a pair of coordinates, an ordered pair.

ALG2019-03



Lecture 19: Page 3

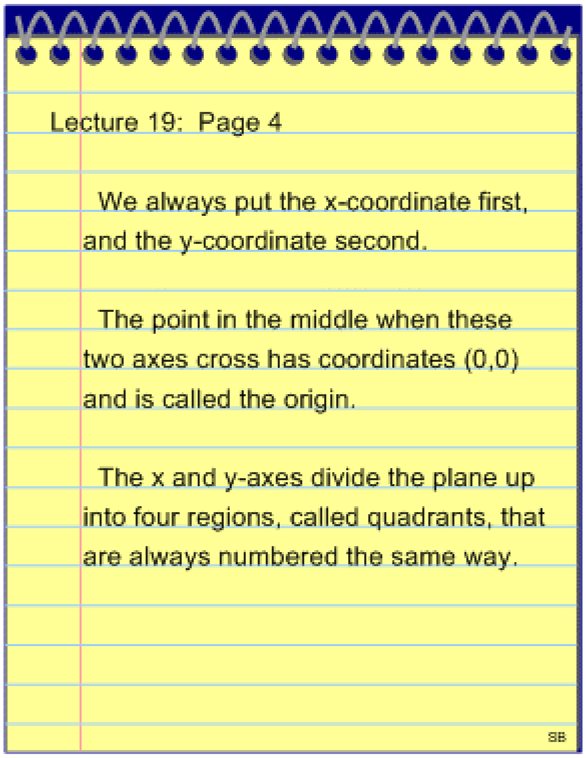
(4, 1) $x = 4, y = 1$

(1, 4) $x = 1, y = 4$

(4, 1) is not the same as (1, 4).

Order is important!

ALG2019-04



Lecture 19: Page 4

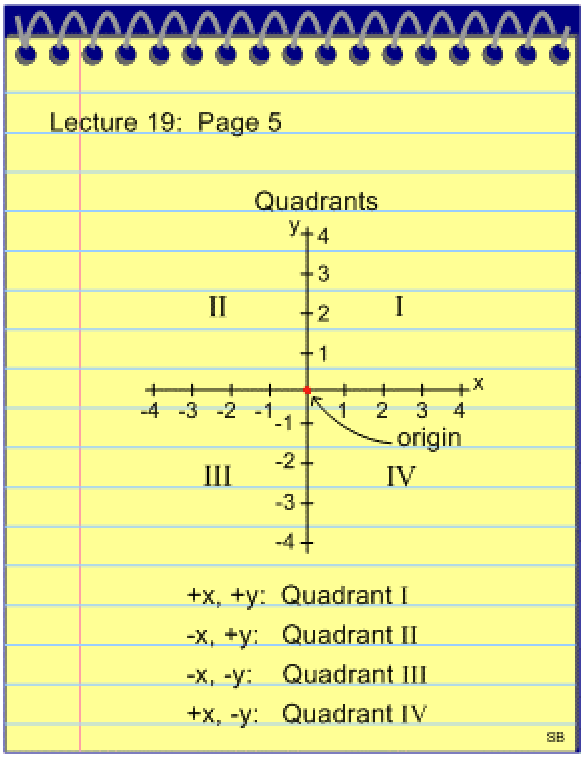
We always put the x-coordinate first, and the y-coordinate second.

The point in the middle when these two axes cross has coordinates (0,0) and is called the origin.

The x and y-axes divide the plane up into four regions, called quadrants, that are always numbered the same way.

Lecture 19 Notes, Continued

ALG2019-05



Lecture 19: Page 5

Quadrants

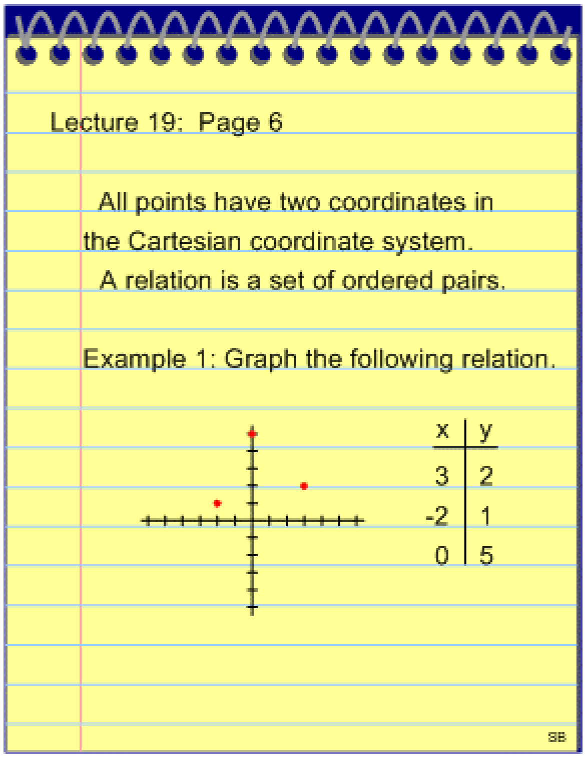
+x, +y: Quadrant I

-x, +y: Quadrant II

-x, -y: Quadrant III

+x, -y: Quadrant IV

ALG2019-06



Lecture 19: Page 6

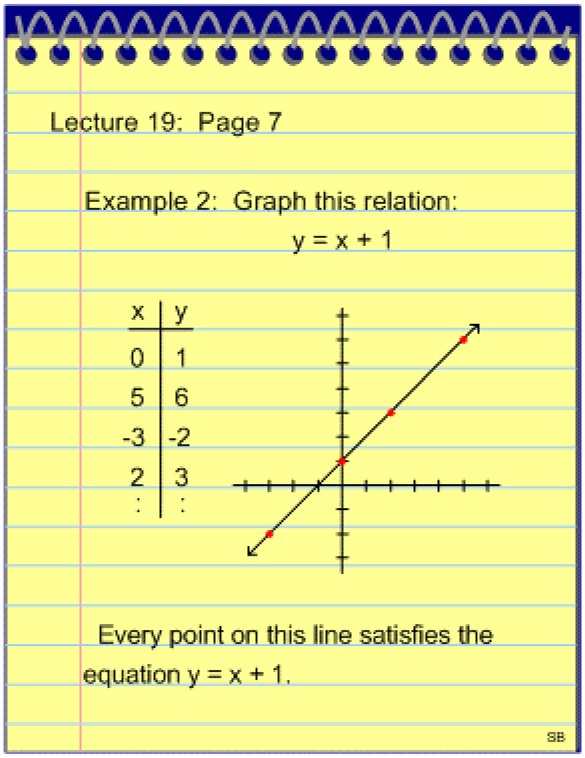
All points have two coordinates in the Cartesian coordinate system.

A relation is a set of ordered pairs.

Example 1: Graph the following relation.

x	y
3	2
-2	1
0	5

ALG2019-07



Lecture 19: Page 7

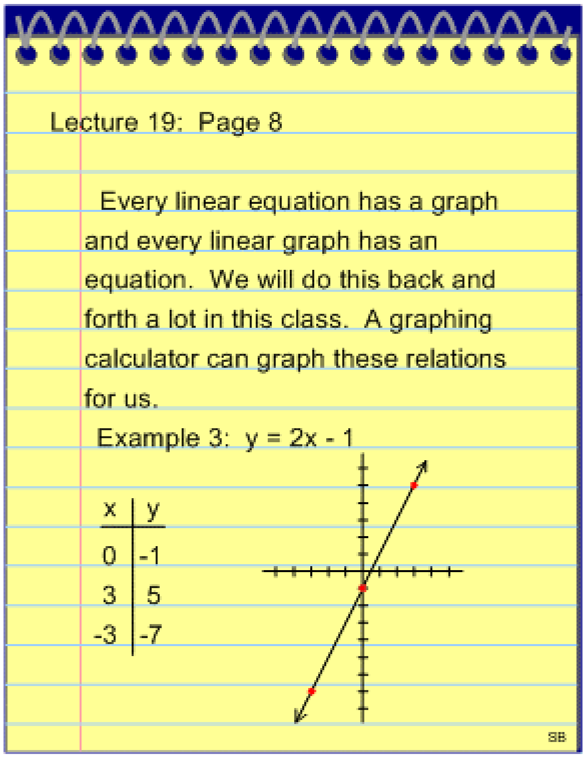
Example 2: Graph this relation:

$y = x + 1$

x	y
0	1
5	6
-3	-2
2	3
:	:

Every point on this line satisfies the equation $y = x + 1$.

ALG2019-08



Lecture 19: Page 8

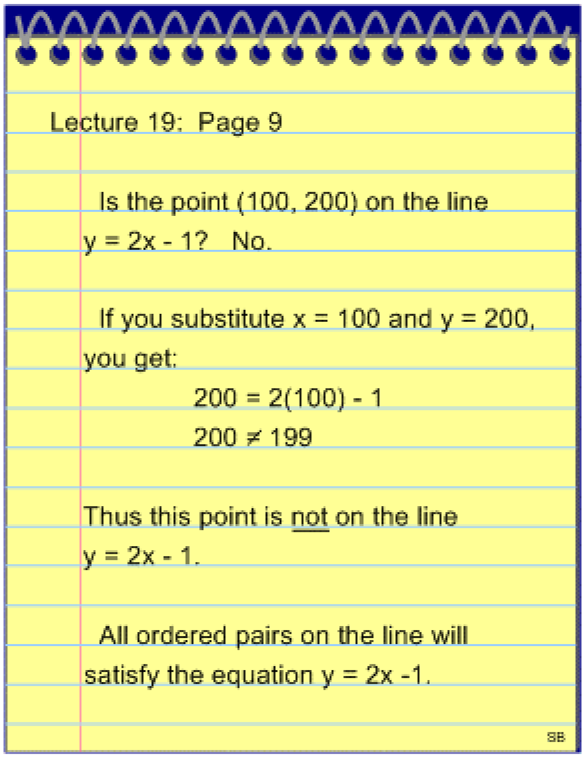
Every linear equation has a graph and every linear graph has an equation. We will do this back and forth a lot in this class. A graphing calculator can graph these relations for us.

Example 3: $y = 2x - 1$

x	y
0	-1
3	5
-3	-7

Lecture 19 Notes, Continued

ALG2019-09



Lecture 19: Page 9

Is the point (100, 200) on the line $y = 2x - 1$? No.

If you substitute $x = 100$ and $y = 200$, you get:

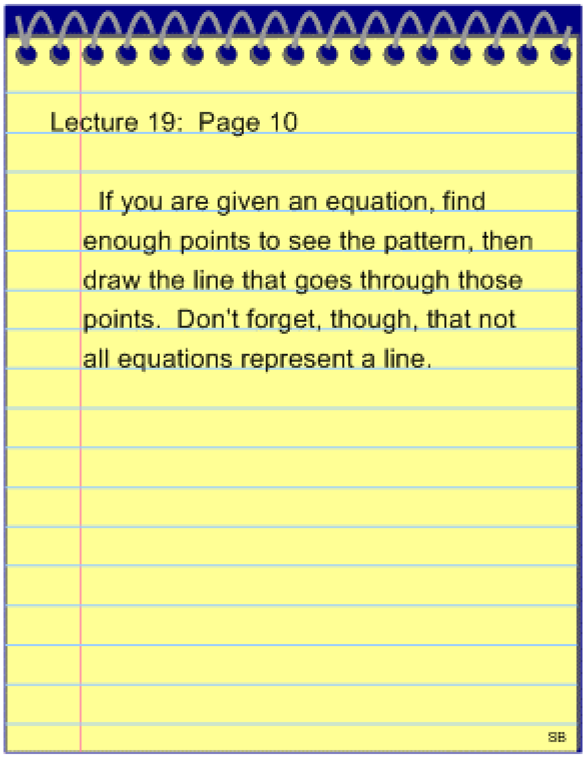
$$200 = 2(100) - 1$$
$$200 \neq 199$$

Thus this point is not on the line $y = 2x - 1$.

All ordered pairs on the line will satisfy the equation $y = 2x - 1$.

SB

ALG2019-10



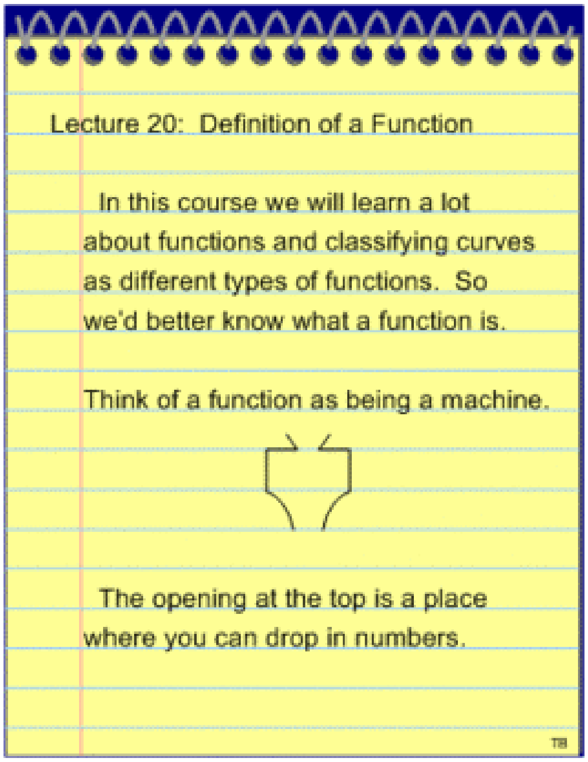
Lecture 19: Page 10

If you are given an equation, find enough points to see the pattern, then draw the line that goes through those points. Don't forget, though, that not all equations represent a line.

SB

Lecture 20 Notes

ALG2020-01



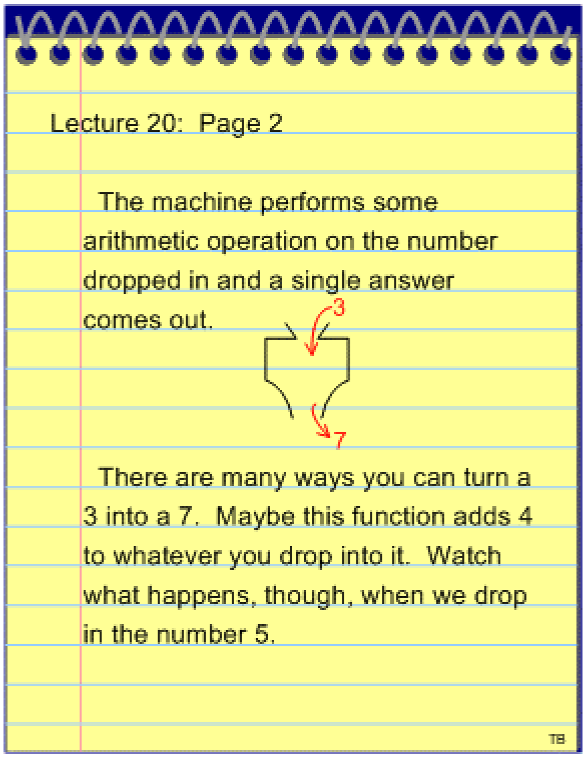
Lecture 20: Definition of a Function

In this course we will learn a lot about functions and classifying curves as different types of functions. So we'd better know what a function is.

Think of a function as being a machine.

The opening at the top is a place where you can drop in numbers.

ALG2020-02

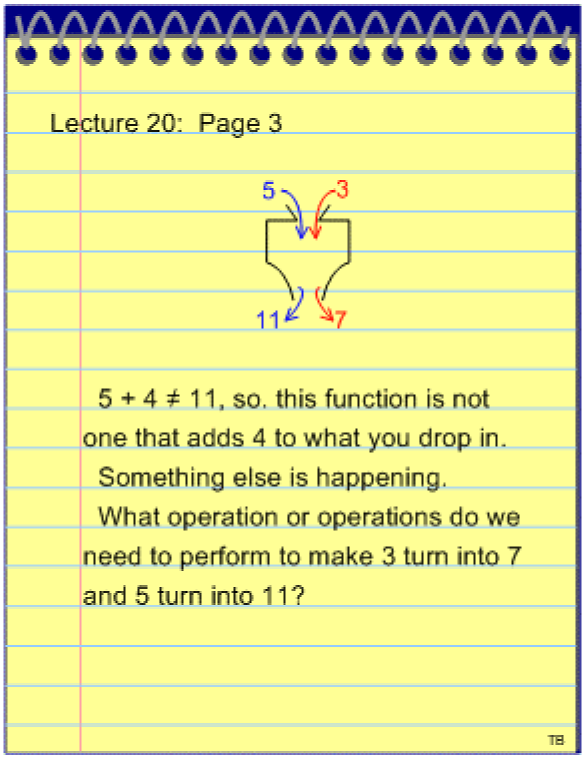


Lecture 20: Page 2

The machine performs some arithmetic operation on the number dropped in and a single answer comes out.

There are many ways you can turn a 3 into a 7. Maybe this function adds 4 to whatever you drop into it. Watch what happens, though, when we drop in the number 5.

ALG2020-03



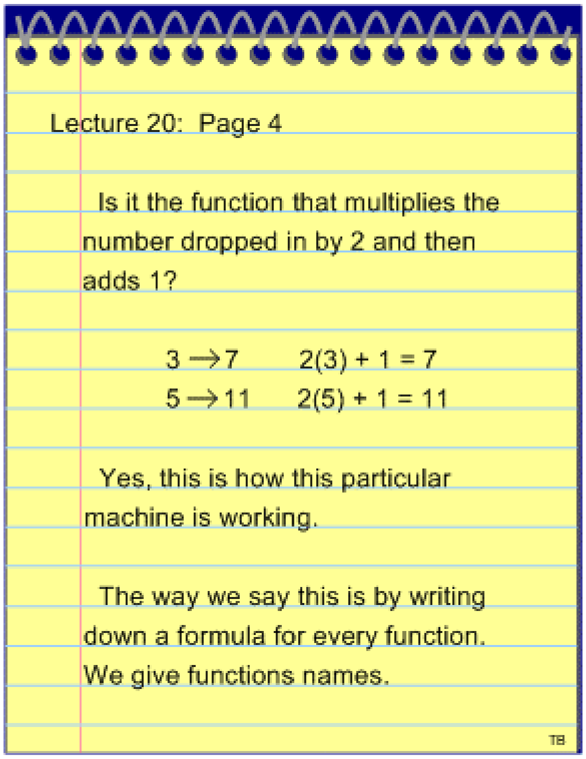
Lecture 20: Page 3

$5 + 4 \neq 11$, so. this function is not one that adds 4 to what you drop in.

Something else is happening.

What operation or operations do we need to perform to make 3 turn into 7 and 5 turn into 11?

ALG2020-04



Lecture 20: Page 4

Is it the function that multiplies the number dropped in by 2 and then adds 1?

$3 \rightarrow 7 \qquad 2(3) + 1 = 7$

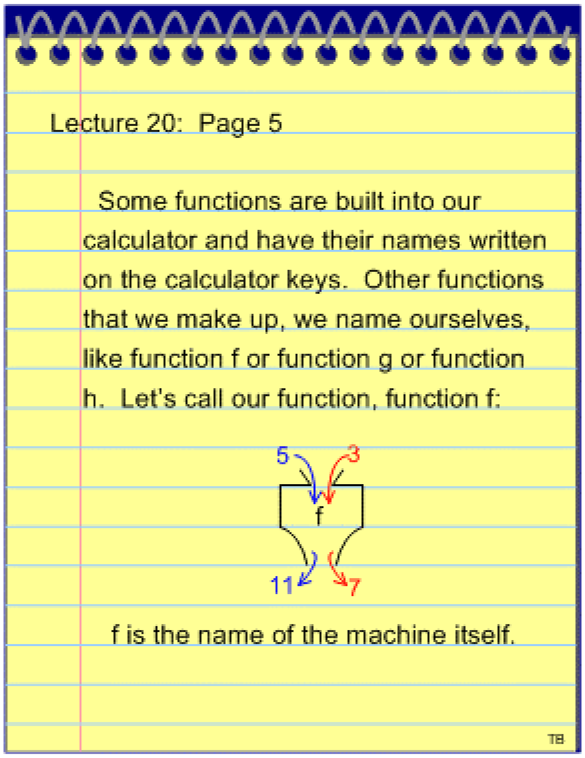
$5 \rightarrow 11 \qquad 2(5) + 1 = 11$

Yes, this is how this particular machine is working.

The way we say this is by writing down a formula for every function. We give functions names.

Lecture 20 Notes

ALG2020-05

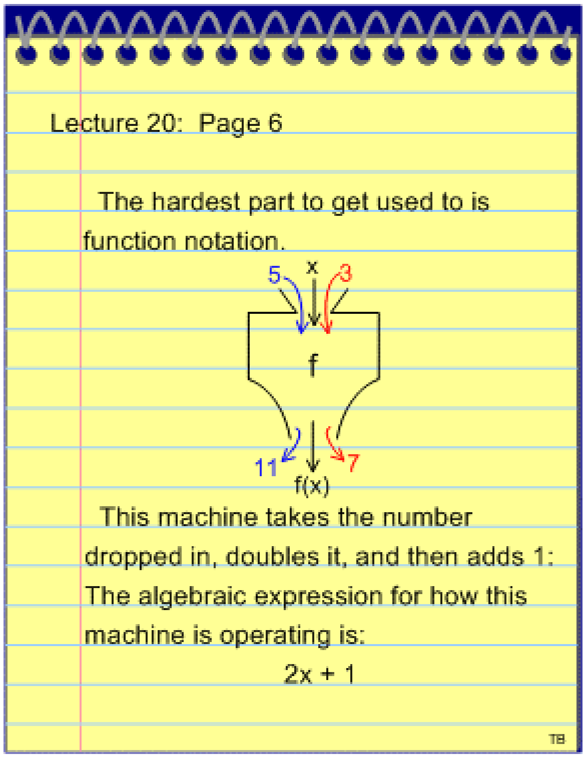


Lecture 20: Page 5

Some functions are built into our calculator and have their names written on the calculator keys. Other functions that we make up, we name ourselves, like function f or function g or function h. Let's call our function, function f:

f is the name of the machine itself.

ALG2020-06



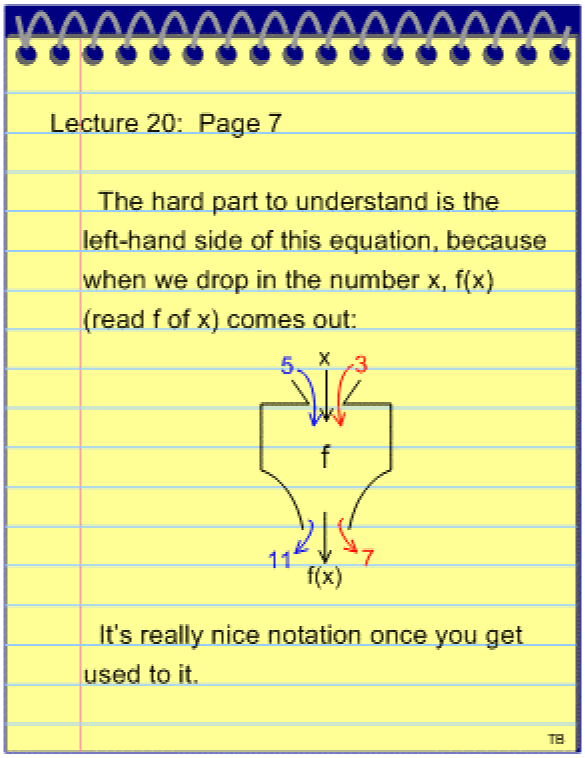
Lecture 20: Page 6

The hardest part to get used to is function notation.

This machine takes the number dropped in, doubles it, and then adds 1: The algebraic expression for how this machine is operating is:

$2x + 1$

ALG2020-07

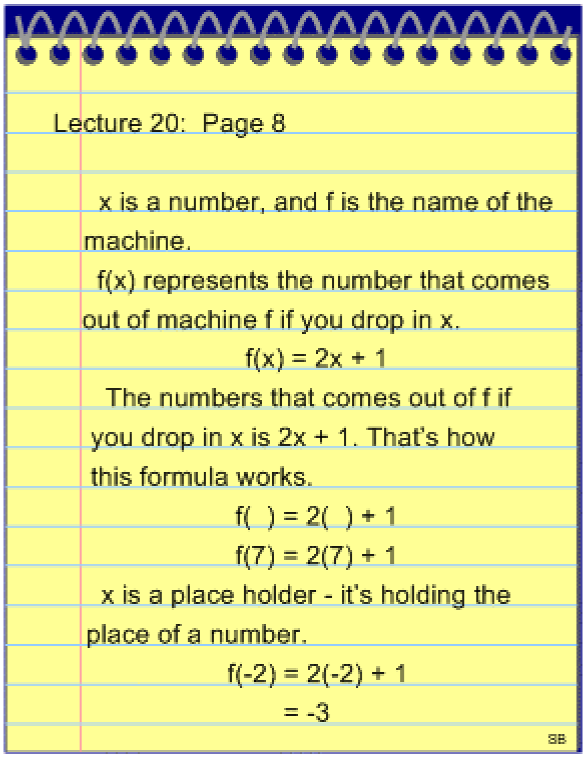


Lecture 20: Page 7

The hard part to understand is the left-hand side of this equation, because when we drop in the number x, f(x) (read f of x) comes out:

It's really nice notation once you get used to it.

ALG2020-08



Lecture 20: Page 8

x is a number, and f is the name of the machine.

f(x) represents the number that comes out of machine f if you drop in x.

$f(x) = 2x + 1$

The numbers that comes out of f if you drop in x is 2x + 1. That's how this formula works.

$f(\ \ ) = 2(\ \ ) + 1$

$f(7) = 2(7) + 1$

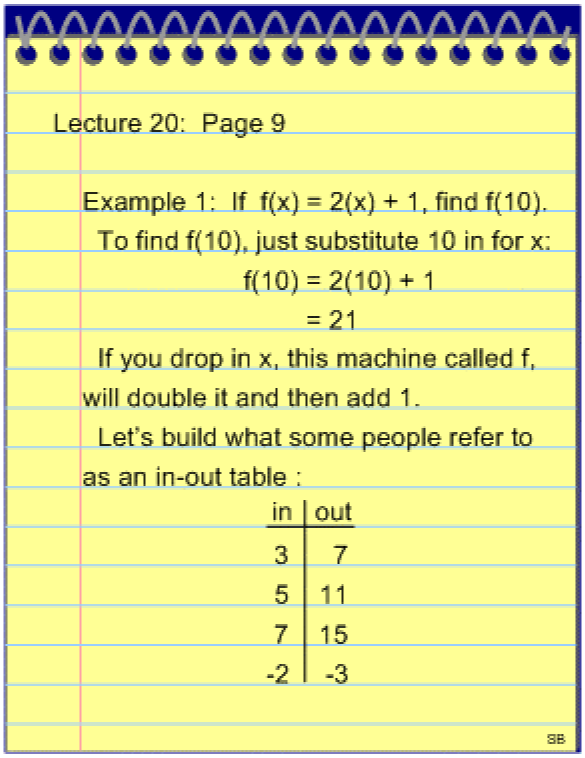
x is a place holder - it's holding the place of a number.

$f(-2) = 2(-2) + 1$

$= -3$

Lecture 20 Notes

ALG2020-09



Lecture 20: Page 9

Example 1: If f(x) = 2(x) + 1, find f(10).

To find f(10), just substitute 10 in for x:

f(10) = 2(10) + 1

= 21

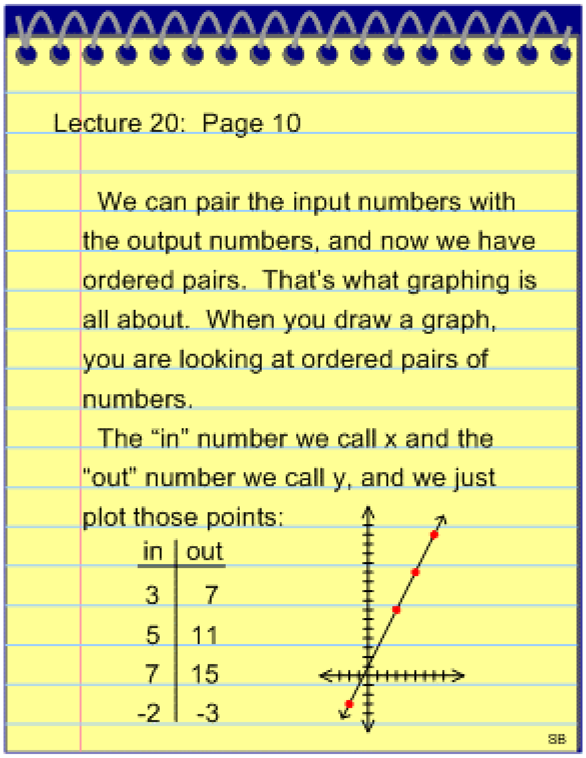
If you drop in x, this machine called f, will double it and then add 1.

Let's build what some people refer to as an in-out table :

in	out
3	7
5	11
7	15
-2	-3

SB

ALG2020-10



Lecture 20: Page 10

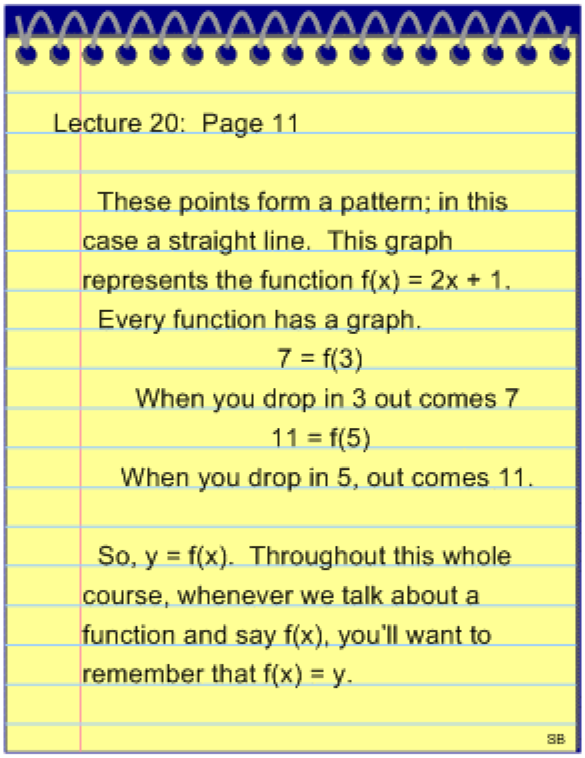
We can pair the input numbers with the output numbers, and now we have ordered pairs. That's what graphing is all about. When you draw a graph, you are looking at ordered pairs of numbers.

The "in" number we call x and the "out" number we call y, and we just plot those points:

in	out
3	7
5	11
7	15
-2	-3

SB

ALG2020-11



Lecture 20: Page 11

These points form a pattern; in this case a straight line. This graph represents the function f(x) = 2x + 1.

Every function has a graph.

7 = f(3)

When you drop in 3 out comes 7

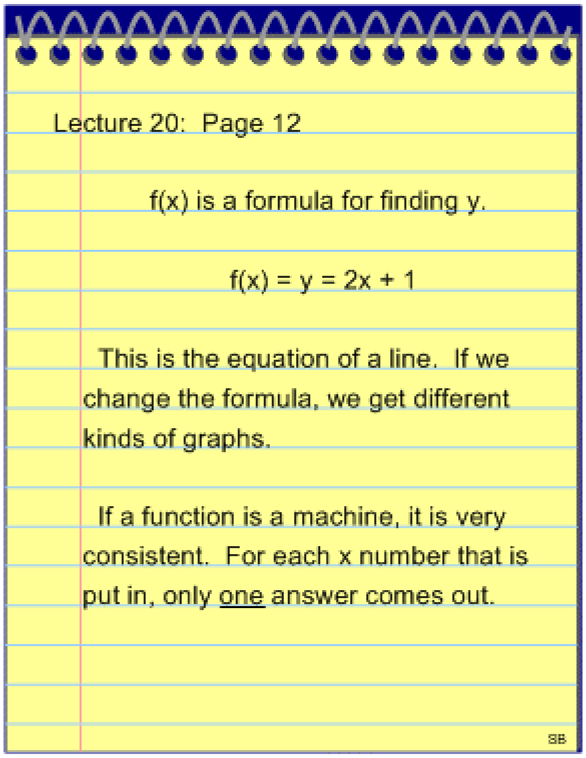
11 = f(5)

When you drop in 5, out comes 11.

So, y = f(x). Throughout this whole course, whenever we talk about a function and say f(x), you'll want to remember that f(x) = y.

SB

ALG2020-12



Lecture 20: Page 12

f(x) is a formula for finding y.

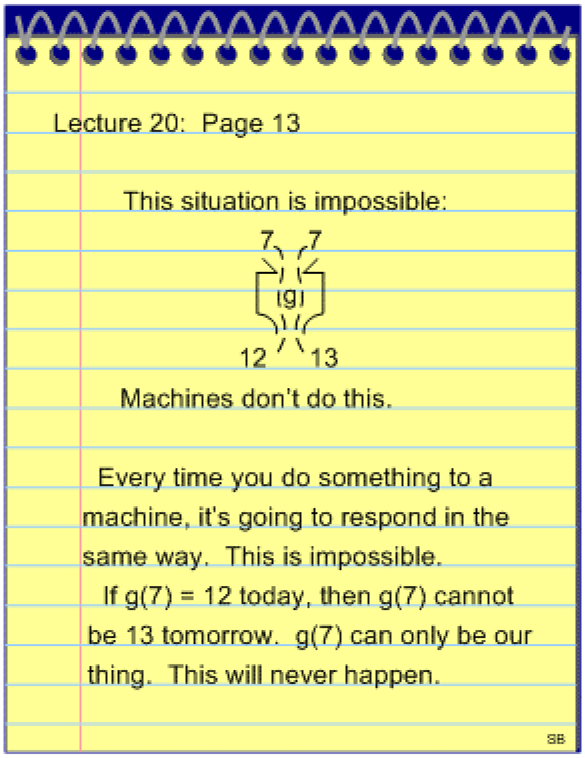
f(x) = y = 2x + 1

This is the equation of a line. If we change the formula, we get different kinds of graphs.

If a function is a machine, it is very consistent. For each x number that is put in, only <u>one</u> answer comes out.

SB

ALG2020-13



Lecture 20: Page 13

This situation is impossible:

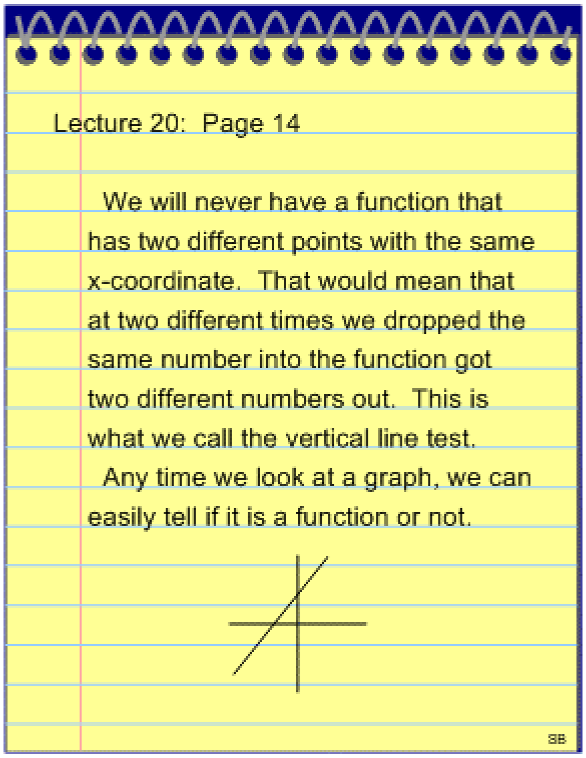
Machines don't do this.

Every time you do something to a machine, it's going to respond in the same way. This is impossible.

If g(7) = 12 today, then g(7) cannot be 13 tomorrow. g(7) can only be our thing. This will never happen.

SB

ALG2020-14



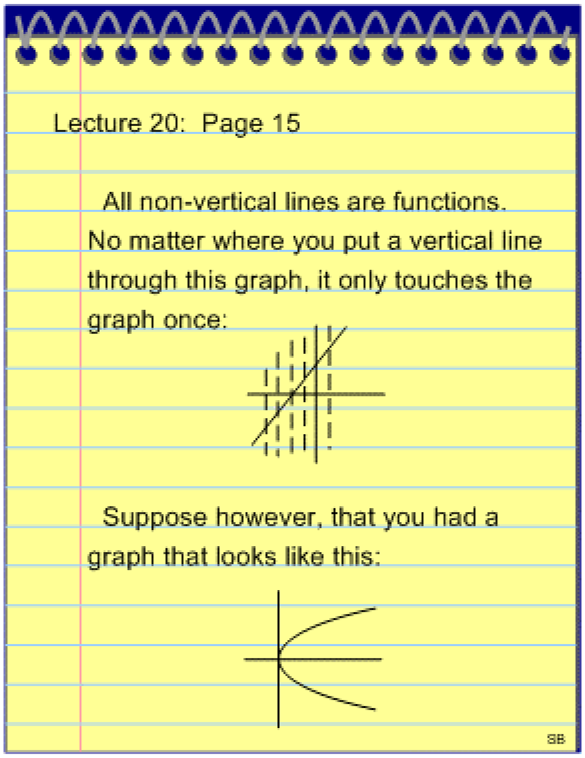
Lecture 20: Page 14

We will never have a function that has two different points with the same x-coordinate. That would mean that at two different times we dropped the same number into the function got two different numbers out. This is what we call the vertical line test.

Any time we look at a graph, we can easily tell if it is a function or not.

SB

ALG2020-15



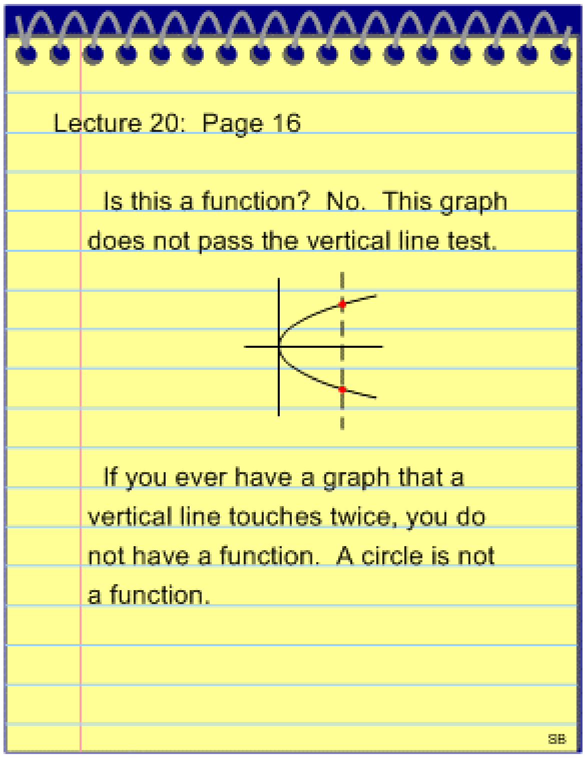
Lecture 20: Page 15

All non-vertical lines are functions. No matter where you put a vertical line through this graph, it only touches the graph once:

Suppose however, that you had a graph that looks like this:

SB

ALG2020-16



Lecture 20: Page 16

Is this a function? No. This graph does not pass the vertical line test.

If you ever have a graph that a vertical line touches twice, you do not have a function. A circle is not a function.

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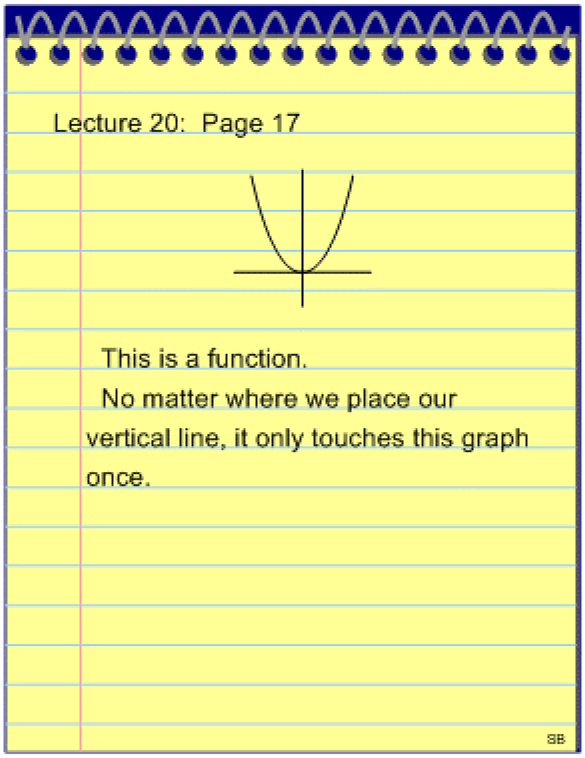

Lecture 20: Page 17

This is a function.

No matter where we place our vertical line, it only touches this graph once.

SB

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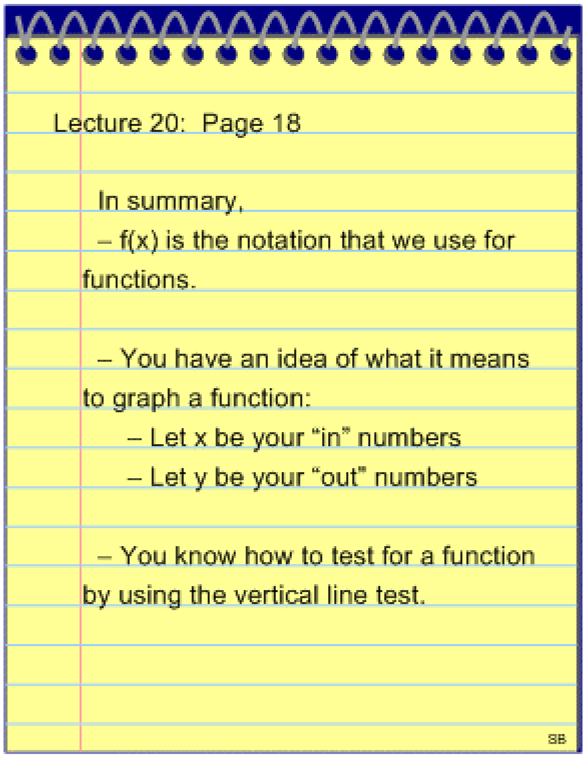

Lecture 20: Page 18

In summary,

– f(x) is the notation that we use for functions.

– You have an idea of what it means to graph a function:

  – Let x be your “in” numbers

  – Let y be your “out” numbers

– You know how to test for a function by using the vertical line test.

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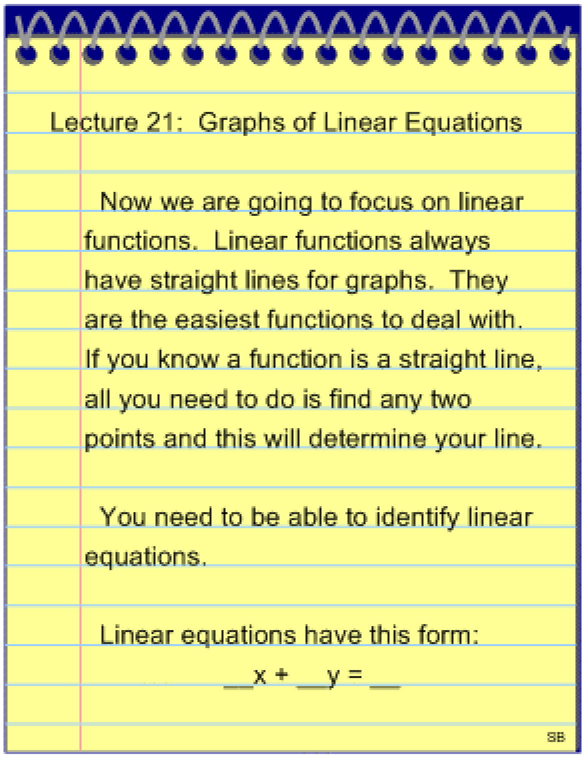
Lecture 21 Notes

ALG2021-01



Lecture 21: Graphs of Linear Equations

Now we are going to focus on linear functions. Linear functions always have straight lines for graphs. They are the easiest functions to deal with. If you know a function is a straight line, all you need to do is find any two points and this will determine your line.

You need to be able to identify linear equations.

Linear equations have this form:

__x + __y = __

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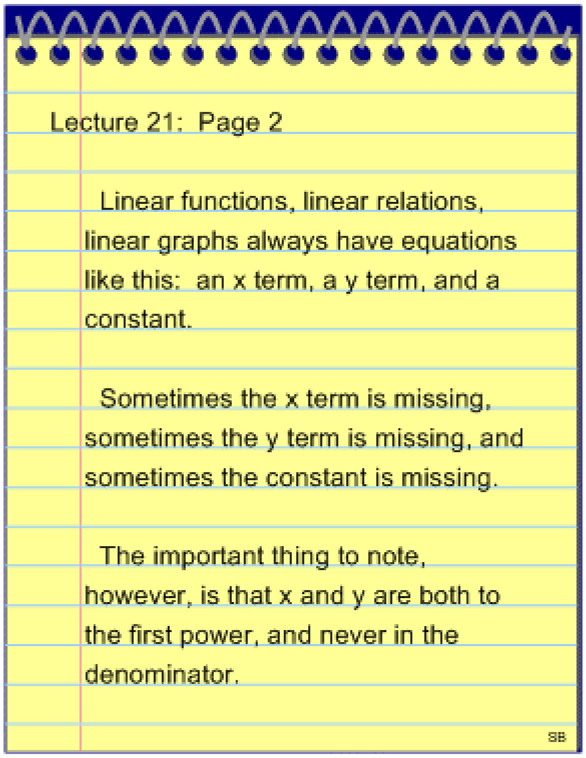

Lecture 21: Page 2

Linear functions, linear relations, linear graphs always have equations like this: an x term, a y term, and a constant.

Sometimes the x term is missing, sometimes the y term is missing, and sometimes the constant is missing.

The important thing to note, however, is that x and y are both to the first power, and never in the denominator.

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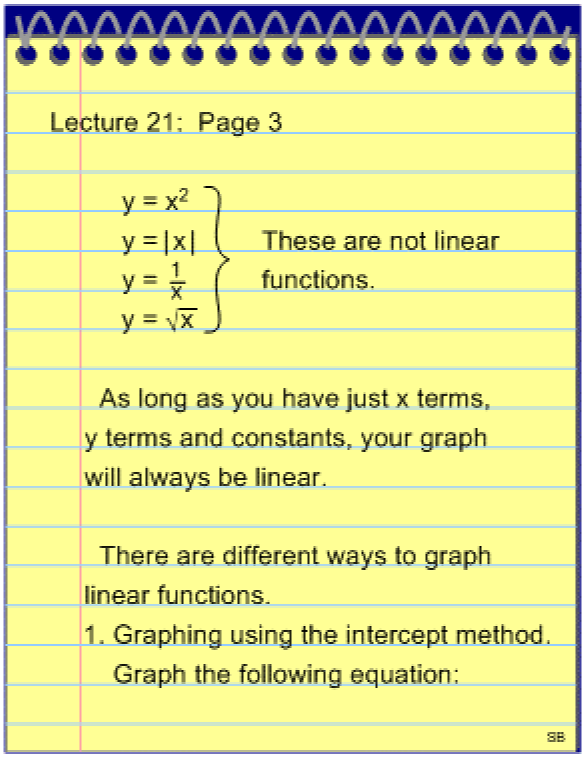

Lecture 21: Page 3

$$\left.\begin{array}{l} y = x^2 \\ y = |x| \\ y = \frac{1}{x} \\ y = \sqrt{x} \end{array}\right\}$$ These are not linear functions.

As long as you have just x terms, y terms and constants, your graph will always be linear.

There are different ways to graph linear functions.

1. Graphing using the intercept method.
   Graph the following equation:

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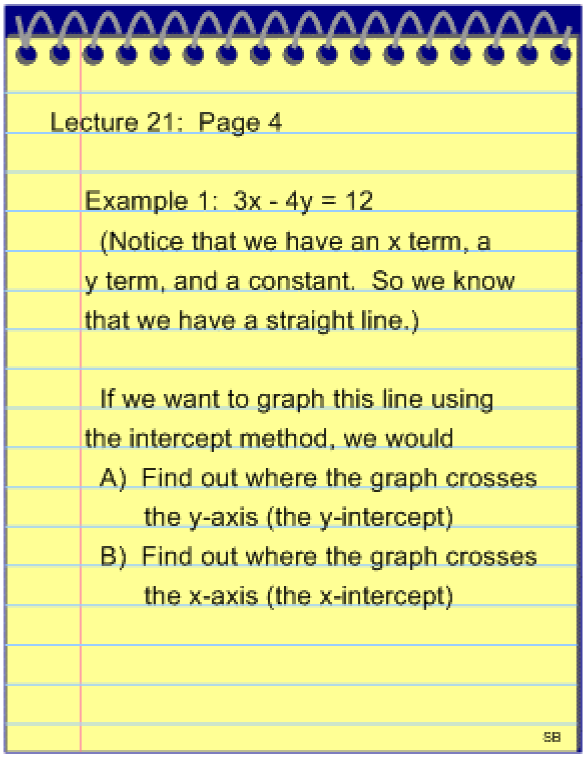

Lecture 21: Page 4

Example 1: $3x - 4y = 12$

(Notice that we have an x term, a y term, and a constant. So we know that we have a straight line.)

If we want to graph this line using the intercept method, we would

A) Find out where the graph crosses the y-axis (the y-intercept)

B) Find out where the graph crosses the x-axis (the x-intercept)

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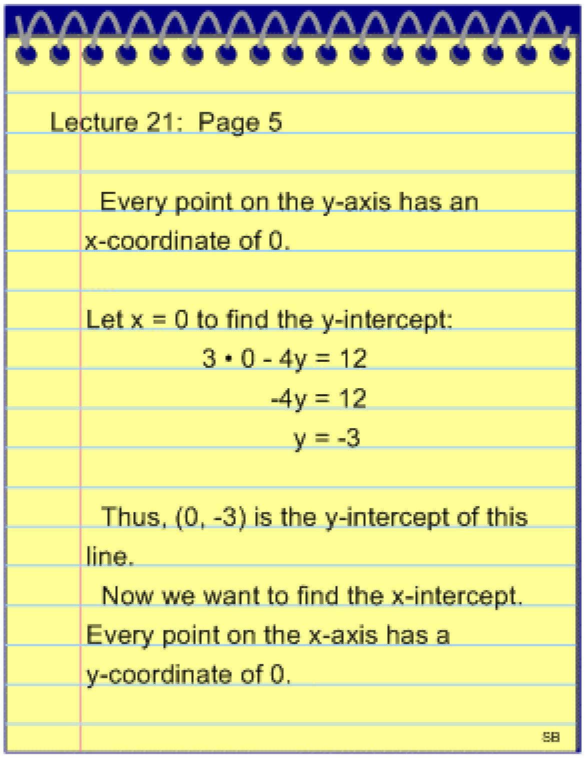
Lecture 21 Notes, Continued

ALG2021-05



Lecture 21: Page 5

Every point on the y-axis has an x-coordinate of 0.

Let x = 0 to find the y-intercept:

$$3 \cdot 0 - 4y = 12$$
$$-4y = 12$$
$$y = -3$$

Thus, (0, -3) is the y-intercept of this line.

Now we want to find the x-intercept. Every point on the x-axis has a y-coordinate of 0.

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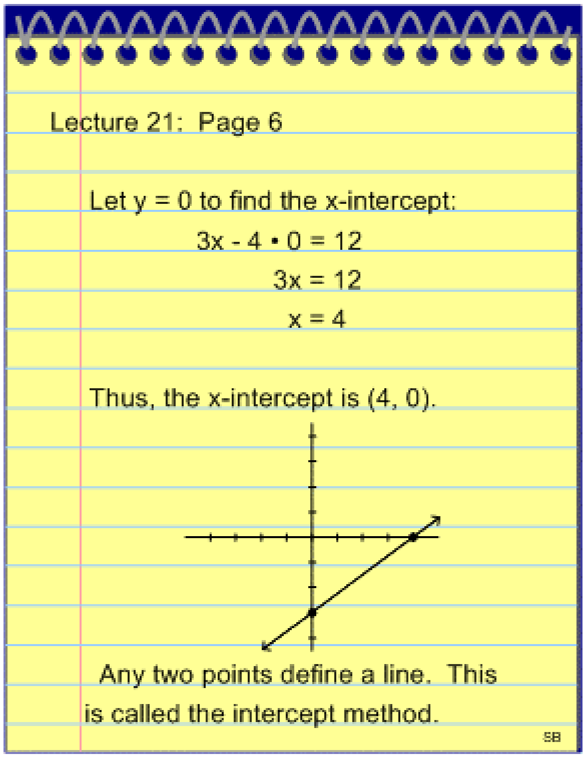
ALG2021-06



Lecture 21: Page 6

Let y = 0 to find the x-intercept:

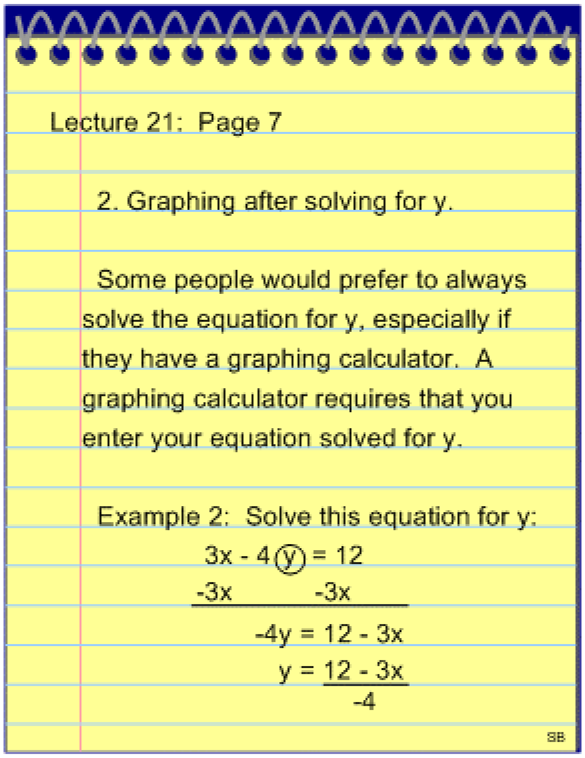
$$3x - 4 \cdot 0 = 12$$
$$3x = 12$$
$$x = 4$$

Thus, the x-intercept is (4, 0).

Any two points define a line. This is called the intercept method.

SB

ALG2021-07



Lecture 21: Page 7

2. Graphing after solving for y.

Some people would prefer to always solve the equation for y, especially if they have a graphing calculator. A graphing calculator requires that you enter your equation solved for y.

Example 2: Solve this equation for y:

$$3x - 4\textcircled{y} = 12$$
$$\underline{-3x \qquad\quad -3x}$$
$$-4y = 12 - 3x$$
$$y = \frac{12 - 3x}{-4}$$

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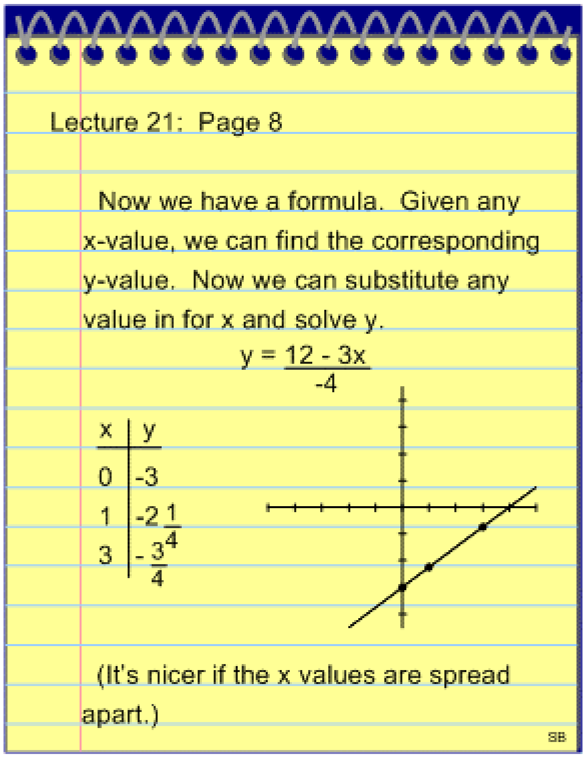
ALG2021-08



Lecture 21: Page 8

Now we have a formula. Given any x-value, we can find the corresponding y-value. Now we can substitute any value in for x and solve y.

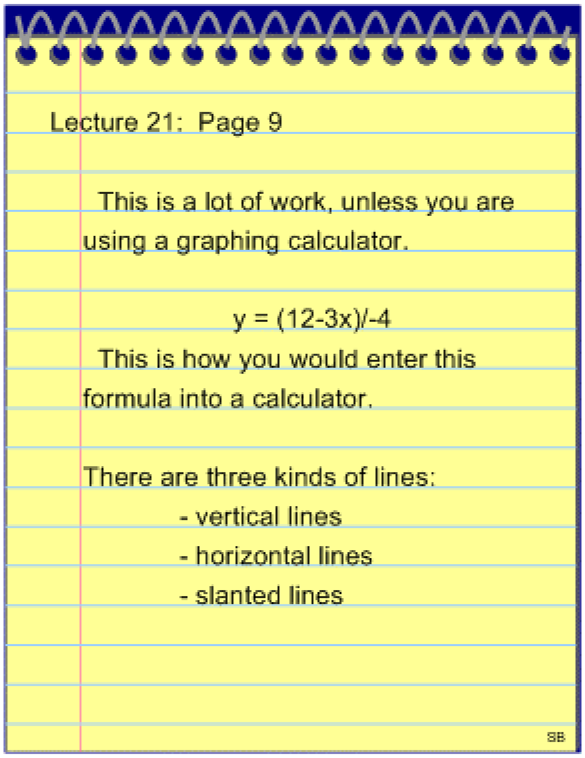
$$y = \frac{12 - 3x}{-4}$$

x	y
0	-3
1	$-2\frac{1}{4}$
3	$-\frac{3}{4}$

(It's nicer if the x values are spread apart.)

SB

ALG2021-09



Lecture 21: Page 9

This is a lot of work, unless you are using a graphing calculator.

y = (12-3x)/-4

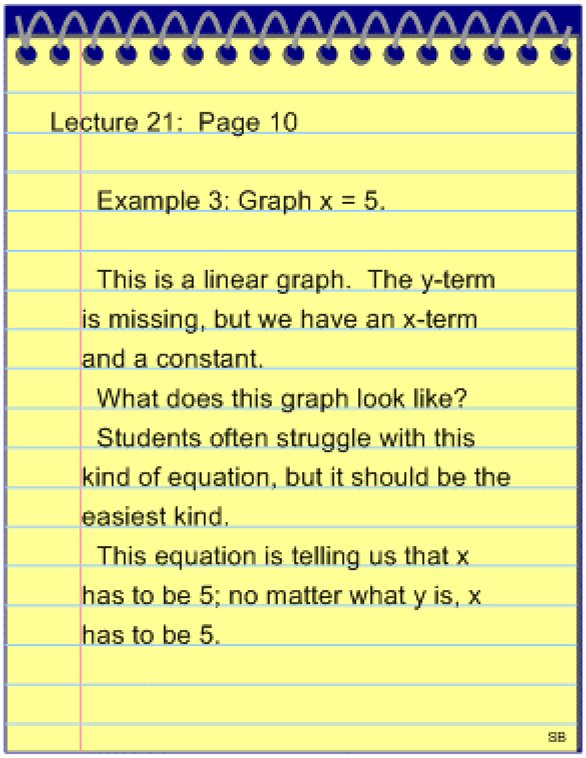
This is how you would enter this formula into a calculator.

There are three kinds of lines:

- vertical lines
- horizontal lines
- slanted lines

SB

ALG2021-10



Lecture 21: Page 10

Example 3: Graph x = 5.

This is a linear graph. The y-term is missing, but we have an x-term and a constant.

What does this graph look like?

Students often struggle with this kind of equation, but it should be the easiest kind.

This equation is telling us that x has to be 5; no matter what y is, x has to be 5.

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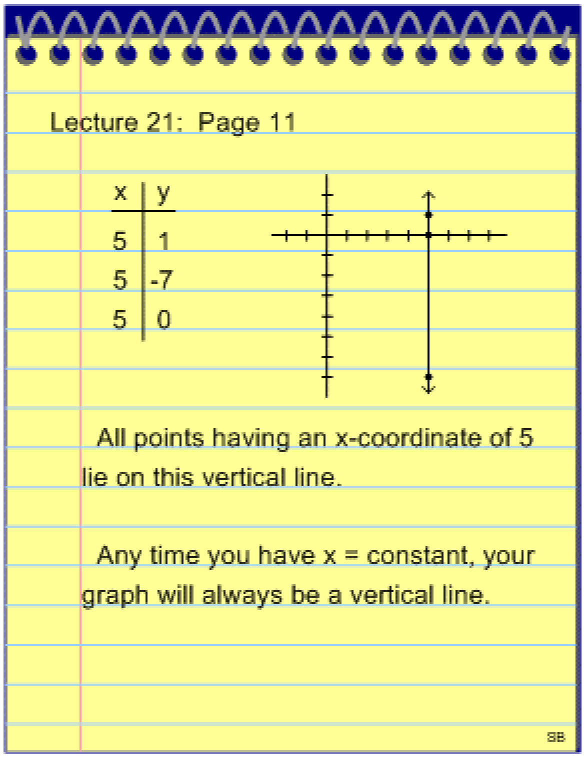
ALG2021-11



Lecture 21: Page 11

x	y
5	1
5	-7
5	0

All points having an x-coordinate of 5 lie on this vertical line.

Any time you have x = constant, your graph will always be a vertical line.

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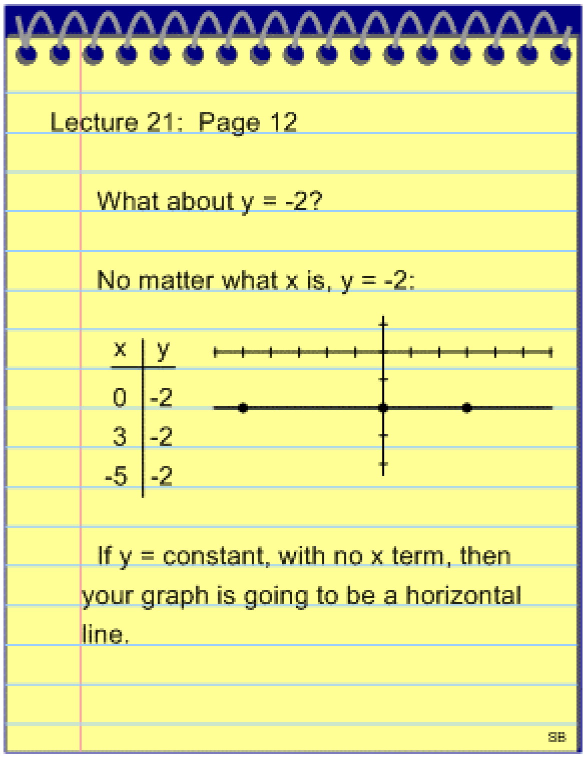
ALG2021-12



Lecture 21: Page 12

What about y = -2?

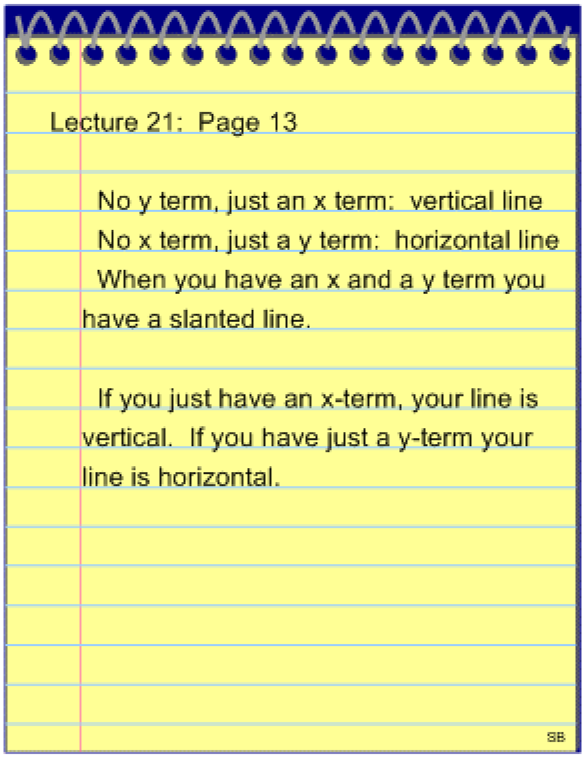
No matter what x is, y = -2:

x	y
0	-2
3	-2
-5	-2

If y = constant, with no x term, then your graph is going to be a horizontal line.

SB

ALG2021-13



Lecture 21: Page 13

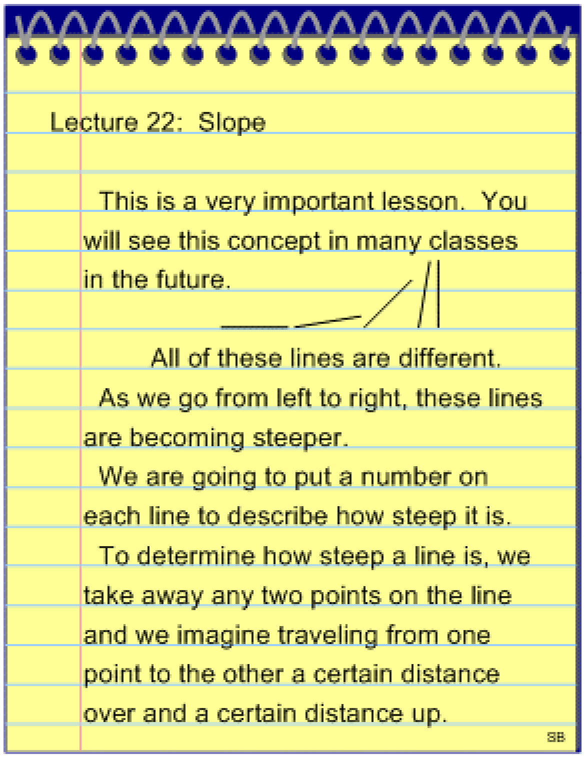
No y term, just an x term: vertical line
No x term, just a y term: horizontal line
When you have an x and a y term you have a slanted line.

If you just have an x-term, your line is vertical. If you have just a y-term your line is horizontal.

SB

Lecture 22 Notes

ALG2022-01



Lecture 22: Slope

This is a very important lesson. You will see this concept in many classes in the future.

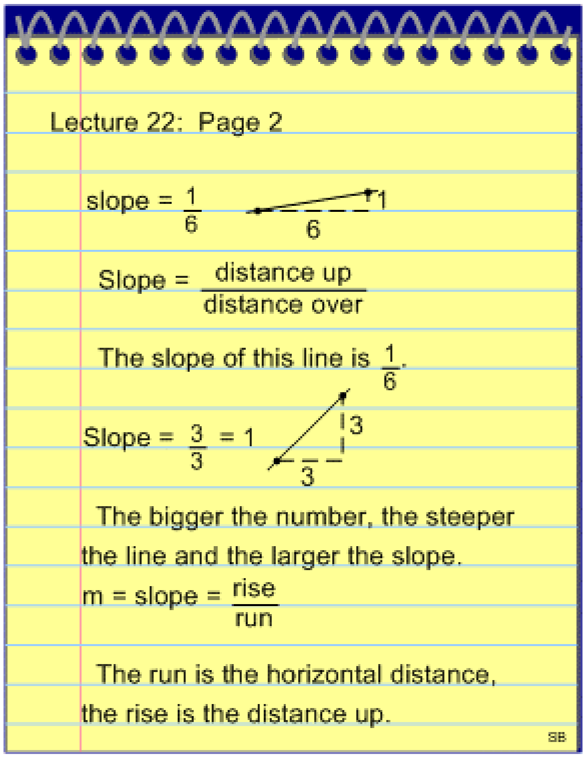
All of these lines are different.

As we go from left to right, these lines are becoming steeper.

We are going to put a number on each line to describe how steep it is.

To determine how steep a line is, we take away any two points on the line and we imagine traveling from one point to the other a certain distance over and a certain distance up.

ALG2022-02



Lecture 22: Page 2

slope = $\frac{1}{6}$

Slope = $\frac{\text{distance up}}{\text{distance over}}$

The slope of this line is $\frac{1}{6}$.

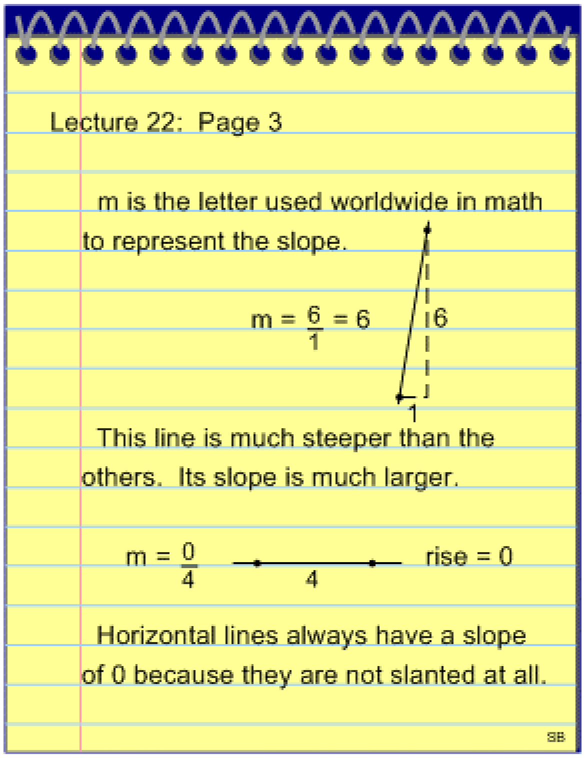
Slope = $\frac{3}{3} = 1$

The bigger the number, the steeper the line and the larger the slope.

m = slope = $\frac{\text{rise}}{\text{run}}$

The run is the horizontal distance, the rise is the distance up.

ALG2022-03



Lecture 22: Page 3

m is the letter used worldwide in math to represent the slope.

m = $\frac{6}{1} = 6$

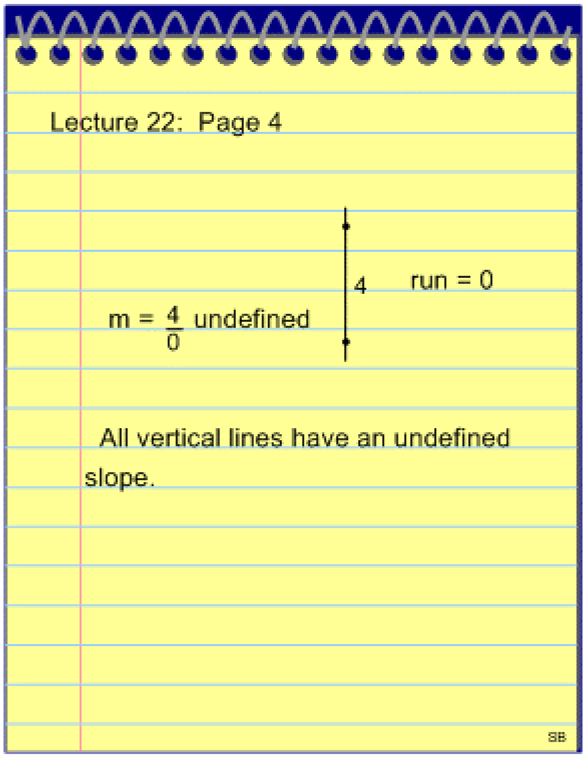
This line is much steeper than the others. Its slope is much larger.

m = $\frac{0}{4}$

rise = 0

Horizontal lines always have a slope of 0 because they are not slanted at all.

ALG2022-04



Lecture 22: Page 4

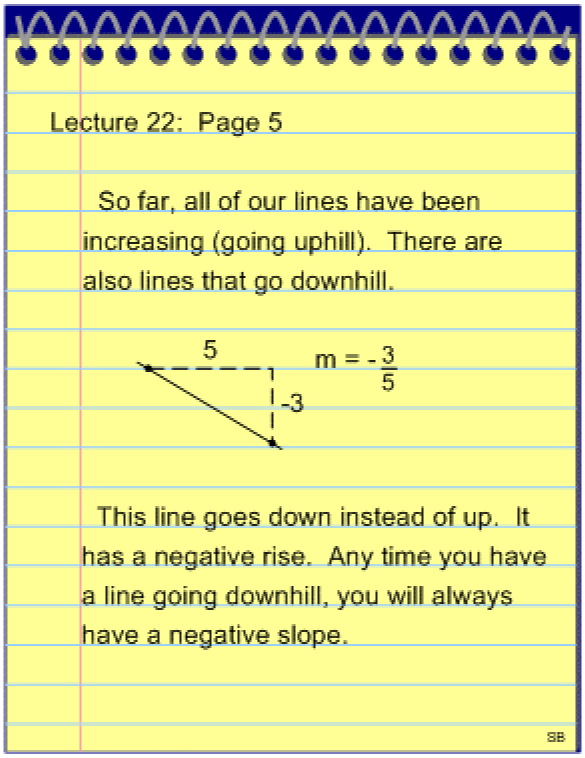
m = $\frac{4}{0}$ undefined

run = 0

All vertical lines have an undefined slope.

Lecture 22 Notes

ALG2022-05



Lecture 22: Page 5

So far, all of our lines have been increasing (going uphill). There are also lines that go downhill.

5

-3

$m = -\frac{3}{5}$

This line goes down instead of up. It has a negative rise. Any time you have a line going downhill, you will always have a negative slope.

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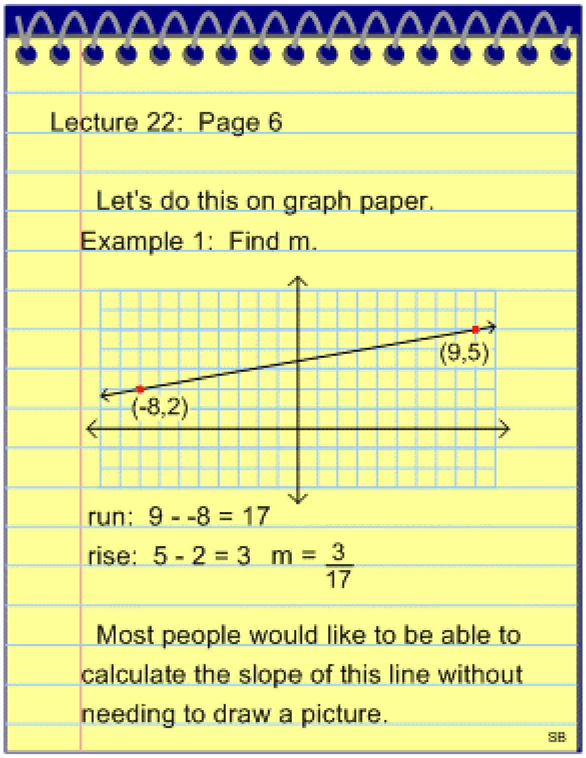
ALG2022-06



Lecture 22: Page 6

Let's do this on graph paper.

Example 1: Find m.

(9,5)

(-8,2)

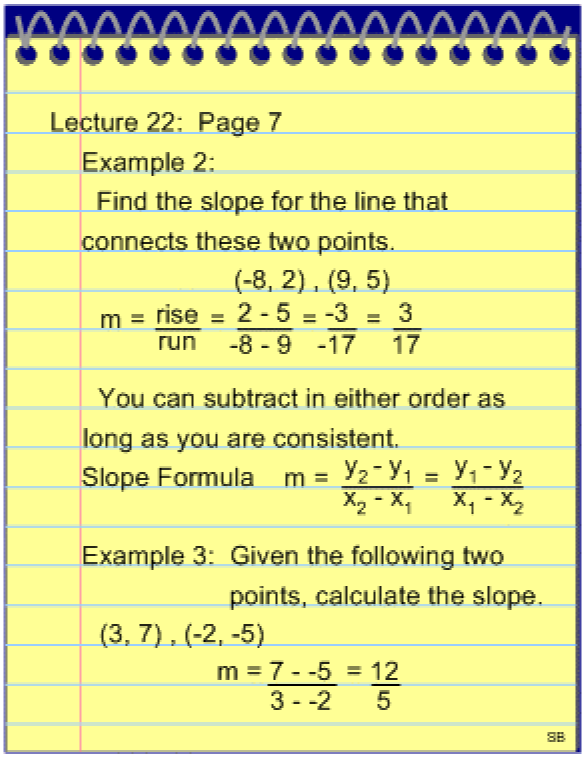
run: 9 - -8 = 17

rise: 5 - 2 = 3 $m = \frac{3}{17}$

Most people would like to be able to calculate the slope of this line without needing to draw a picture.

SB

ALG2022-07



Lecture 22: Page 7

Example 2:

Find the slope for the line that connects these two points.

(-8, 2) , (9, 5)

$$m = \frac{rise}{run} = \frac{2 - 5}{-8 - 9} = \frac{-3}{-17} = \frac{3}{17}$$

You can subtract in either order as long as you are consistent.

Slope Formula $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$

Example 3: Given the following two points, calculate the slope.

(3, 7) , (-2, -5)

$$m = \frac{7 - -5}{3 - -2} = \frac{12}{5}$$

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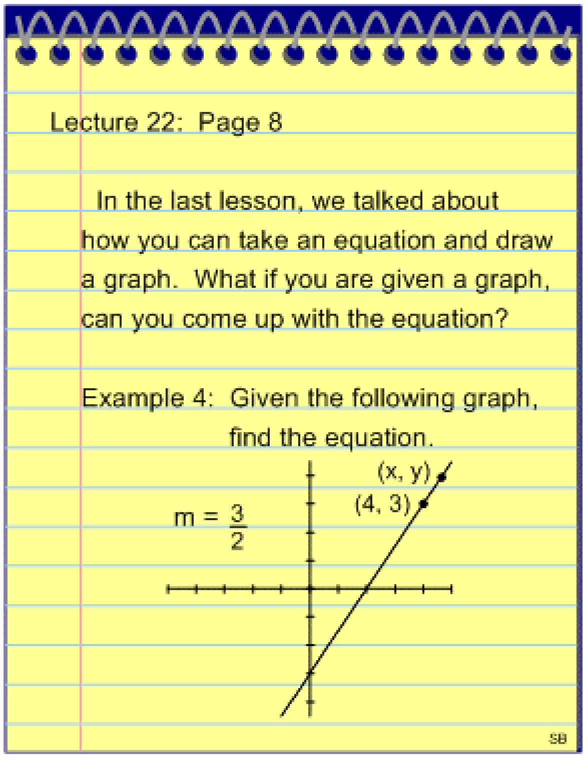
ALG2022-08



Lecture 22: Page 8

In the last lesson, we talked about how you can take an equation and draw a graph. What if you are given a graph, can you come up with the equation?

Example 4: Given the following graph, find the equation.

$m = \frac{3}{2}$

(x, y)

(4, 3)

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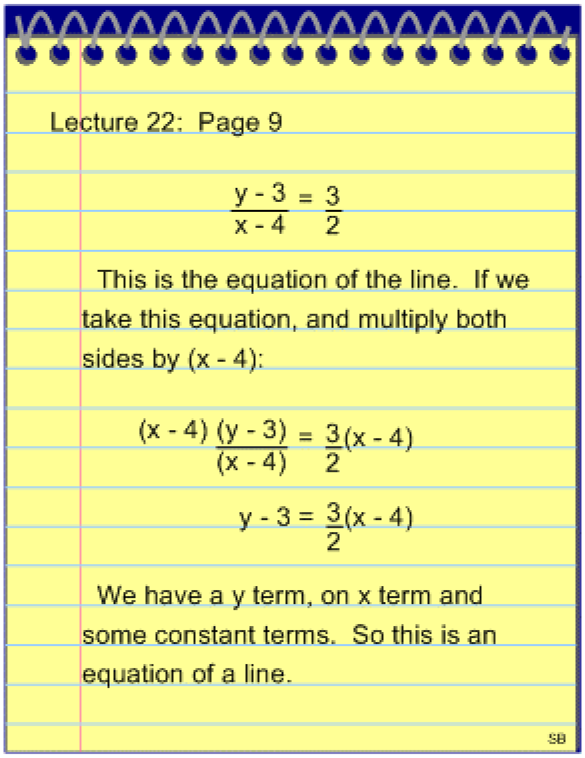
ALG2022-09



Lecture 22: Page 9

$$\frac{y-3}{x-4} = \frac{3}{2}$$

This is the equation of the line. If we take this equation, and multiply both sides by (x - 4):

$$(x-4)\frac{(y-3)}{(x-4)} = \frac{3}{2}(x-4)$$

$$y - 3 = \frac{3}{2}(x-4)$$

We have a y term, on x term and some constant terms. So this is an equation of a line.

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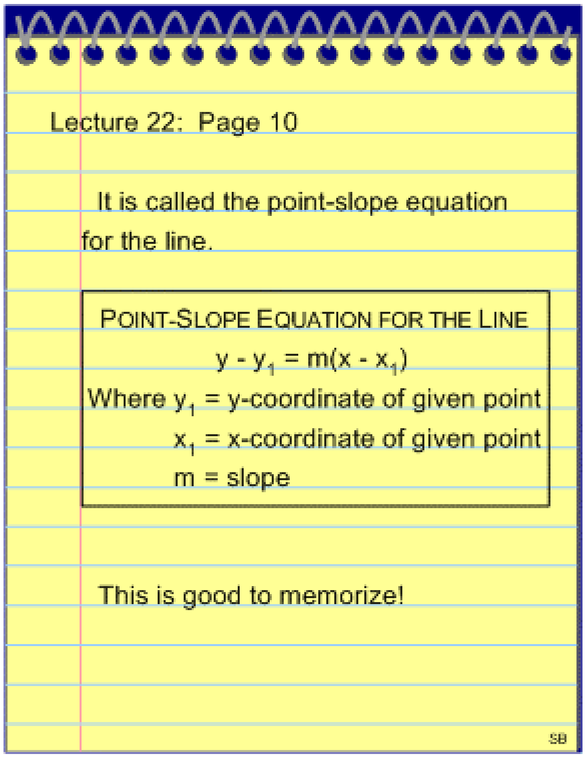
ALG2022-10



Lecture 22: Page 10

It is called the point-slope equation for the line.

POINT-SLOPE EQUATION FOR THE LINE

$$y - y_1 = m(x - x_1)$$

Where y_1 = y-coordinate of given point

x_1 = x-coordinate of given point

m = slope

This is good to memorize!

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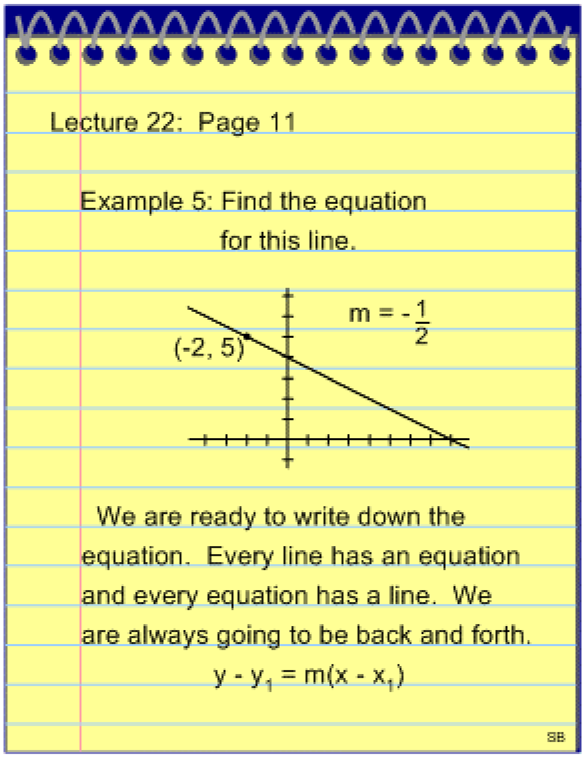
ALG2022-11



Lecture 22: Page 11

Example 5: Find the equation for this line.

We are ready to write down the equation. Every line has an equation and every equation has a line. We are always going to be back and forth.

$$y - y_1 = m(x - x_1)$$

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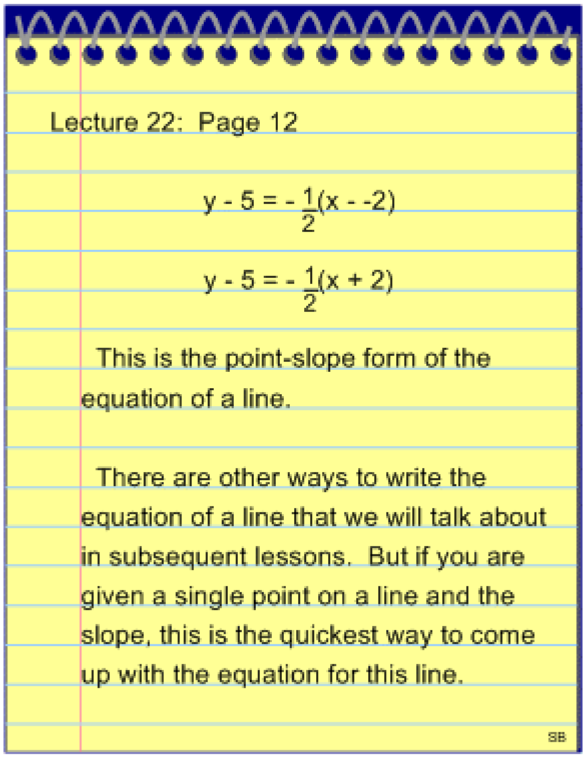
ALG2022-12



Lecture 22: Page 12

$$y - 5 = -\frac{1}{2}(x - -2)$$

$$y - 5 = -\frac{1}{2}(x + 2)$$

This is the point-slope form of the equation of a line.

There are other ways to write the equation of a line that we will talk about in subsequent lessons. But if you are given a single point on a line and the slope, this is the quickest way to come up with the equation for this line.

SB

Lecture 23 Notes

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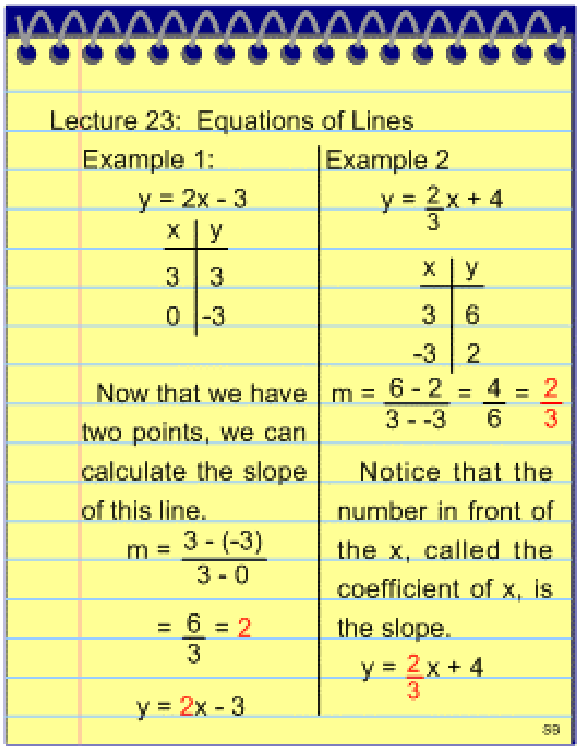

Lecture 23: Equations of Lines

Example 1:

$y = 2x - 3$

x	y
3	3
0	-3

Now that we have two points, we can calculate the slope of this line.

$$m = \frac{3 - (-3)}{3 - 0}$$

$$= \frac{6}{3} = 2$$

$y = 2x - 3$

Example 2

$y = \frac{2}{3}x + 4$

x	y
3	6
-3	2

$$m = \frac{6 - 2}{3 - -3} = \frac{4}{6} = \frac{2}{3}$$

Notice that the number in front of the x, called the coefficient of x, is the slope.

$y = \frac{2}{3}x + 4$

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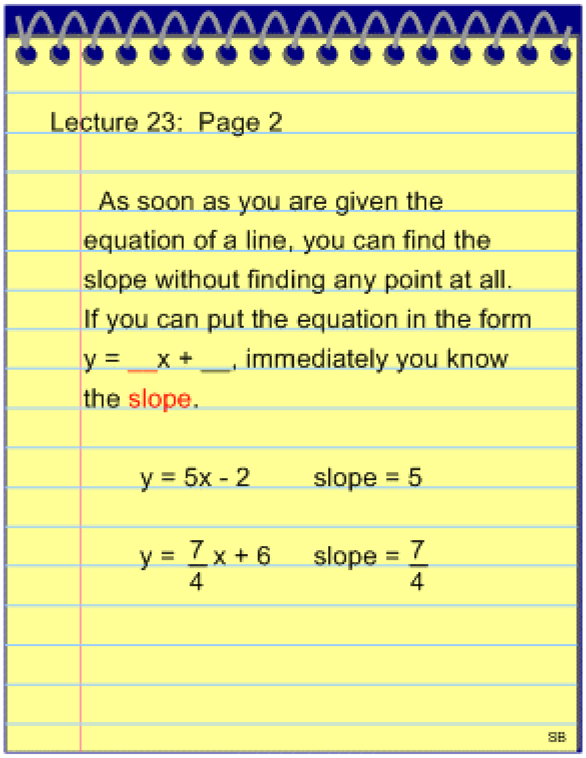


Lecture 23: Page 2

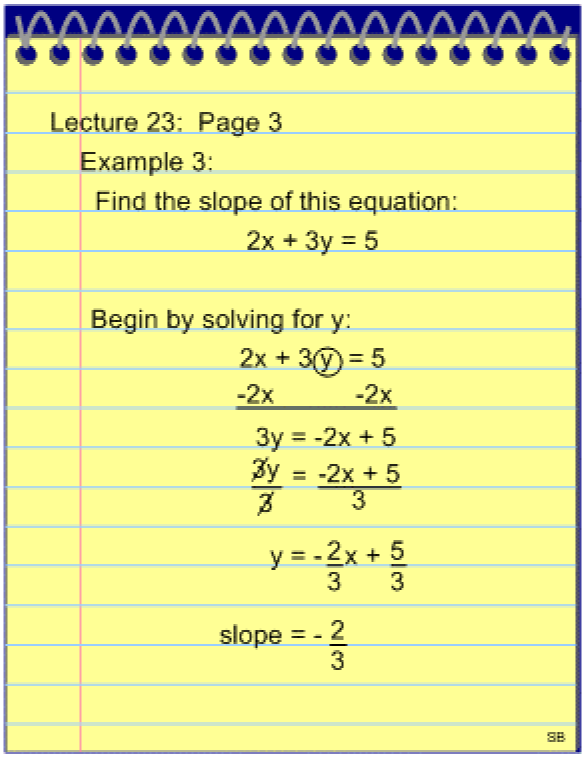
As soon as you are given the equation of a line, you can find the slope without finding any point at all. If you can put the equation in the form $y = \_\_x + \_\_$, immediately you know the slope.

$y = 5x - 2$ slope = 5

$y = \frac{7}{4}x + 6$ slope = $\frac{7}{4}$

SB

ALG2023-03



Lecture 23: Page 3

Example 3:

Find the slope of this equation:

$2x + 3y = 5$

Begin by solving for y:

$$2x + 3\textcircled{y} = 5$$

$$\underline{-2x \qquad -2x}$$

$$3y = -2x + 5$$

$$\frac{\cancel{3}y}{\cancel{3}} = \frac{-2x + 5}{3}$$

$$y = -\frac{2}{3}x + \frac{5}{3}$$

$$\text{slope} = -\frac{2}{3}$$

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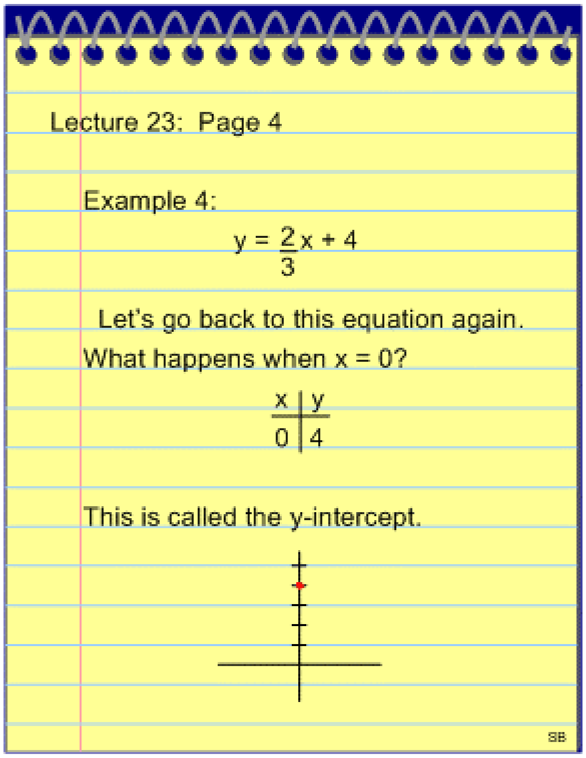


Lecture 23: Page 4

Example 4:

$y = \frac{2}{3}x + 4$

Let's go back to this equation again. What happens when x = 0?

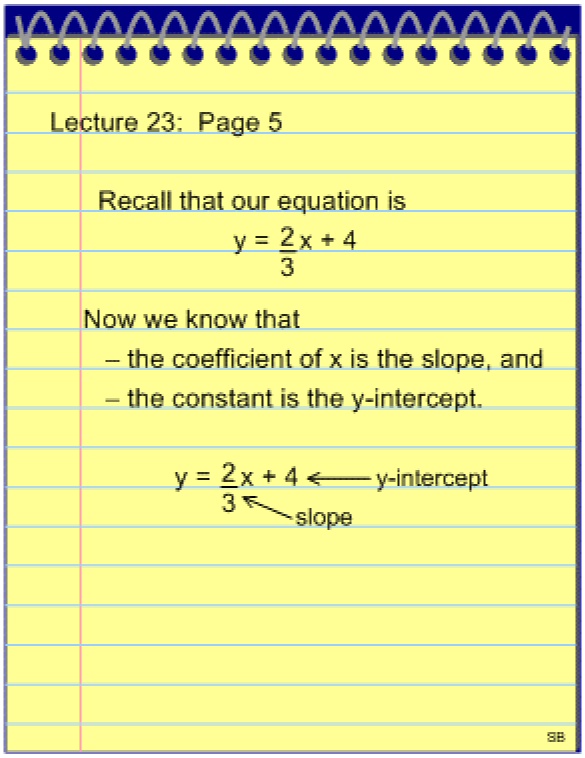
x	y
0	4

This is called the y-intercept.

SB

Lecture 23 Notes, Continued

ALG2023-05



Lecture 23: Page 5

Recall that our equation is

$$y = \frac{2}{3}x + 4$$

Now we know that

- the coefficient of x is the slope, and
- the constant is the y-intercept.

$$y = \underset{\uparrow \text{slope}}{\frac{2}{3}}x + 4 \leftarrow \text{y-intercept}$$

SB

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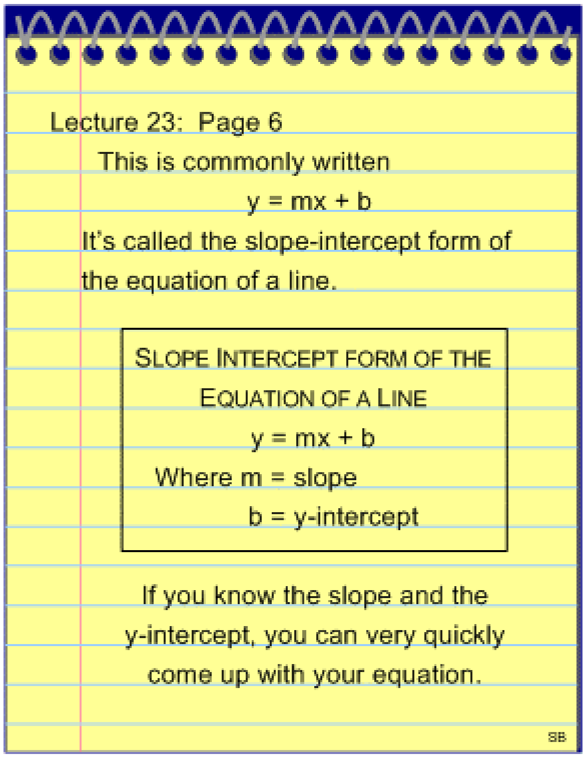

Lecture 23: Page 6

This is commonly written

$$y = mx + b$$

It's called the slope-intercept form of the equation of a line.

SLOPE INTERCEPT FORM OF THE EQUATION OF A LINE
$y = mx + b$
Where m = slope
b = y-intercept

If you know the slope and the y-intercept, you can very quickly come up with your equation.

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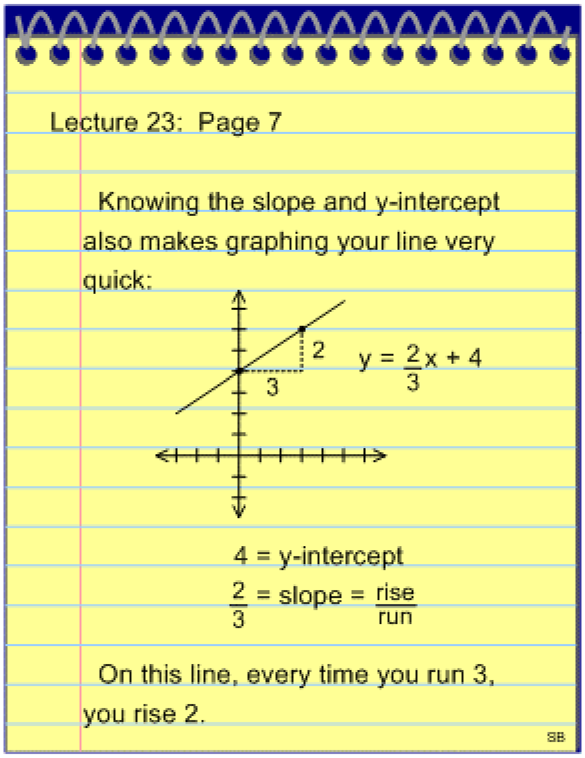

Lecture 23: Page 7

Knowing the slope and y-intercept also makes graphing your line very quick:

$$y = \frac{2}{3}x + 4$$

4 = y-intercept

$$\frac{2}{3} = \text{slope} = \frac{\text{rise}}{\text{run}}$$

On this line, every time you run 3, you rise 2.

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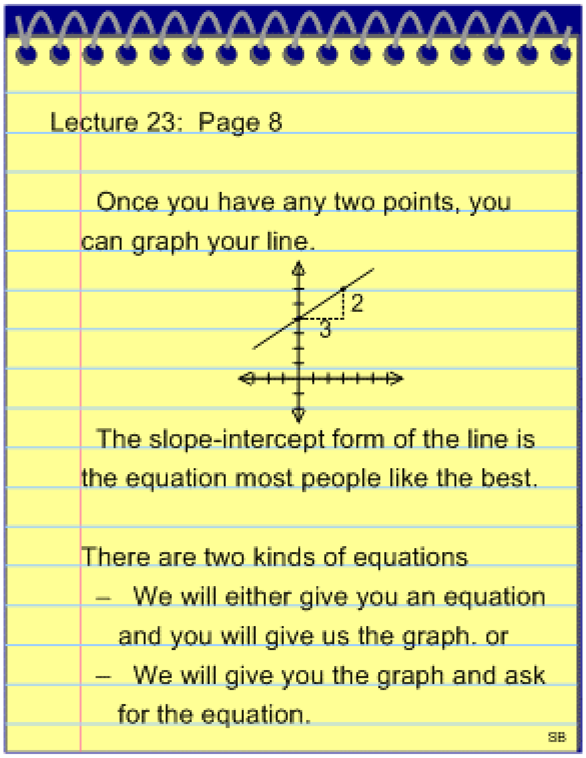

Lecture 23: Page 8

Once you have any two points, you can graph your line.

The slope-intercept form of the line is the equation most people like the best.

There are two kinds of equations

- We will either give you an equation and you will give us the graph, or
- We will give you the graph and ask for the equation.

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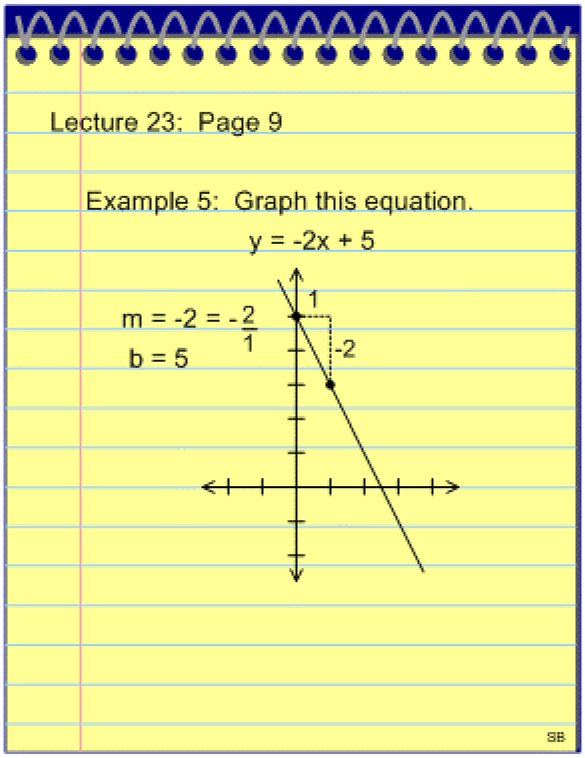


Lecture 23: Page 9

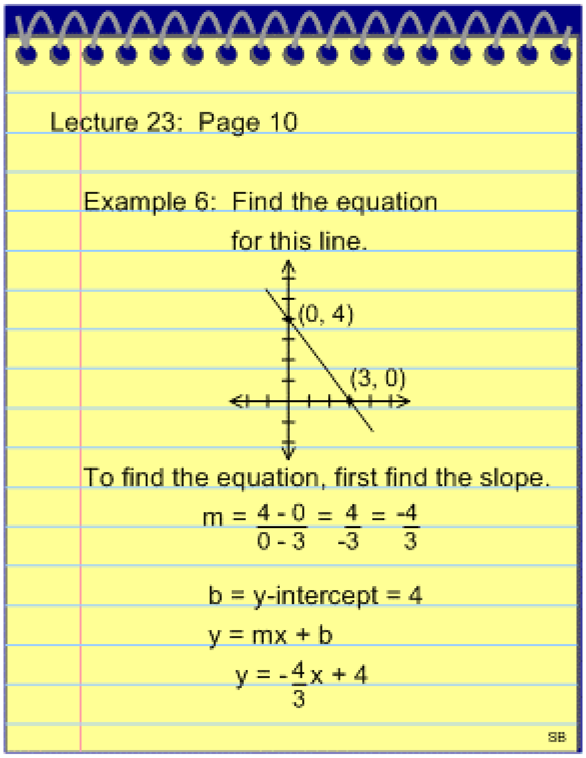
Example 5: Graph this equation.

$y = -2x + 5$

$m = -2 = -\frac{2}{1}$

$b = 5$

ALG2023-10



Lecture 23: Page 10

Example 6: Find the equation for this line.

To find the equation, first find the slope.

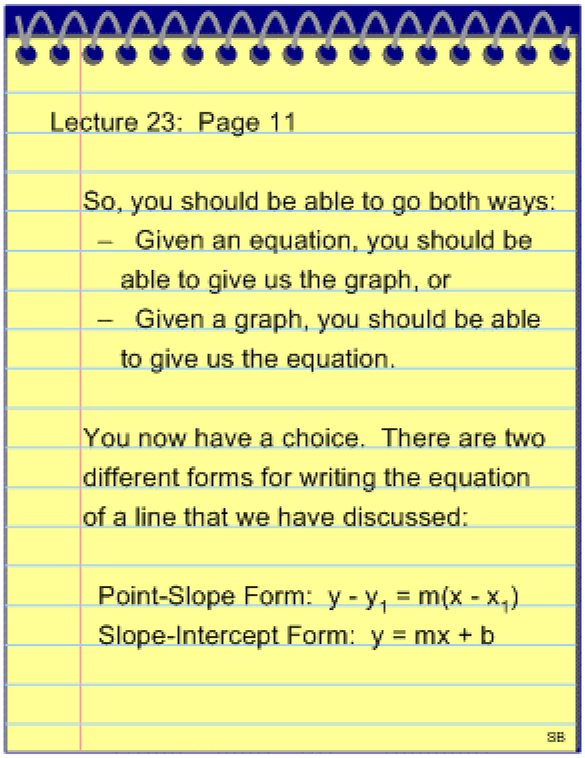
$m = \frac{4 - 0}{0 - 3} = \frac{4}{-3} = \frac{-4}{3}$

b = y-intercept = 4

$y = mx + b$

$y = -\frac{4}{3}x + 4$

ALG2023-11



Lecture 23: Page 11

So, you should be able to go both ways:

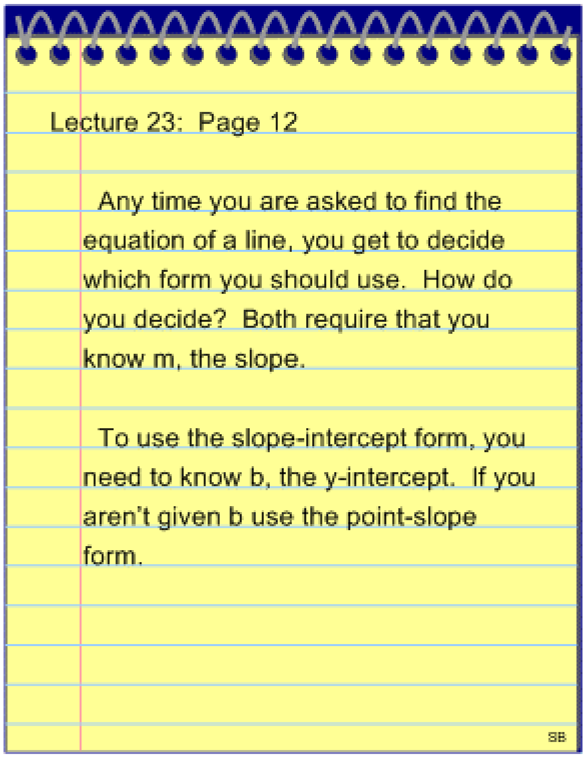
- Given an equation, you should be able to give us the graph, or
- Given a graph, you should be able to give us the equation.

You now have a choice. There are two different forms for writing the equation of a line that we discussed:

Point-Slope Form: $y - y_1 = m(x - x_1)$

Slope-Intercept Form: $y = mx + b$

ALG2023-12



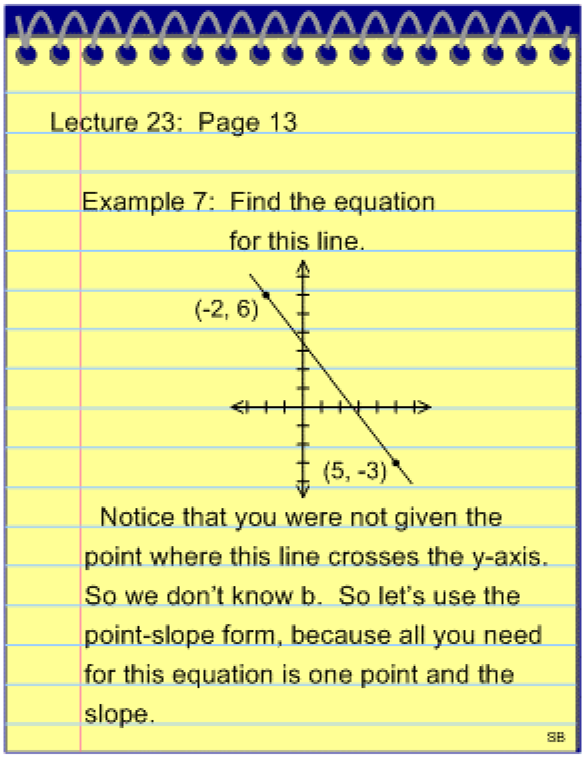
Lecture 23: Page 12

Any time you are asked to find the equation of a line, you get to decide which form you should use. How do you decide? Both require that you know m, the slope.

To use the slope-intercept form, you need to know b, the y-intercept. If you aren't given b use the point-slope form.

Lecture 23 Notes, Continued

ALG2023-13



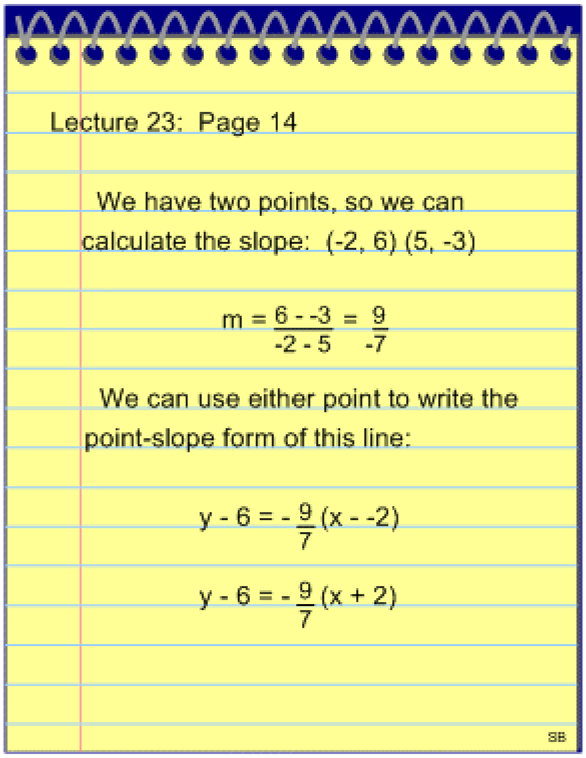
Lecture 23: Page 13

Example 7: Find the equation for this line.

Notice that you were not given the point where this line crosses the y-axis. So we don't know b. So let's use the point-slope form, because all you need for this equation is one point and the slope.

SB

ALG2023-14



Lecture 23: Page 14

We have two points, so we can calculate the slope: (-2, 6) (5, -3)

$$m = \frac{6 - -3}{-2 - 5} = \frac{9}{-7}$$

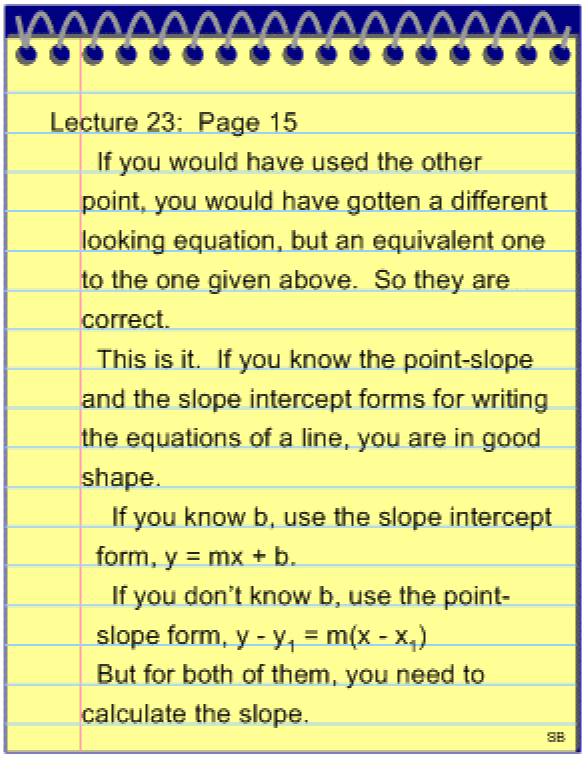
We can use either point to write the point-slope form of this line:

$$y - 6 = -\frac{9}{7}(x - -2)$$

$$y - 6 = -\frac{9}{7}(x + 2)$$

SB

ALG2023-15



Lecture 23: Page 15

If you would have used the other point, you would have gotten a different looking equation, but an equivalent one to the one given above. So they are correct.

This is it. If you know the point-slope and the slope intercept forms for writing the equations of a line, you are in good shape.

If you know b, use the slope intercept form, $y = mx + b$.

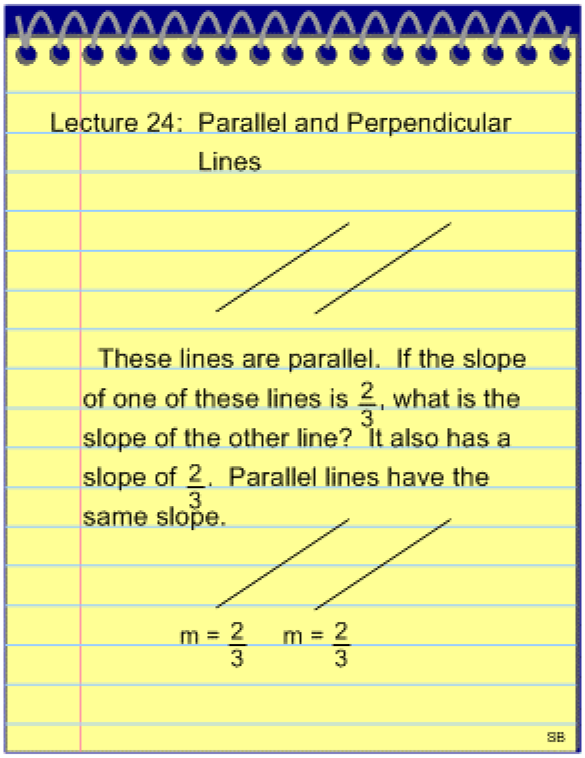
If you don't know b, use the point-slope form, $y - y_1 = m(x - x_1)$

But for both of them, you need to calculate the slope.

SB

Lecture 24 Notes

ALG2024-01



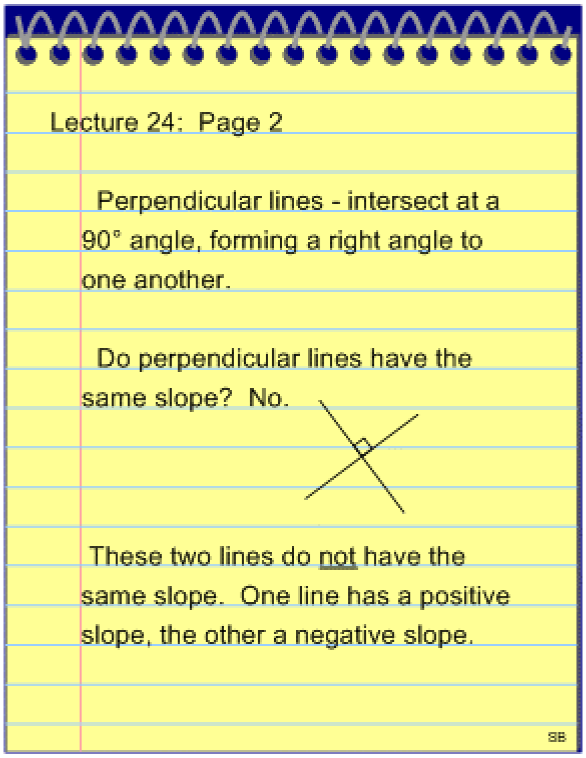
Lecture 24: Parallel and Perpendicular Lines

These lines are parallel. If the slope of one of these lines is $\frac{2}{3}$, what is the slope of the other line? It also has a slope of $\frac{2}{3}$. Parallel lines have the same slope.

$m = \frac{2}{3}$ $m = \frac{2}{3}$

SB

ALG2024-02



Lecture 24: Page 2

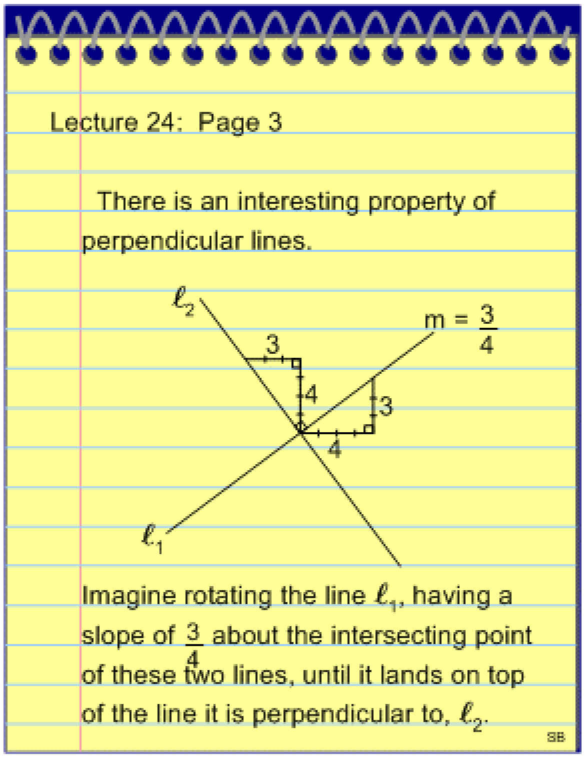
Perpendicular lines - intersect at a 90° angle, forming a right angle to one another.

Do perpendicular lines have the same slope? No.

These two lines do not have the same slope. One line has a positive slope, the other a negative slope.

SB

ALG2024-03



Lecture 24: Page 3

There is an interesting property of perpendicular lines.

Imagine rotating the line ℓ_1, having a slope of $\frac{3}{4}$ about the intersecting point of these two lines, until it lands on top of the line it is perpendicular to, ℓ_2.

SB

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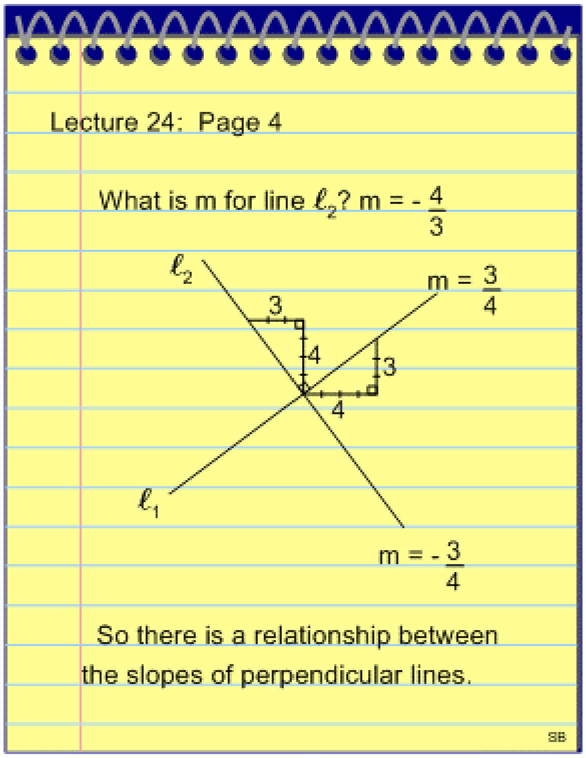


Lecture 24: Page 4

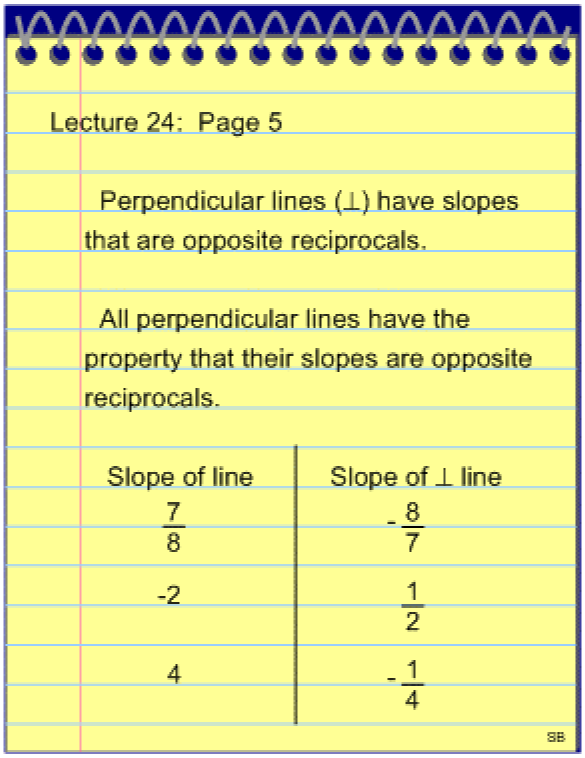
What is m for line ℓ_2? $m = -\frac{4}{3}$

So there is a relationship between the slopes of perpendicular lines.

SB

Lecture 24 Notes, Continued

ALG2024-05



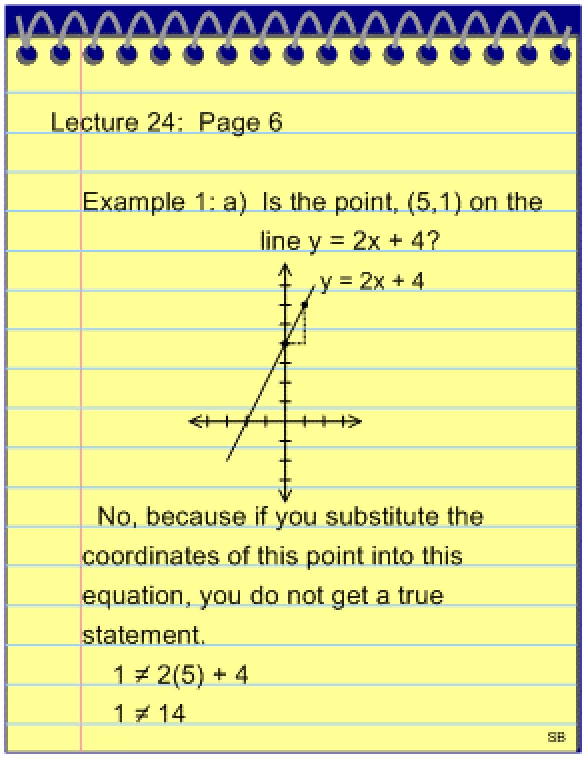
Lecture 24: Page 5

Perpendicular lines (⊥) have slopes that are opposite reciprocals.

All perpendicular lines have the property that their slopes are opposite reciprocals.

Slope of line	Slope of ⊥ line
$\frac{7}{8}$	$-\frac{8}{7}$
-2	$\frac{1}{2}$
4	$-\frac{1}{4}$

ALG2024-06



Lecture 24: Page 6

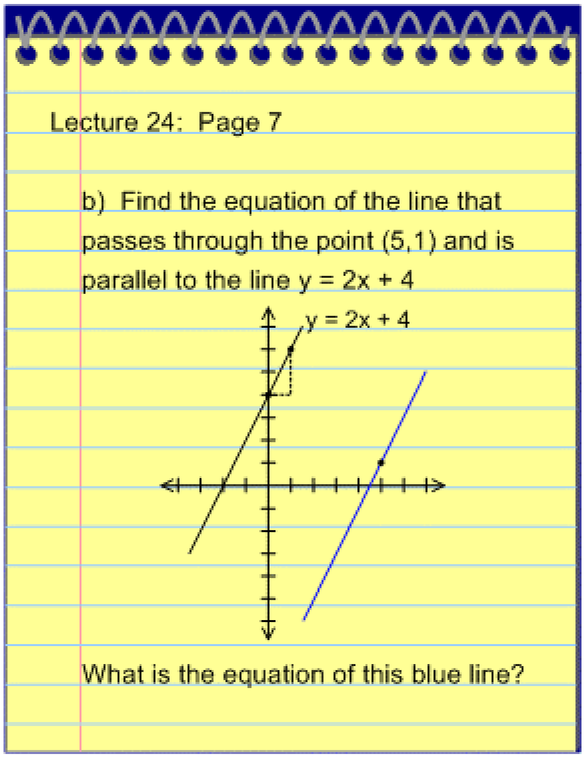
Example 1: a) Is the point, (5,1) on the line y = 2x + 4?

No, because if you substitute the coordinates of this point into this equation, you do not get a true statement.

$1 \neq 2(5) + 4$

$1 \neq 14$

ALG2024-07



Lecture 24: Page 7

b) Find the equation of the line that passes through the point (5,1) and is parallel to the line y = 2x + 4

What is the equation of this blue line?

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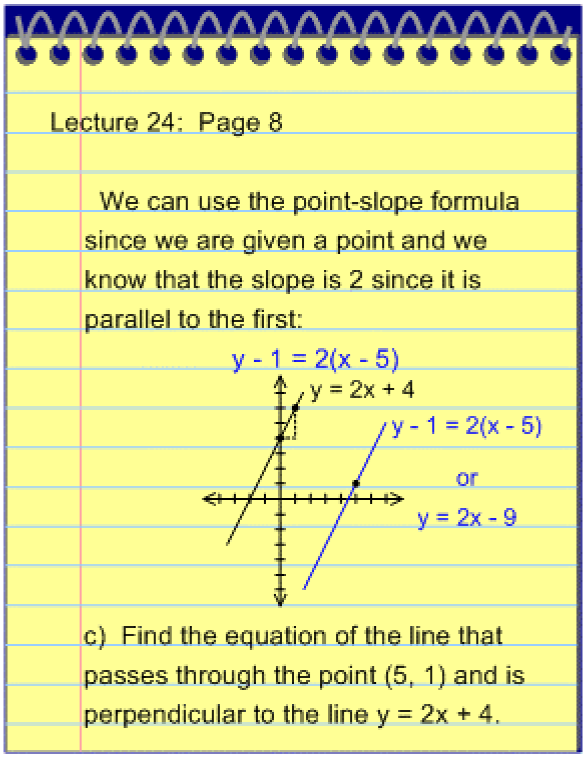


Lecture 24: Page 8

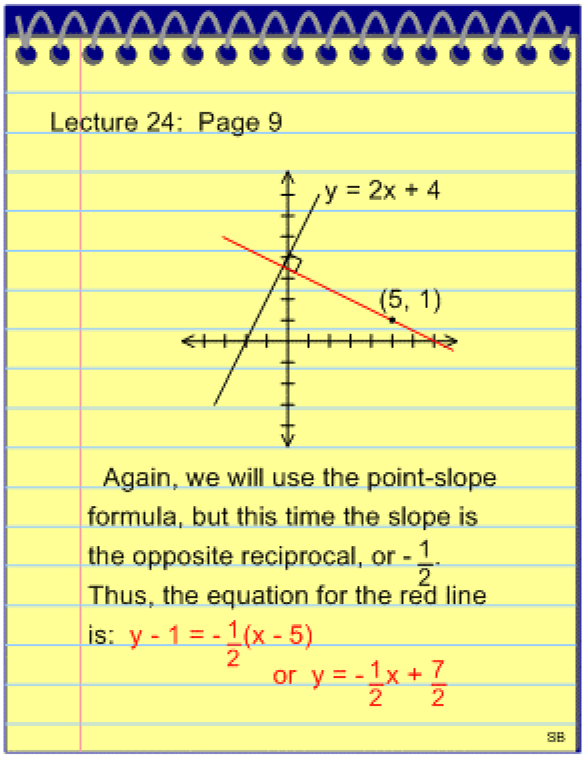
We can use the point-slope formula since we are given a point and we know that the slope is 2 since it is parallel to the first:

$y - 1 = 2(x - 5)$

c) Find the equation of the line that passes through the point (5, 1) and is perpendicular to the line y = 2x + 4.

Lecture 24 Notes, Continued

ALG2024-09

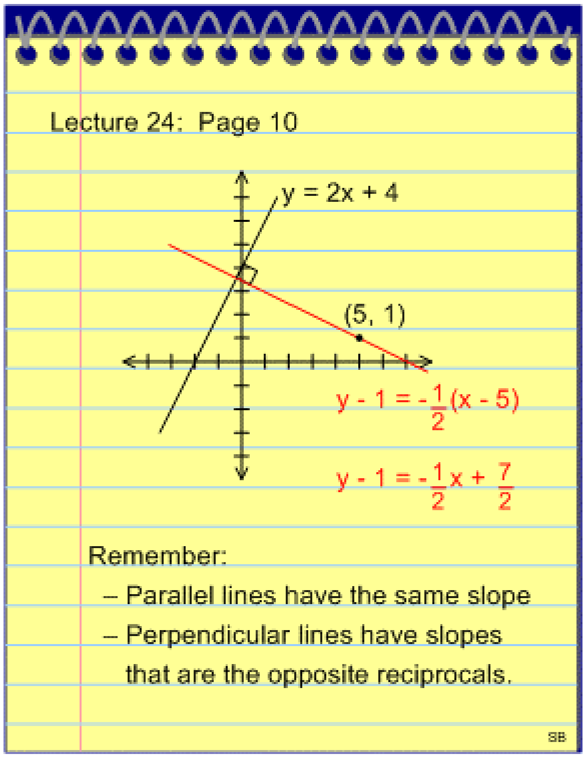


Lecture 24: Page 9

Again, we will use the point-slope formula, but this time the slope is the opposite reciprocal, or $-\frac{1}{2}$. Thus, the equation for the red line is: $y - 1 = -\frac{1}{2}(x - 5)$

or $y = -\frac{1}{2}x + \frac{7}{2}$

ALG2024-10



Lecture 24: Page 10

$y - 1 = -\frac{1}{2}(x - 5)$

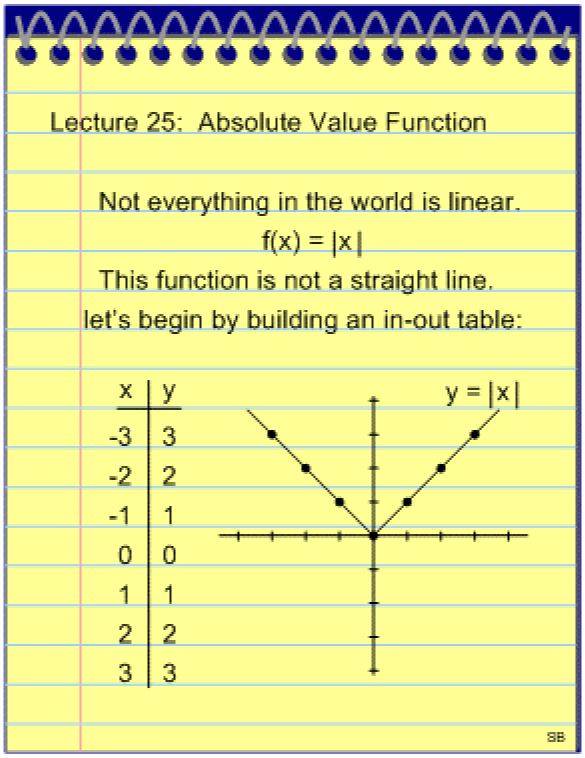
$y - 1 = -\frac{1}{2}x + \frac{7}{2}$

Remember:

- Parallel lines have the same slope
- Perpendicular lines have slopes that are the opposite reciprocals.

Lecture 25 Notes

ALG2025-01



Lecture 25: Absolute Value Function

Not everything in the world is linear.

$f(x) = |x|$

This function is not a straight line.

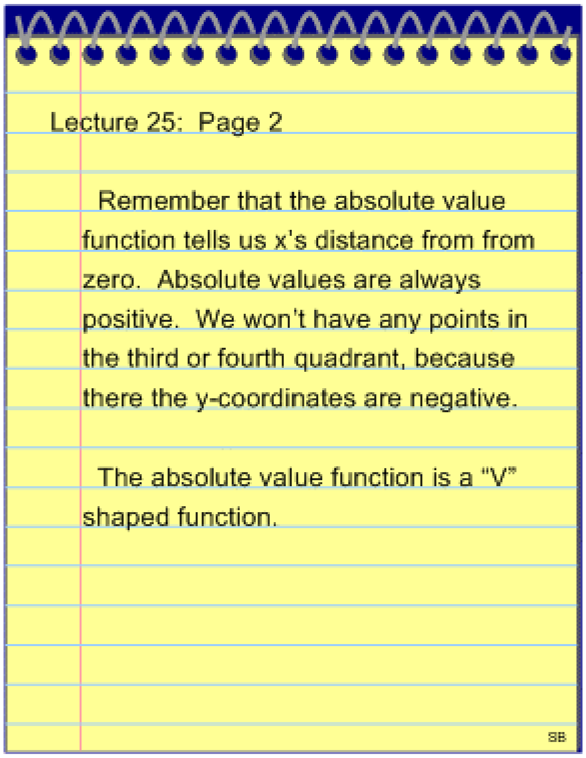
let's begin by building an in-out table:

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

$y = |x|$

SB

ALG2025-02



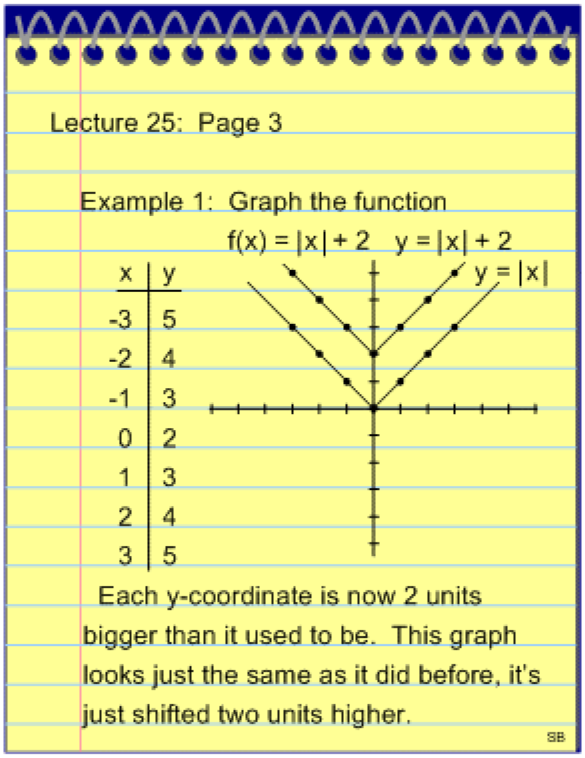
Lecture 25: Page 2

Remember that the absolute value function tells us x's distance from from zero. Absolute values are always positive. We won't have any points in the third or fourth quadrant, because there the y-coordinates are negative.

The absolute value function is a "V" shaped function.

SB

ALG2025-03



Lecture 25: Page 3

Example 1: Graph the function

$f(x) = |x| + 2$ $y = |x| + 2$

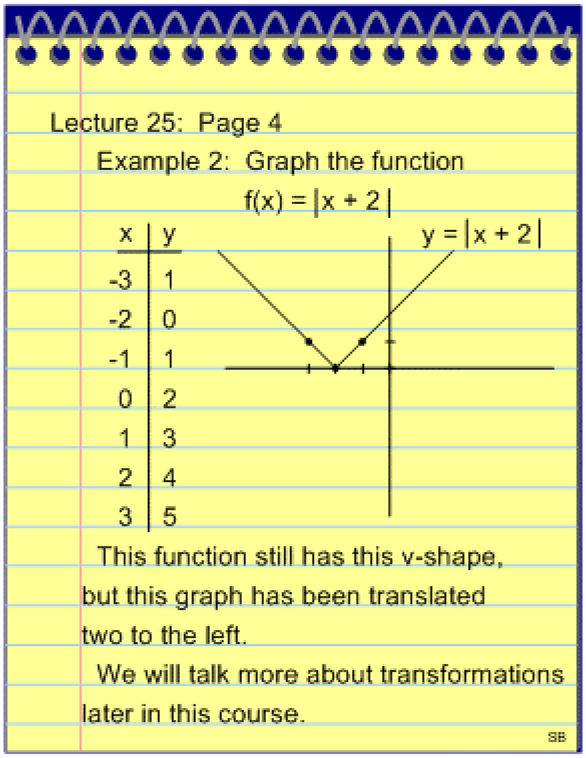
x	y
-3	5
-2	4
-1	3
0	2
1	3
2	4
3	5

$y = |x|$

Each y-coordinate is now 2 units bigger than it used to be. This graph looks just the same as it did before, it's just shifted two units higher.

SB

ALG2025-04



Lecture 25: Page 4

Example 2: Graph the function

$f(x) = |x + 2|$

x	y
-3	1
-2	0
-1	1
0	2
1	3
2	4
3	5

$y = |x + 2|$

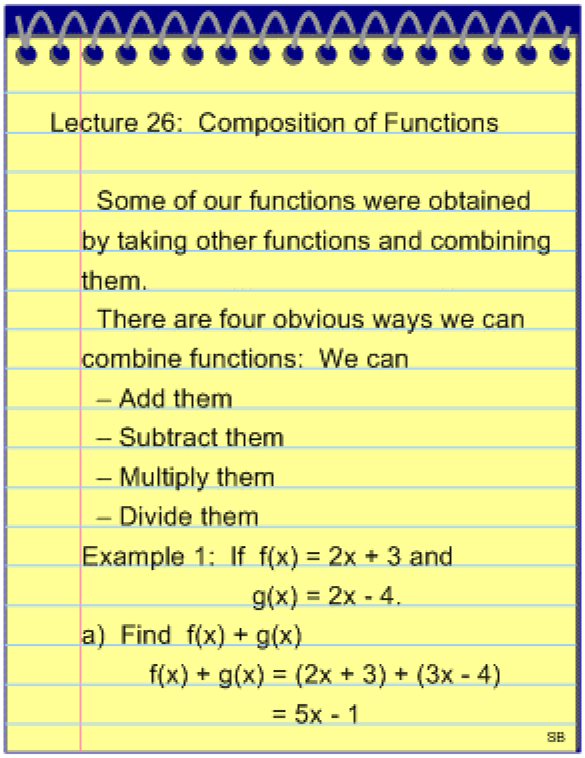
This function still has this v-shape, but this graph has been translated two to the left.

We will talk more about transformations later in this course.

SB

Lecture 26 Notes

ALG2026-01



Lecture 26: Composition of Functions

Some of our functions were obtained by taking other functions and combining them.

There are four obvious ways we can combine functions: We can

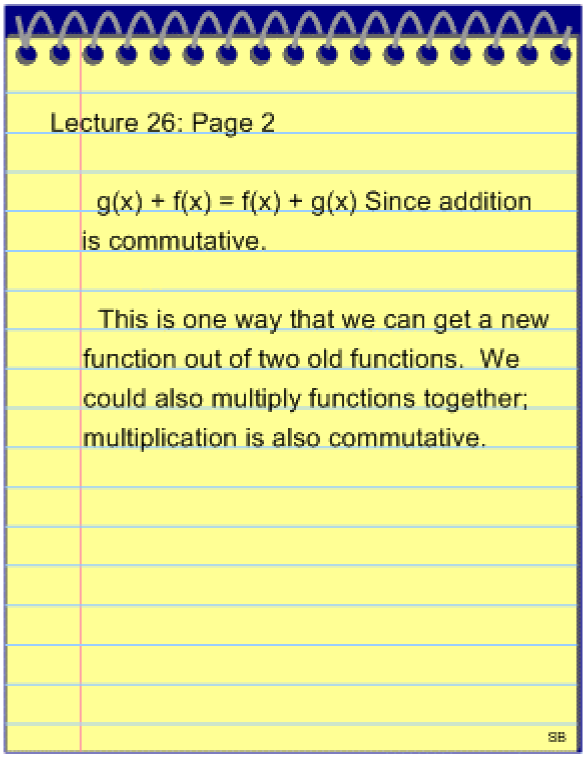
- Add them
- Subtract them
- Multiply them
- Divide them

Example 1: If $f(x) = 2x + 3$ and
$g(x) = 2x - 4$.

a) Find $f(x) + g(x)$

$f(x) + g(x) = (2x + 3) + (3x - 4)$
$= 5x - 1$

ALG2026-02

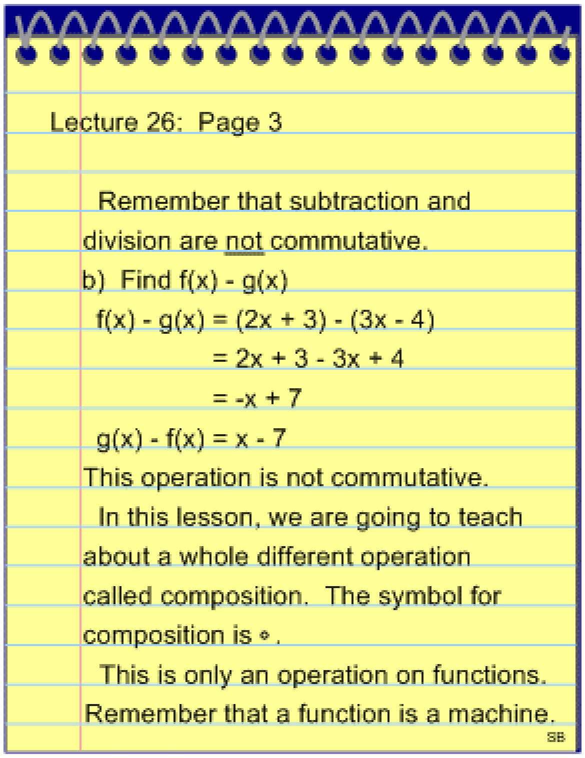


Lecture 26: Page 2

$g(x) + f(x) = f(x) + g(x)$ Since addition is commutative.

This is one way that we can get a new function out of two old functions. We could also multiply functions together; multiplication is also commutative.

ALG2026-03



Lecture 26: Page 3

Remember that subtraction and division are not commutative.

b) Find $f(x) - g(x)$

$f(x) - g(x) = (2x + 3) - (3x - 4)$
$= 2x + 3 - 3x + 4$
$= -x + 7$

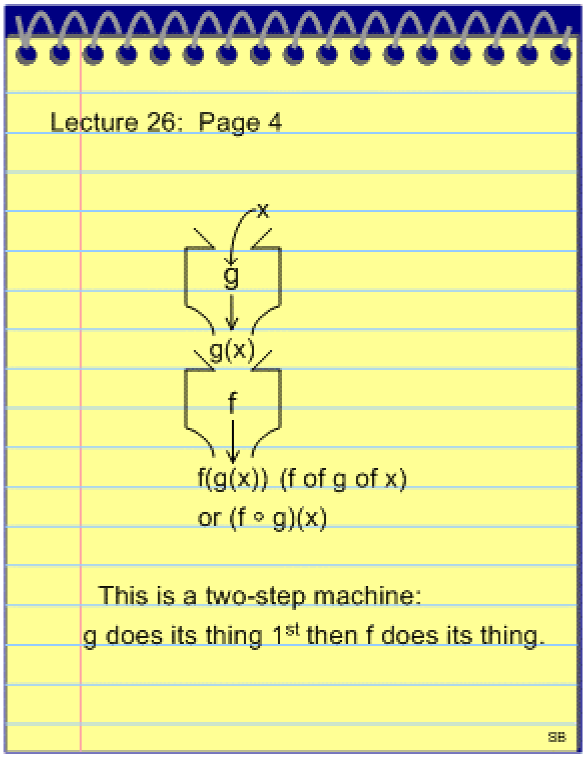
$g(x) - f(x) = x - 7$

This operation is not commutative.

In this lesson, we are going to teach about a whole different operation called composition. The symbol for composition is $\circ$.

This is only an operation on functions. Remember that a function is a machine.

ALG2026-04



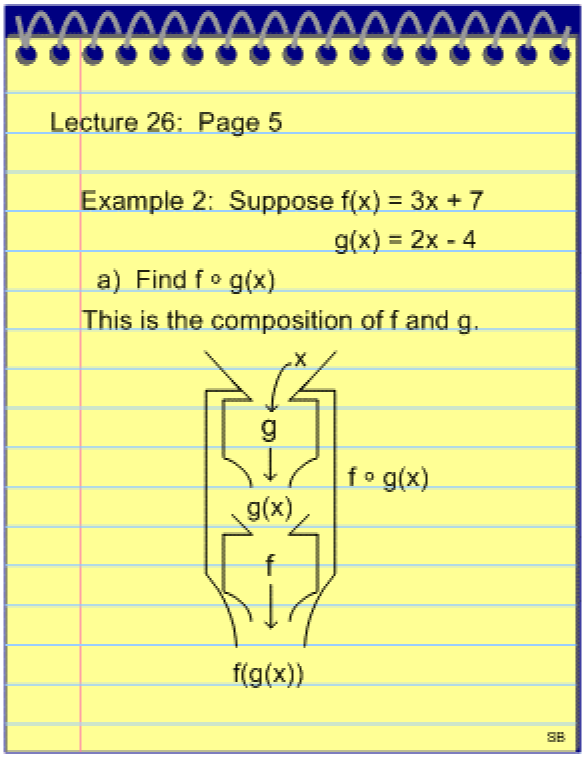
Lecture 26: Page 4

$f(g(x))$ (f of g of x)
or $(f \circ g)(x)$

This is a two-step machine:
g does its thing 1st then f does its thing.

Lecture 26 Notes, Continued

ALG2026-05



Lecture 26: Page 5

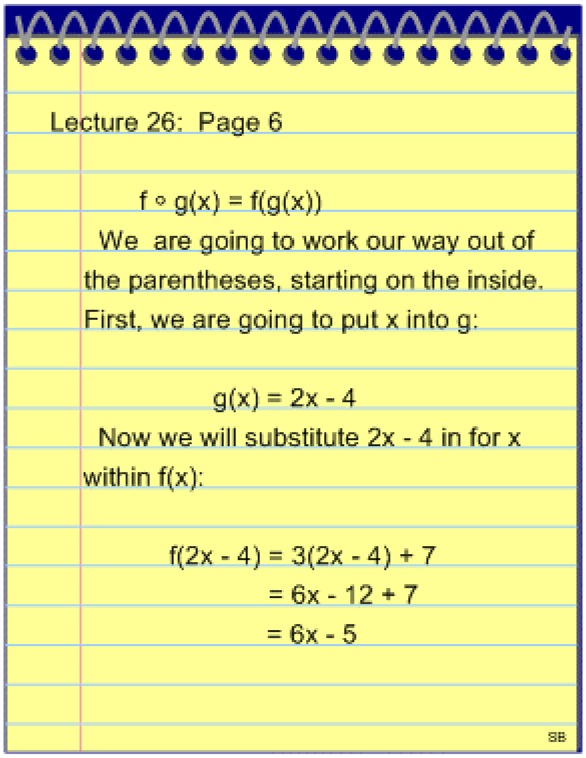
Example 2: Suppose $f(x) = 3x + 7$

$g(x) = 2x - 4$

a) Find $f \circ g(x)$

This is the composition of f and g.

ALG2026-06



Lecture 26: Page 6

$f \circ g(x) = f(g(x))$

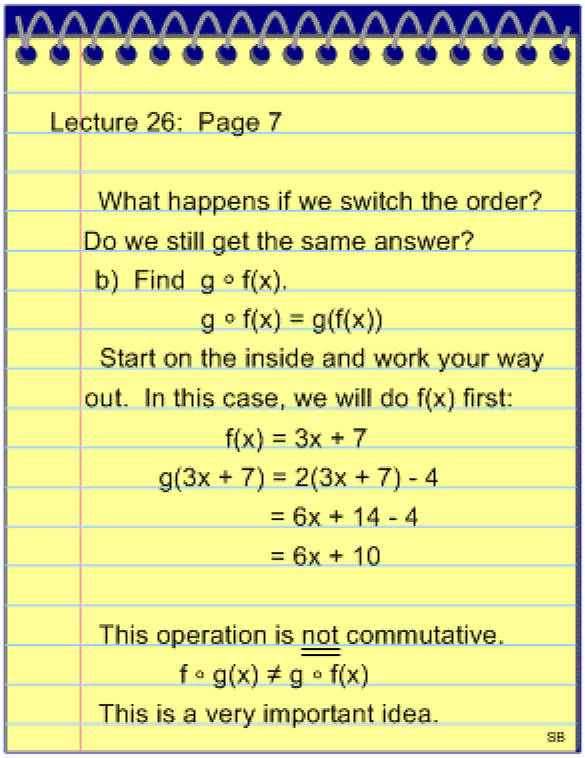
We are going to work our way out of the parentheses, starting on the inside. First, we are going to put x into g:

$g(x) = 2x - 4$

Now we will substitute 2x - 4 in for x within f(x):

$$\begin{aligned} f(2x - 4) &= 3(2x - 4) + 7 \\ &= 6x - 12 + 7 \\ &= 6x - 5 \end{aligned}$$

ALG2026-07



Lecture 26: Page 7

What happens if we switch the order? Do we still get the same answer?

b) Find $g \circ f(x)$.

$g \circ f(x) = g(f(x))$

Start on the inside and work your way out. In this case, we will do f(x) first:

$f(x) = 3x + 7$

$$\begin{aligned} g(3x + 7) &= 2(3x + 7) - 4 \\ &= 6x + 14 - 4 \\ &= 6x + 10 \end{aligned}$$

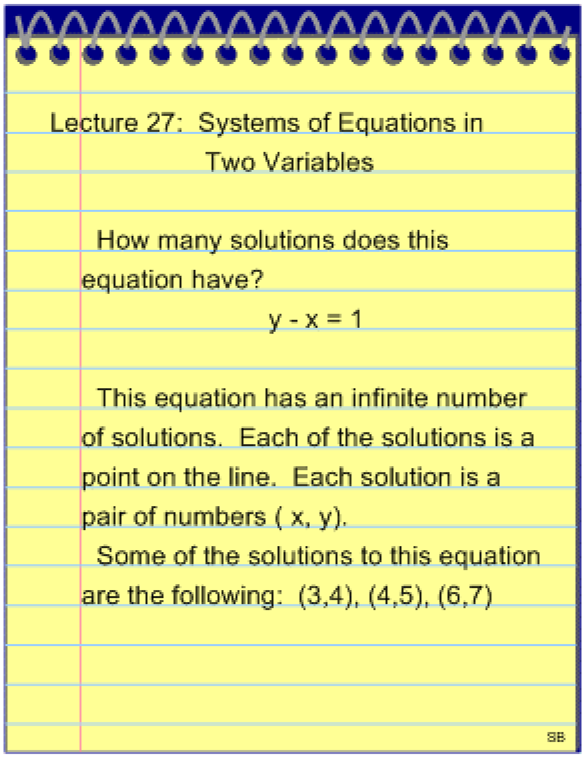
This operation is $\underline{\underline{\text{not}}}$ commutative.

$f \circ g(x) \neq g \circ f(x)$

This is a very important idea.

Lecture 27 Notes

ALG2027-01



Lecture 27: Systems of Equations in Two Variables

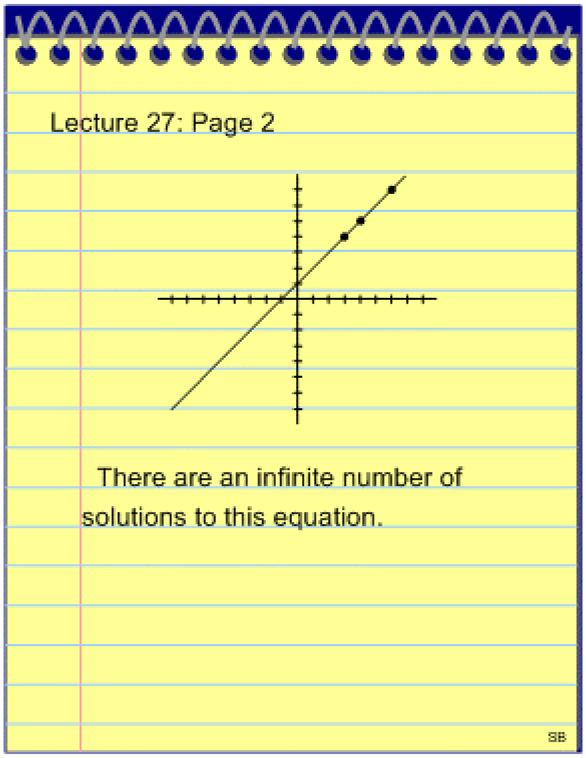
How many solutions does this equation have?

$y - x = 1$

This equation has an infinite number of solutions. Each of the solutions is a point on the line. Each solution is a pair of numbers (x, y).

Some of the solutions to this equation are the following: (3,4), (4,5), (6,7)

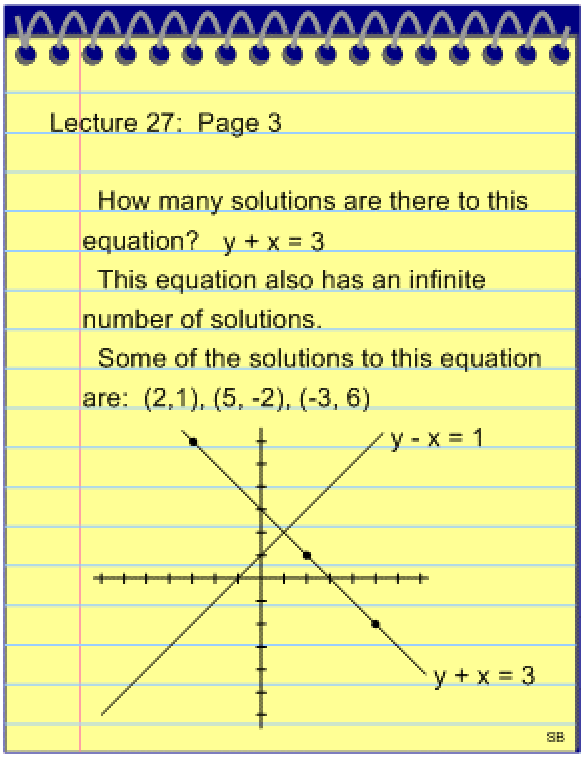
ALG2027-02



Lecture 27: Page 2

There are an infinite number of solutions to this equation.

ALG2027-03



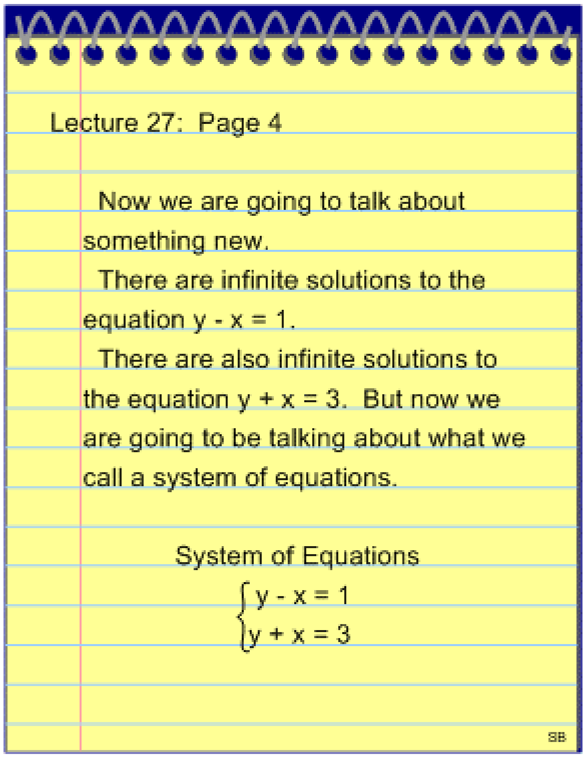
Lecture 27: Page 3

How many solutions are there to this equation? $y + x = 3$

This equation also has an infinite number of solutions.

Some of the solutions to this equation are: (2,1), (5, -2), (-3, 6)

ALG2027-04



Lecture 27: Page 4

Now we are going to talk about something new.

There are infinite solutions to the equation y - x = 1.

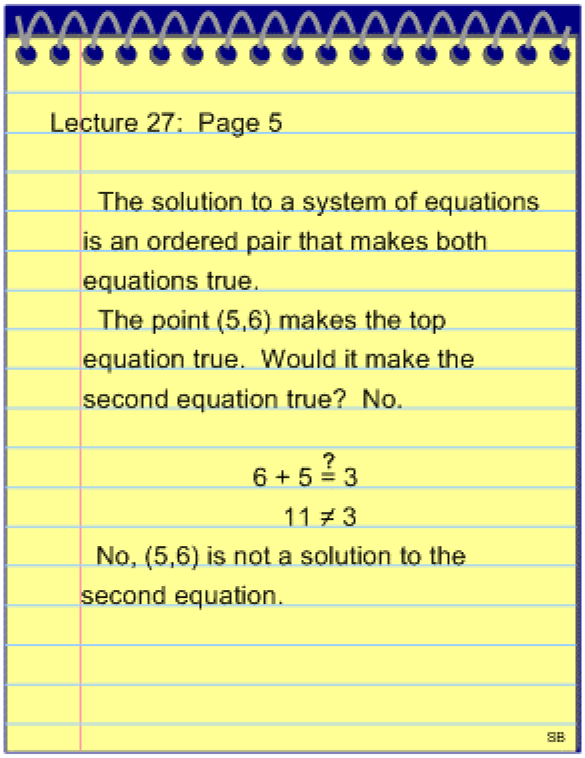
There are also infinite solutions to the equation y + x = 3. But now we are going to be talking about what we call a system of equations.

System of Equations

$$\begin{cases} y - x = 1 \\ y + x = 3 \end{cases}$$

Lecture 27 Notes, Continued

ALG2027-05



Lecture 27: Page 5

The solution to a system of equations is an ordered pair that makes both equations true.

The point (5,6) makes the top equation true. Would it make the second equation true? No.

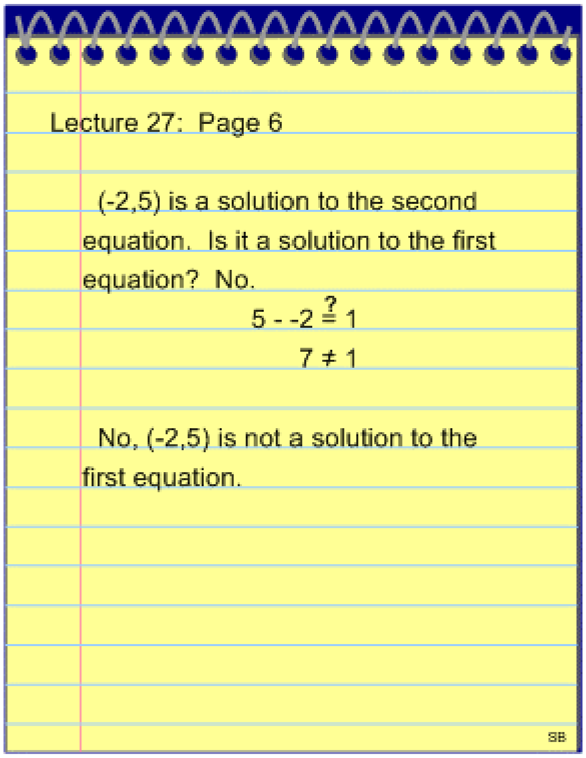
$$6 + 5 \overset{?}{=} 3$$

$$11 \neq 3$$

No, (5,6) is not a solution to the second equation.

SB

ALG2027-06



Lecture 27: Page 6

(-2,5) is a solution to the second equation. Is it a solution to the first equation? No.

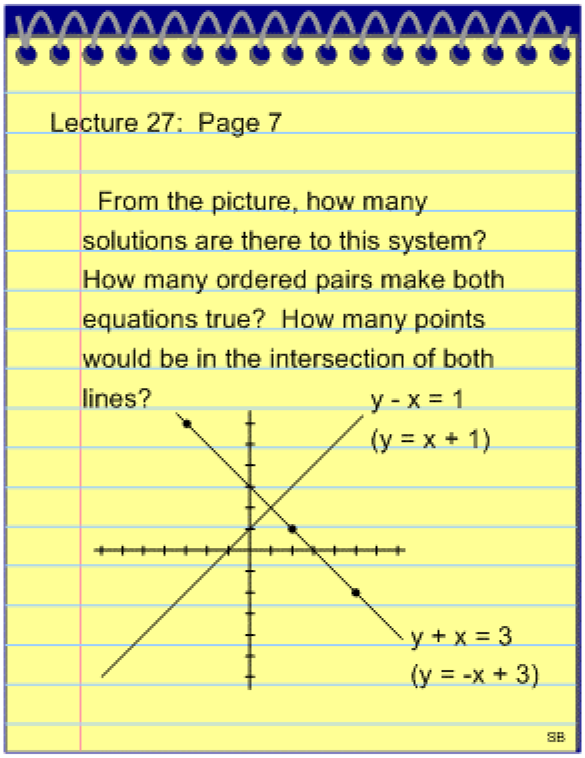
$$5 - -2 \overset{?}{=} 1$$

$$7 \neq 1$$

No, (-2,5) is not a solution to the first equation.

SB

ALG2027-07



Lecture 27: Page 7

From the picture, how many solutions are there to this system? How many ordered pairs make both equations true? How many points would be in the intersection of both lines?

y - x = 1

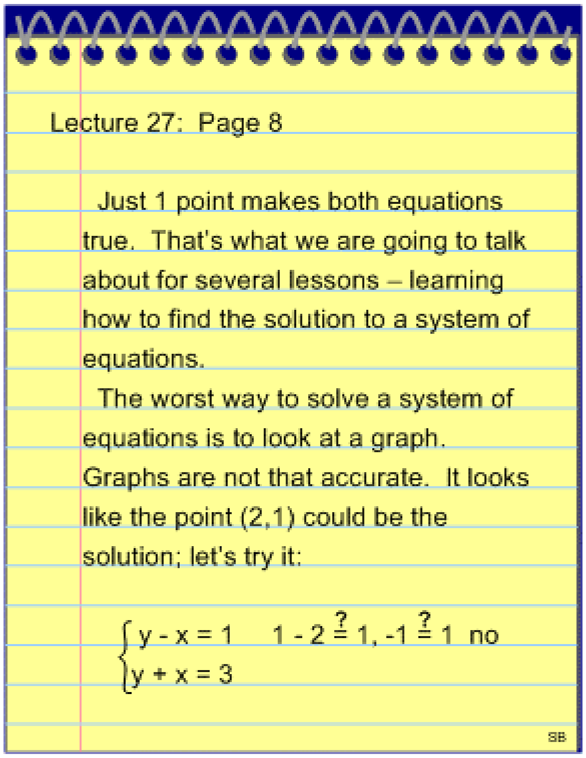
(y = x + 1)

y + x = 3

(y = -x + 3)

SB

ALG2027-08



Lecture 27: Page 8

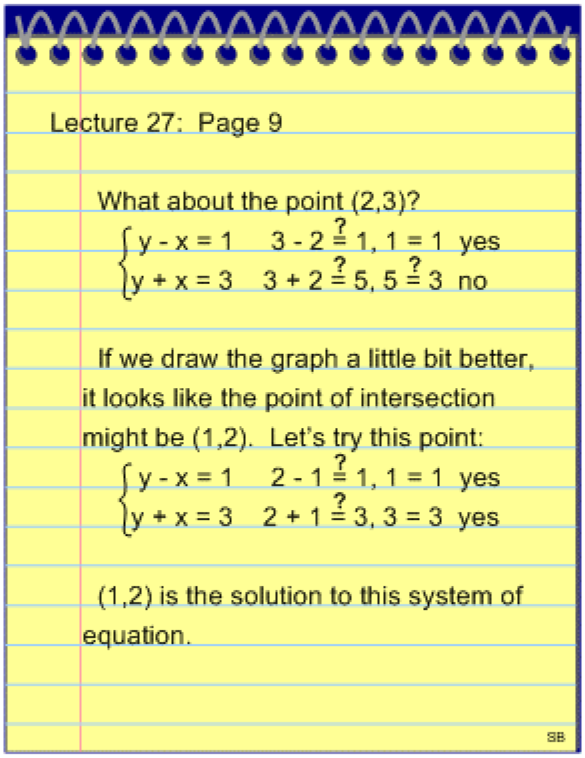
Just 1 point makes both equations true. That's what we are going to talk about for several lessons – learning how to find the solution to a system of equations.

The worst way to solve a system of equations is to look at a graph. Graphs are not that accurate. It looks like the point (2,1) could be the solution; let's try it:

$$\begin{cases} y - x = 1 \\ y + x = 3 \end{cases} \quad 1 - 2 \overset{?}{=} 1, -1 \overset{?}{=} 1 \text{ no}$$

SB

ALG2027-09



Lecture 27: Page 9

What about the point (2,3)?

$$\begin{cases} y - x = 1 & 3 - 2 \overset{?}{=} 1,\ 1 = 1 \text{ yes} \\ y + x = 3 & 3 + 2 \overset{?}{=} 5,\ 5 \overset{?}{=} 3 \text{ no} \end{cases}$$

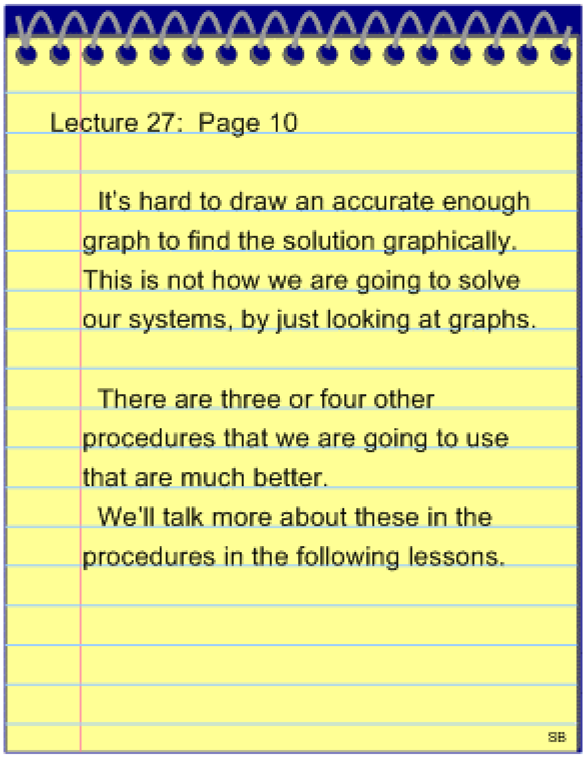
If we draw the graph a little bit better, it looks like the point of intersection might be (1,2). Let's try this point:

$$\begin{cases} y - x = 1 & 2 - 1 \overset{?}{=} 1,\ 1 = 1 \text{ yes} \\ y + x = 3 & 2 + 1 \overset{?}{=} 3,\ 3 = 3 \text{ yes} \end{cases}$$

(1,2) is the solution to this system of equation.

SB

ALG2027-10



Lecture 27: Page 10

It's hard to draw an accurate enough graph to find the solution graphically. This is not how we are going to solve our systems, by just looking at graphs.

There are three or four other procedures that we are going to use that are much better.

We'll talk more about these in the procedures in the following lessons.

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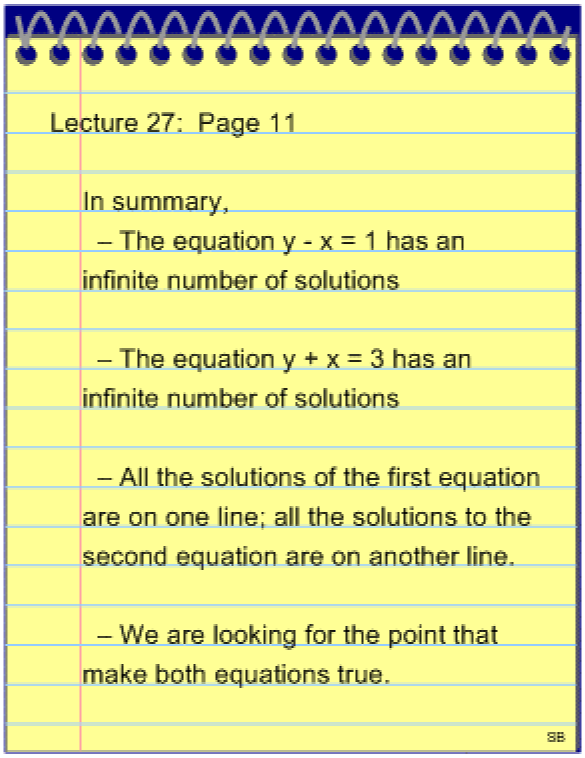


Lecture 27: Page 11

In summary,

– The equation y - x = 1 has an infinite number of solutions

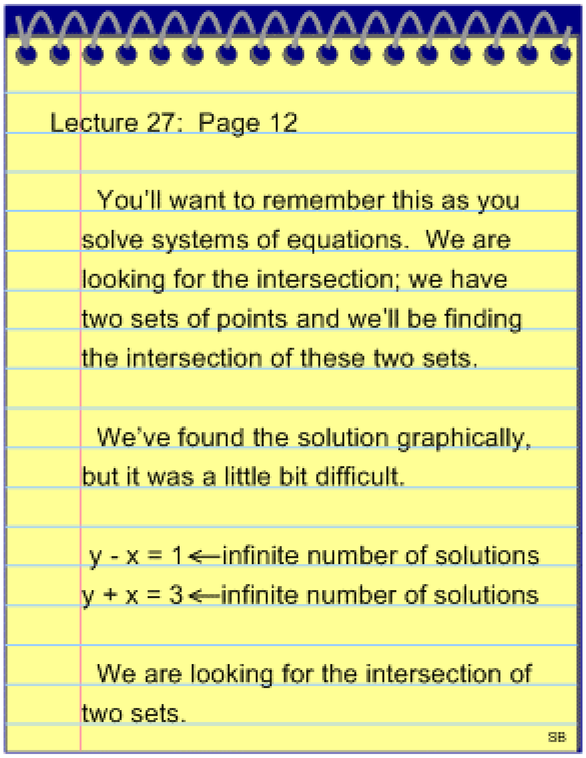
– The equation y + x = 3 has an infinite number of solutions

– All the solutions of the first equation are on one line; all the solutions to the second equation are on another line.

– We are looking for the point that make both equations true.

SB

ALG2027-12



Lecture 27: Page 12

You'll want to remember this as you solve systems of equations. We are looking for the intersection; we have two sets of points and we'll be finding the intersection of these two sets.

We've found the solution graphically, but it was a little bit difficult.

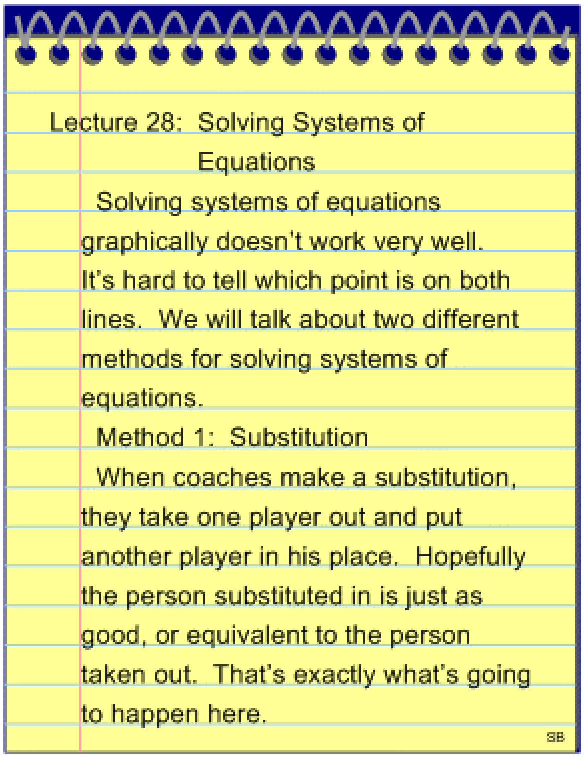
y - x = 1 ⟵ infinite number of solutions
y + x = 3 ⟵ infinite number of solutions

We are looking for the intersection of two sets.

SB

Lecture 28 Notes

ALG2028-01



Lecture 28: Solving Systems of Equations

Solving systems of equations graphically doesn't work very well. It's hard to tell which point is on both lines. We will talk about two different methods for solving systems of equations.

Method 1: Substitution

When coaches make a substitution, they take one player out and put another player in his place. Hopefully the person substituted in is just as good, or equivalent to the person taken out. That's exactly what's going to happen here.

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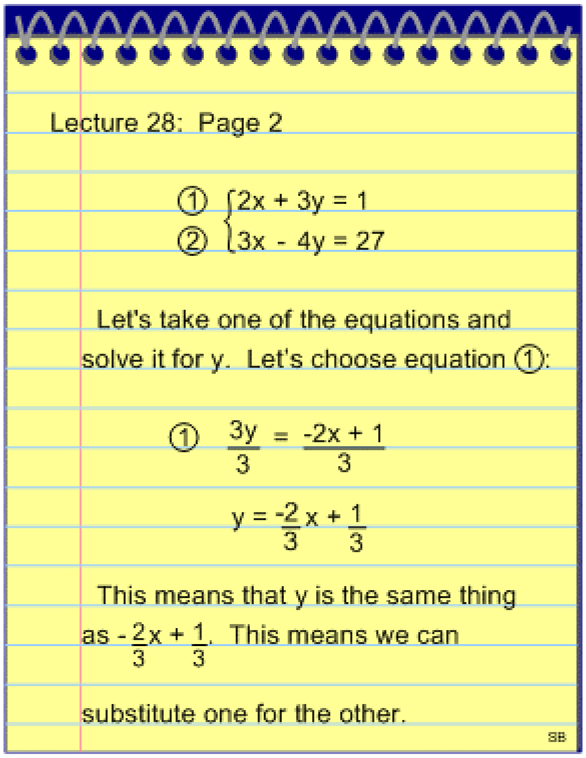


Lecture 28: Page 2

$$\begin{array}{l} \textcircled{1} \\ \textcircled{2} \end{array} \left\{ \begin{array}{l} 2x + 3y = 1 \\ 3x - 4y = 27 \end{array} \right.$$

Let's take one of the equations and solve it for y. Let's choose equation ①:

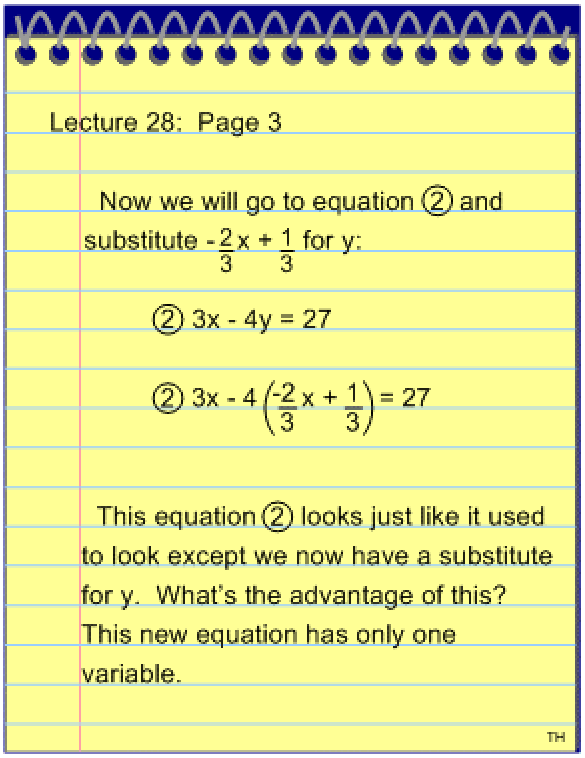
$$\textcircled{1} \quad \frac{3y}{3} = \frac{-2x + 1}{3}$$

$$y = \frac{-2}{3}x + \frac{1}{3}$$

This means that y is the same thing as $-\frac{2}{3}x + \frac{1}{3}$. This means we can substitute one for the other.

SB

ALG2028-03



Lecture 28: Page 3

Now we will go to equation ② and substitute $-\frac{2}{3}x + \frac{1}{3}$ for y:

$$\textcircled{2} \quad 3x - 4y = 27$$

$$\textcircled{2} \quad 3x - 4\left(\frac{-2}{3}x + \frac{1}{3}\right) = 27$$

This equation ② looks just like it used to look except we now have a substitute for y. What's the advantage of this? This new equation has only one variable.

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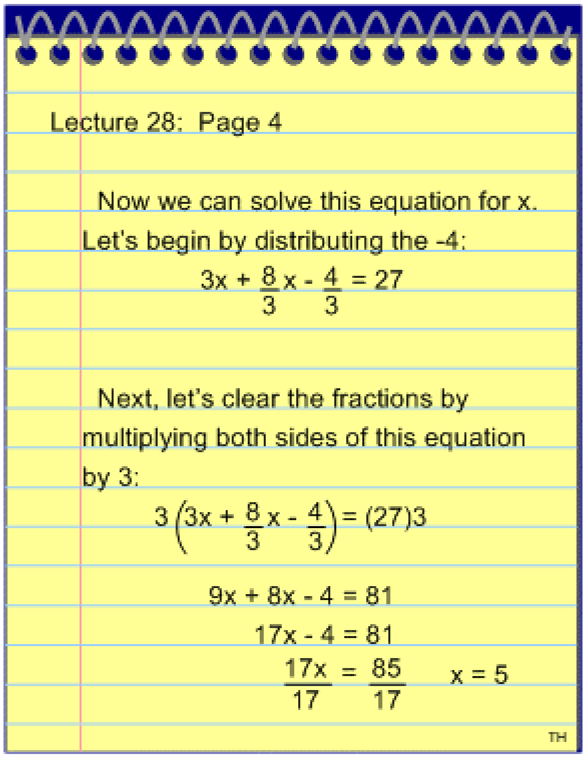


Lecture 28: Page 4

Now we can solve this equation for x. Let's begin by distributing the -4:

$$3x + \frac{8}{3}x - \frac{4}{3} = 27$$

Next, let's clear the fractions by multiplying both sides of this equation by 3:

$$3\left(3x + \frac{8}{3}x - \frac{4}{3}\right) = (27)3$$

$$9x + 8x - 4 = 81$$

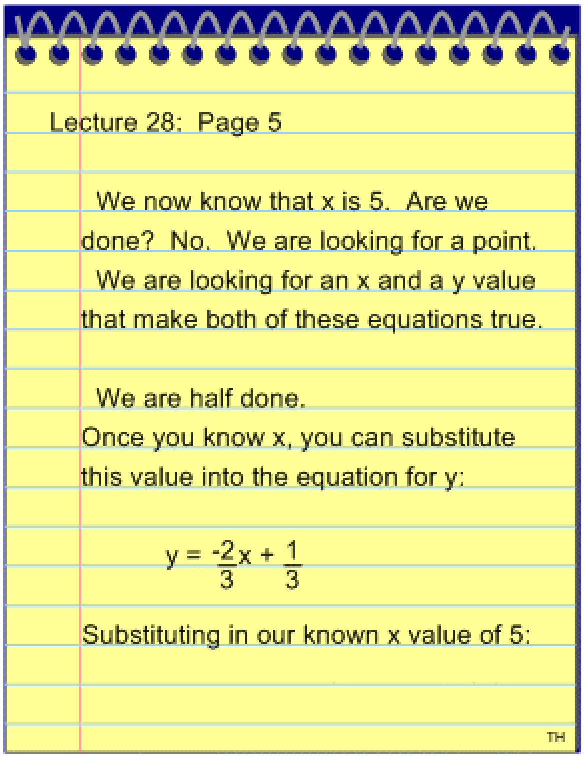
$$17x - 4 = 81$$

$$\frac{17x}{17} = \frac{85}{17} \qquad x = 5$$

TH

Lecture 28 Notes, Continued

ALG2028-05



Lecture 28: Page 5

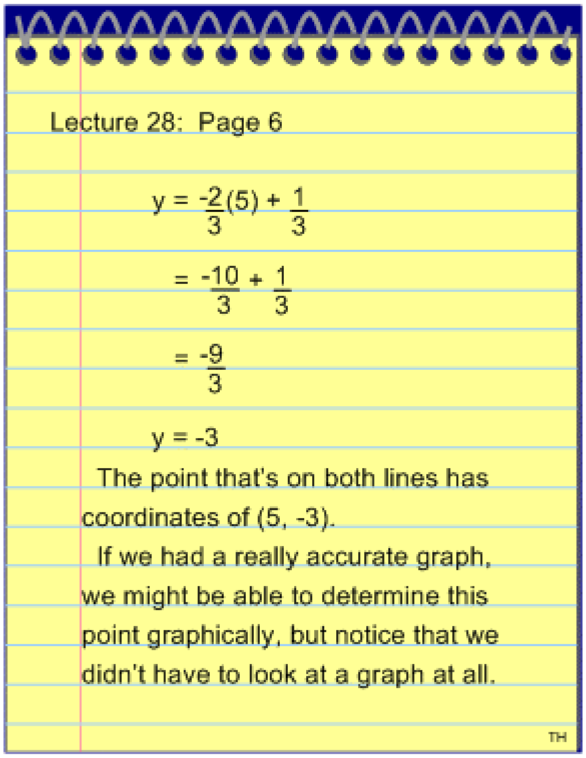
We now know that x is 5. Are we done? No. We are looking for a point. We are looking for an x and a y value that make both of these equations true.

We are half done.
Once you know x, you can substitute this value into the equation for y:

$$y = \frac{-2}{3}x + \frac{1}{3}$$

Substituting in our known x value of 5:

ALG2028-06



Lecture 28: Page 6

$$y = \frac{-2}{3}(5) + \frac{1}{3}$$

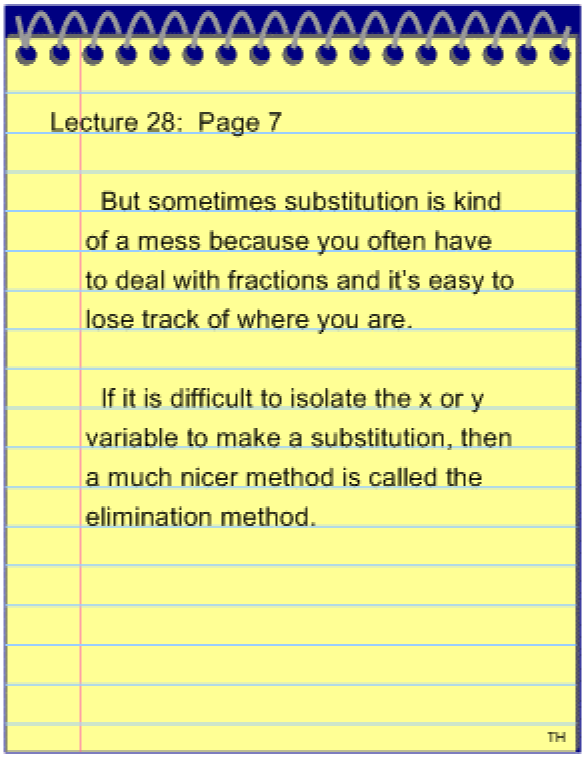
$$= \frac{-10}{3} + \frac{1}{3}$$

$$= \frac{-9}{3}$$

$$y = -3$$

The point that's on both lines has coordinates of (5, -3).
If we had a really accurate graph, we might be able to determine this point graphically, but notice that we didn't have to look at a graph at all.

ALG2028-07



Lecture 28: Page 7

But sometimes substitution is kind of a mess because you often have to deal with fractions and it's easy to lose track of where you are.

If it is difficult to isolate the x or y variable to make a substitution, then a much nicer method is called the elimination method.

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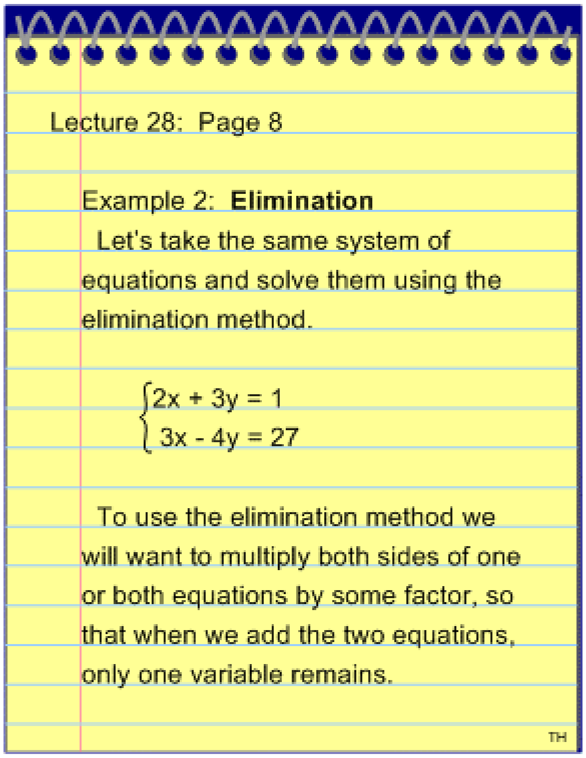


Lecture 28: Page 8

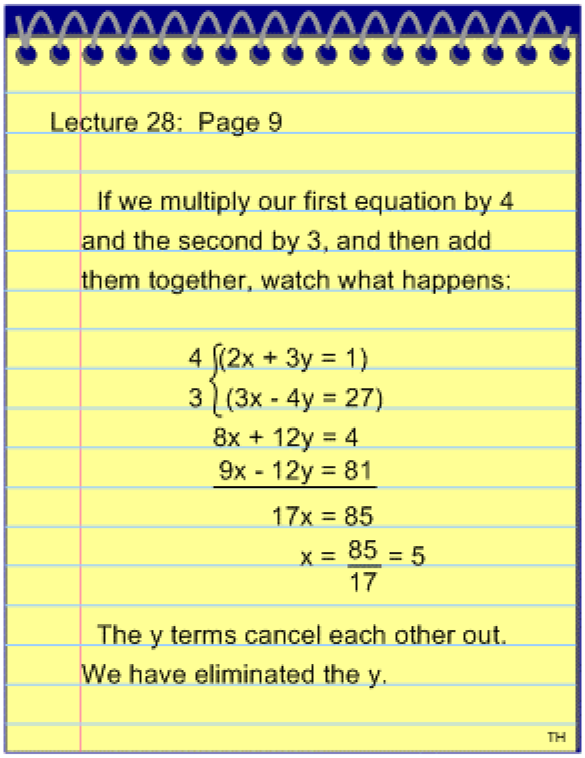
Example 2: **Elimination**
Let's take the same system of equations and solve them using the elimination method.

$$\begin{cases} 2x + 3y = 1 \\ 3x - 4y = 27 \end{cases}$$

To use the elimination method we will want to multiply both sides of one or both equations by some factor, so that when we add the two equations, only one variable remains.

Lecture 28 Notes, Continued

ALG2028-09



Lecture 28: Page 9

If we multiply our first equation by 4 and the second by 3, and then add them together, watch what happens:

$$\begin{array}{r}4\\3\end{array}\left\{\begin{array}{l}(2x + 3y = 1)\\(3x - 4y = 27)\end{array}\right.$$

$$\begin{array}{r}8x + 12y = 4\\ \underline{9x - 12y = 81}\\ 17x = 85\end{array}$$

$$x = \frac{85}{17} = 5$$

The y terms cancel each other out. We have eliminated the y.

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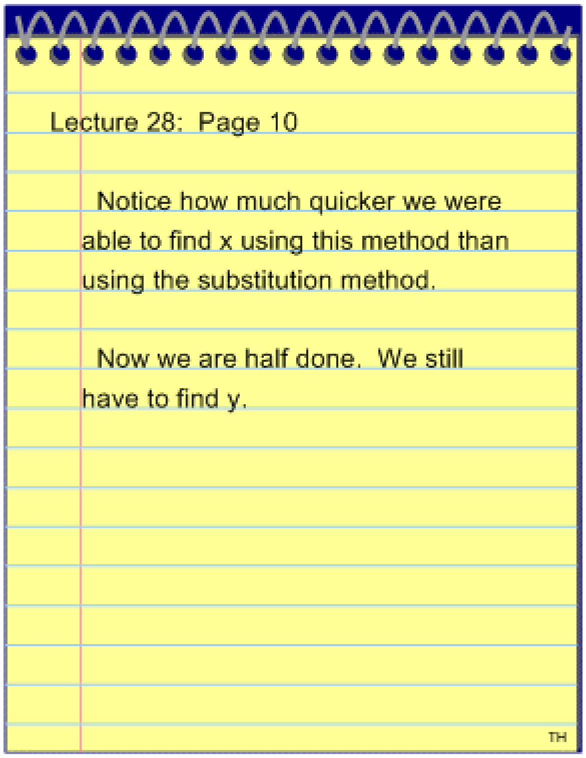
ALG2028-10



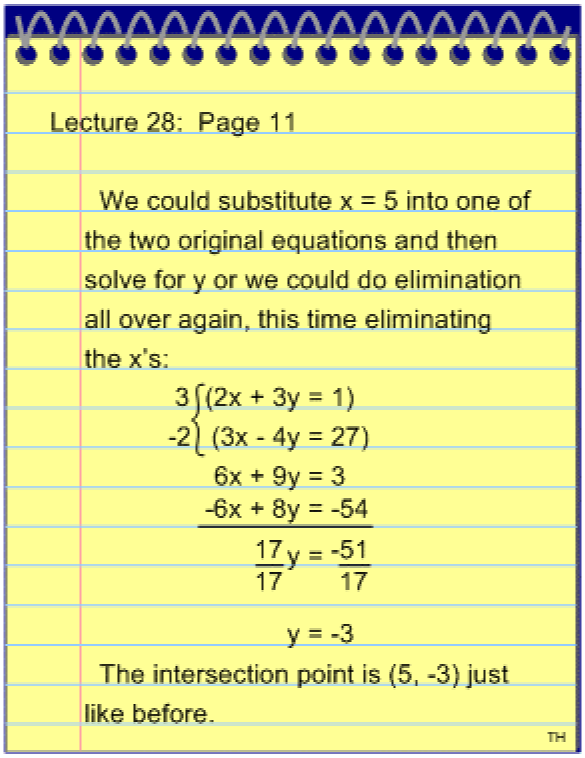
Lecture 28: Page 10

Notice how much quicker we were able to find x using this method than using the substitution method.

Now we are half done. We still have to find y.

TH

ALG2028-11



Lecture 28: Page 11

We could substitute x = 5 into one of the two original equations and then solve for y or we could do elimination all over again, this time eliminating the x's:

$$\begin{array}{r}3\\-2\end{array}\left\{\begin{array}{l}(2x + 3y = 1)\\(3x - 4y = 27)\end{array}\right.$$

$$\begin{array}{r}6x + 9y = 3\\ \underline{-6x + 8y = -54}\end{array}$$

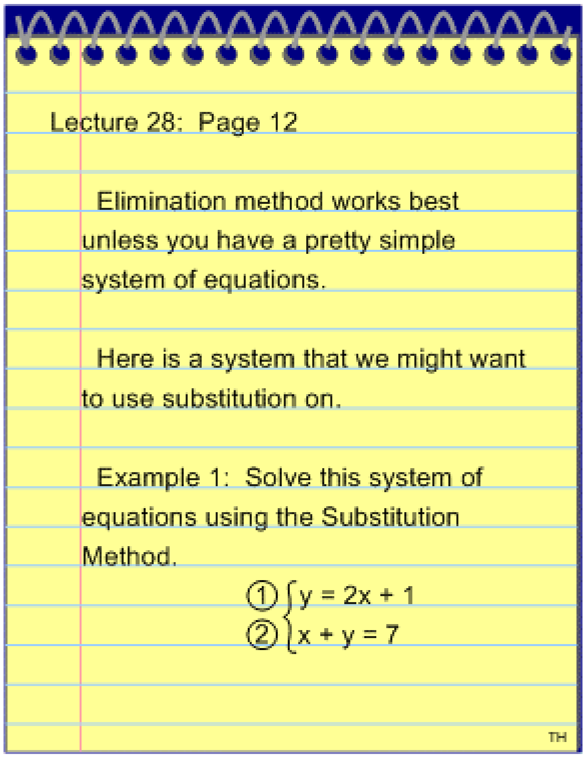
$$\frac{17}{17}y = \frac{-51}{17}$$

$$y = -3$$

The intersection point is (5, -3) just like before.

TH

ALG2028-12



Lecture 28: Page 12

Elimination method works best unless you have a pretty simple system of equations.

Here is a system that we might want to use substitution on.

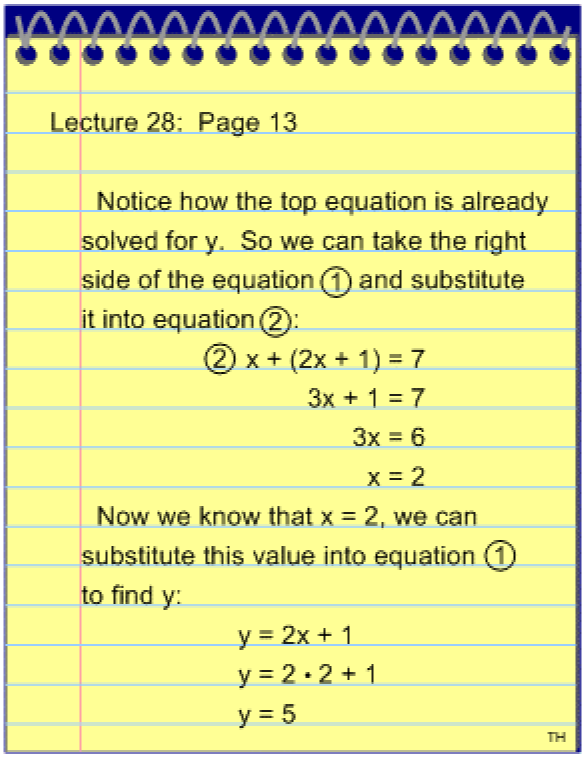
Example 1: Solve this system of equations using the Substitution Method.

$$\begin{array}{r}①\\②\end{array}\left\{\begin{array}{l}y = 2x + 1\\x + y = 7\end{array}\right.$$

TH

Lecture 28 Notes, Continued

ALG2028-13



Lecture 28: Page 13

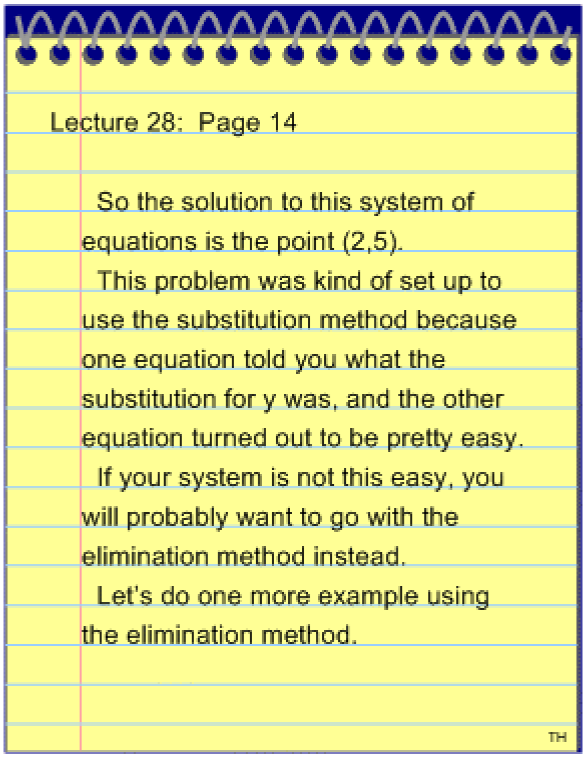
Notice how the top equation is already solved for y. So we can take the right side of the equation ① and substitute it into equation ②:

$$② \quad x + (2x + 1) = 7$$
$$3x + 1 = 7$$
$$3x = 6$$
$$x = 2$$

Now we know that x = 2, we can substitute this value into equation ① to find y:

$$y = 2x + 1$$
$$y = 2 \cdot 2 + 1$$
$$y = 5$$

ALG2028-14



Lecture 28: Page 14

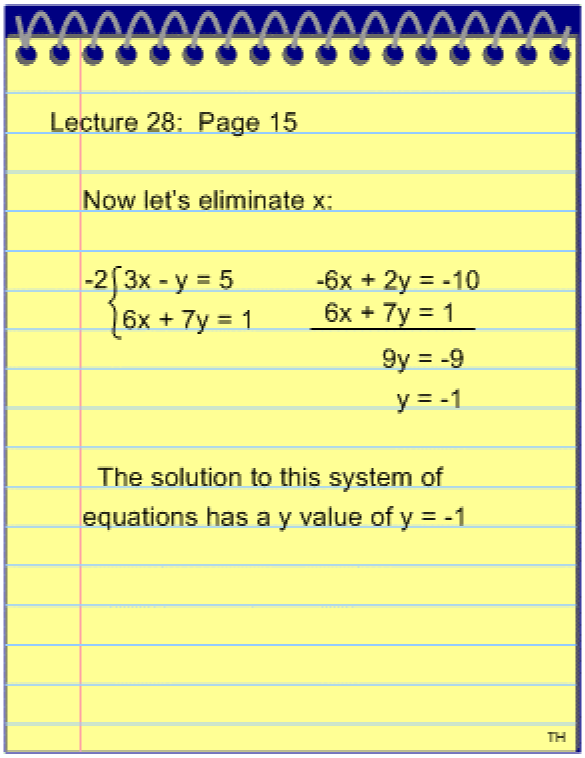
So the solution to this system of equations is the point (2,5).

This problem was kind of set up to use the substitution method because one equation told you what the substitution for y was, and the other equation turned out to be pretty easy.

If your system is not this easy, you will probably want to go with the elimination method instead.

Let's do one more example using the elimination method.

ALG2028-015



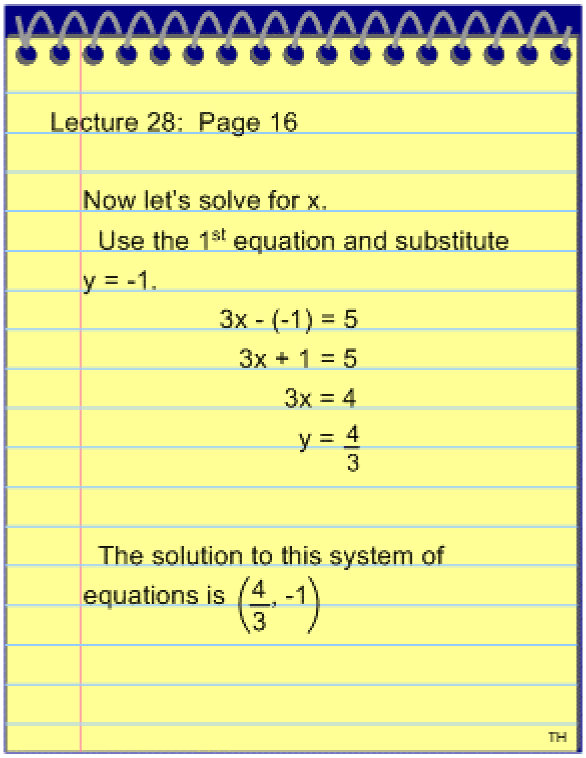
Lecture 28: Page 15

Now let's eliminate x:

$$-2\begin{cases}3x - y = 5\\ 6x + 7y = 1\end{cases} \qquad \begin{array}{r}-6x + 2y = -10\\ \underline{6x + 7y = 1}\\ 9y = -9\\ y = -1\end{array}$$

The solution to this system of equations has a y value of y = -1

ALG2028-16



Lecture 28: Page 16

Now let's solve for x.

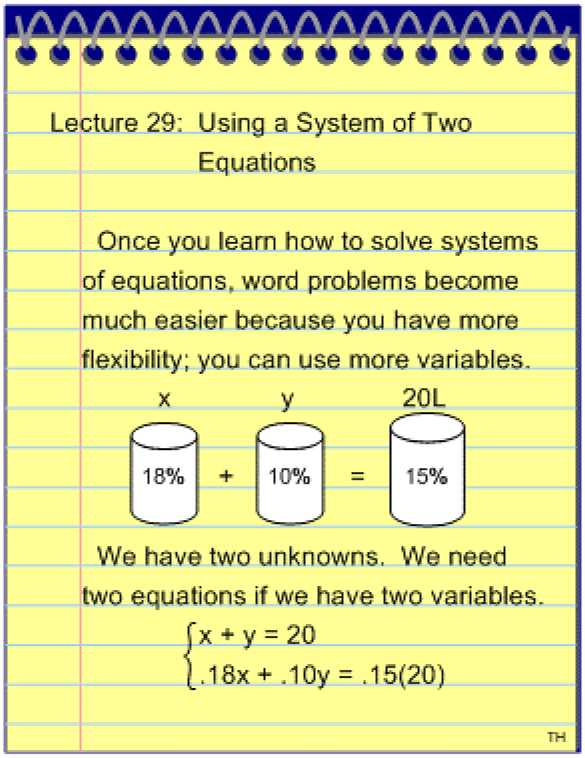
Use the 1^{st} equation and substitute y = -1.

$$3x - (-1) = 5$$
$$3x + 1 = 5$$
$$3x = 4$$
$$y = \frac{4}{3}$$

The solution to this system of equations is $\left(\frac{4}{3}, -1\right)$

Lecture 29 Notes

ALG2029-01



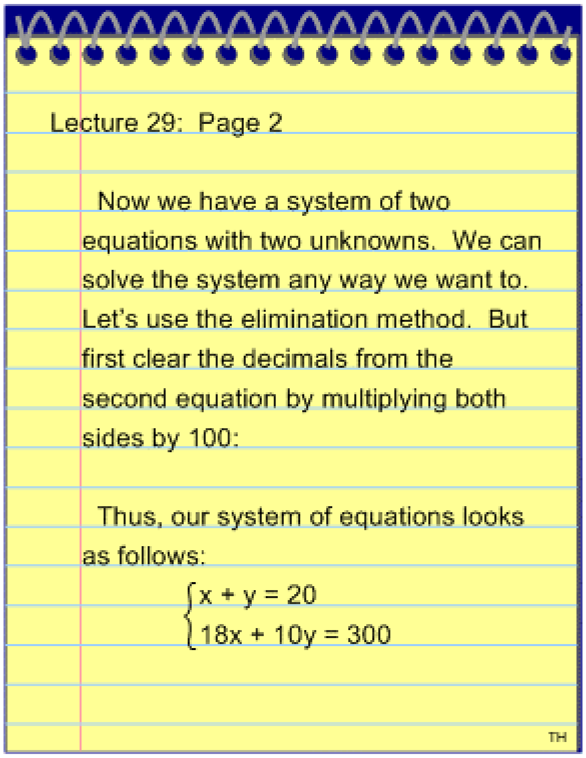
Lecture 29: Using a System of Two Equations

Once you learn how to solve systems of equations, word problems become much easier because you have more flexibility; you can use more variables.

We have two unknowns. We need two equations if we have two variables.

$$\begin{cases} x + y = 20 \\ .18x + .10y = .15(20) \end{cases}$$

ALG2029-02



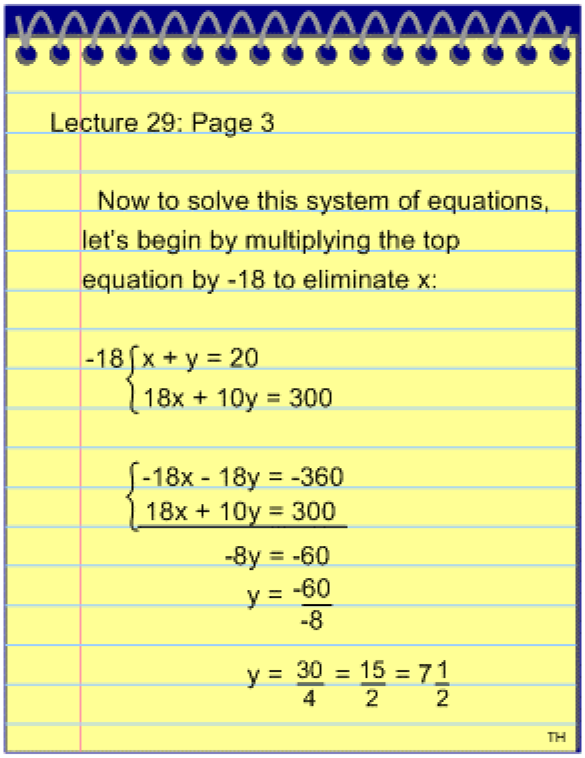
Lecture 29: Page 2

Now we have a system of two equations with two unknowns. We can solve the system any way we want to. Let's use the elimination method. But first clear the decimals from the second equation by multiplying both sides by 100:

Thus, our system of equations looks as follows:

$$\begin{cases} x + y = 20 \\ 18x + 10y = 300 \end{cases}$$

ALG2029-03



Lecture 29: Page 3

Now to solve this system of equations, let's begin by multiplying the top equation by -18 to eliminate x:

$$-18\begin{cases} x + y = 20 \\ 18x + 10y = 300 \end{cases}$$

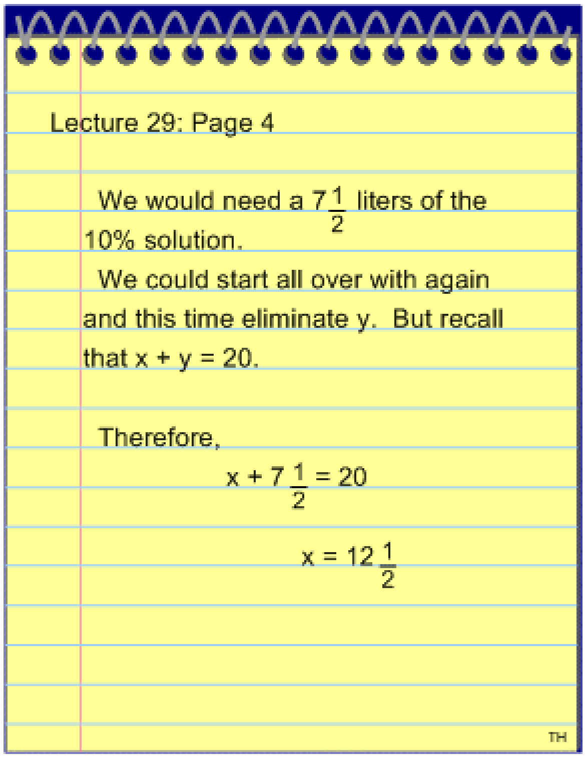
$$\begin{cases} -18x - 18y = -360 \\ \underline{18x + 10y = 300} \end{cases}$$

$$-8y = -60$$

$$y = \frac{-60}{-8}$$

$$y = \frac{30}{4} = \frac{15}{2} = 7\frac{1}{2}$$

ALG2029-04



Lecture 29: Page 4

We would need a $7\frac{1}{2}$ liters of the 10% solution.

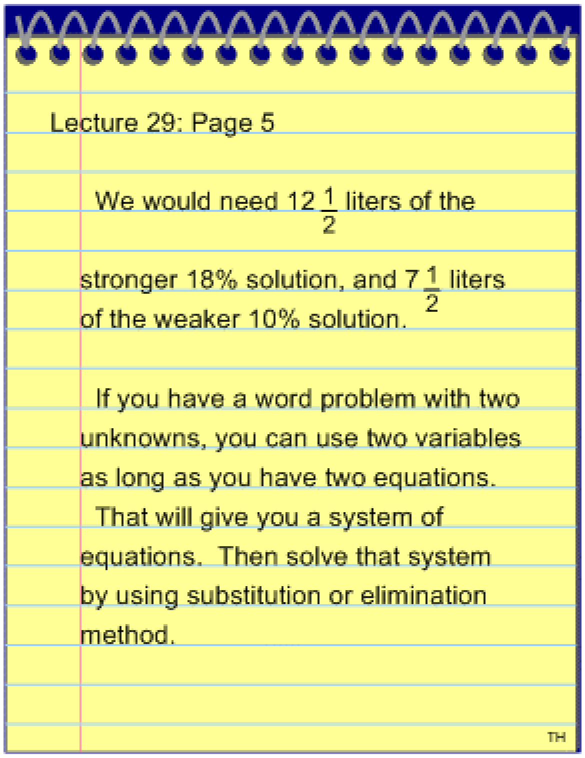
We could start all over with again and this time eliminate y. But recall that x + y = 20.

Therefore,

$$x + 7\frac{1}{2} = 20$$

$$x = 12\frac{1}{2}$$

ALG2029-05



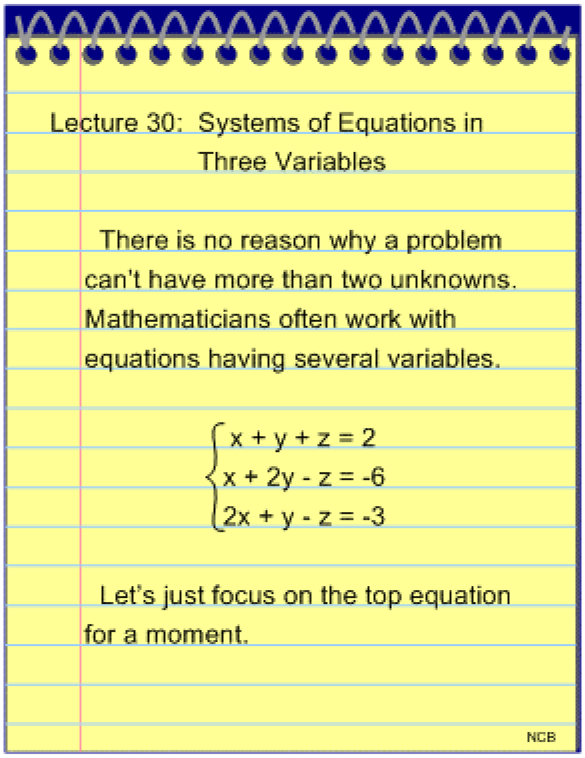
Lecture 29: Page 5

We would need 12 $\frac{1}{2}$ liters of the stronger 18% solution, and 7 $\frac{1}{2}$ liters of the weaker 10% solution.

If you have a word problem with two unknowns, you can use two variables as long as you have two equations. That will give you a system of equations. Then solve that system by using substitution or elimination method.

Lecture 30 Notes

ALG2030-01



Lecture 30: Systems of Equations in Three Variables

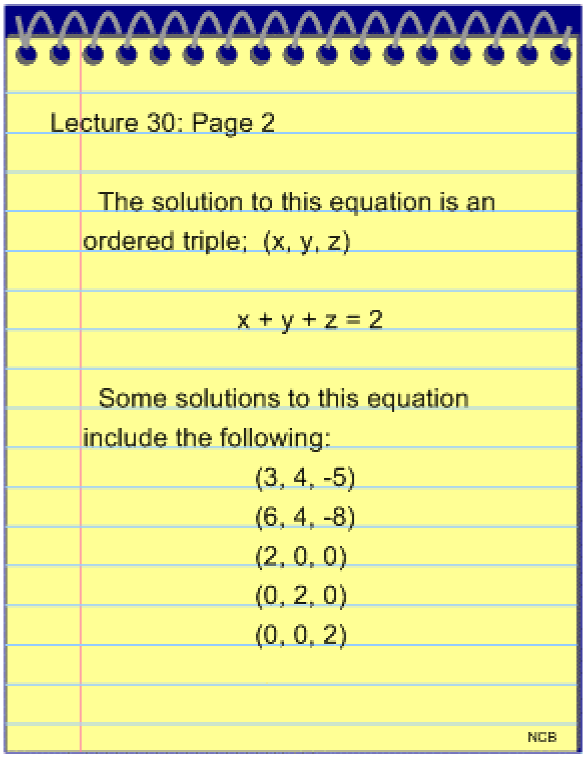
There is no reason why a problem can't have more than two unknowns. Mathematicians often work with equations having several variables.

$$\begin{cases} x + y + z = 2 \\ x + 2y - z = -6 \\ 2x + y - z = -3 \end{cases}$$

Let's just focus on the top equation for a moment.

NCB

ALG2030-02



Lecture 30: Page 2

The solution to this equation is an ordered triple; (x, y, z)

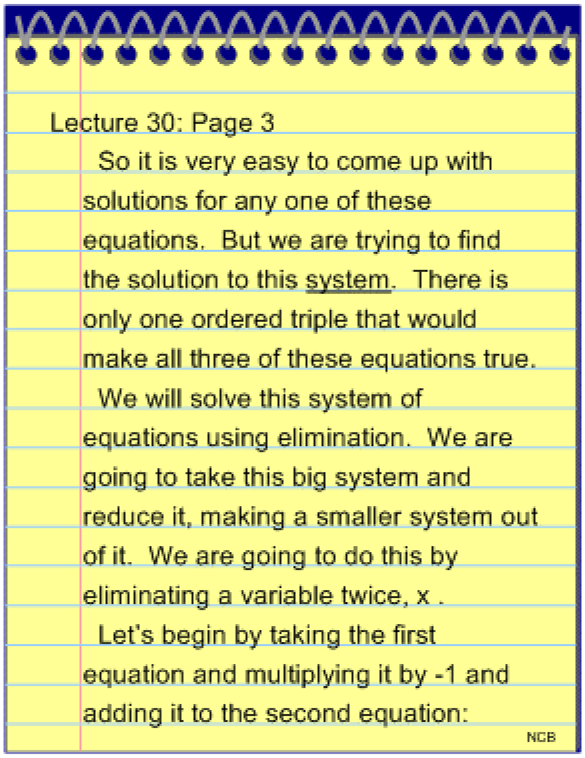
$$x + y + z = 2$$

Some solutions to this equation include the following:

(3, 4, -5)
(6, 4, -8)
(2, 0, 0)
(0, 2, 0)
(0, 0, 2)

NCB

ALG2030-03



Lecture 30: Page 3

So it is very easy to come up with solutions for any one of these equations. But we are trying to find the solution to this system. There is only one ordered triple that would make all three of these equations true.

We will solve this system of equations using elimination. We are going to take this big system and reduce it, making a smaller system out of it. We are going to do this by eliminating a variable twice, x .

Let's begin by taking the first equation and multiplying it by -1 and adding it to the second equation:

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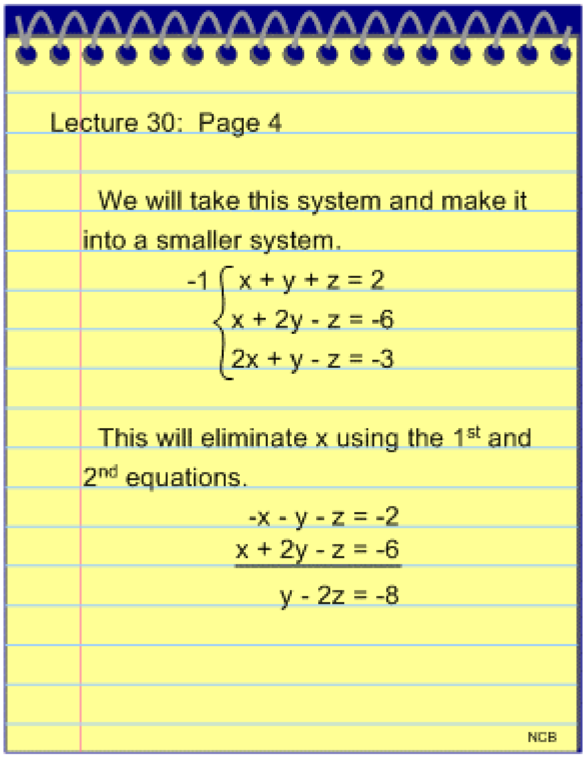


Lecture 30: Page 4

We will take this system and make it into a smaller system.

$$-1\begin{cases} x + y + z = 2 \\ x + 2y - z = -6 \\ 2x + y - z = -3 \end{cases}$$

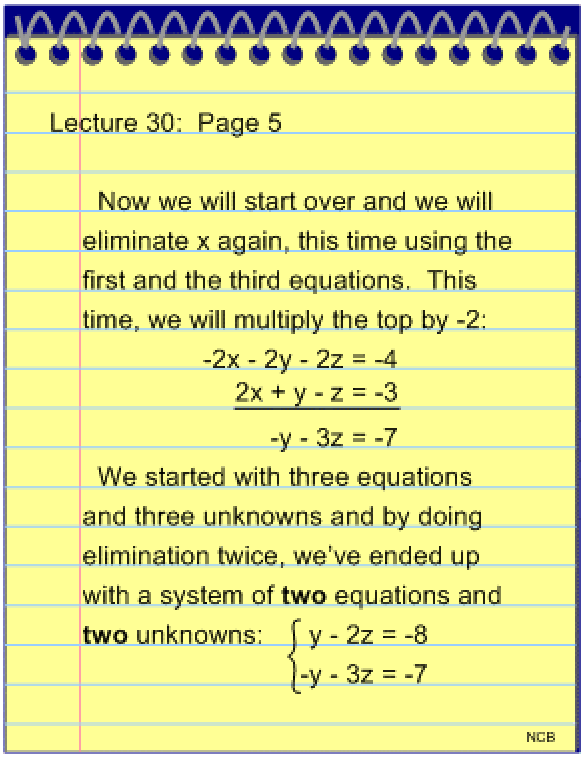
This will eliminate x using the 1st and 2nd equations.

$$\begin{array}{r} -x - y - z = -2 \\ \underline{x + 2y - z = -6} \\ y - 2z = -8 \end{array}$$

NCB

Lecture 30 Notes

ALG2030-05



Lecture 30: Page 5

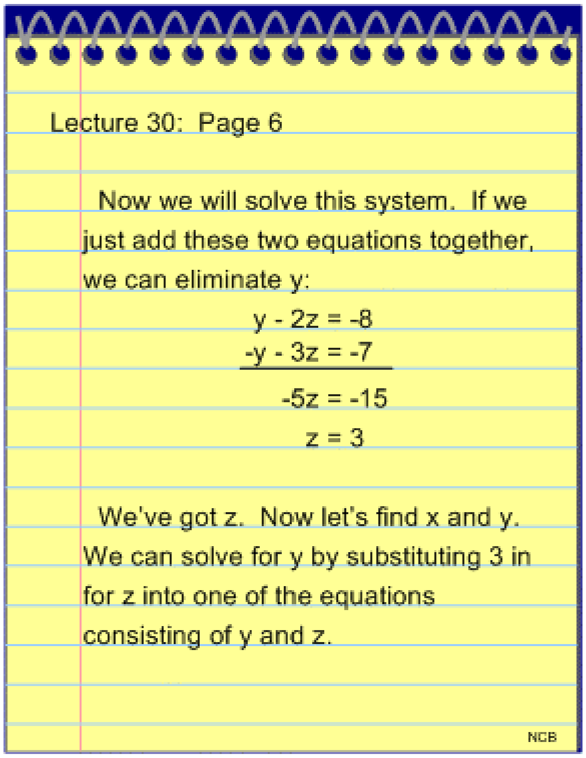
Now we will start over and we will eliminate x again, this time using the first and the third equations. This time, we will multiply the top by -2:

$$\begin{array}{r} -2x - 2y - 2z = -4 \\ \underline{2x + y - z = -3} \\ -y - 3z = -7 \end{array}$$

We started with three equations and three unknowns and by doing elimination twice, we've ended up with a system of **two** equations and **two** unknowns: $\begin{cases} y - 2z = -8 \\ -y - 3z = -7 \end{cases}$

NCB

ALG2030-06



Lecture 30: Page 6

Now we will solve this system. If we just add these two equations together, we can eliminate y:

$$\begin{array}{r} y - 2z = -8 \\ \underline{-y - 3z = -7} \\ -5z = -15 \\ z = 3 \end{array}$$

We've got z. Now let's find x and y. We can solve for y by substituting 3 in for z into one of the equations consisting of y and z.

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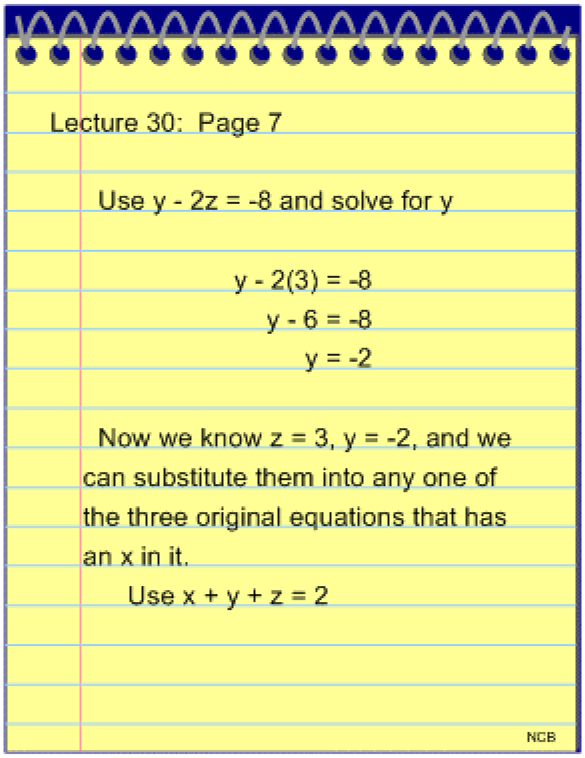

Lecture 30: Page 7

Use $y - 2z = -8$ and solve for y

$$\begin{array}{r} y - 2(3) = -8 \\ y - 6 = -8 \\ y = -2 \end{array}$$

Now we know $z = 3$, $y = -2$, and we can substitute them into any one of the three original equations that has an x in it.

Use $x + y + z = 2$

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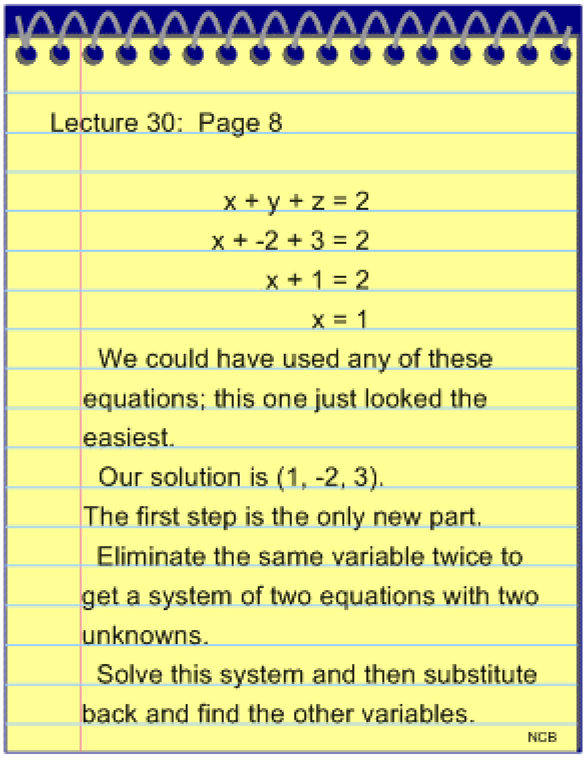

Lecture 30: Page 8

$$\begin{array}{r} x + y + z = 2 \\ x + -2 + 3 = 2 \\ x + 1 = 2 \\ x = 1 \end{array}$$

We could have used any of these equations; this one just looked the easiest.

Our solution is (1, -2, 3).

The first step is the only new part.

Eliminate the same variable twice to get a system of two equations with two unknowns.

Solve this system and then substitute back and find the other variables.

NCB

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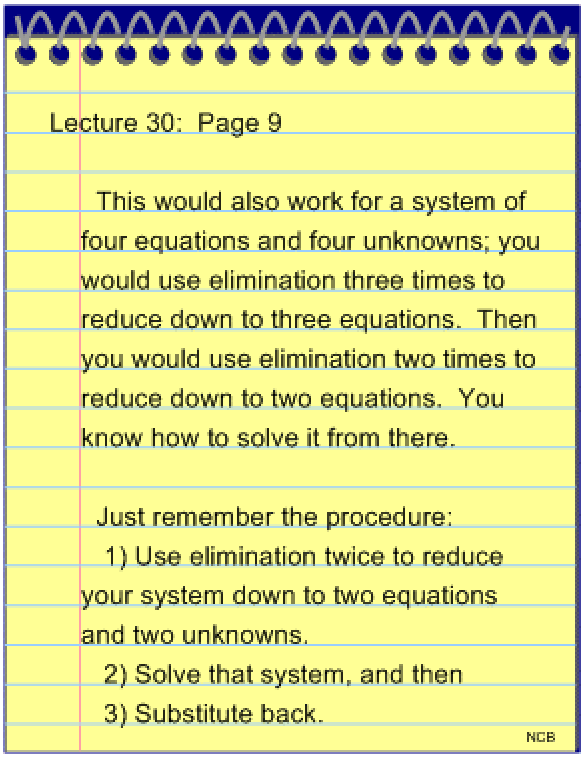


Lecture 30: Page 9

This would also work for a system of four equations and four unknowns; you would use elimination three times to reduce down to three equations. Then you would use elimination two times to reduce down to two equations. You know how to solve it from there.

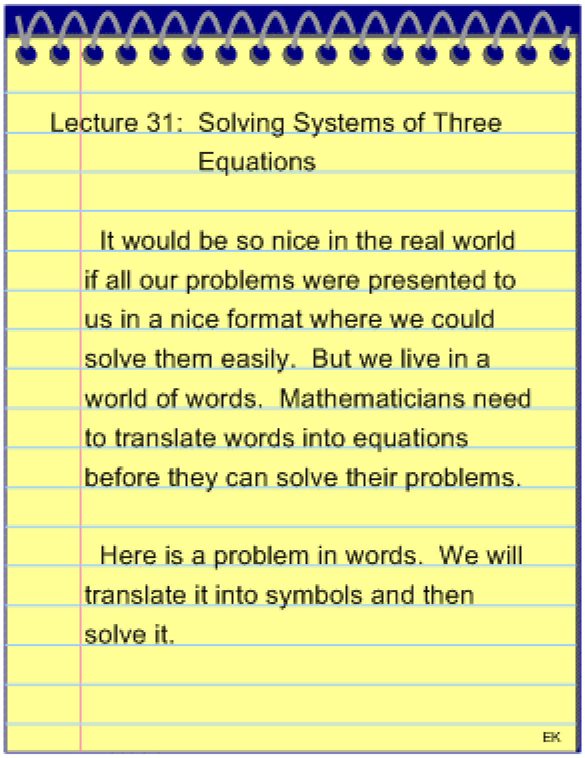
Just remember the procedure:

1) Use elimination twice to reduce your system down to two equations and two unknowns.
2) Solve that system, and then
3) Substitute back.

NCB

Lecture 31 Notes

ALG2031-01



Lecture 31: Solving Systems of Three Equations

It would be so nice in the real world if all our problems were presented to us in a nice format where we could solve them easily. But we live in a world of words. Mathematicians need to translate words into equations before they can solve their problems.

Here is a problem in words. We will translate it into symbols and then solve it.

EK

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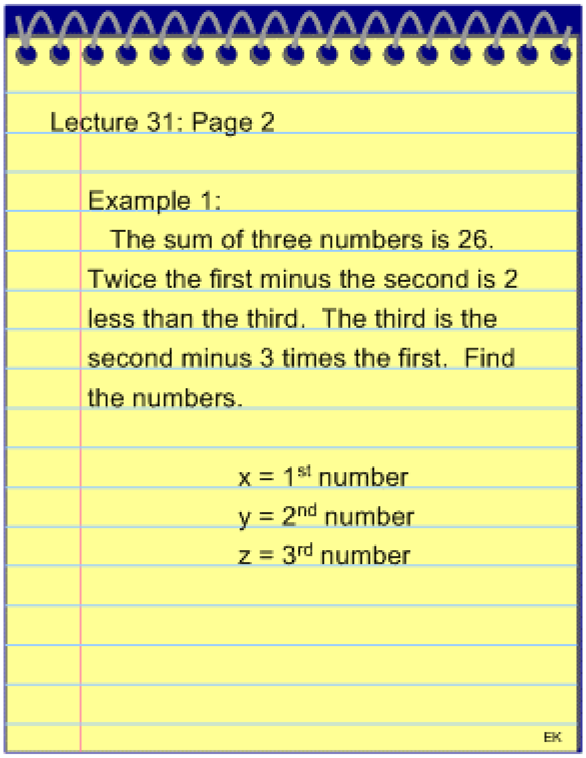


Lecture 31: Page 2

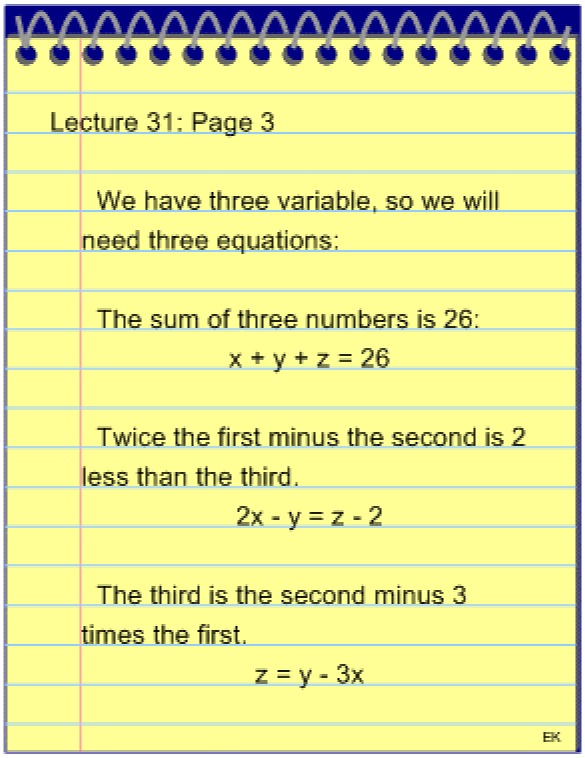
Example 1:

The sum of three numbers is 26. Twice the first minus the second is 2 less than the third. The third is the second minus 3 times the first. Find the numbers.

$x = 1^{st}$ number
$y = 2^{nd}$ number
$z = 3^{rd}$ number

EK

ALG2031-03



Lecture 31: Page 3

We have three variable, so we will need three equations:

The sum of three numbers is 26:

$$x + y + z = 26$$

Twice the first minus the second is 2 less than the third.

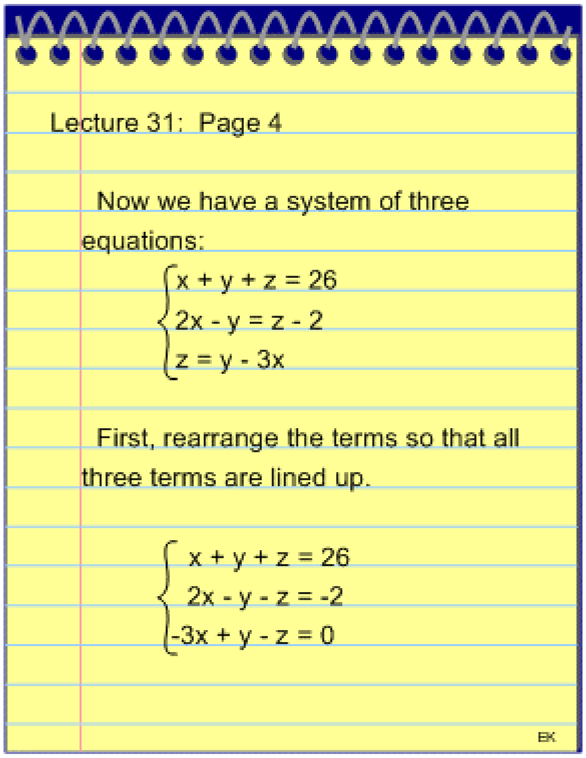
$$2x - y = z - 2$$

The third is the second minus 3 times the first.

$$z = y - 3x$$

EK

ALG2031-04



Lecture 31: Page 4

Now we have a system of three equations:

$$\begin{cases} x + y + z = 26 \\ 2x - y = z - 2 \\ z = y - 3x \end{cases}$$

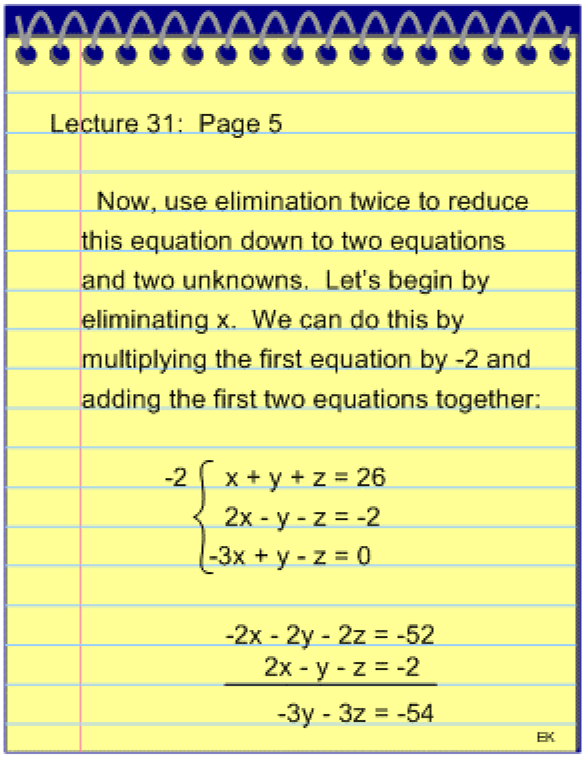
First, rearrange the terms so that all three terms are lined up.

$$\begin{cases} x + y + z = 26 \\ 2x - y - z = -2 \\ -3x + y - z = 0 \end{cases}$$

EK

Lecture 31 Notes, Continued

ALG2031-05



Lecture 31: Page 5

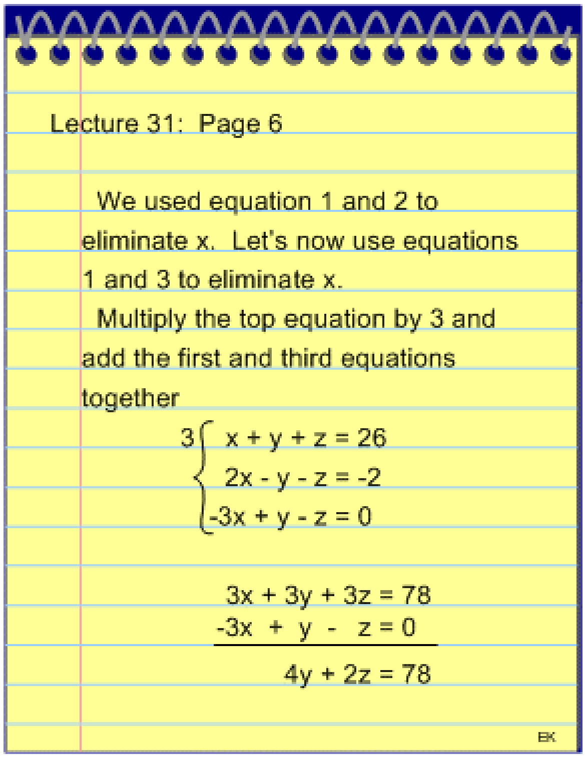
Now, use elimination twice to reduce this equation down to two equations and two unknowns. Let's begin by eliminating x. We can do this by multiplying the first equation by -2 and adding the first two equations together:

$$-2\begin{cases} x + y + z = 26 \\ 2x - y - z = -2 \\ -3x + y - z = 0 \end{cases}$$

$$\begin{array}{r} -2x - 2y - 2z = -52 \\ 2x - y - z = -2 \\ \hline -3y - 3z = -54 \end{array}$$

EK

ALG2031-06



Lecture 31: Page 6

We used equation 1 and 2 to eliminate x. Let's now use equations 1 and 3 to eliminate x.

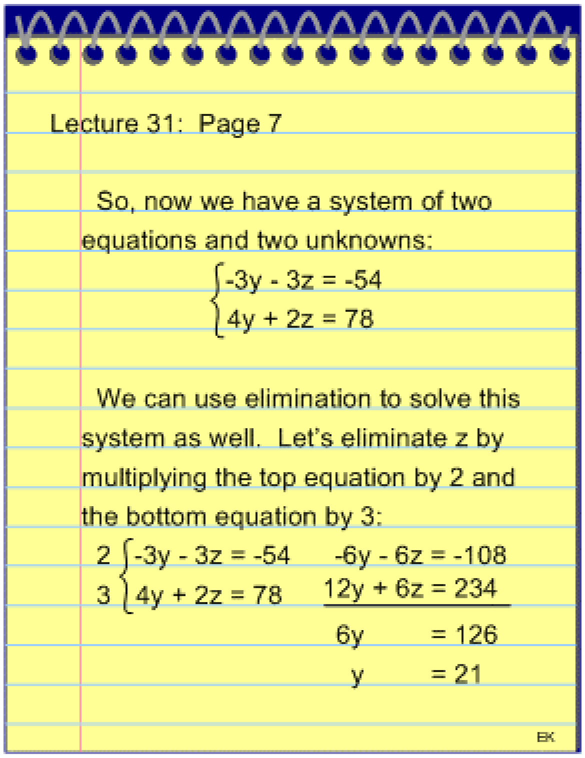
Multiply the top equation by 3 and add the first and third equations together

$$3\begin{cases} x + y + z = 26 \\ 2x - y - z = -2 \\ -3x + y - z = 0 \end{cases}$$

$$\begin{array}{r} 3x + 3y + 3z = 78 \\ -3x + y - z = 0 \\ \hline 4y + 2z = 78 \end{array}$$

EK

ALG2031-07



Lecture 31: Page 7

So, now we have a system of two equations and two unknowns:

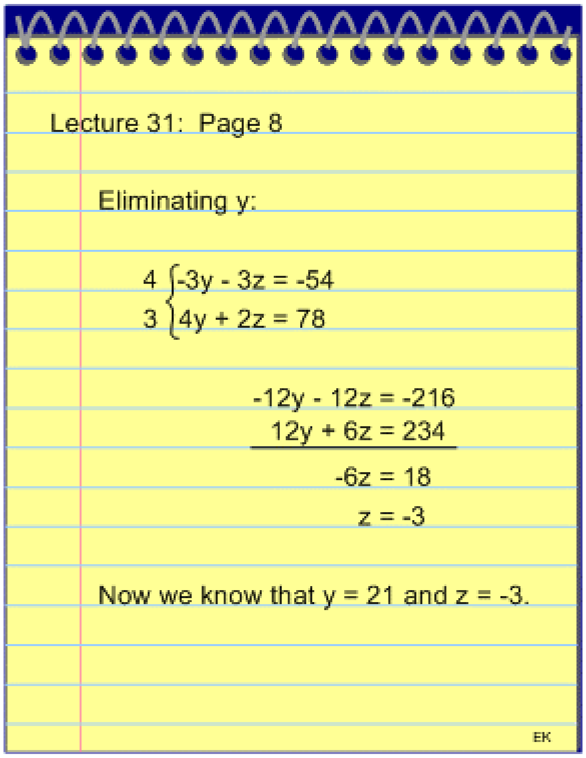
$$\begin{cases} -3y - 3z = -54 \\ 4y + 2z = 78 \end{cases}$$

We can use elimination to solve this system as well. Let's eliminate z by multiplying the top equation by 2 and the bottom equation by 3:

$$\begin{array}{r} 2 \\ 3 \end{array}\begin{cases} -3y - 3z = -54 \\ 4y + 2z = 78 \end{cases} \qquad \begin{array}{r} -6y - 6z = -108 \\ 12y + 6z = 234 \\ \hline 6y \quad = 126 \\ y \quad = 21 \end{array}$$

EK

ALG2031-08



Lecture 31: Page 8

Eliminating y:

$$\begin{array}{r} 4 \\ 3 \end{array}\begin{cases} -3y - 3z = -54 \\ 4y + 2z = 78 \end{cases}$$

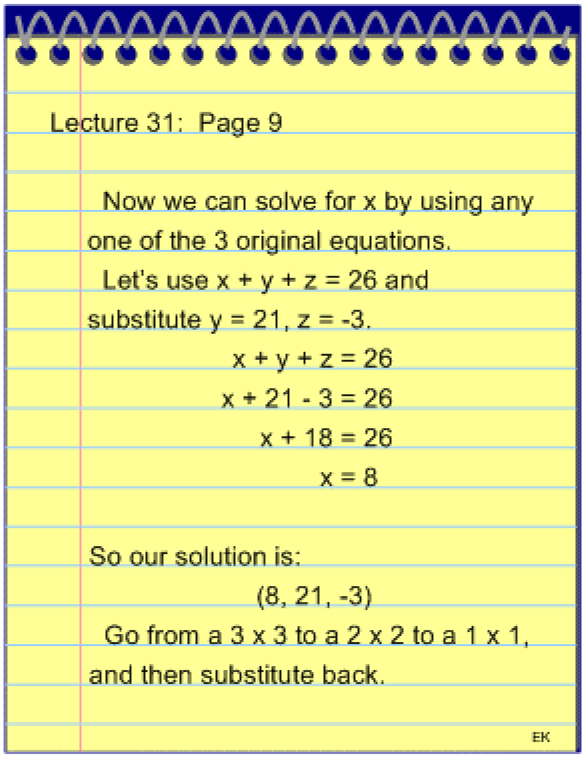
$$\begin{array}{r} -12y - 12z = -216 \\ 12y + 6z = 234 \\ \hline -6z = 18 \\ z = -3 \end{array}$$

Now we know that y = 21 and z = -3.

EK

Lecture 31 Notes, Continued

ALG2031-09



Lecture 31: Page 9

Now we can solve for x by using any one of the 3 original equations.

Let's use $x + y + z = 26$ and substitute $y = 21$, $z = -3$.

$$x + y + z = 26$$
$$x + 21 - 3 = 26$$
$$x + 18 = 26$$
$$x = 8$$

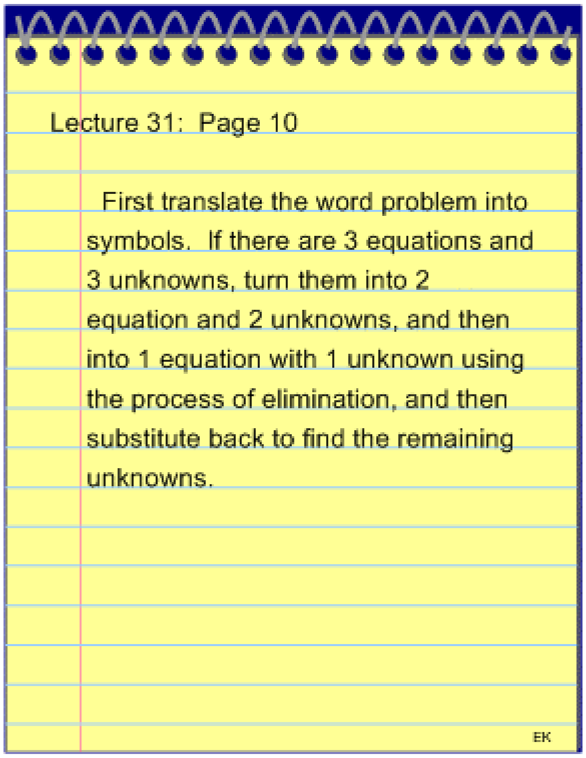
So our solution is:

$(8, 21, -3)$

Go from a 3 x 3 to a 2 x 2 to a 1 x 1, and then substitute back.

EK

ALG2031-10



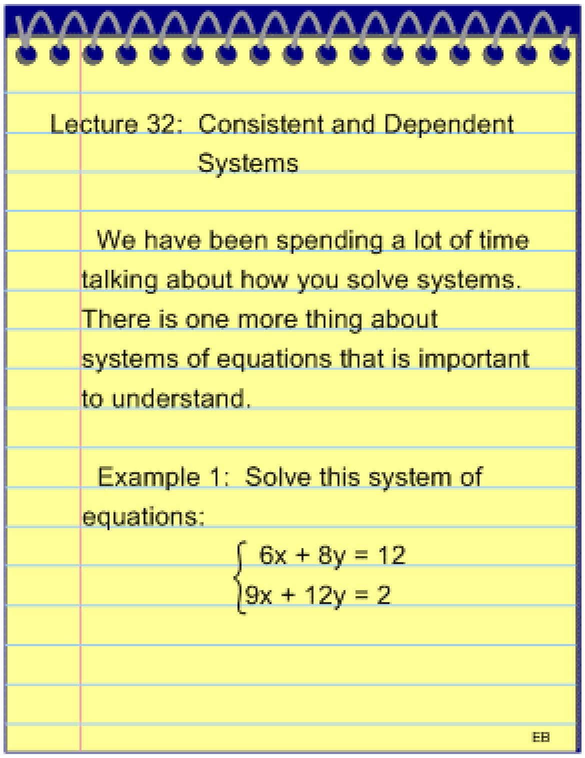
Lecture 31: Page 10

First translate the word problem into symbols. If there are 3 equations and 3 unknowns, turn them into 2 equation and 2 unknowns, and then into 1 equation with 1 unknown using the process of elimination, and then substitute back to find the remaining unknowns.

EK

Lecture 32 Notes

ALG2032-01



Lecture 32: Consistent and Dependent Systems

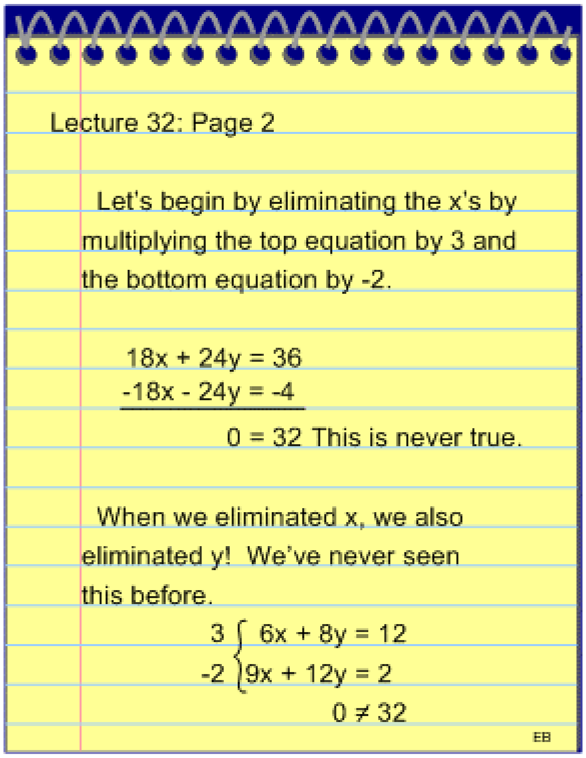
We have been spending a lot of time talking about how you solve systems. There is one more thing about systems of equations that is important to understand.

Example 1: Solve this system of equations:

$$\begin{cases} 6x + 8y = 12 \\ 9x + 12y = 2 \end{cases}$$

EB

ALG2032-02



Lecture 32: Page 2

Let's begin by eliminating the x's by multiplying the top equation by 3 and the bottom equation by -2.

$$\begin{array}{r} 18x + 24y = 36 \\ \underline{-18x - 24y = -4} \\ 0 = 32 \end{array}$$ This is never true.

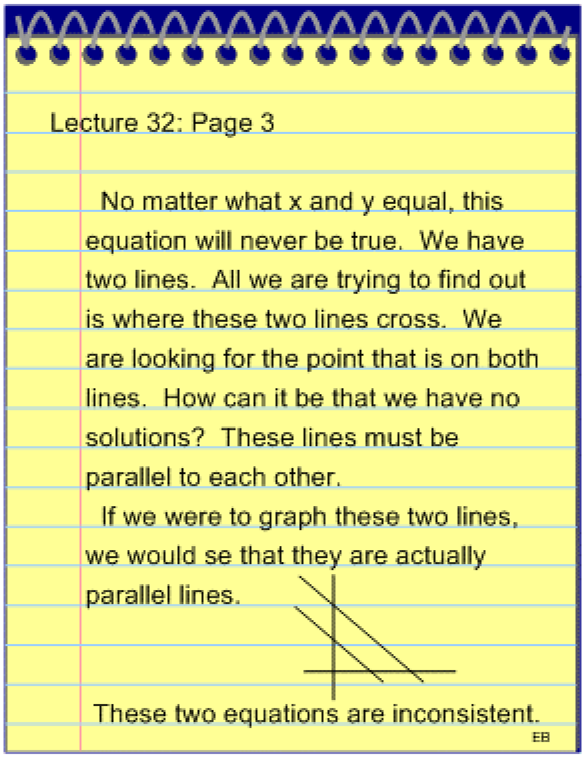
When we eliminated x, we also eliminated y! We've never seen this before.

$$\begin{array}{r} 3 \\ -2 \end{array} \begin{cases} 6x + 8y = 12 \\ 9x + 12y = 2 \end{cases}$$

$$0 \neq 32$$

EB

ALG2032-03



Lecture 32: Page 3

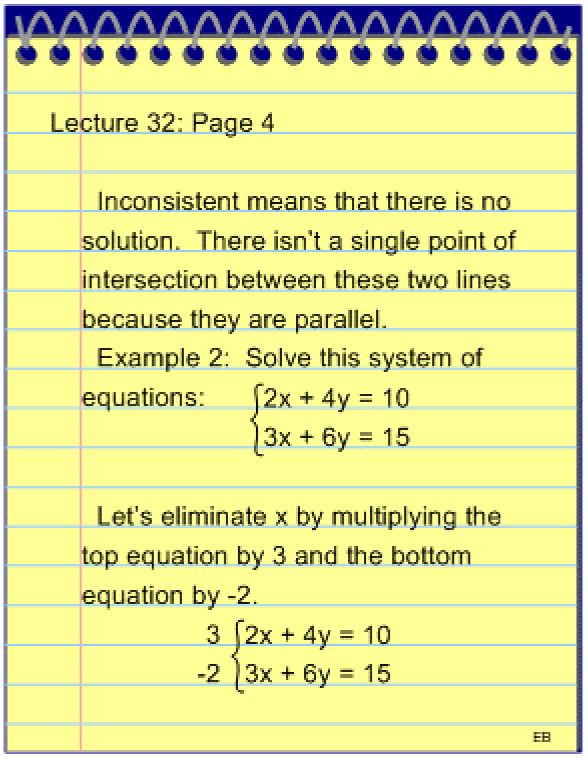
No matter what x and y equal, this equation will never be true. We have two lines. All we are trying to find out is where these two lines cross. We are looking for the point that is on both lines. How can it be that we have no solutions? These lines must be parallel to each other.

If we were to graph these two lines, we would se that they are actually parallel lines.

These two equations are inconsistent.

EB

ALG2032-04



Lecture 32: Page 4

Inconsistent means that there is no solution. There isn't a single point of intersection between these two lines because they are parallel.

Example 2: Solve this system of equations: $\begin{cases} 2x + 4y = 10 \\ 3x + 6y = 15 \end{cases}$

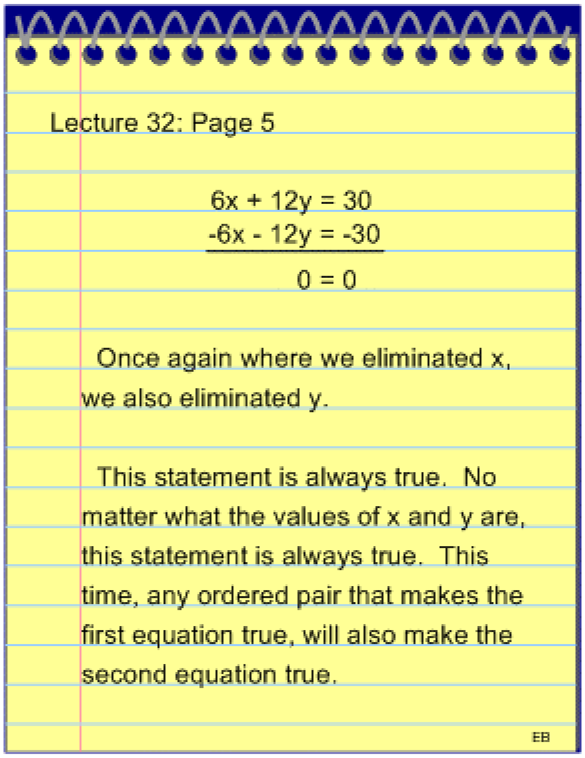
Let's eliminate x by multiplying the top equation by 3 and the bottom equation by -2.

$$\begin{array}{r} 3 \\ -2 \end{array} \begin{cases} 2x + 4y = 10 \\ 3x + 6y = 15 \end{cases}$$

EB

Lecture 32 Notes

ALG2032-05



Lecture 32: Page 5

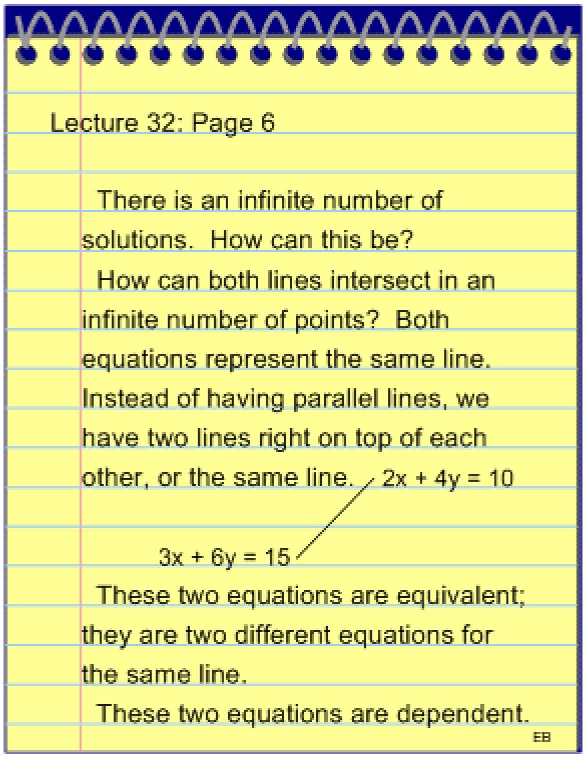
$$\begin{array}{r} 6x + 12y = 30 \\ -6x - 12y = -30 \\ \hline 0 = 0 \end{array}$$

Once again where we eliminated x, we also eliminated y.

This statement is always true. No matter what the values of x and y are, this statement is always true. This time, any ordered pair that makes the first equation true, will also make the second equation true.

EB

ALG2032-06



Lecture 32: Page 6

There is an infinite number of solutions. How can this be?

How can both lines intersect in an infinite number of points? Both equations represent the same line. Instead of having parallel lines, we have two lines right on top of each other, or the same line.

$2x + 4y = 10$

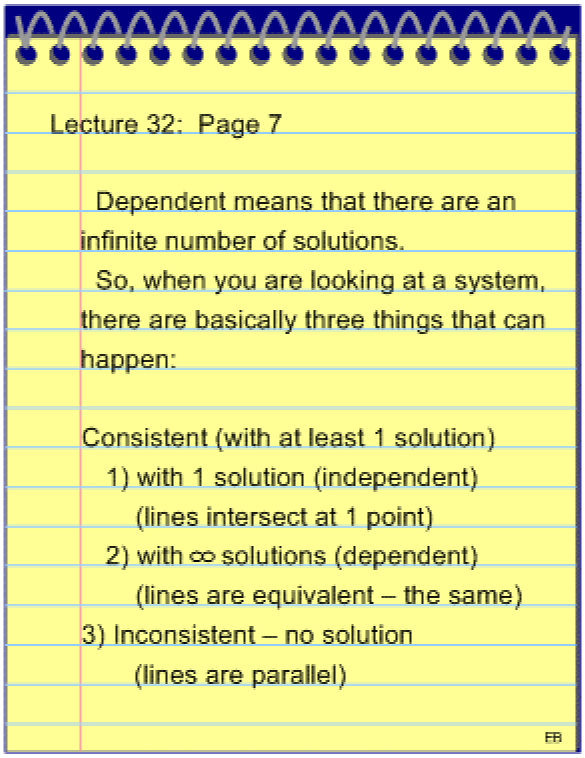
$3x + 6y = 15$

These two equations are equivalent; they are two different equations for the same line.

These two equations are dependent.

EB

ALG2032-07



Lecture 32: Page 7

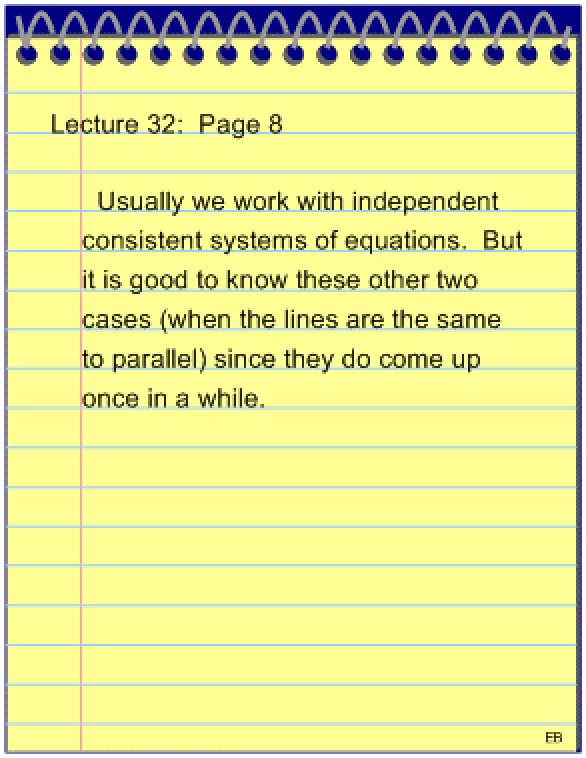
Dependent means that there are an infinite number of solutions.

So, when you are looking at a system, there are basically three things that can happen:

Consistent (with at least 1 solution)
  1) with 1 solution (independent)
    (lines intersect at 1 point)
  2) with ∞ solutions (dependent)
    (lines are equivalent – the same)
3) Inconsistent – no solution
    (lines are parallel)

EB

ALG2032-08



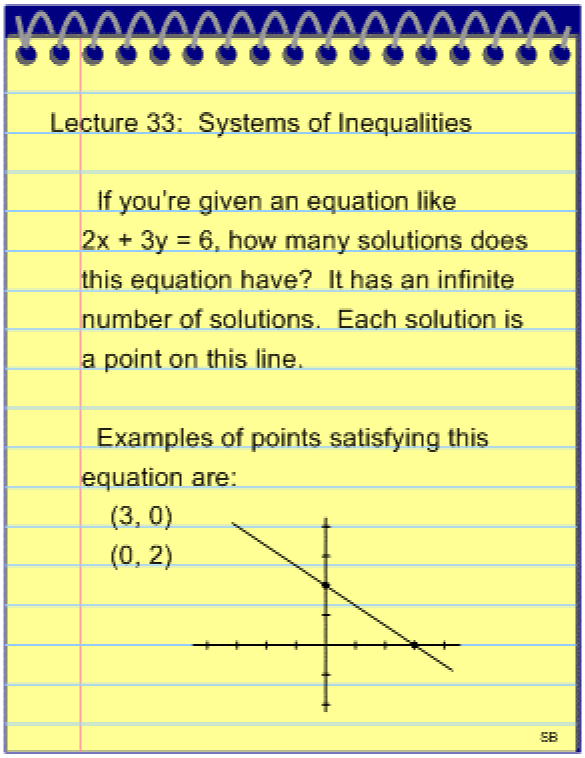
Lecture 32: Page 8

Usually we work with independent consistent systems of equations. But it is good to know these other two cases (when the lines are the same to parallel) since they do come up once in a while.

EB

Lecture 33 Notes

ALG2033-01



Lecture 33: Systems of Inequalities

If you're given an equation like $2x + 3y = 6$, how many solutions does this equation have? It has an infinite number of solutions. Each solution is a point on this line.

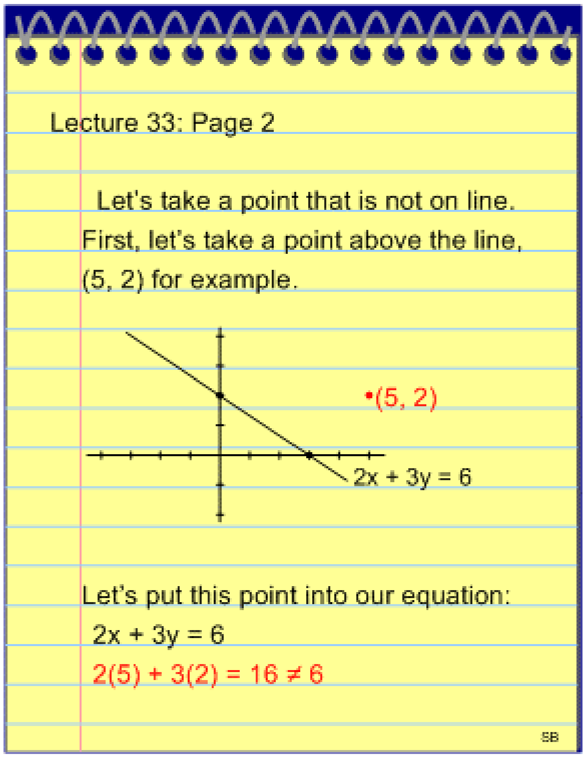
Examples of points satisfying this equation are:

(3, 0)

(0, 2)

SB

ALG2033-02



Lecture 33: Page 2

Let's take a point that is not on line. First, let's take a point above the line, (5, 2) for example.

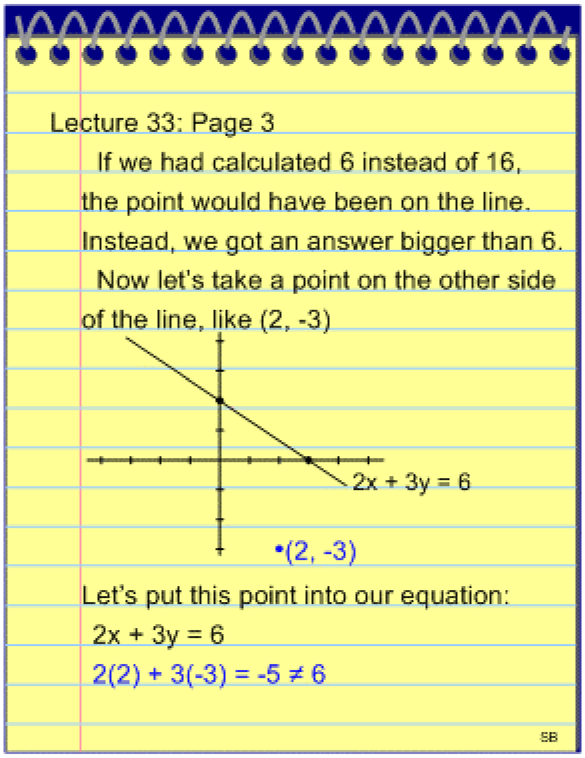
Let's put this point into our equation:

$2x + 3y = 6$

$2(5) + 3(2) = 16 \neq 6$

SB

ALG2033-03



Lecture 33: Page 3

If we had calculated 6 instead of 16, the point would have been on the line. Instead, we got an answer bigger than 6.

Now let's take a point on the other side of the line, like (2, -3)

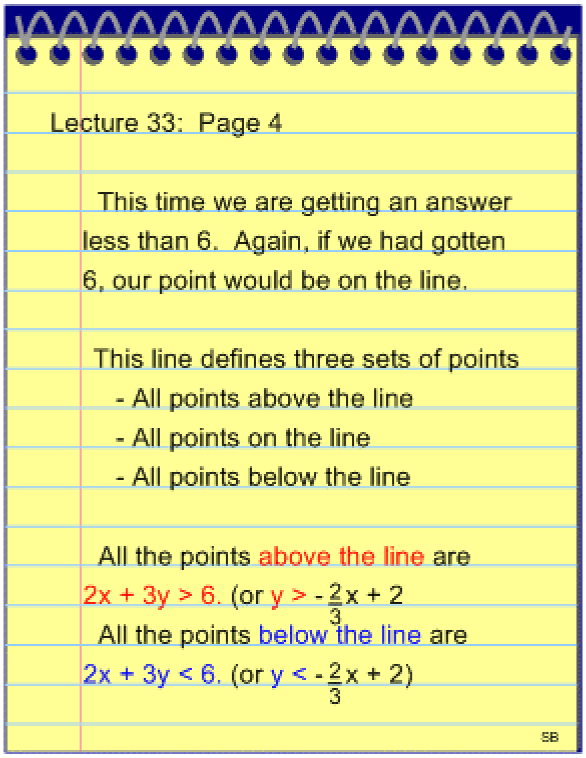
Let's put this point into our equation:

$2x + 3y = 6$

$2(2) + 3(-3) = -5 \neq 6$

SB

ALG2033-04



Lecture 33: Page 4

This time we are getting an answer less than 6. Again, if we had gotten 6, our point would be on the line.

This line defines three sets of points

- All points above the line
- All points on the line
- All points below the line

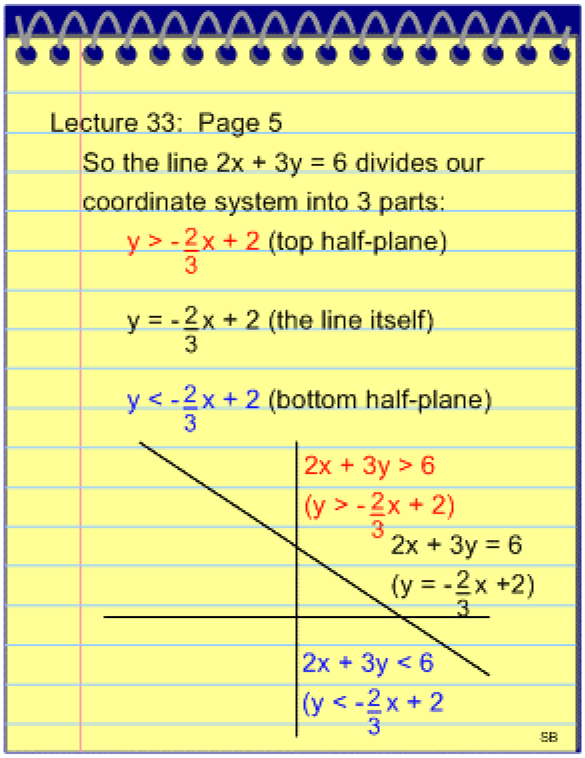
All the points above the line are $2x + 3y > 6$. (or $y > -\frac{2}{3}x + 2$

All the points below the line are $2x + 3y < 6$. (or $y < -\frac{2}{3}x + 2$)

SB

Lecture 33 Notes, Continued

ALG2033-05



Lecture 33: Page 5

So the line $2x + 3y = 6$ divides our coordinate system into 3 parts:

$y > -\frac{2}{3}x + 2$ (top half-plane)

$y = -\frac{2}{3}x + 2$ (the line itself)

$y < -\frac{2}{3}x + 2$ (bottom half-plane)

$2x + 3y > 6$

$(y > -\frac{2}{3}x + 2)$

$2x + 3y = 6$

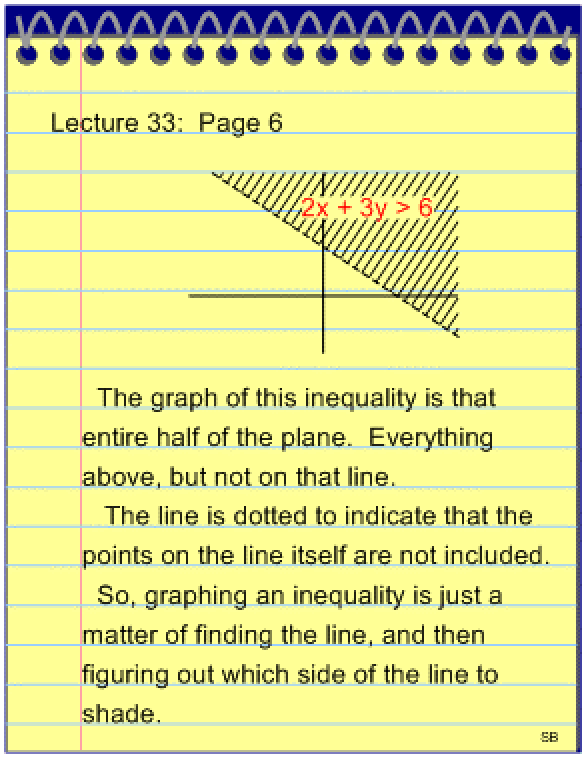
$(y = -\frac{2}{3}x + 2)$

$2x + 3y < 6$

$(y < -\frac{2}{3}x + 2$

SB

ALG2033-06



Lecture 33: Page 6

$2x + 3y > 6$

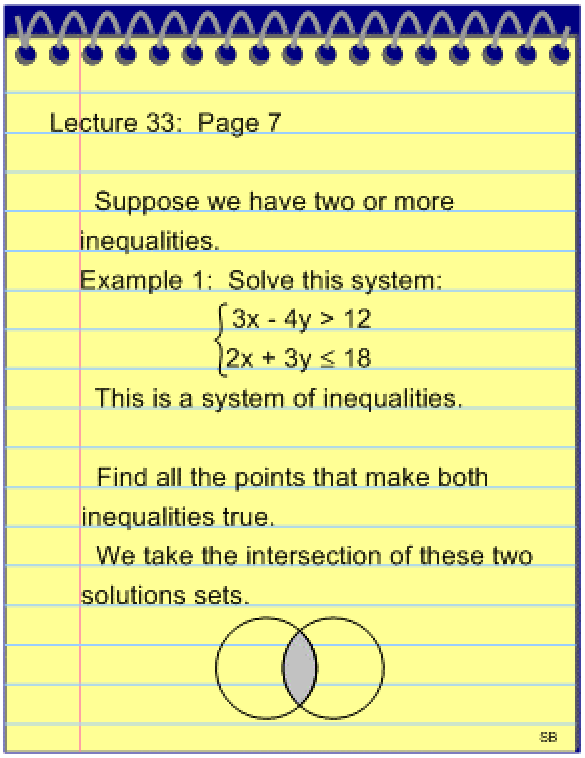
The graph of this inequality is that entire half of the plane. Everything above, but not on that line.

The line is dotted to indicate that the points on the line itself are not included.

So, graphing an inequality is just a matter of finding the line, and then figuring out which side of the line to shade.

SB

ALG2033-07



Lecture 33: Page 7

Suppose we have two or more inequalities.

Example 1: Solve this system:

$$\begin{cases} 3x - 4y > 12 \\ 2x + 3y \leq 18 \end{cases}$$

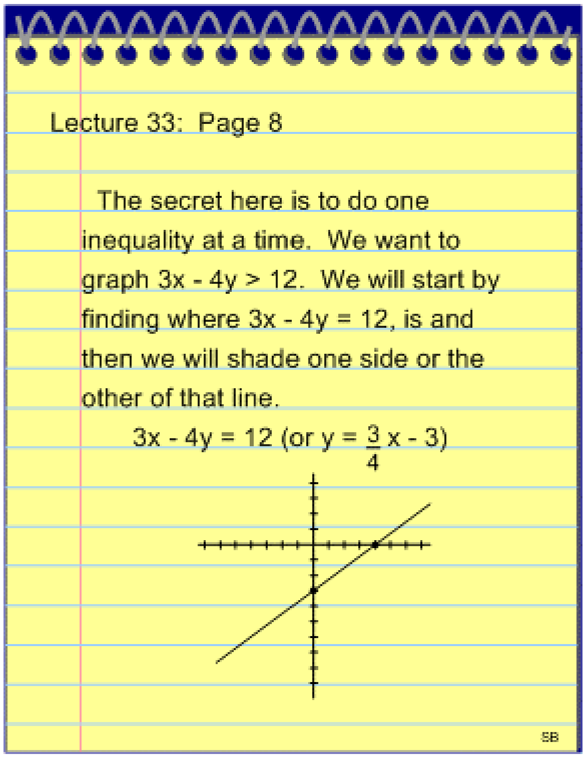
This is a system of inequalities.

Find all the points that make both inequalities true.

We take the intersection of these two solutions sets.

SB

ALG2033-08



Lecture 33: Page 8

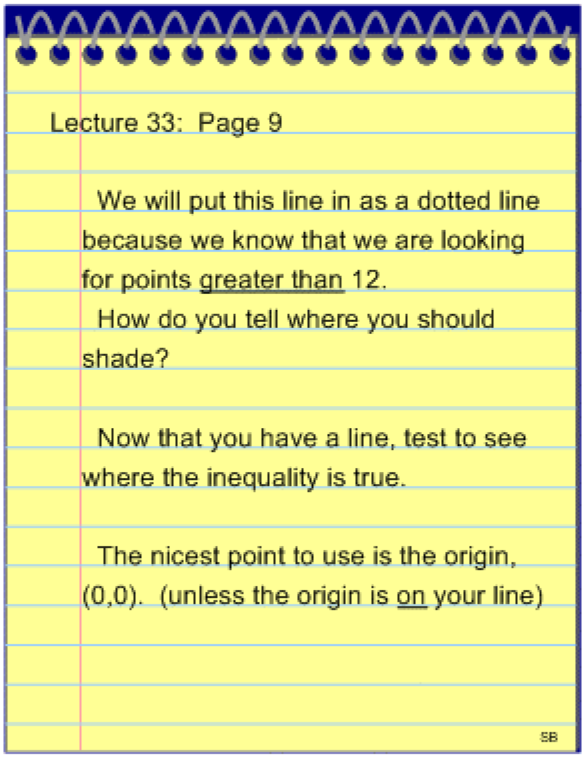
The secret here is to do one inequality at a time. We want to graph $3x - 4y > 12$. We will start by finding where $3x - 4y = 12$, is and then we will shade one side or the other of that line.

$3x - 4y = 12$ (or $y = \frac{3}{4}x - 3$)

SB

Lecture 33 Notes, Continued

ALG2033-09



Lecture 33: Page 9

We will put this line in as a dotted line because we know that we are looking for points greater than 12.

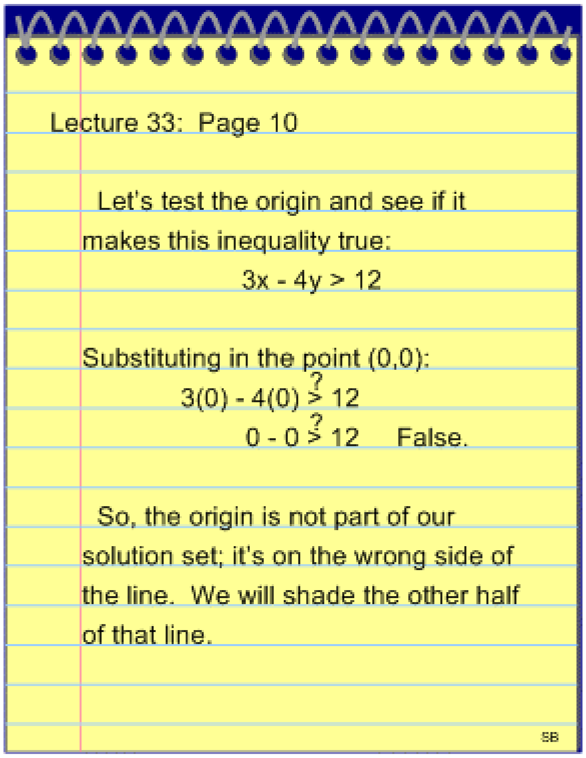
How do you tell where you should shade?

Now that you have a line, test to see where the inequality is true.

The nicest point to use is the origin, (0,0). (unless the origin is on your line)

SB

ALG2033-10



Lecture 33: Page 10

Let's test the origin and see if it makes this inequality true:

$$3x - 4y > 12$$

Substituting in the point (0,0):

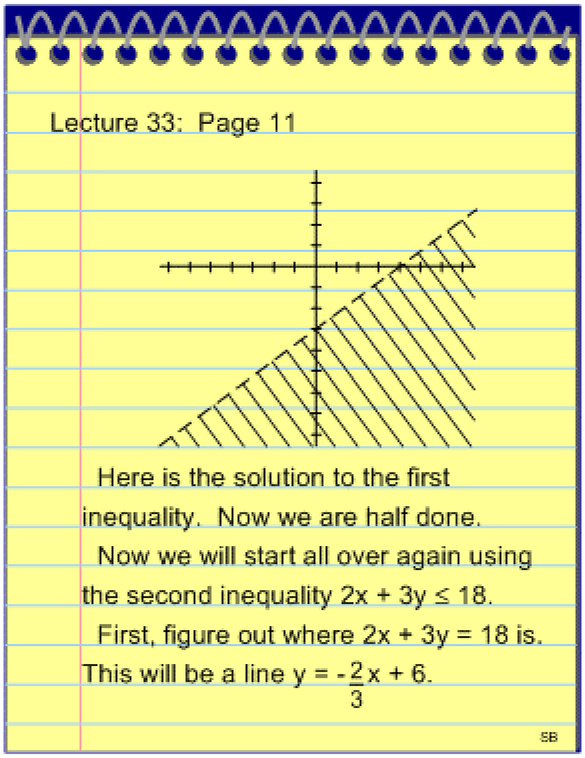
$$3(0) - 4(0) \overset{?}{>} 12$$

$$0 - 0 \overset{?}{>} 12 \quad \text{False.}$$

So, the origin is not part of our solution set; it's on the wrong side of the line. We will shade the other half of that line.

SB

ALG2033-11



Lecture 33: Page 11

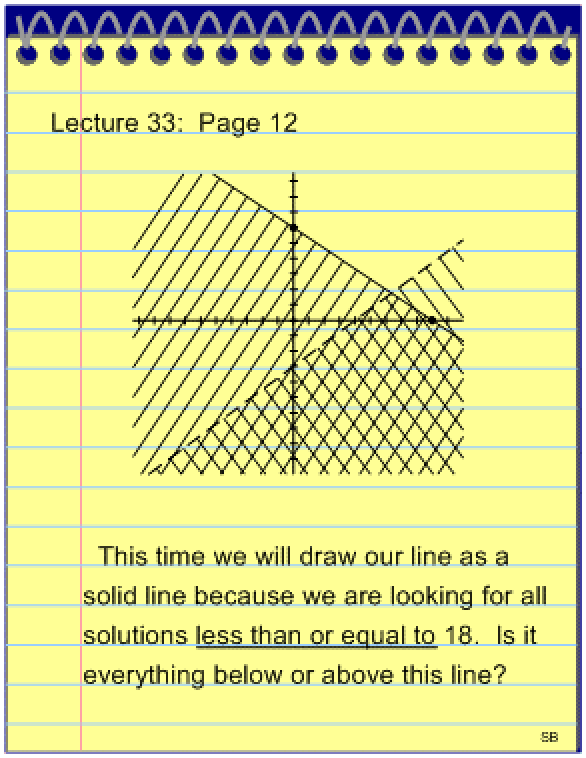
Here is the solution to the first inequality. Now we are half done.

Now we will start all over again using the second inequality $2x + 3y \leq 18$.

First, figure out where $2x + 3y = 18$ is. This will be a line $y = -\frac{2}{3}x + 6$.

SB

ALG2033-12



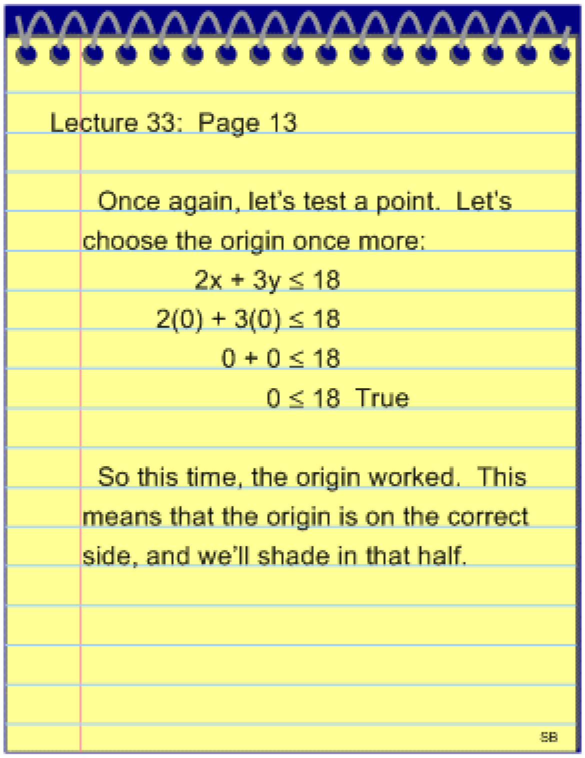
Lecture 33: Page 12

This time we will draw our line as a solid line because we are looking for all solutions less than or equal to 18. Is it everything below or above this line?

SB

Lecture 33 Notes, Continued

ALG2033-13



Lecture 33: Page 13

Once again, let's test a point. Let's choose the origin once more:

$$2x + 3y \le 18$$

$$2(0) + 3(0) \le 18$$

$$0 + 0 \le 18$$

$$0 \le 18 \text{ True}$$

So this time, the origin worked. This means that the origin is on the correct side, and we'll shade in that half.

SB

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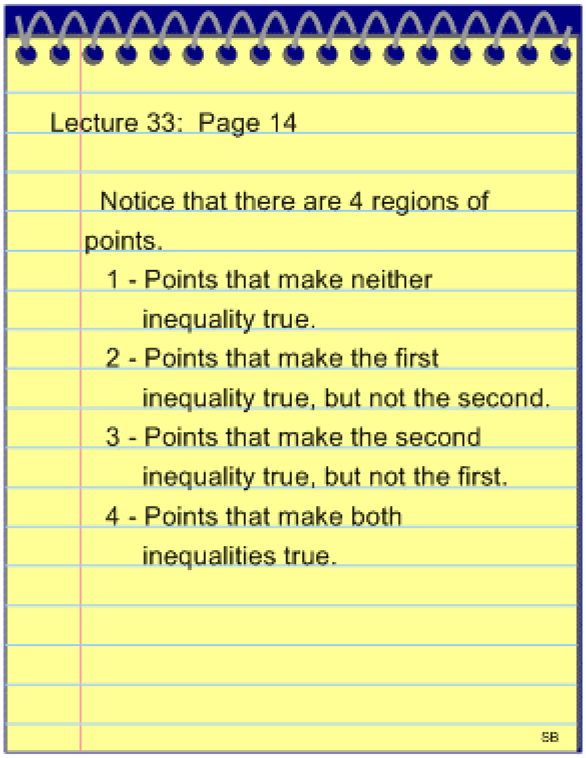


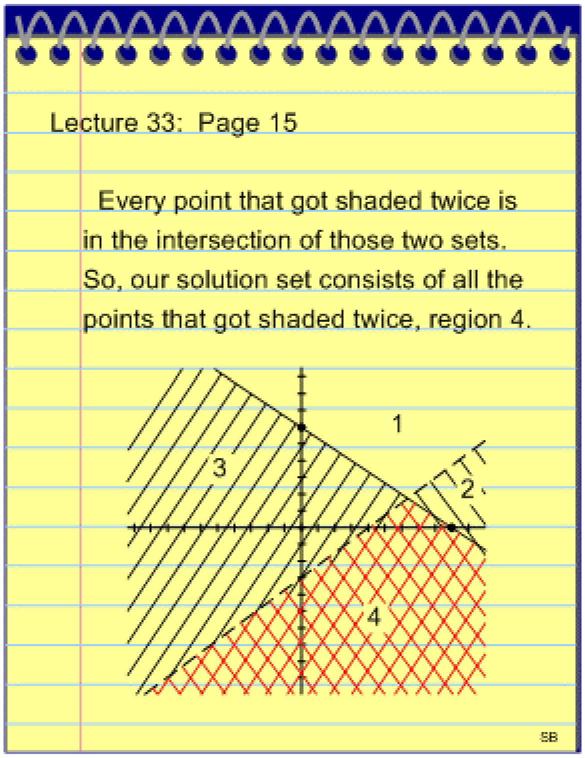
Lecture 33: Page 14

Notice that there are 4 regions of points.

1 - Points that make neither inequality true.
2 - Points that make the first inequality true, but not the second.
3 - Points that make the second inequality true, but not the first.
4 - Points that make both inequalities true.

SB

ALG2033-15



Lecture 33: Page 15

Every point that got shaded twice is in the intersection of those two sets. So, our solution set consists of all the points that got shaded twice, region 4.

SB

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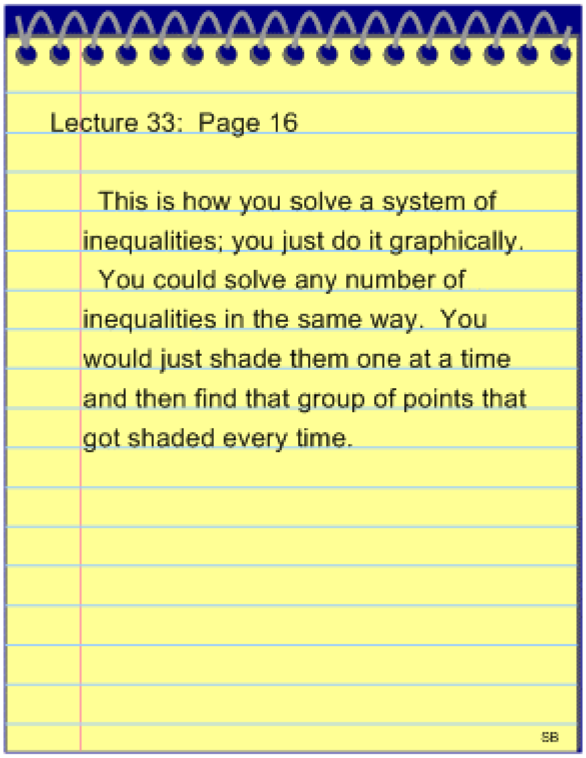
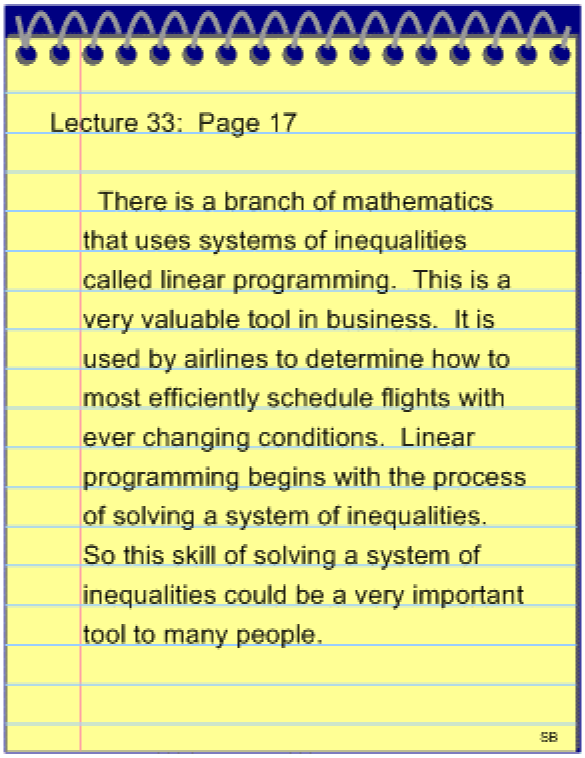


Lecture 33: Page 16

This is how you solve a system of inequalities; you just do it graphically. You could solve any number of inequalities in the same way. You would just shade them one at a time and then find that group of points that got shaded every time.

SB

ALG2033-17



Lecture 33: Page 17

There is a branch of mathematics that uses systems of inequalities called linear programming. This is a very valuable tool in business. It is used by airlines to determine how to most efficiently schedule flights with ever changing conditions. Linear programming begins with the process of solving a system of inequalities. So this skill of solving a system of inequalities could be a very important tool to many people.

SB

Lecture 34 Notes

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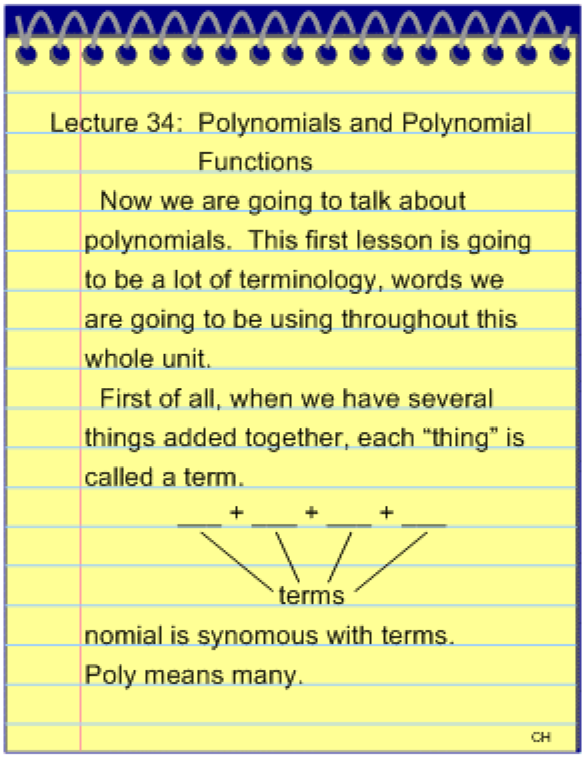

Lecture 34: Polynomials and Polynomial Functions

Now we are going to talk about polynomials. This first lesson is going to be a lot of terminology, words we are going to be using throughout this whole unit.

First of all, when we have several things added together, each "thing" is called a term.

___ + ___ + ___ + ___

terms

nomial is synomous with terms.

Poly means many.

CH

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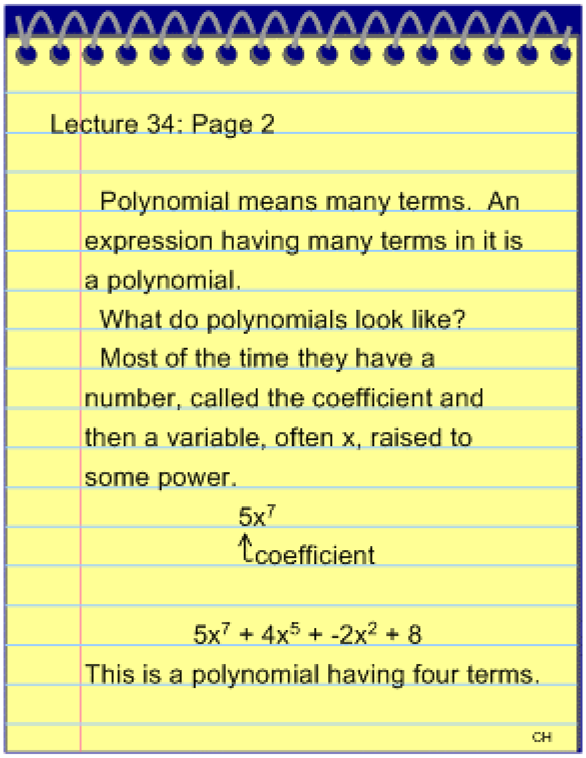

Lecture 34: Page 2

Polynomial means many terms. An expression having many terms in it is a polynomial.

What do polynomials look like? Most of the time they have a number, called the coefficient and then a variable, often x, raised to some power.

$5x^7$

↑coefficient

$5x^7 + 4x^5 + -2x^2 + 8$

This is a polynomial having four terms.

CH

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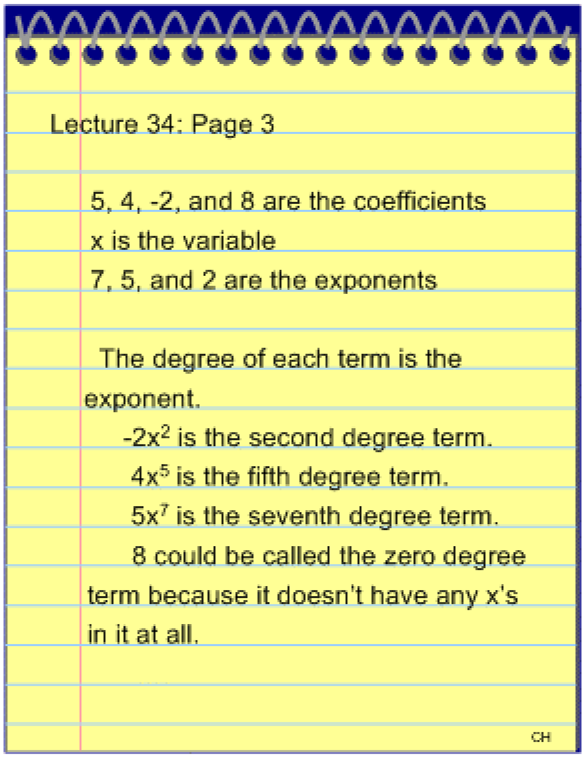

Lecture 34: Page 3

5, 4, -2, and 8 are the coefficients

x is the variable

7, 5, and 2 are the exponents

The degree of each term is the exponent.

$-2x^2$ is the second degree term.

$4x^5$ is the fifth degree term.

$5x^7$ is the seventh degree term.

8 could be called the zero degree term because it doesn't have any x's in it at all.

CH

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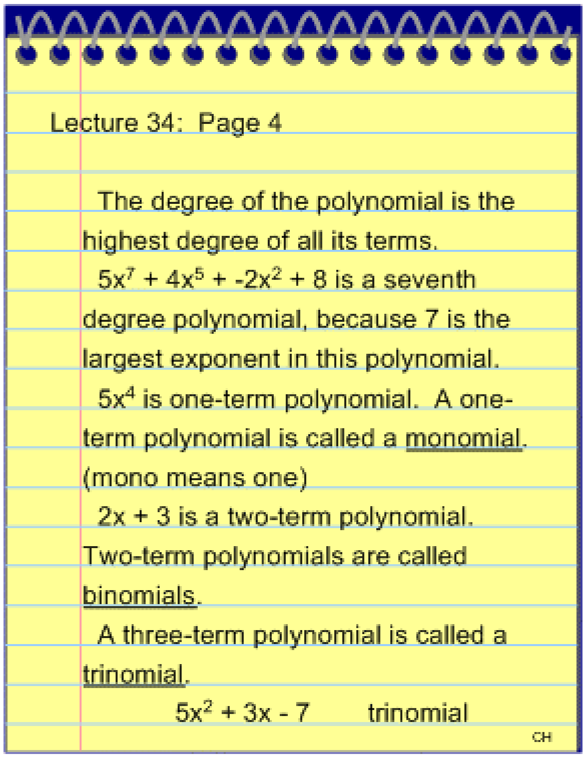


Lecture 34: Page 4

The degree of the polynomial is the highest degree of all its terms.

$5x^7 + 4x^5 + -2x^2 + 8$ is a seventh degree polynomial, because 7 is the largest exponent in this polynomial.

$5x^4$ is one-term polynomial. A one-term polynomial is called a monomial. (mono means one)

$2x + 3$ is a two-term polynomial. Two-term polynomials are called binomials.

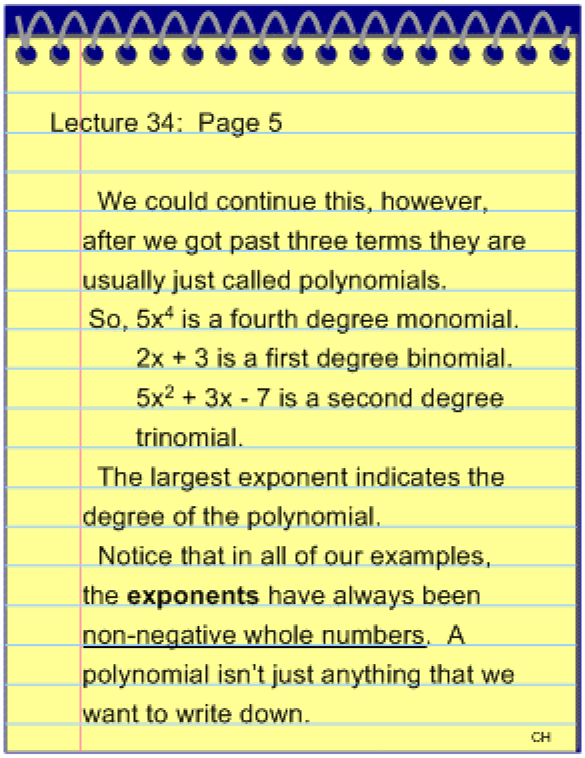
A three-term polynomial is called a trinomial.

$5x^2 + 3x - 7$ trinomial

CH

Lecture 34 Notes, Continued

ALG2034-05



Lecture 34: Page 5

We could continue this, however, after we got past three terms they are usually just called polynomials.

So, $5x^4$ is a fourth degree monomial.

$2x + 3$ is a first degree binomial.

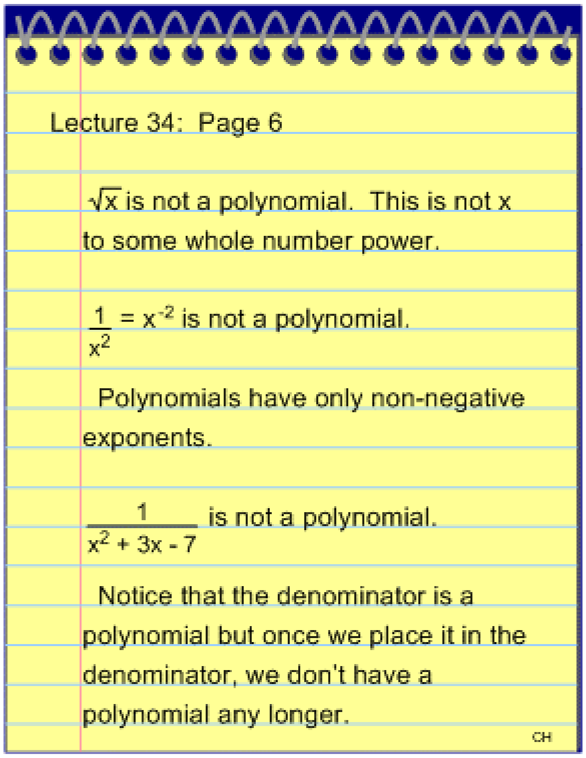
$5x^2 + 3x - 7$ is a second degree trinomial.

The largest exponent indicates the degree of the polynomial.

Notice that in all of our examples, the **exponents** have always been non-negative whole numbers. A polynomial isn't just anything that we want to write down.

CH

ALG2034-06



Lecture 34: Page 6

$\sqrt{x}$ is not a polynomial. This is not x to some whole number power.

$\frac{1}{x^2} = x^{-2}$ is not a polynomial.

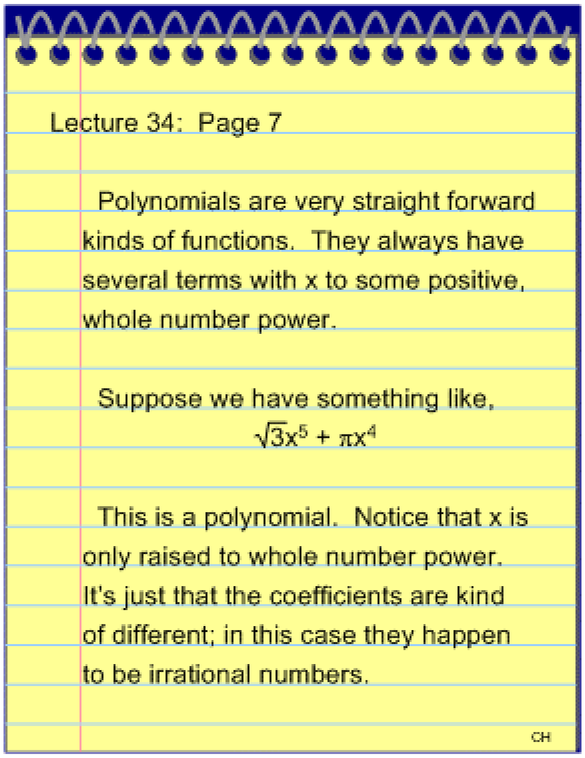
Polynomials have only non-negative exponents.

$\frac{1}{x^2 + 3x - 7}$ is not a polynomial.

Notice that the denominator is a polynomial but once we place it in the denominator, we don't have a polynomial any longer.

CH

ALG2034-07



Lecture 34: Page 7

Polynomials are very straight forward kinds of functions. They always have several terms with x to some positive, whole number power.

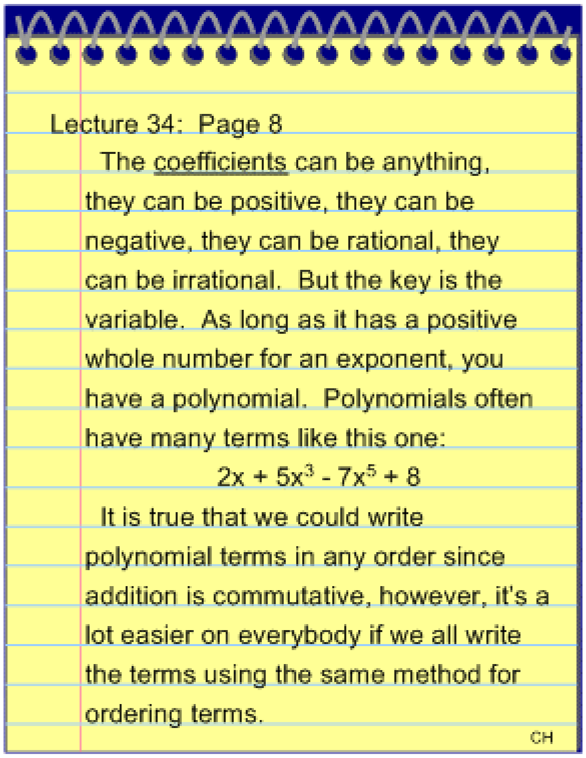
Suppose we have something like,

$$\sqrt{3}x^5 + \pi x^4$$

This is a polynomial. Notice that x is only raised to whole number power. It's just that the coefficients are kind of different; in this case they happen to be irrational numbers.

CH

ALG2034-08



Lecture 34: Page 8

The coefficients can be anything, they can be positive, they can be negative, they can be rational, they can be irrational. But the key is the variable. As long as it has a positive whole number for an exponent, you have a polynomial. Polynomials often have many terms like this one:

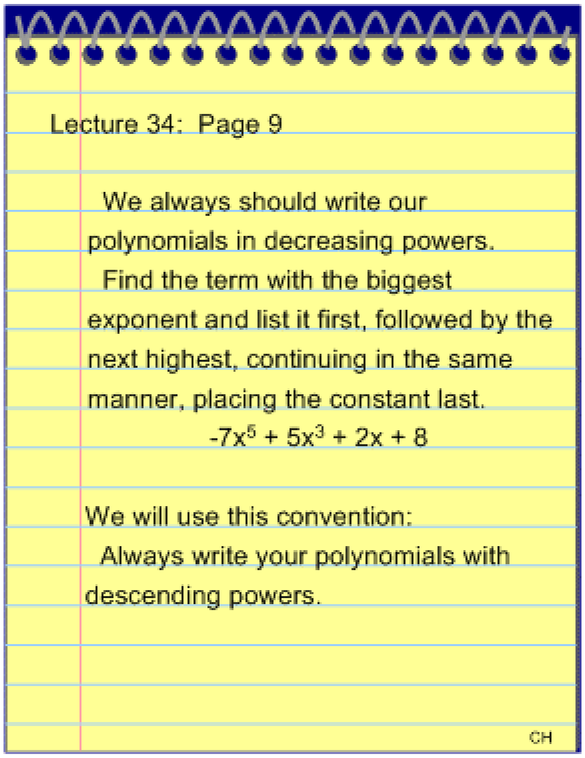
$$2x + 5x^3 - 7x^5 + 8$$

It is true that we could write polynomial terms in any order since addition is commutative, however, it's a lot easier on everybody if we all write the terms using the same method for ordering terms.

CH

Lecture 34 Notes, Continued

ALG2034-09



Lecture 34: Page 9

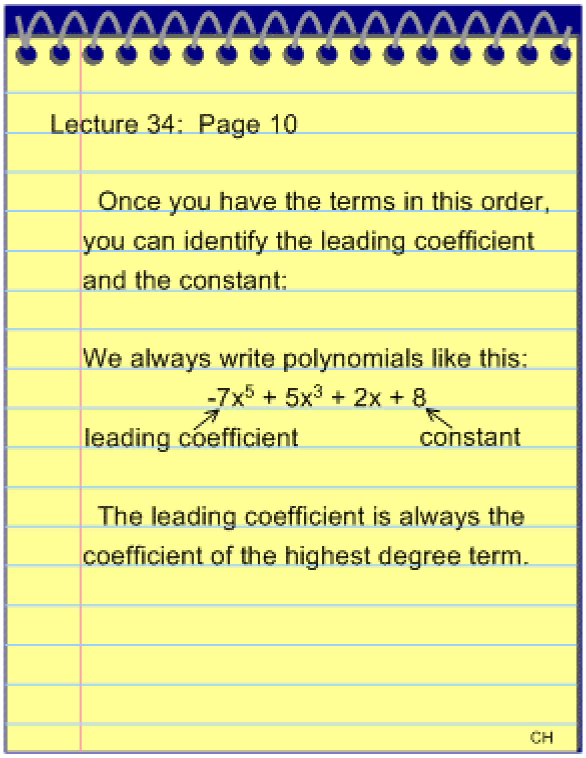
We always should write our polynomials in decreasing powers. Find the term with the biggest exponent and list it first, followed by the next highest, continuing in the same manner, placing the constant last.

$-7x^5 + 5x^3 + 2x + 8$

We will use this convention: Always write your polynomials with descending powers.

CH

ALG2034-10



Lecture 34: Page 10

Once you have the terms in this order, you can identify the leading coefficient and the constant:

We always write polynomials like this:

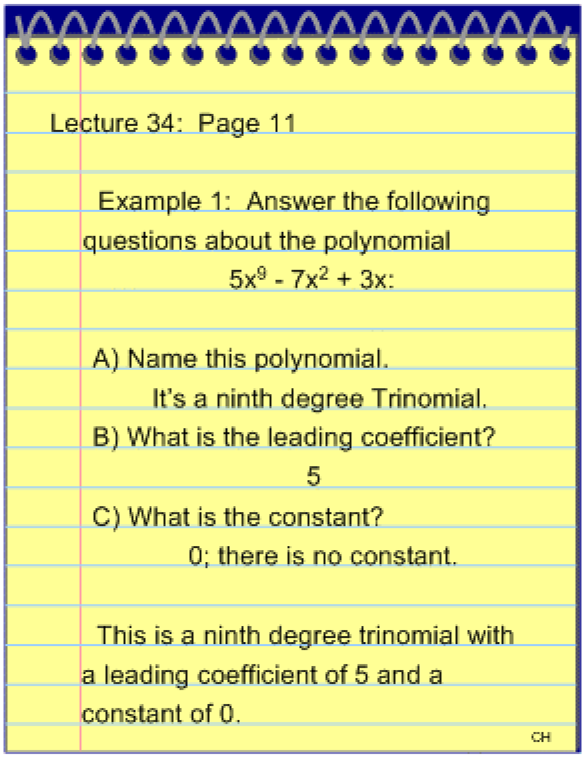
$-7x^5 + 5x^3 + 2x + 8$

leading coefficient constant

The leading coefficient is always the coefficient of the highest degree term.

CH

ALG2034-11



Lecture 34: Page 11

Example 1: Answer the following questions about the polynomial

$5x^9 - 7x^2 + 3x$:

A) Name this polynomial.

It's a ninth degree Trinomial.

B) What is the leading coefficient?

5

C) What is the constant?

0; there is no constant.

This is a ninth degree trinomial with a leading coefficient of 5 and a constant of 0.

CH

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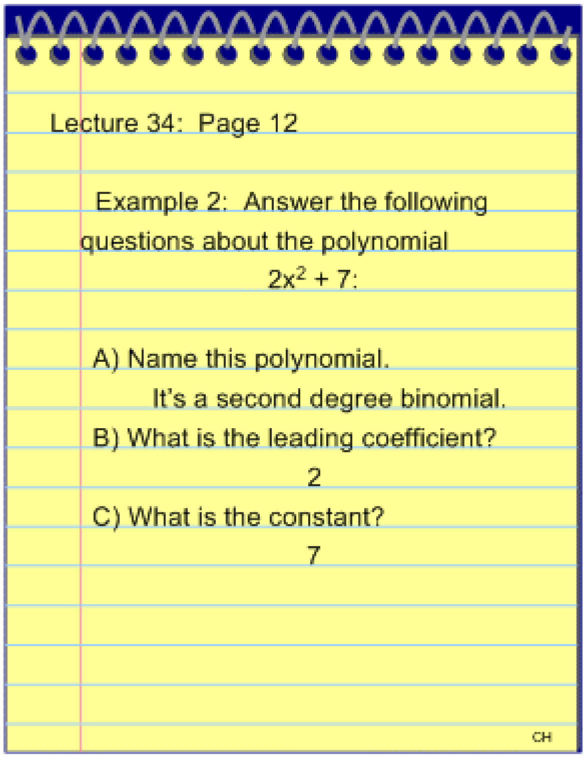


Lecture 34: Page 12

Example 2: Answer the following questions about the polynomial

$2x^2 + 7$:

A) Name this polynomial.

It's a second degree binomial.

B) What is the leading coefficient?

2

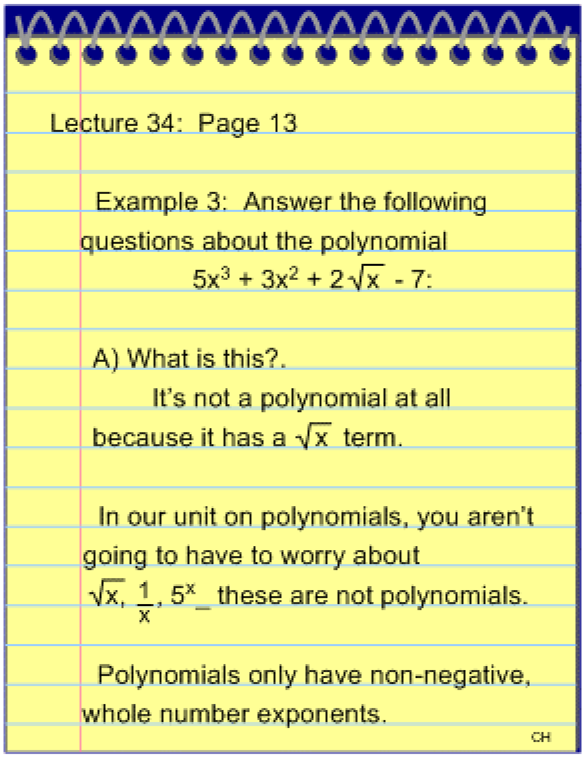
C) What is the constant?

7

CH

Lecture 34 Notes, Continued

ALG2034-13



Lecture 34: Page 13

Example 3: Answer the following questions about the polynomial

$$5x^3 + 3x^2 + 2\sqrt{x} - 7:$$

A) What is this?.

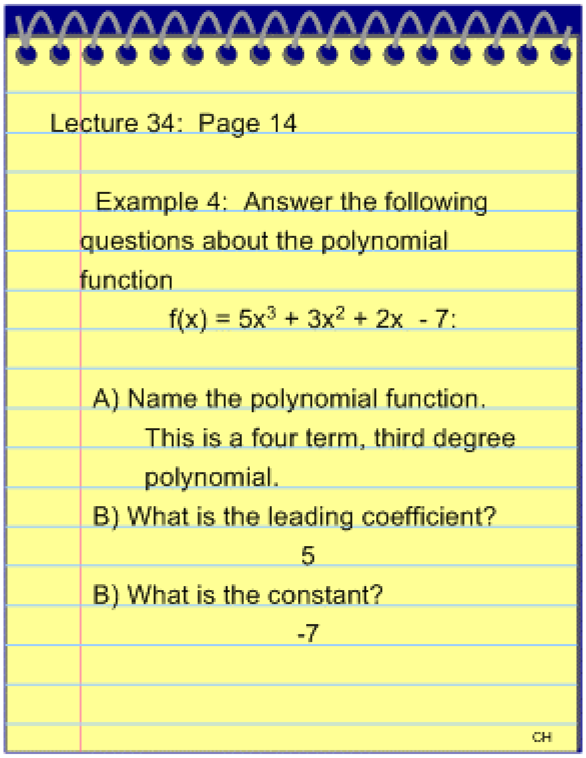
It's not a polynomial at all because it has a $\sqrt{x}$ term.

In our unit on polynomials, you aren't going to have to worry about $\sqrt{x}$, $\frac{1}{x}$, 5^x_ these are not polynomials.

Polynomials only have non-negative, whole number exponents.

CH

ALG2034-14



Lecture 34: Page 14

Example 4: Answer the following questions about the polynomial function

$$f(x) = 5x^3 + 3x^2 + 2x - 7:$$

A) Name the polynomial function.

This is a four term, third degree polynomial.

B) What is the leading coefficient?

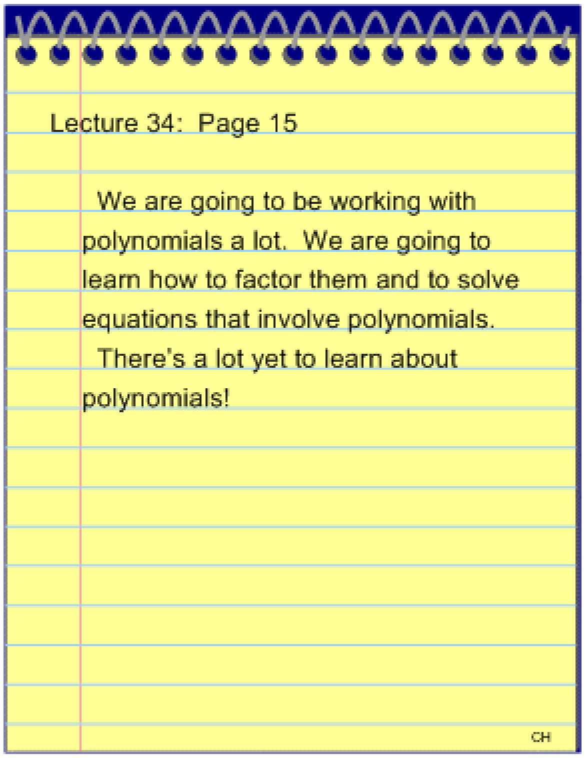
5

B) What is the constant?

-7

CH

ALG2034-15



Lecture 34: Page 15

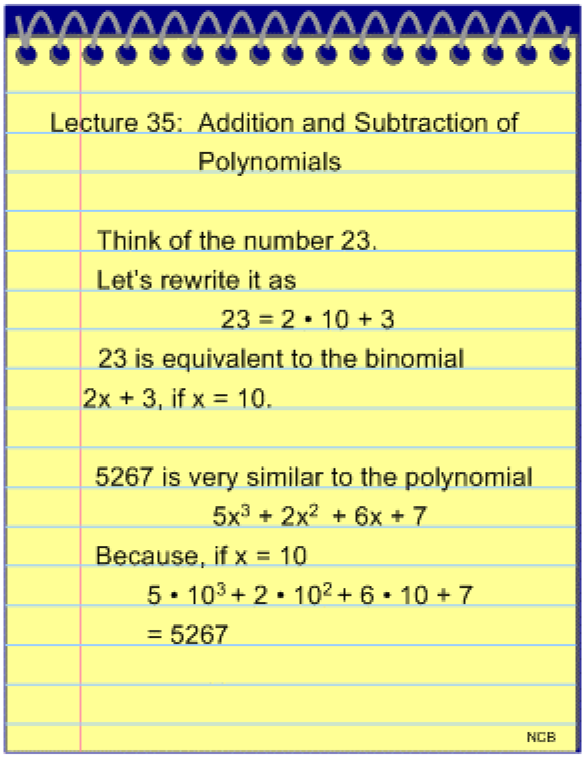
We are going to be working with polynomials a lot. We are going to learn how to factor them and to solve equations that involve polynomials.

There's a lot yet to learn about polynomials!

CH

Lecture 35 Notes

ALG2035-01



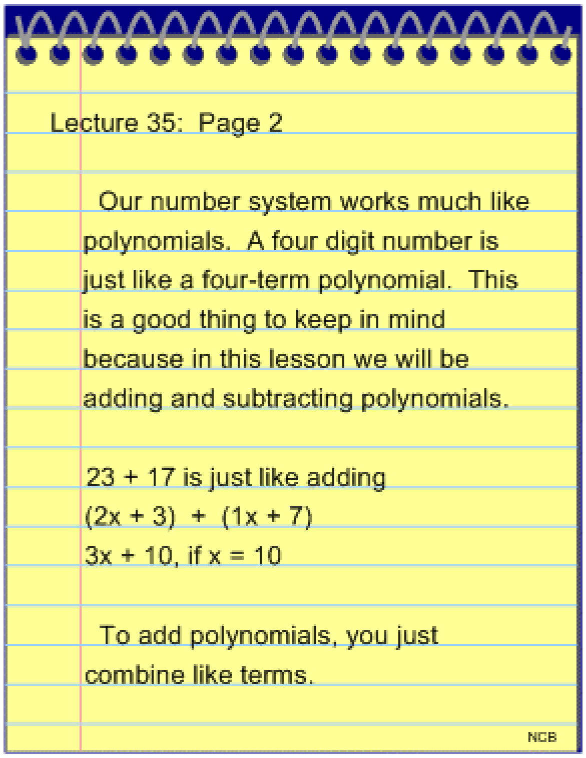
Lecture 35: Addition and Subtraction of Polynomials

Think of the number 23.
Let's rewrite it as
$23 = 2 \cdot 10 + 3$
23 is equivalent to the binomial
$2x + 3$, if $x = 10$.

5267 is very similar to the polynomial
$5x^3 + 2x^2 + 6x + 7$
Because, if $x = 10$
$5 \cdot 10^3 + 2 \cdot 10^2 + 6 \cdot 10 + 7$
$= 5267$

NCB

ALG2035-02



Lecture 35: Page 2

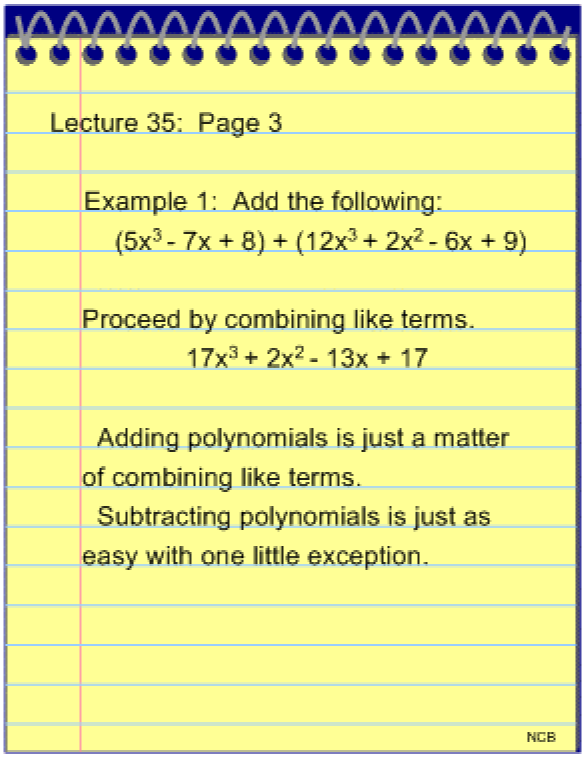
Our number system works much like polynomials. A four digit number is just like a four-term polynomial. This is a good thing to keep in mind because in this lesson we will be adding and subtracting polynomials.

23 + 17 is just like adding
$(2x + 3) + (1x + 7)$
$3x + 10$, if $x = 10$

To add polynomials, you just combine like terms.

NCB

ALG2035-03



Lecture 35: Page 3

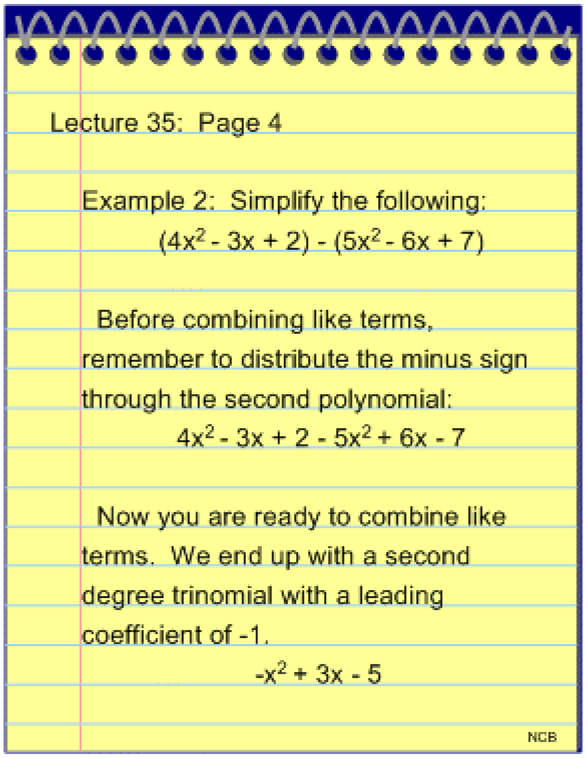
Example 1: Add the following:
$(5x^3 - 7x + 8) + (12x^3 + 2x^2 - 6x + 9)$

Proceed by combining like terms.
$17x^3 + 2x^2 - 13x + 17$

Adding polynomials is just a matter of combining like terms.
Subtracting polynomials is just as easy with one little exception.

NCB

ALG2035-04



Lecture 35: Page 4

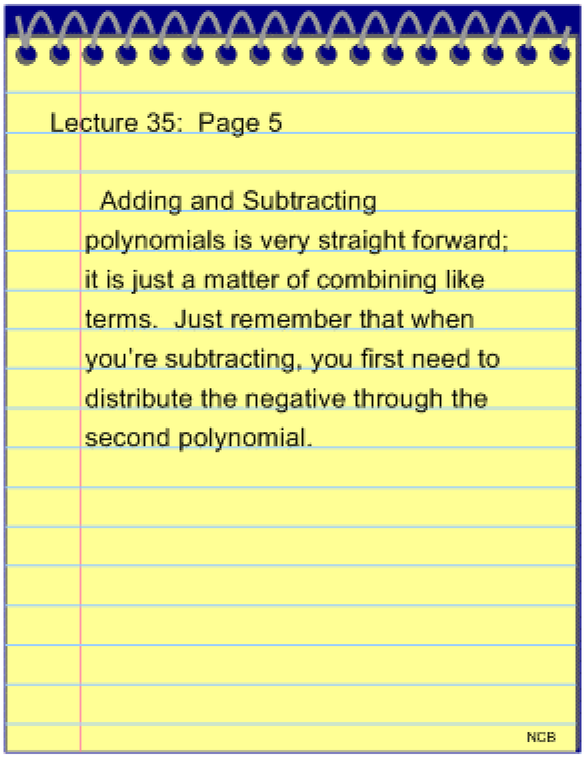
Example 2: Simplify the following:
$(4x^2 - 3x + 2) - (5x^2 - 6x + 7)$

Before combining like terms, remember to distribute the minus sign through the second polynomial:
$4x^2 - 3x + 2 - 5x^2 + 6x - 7$

Now you are ready to combine like terms. We end up with a second degree trinomial with a leading coefficient of -1.
$-x^2 + 3x - 5$

NCB

ALG2035-05



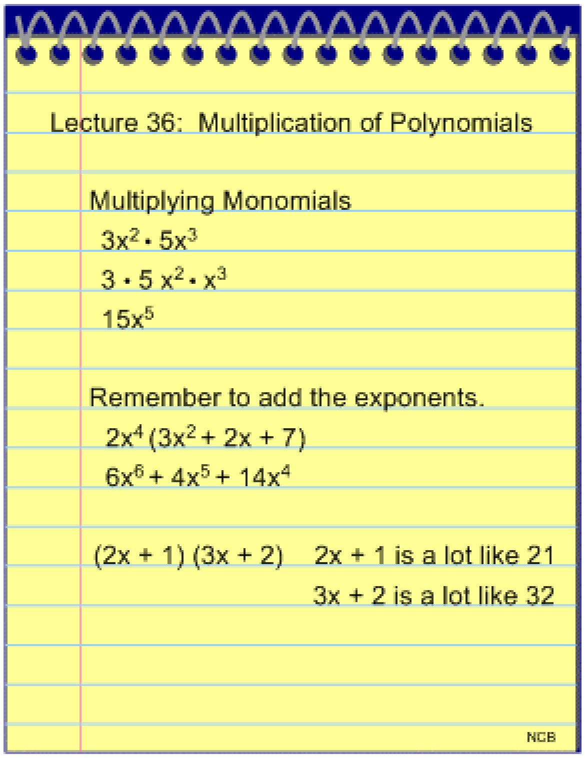
Lecture 35: Page 5

Adding and Subtracting polynomials is very straight forward; it is just a matter of combining like terms. Just remember that when you're subtracting, you first need to distribute the negative through the second polynomial.

NCB

Lecture 36 Notes

ALG2036-01



Lecture 36: Multiplication of Polynomials

Multiplying Monomials

$3x^2 \cdot 5x^3$

$3 \cdot 5\ x^2 \cdot x^3$

$15x^5$

Remember to add the exponents.

$2x^4\,(3x^2 + 2x + 7)$

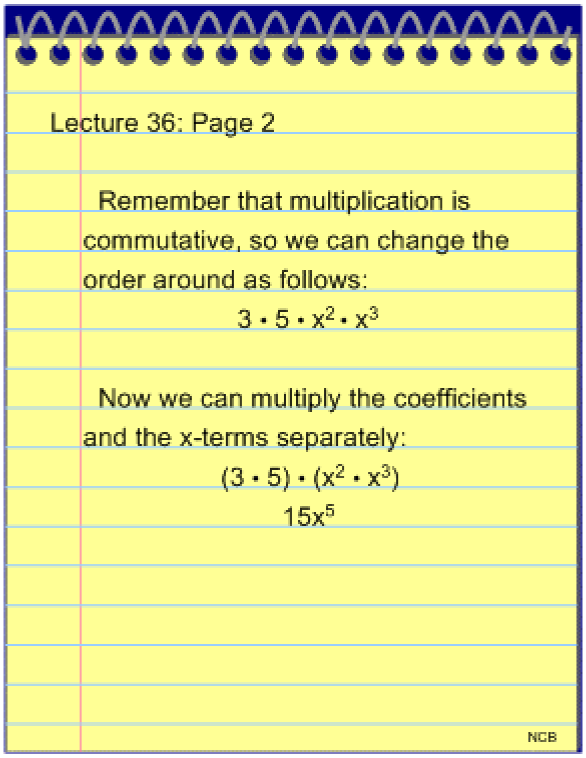
$6x^6 + 4x^5 + 14x^4$

$(2x + 1)\ (3x + 2)$ $2x + 1$ is a lot like 21

$3x + 2$ is a lot like 32

NCB

ALG2036-02



Lecture 36: Page 2

Remember that multiplication is commutative, so we can change the order around as follows:

$3 \cdot 5 \cdot x^2 \cdot x^3$

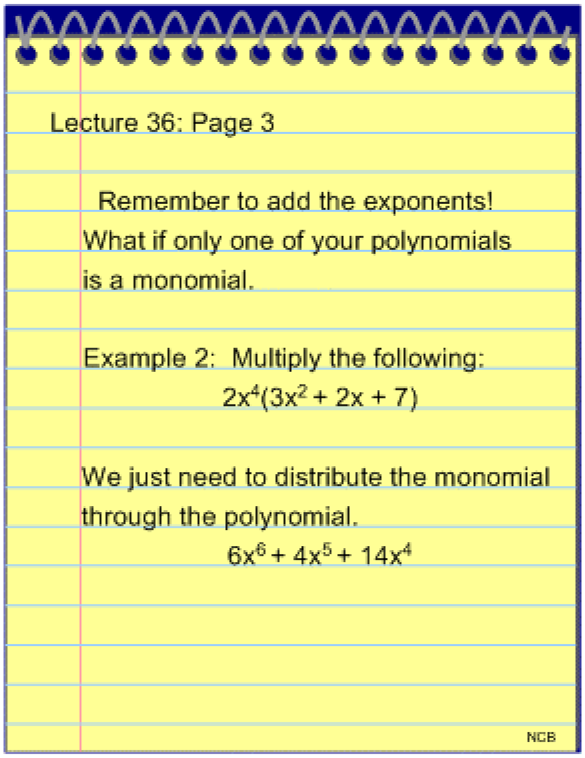
Now we can multiply the coefficients and the x-terms separately:

$(3 \cdot 5) \cdot (x^2 \cdot x^3)$

$15x^5$

NCB

ALG2036-03



Lecture 36: Page 3

Remember to add the exponents!

What if only one of your polynomials is a monomial.

Example 2: Multiply the following:

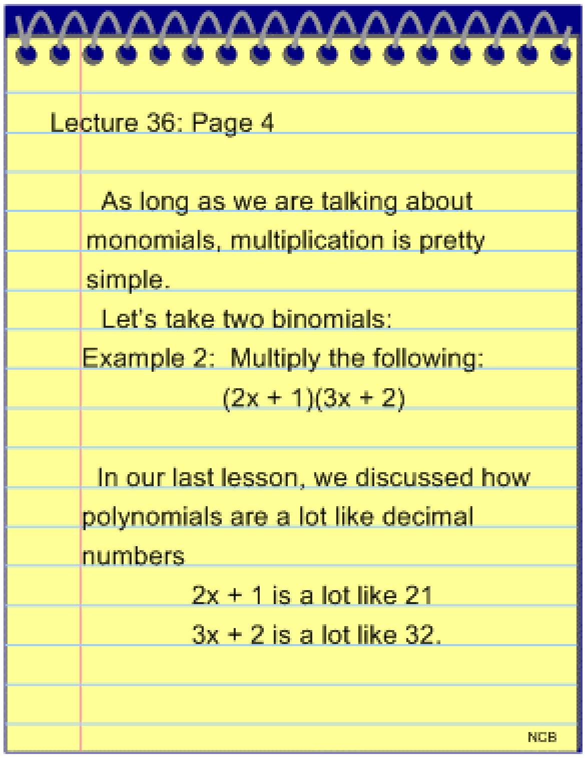
$2x^4(3x^2 + 2x + 7)$

We just need to distribute the monomial through the polynomial.

$6x^6 + 4x^5 + 14x^4$

NCB

ALG2036-04



Lecture 36: Page 4

As long as we are talking about monomials, multiplication is pretty simple.

Let's take two binomials:

Example 2: Multiply the following:

$(2x + 1)(3x + 2)$

In our last lesson, we discussed how polynomials are a lot like decimal numbers

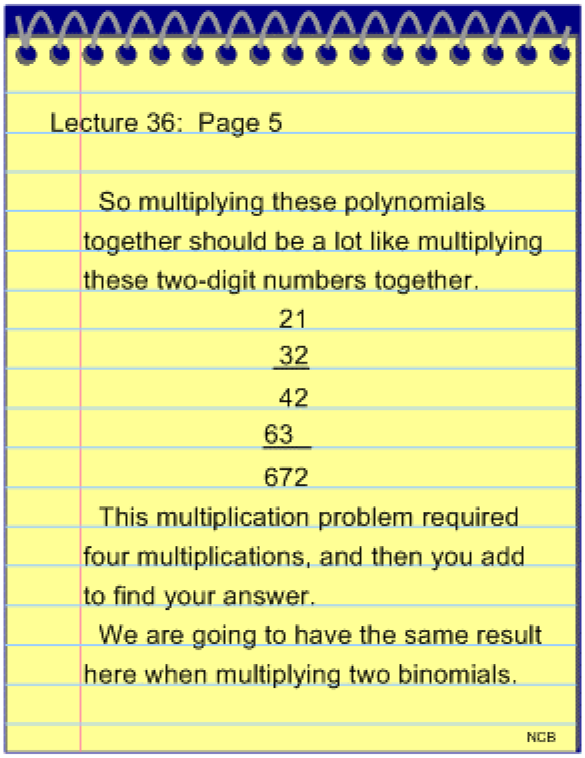
$2x + 1$ is a lot like 21

$3x + 2$ is a lot like 32.

NCB

Lecture 36 Notes, Continued

ALG2036-05



Lecture 36: Page 5

So multiplying these polynomials together should be a lot like multiplying these two-digit numbers together.

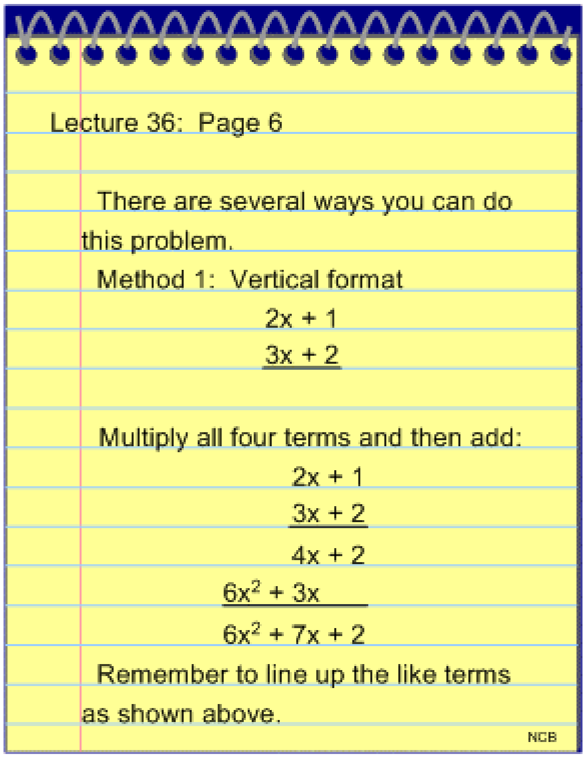
$$\begin{array}{r} 21 \\ \underline{32} \\ 42 \\ \underline{63} \\ 672 \end{array}$$

This multiplication problem required four multiplications, and then you add to find your answer.

We are going to have the same result here when multiplying two binomials.

NCB

ALG2036-06



Lecture 36: Page 6

There are several ways you can do this problem.

Method 1: Vertical format

$$\begin{array}{r} 2x + 1 \\ \underline{3x + 2} \end{array}$$

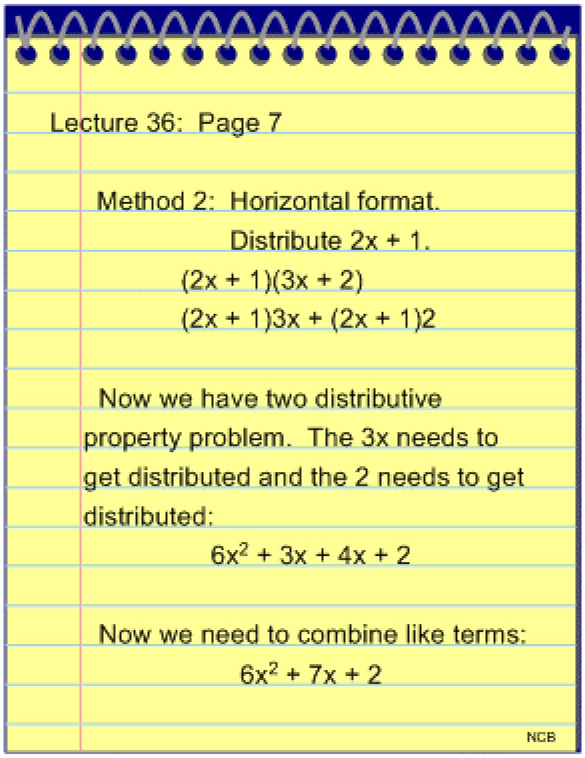
Multiply all four terms and then add:

$$\begin{array}{r} 2x + 1 \\ \underline{3x + 2} \\ 4x + 2 \\ \underline{6x^2 + 3x} \\ 6x^2 + 7x + 2 \end{array}$$

Remember to line up the like terms as shown above.

NCB

ALG2036-07



Lecture 36: Page 7

Method 2: Horizontal format.

Distribute 2x + 1.

$(2x + 1)(3x + 2)$

$(2x + 1)3x + (2x + 1)2$

Now we have two distributive property problem. The 3x needs to get distributed and the 2 needs to get distributed:

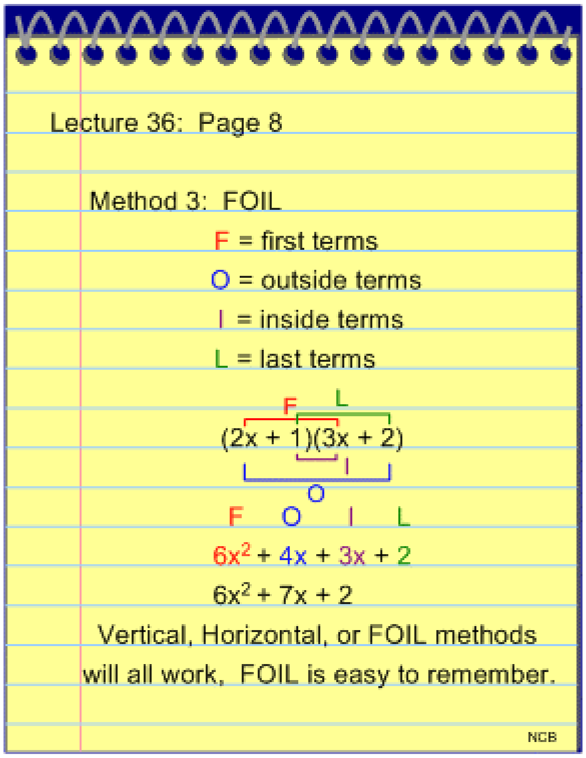
$6x^2 + 3x + 4x + 2$

Now we need to combine like terms:

$6x^2 + 7x + 2$

NCB

ALG2036-08



Lecture 36: Page 8

Method 3: FOIL

F = first terms

O = outside terms

I = inside terms

L = last terms

F L

(2x + 1)(3x + 2)

I

O

F O I L

$6x^2 + 4x + 3x + 2$

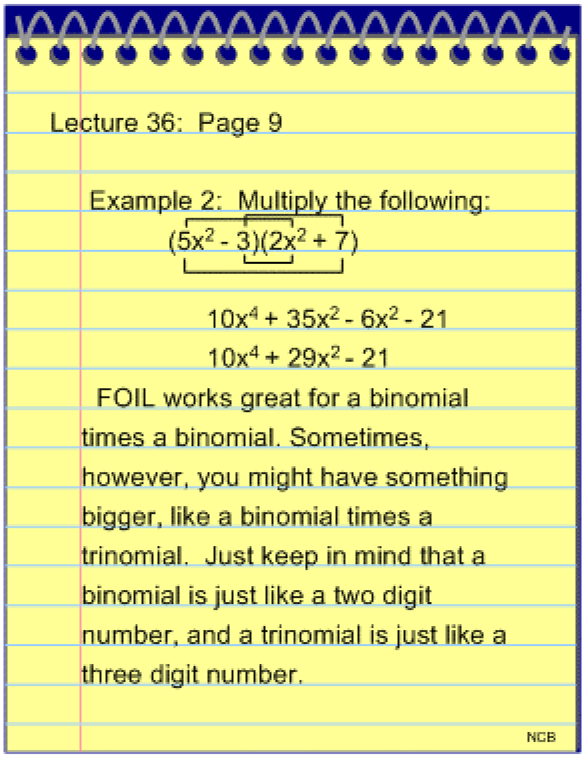
$6x^2 + 7x + 2$

Vertical, Horizontal, or FOIL methods will all work. FOIL is easy to remember.

NCB

Lecture 36 Notes, Continued

ALG2036-09



Lecture 36: Page 9

Example 2: Multiply the following:

$$(5x^2 - 3)(2x^2 + 7)$$

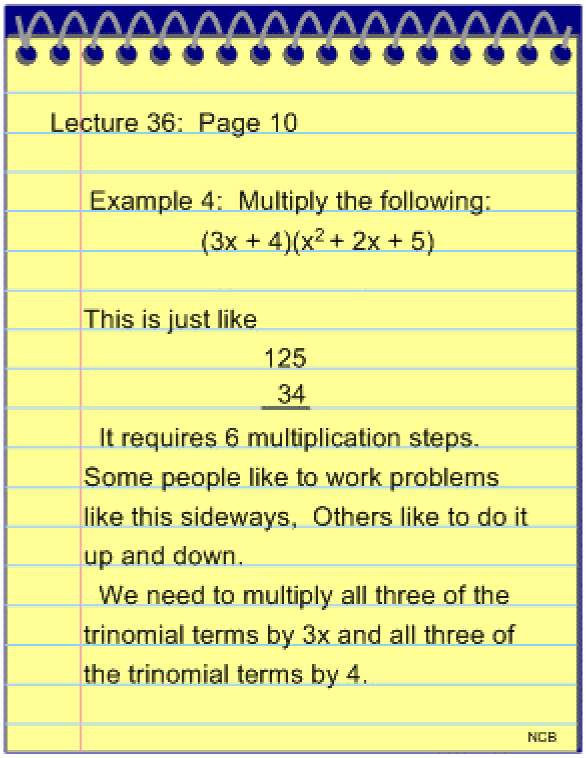
$$10x^4 + 35x^2 - 6x^2 - 21$$

$$10x^4 + 29x^2 - 21$$

FOIL works great for a binomial times a binomial. Sometimes, however, you might have something bigger, like a binomial times a trinomial. Just keep in mind that a binomial is just like a two digit number, and a trinomial is just like a three digit number.

NCB

ALG2036-10



Lecture 36: Page 10

Example 4: Multiply the following:

$$(3x + 4)(x^2 + 2x + 5)$$

This is just like

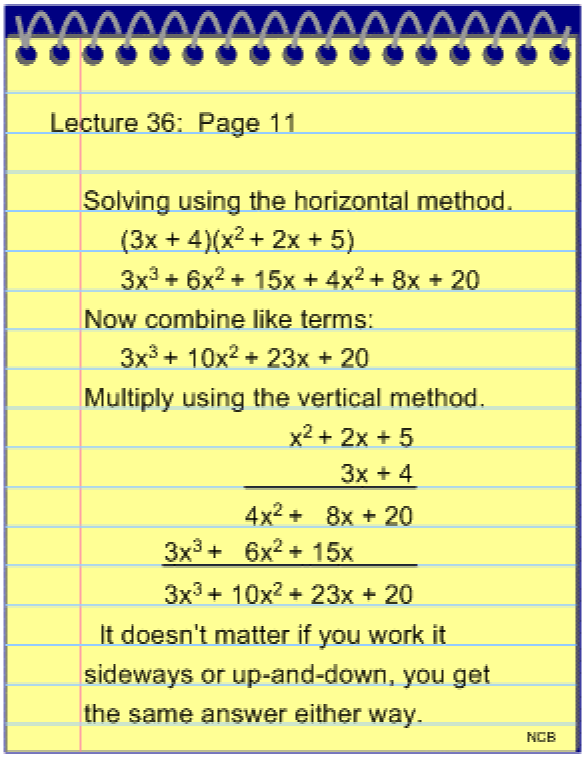
$$\begin{array}{r} 125 \\ \underline{34} \end{array}$$

It requires 6 multiplication steps. Some people like to work problems like this sideways, Others like to do it up and down.

We need to multiply all three of the trinomial terms by 3x and all three of the trinomial terms by 4.

NCB

ALG2036-11



Lecture 36: Page 11

Solving using the horizontal method.

$$(3x + 4)(x^2 + 2x + 5)$$

$$3x^3 + 6x^2 + 15x + 4x^2 + 8x + 20$$

Now combine like terms:

$$3x^3 + 10x^2 + 23x + 20$$

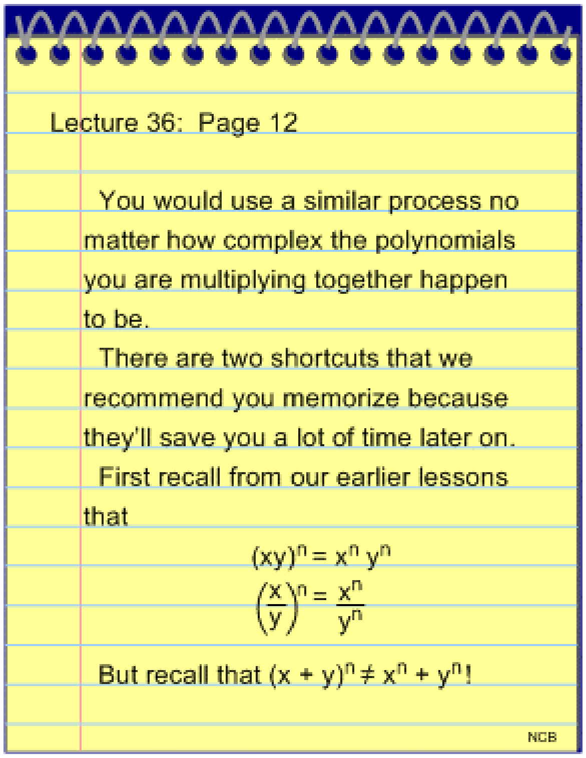
Multiply using the vertical method.

$$\begin{array}{r} x^2 + 2x + 5 \\ \underline{3x + 4} \\ 4x^2 + 8x + 20 \\ \underline{3x^3 + 6x^2 + 15x } \\ 3x^3 + 10x^2 + 23x + 20 \end{array}$$

It doesn't matter if you work it sideways or up-and-down, you get the same answer either way.

NCB

ALG2036-12



Lecture 36: Page 12

You would use a similar process no matter how complex the polynomials you are multiplying together happen to be.

There are two shortcuts that we recommend you memorize because they'll save you a lot of time later on.

First recall from our earlier lessons that

$$(xy)^n = x^n y^n$$

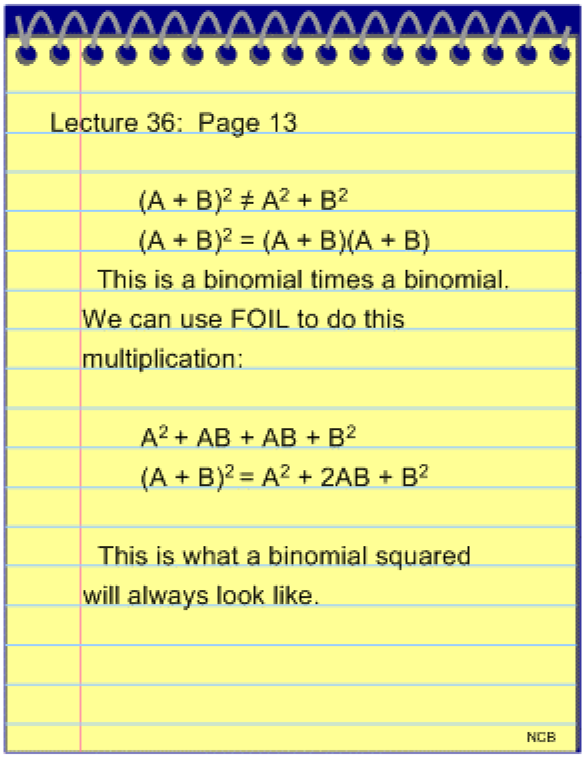
$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

But recall that $(x + y)^n \neq x^n + y^n$!

NCB

Lecture 36 Notes, Continued

ALG2036-13



Lecture 36: Page 13

$(A + B)^2 \neq A^2 + B^2$

$(A + B)^2 = (A + B)(A + B)$

This is a binomial times a binomial. We can use FOIL to do this multiplication:

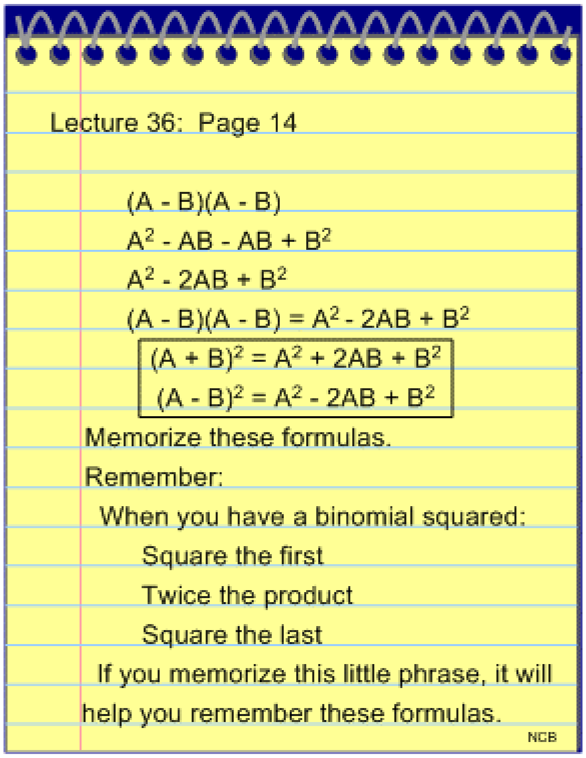
$A^2 + AB + AB + B^2$

$(A + B)^2 = A^2 + 2AB + B^2$

This is what a binomial squared will always look like.

NCB

ALG2036-14



Lecture 36: Page 14

$(A - B)(A - B)$

$A^2 - AB - AB + B^2$

$A^2 - 2AB + B^2$

$(A - B)(A - B) = A^2 - 2AB + B^2$

$$\boxed{\begin{array}{c}(A + B)^2 = A^2 + 2AB + B^2\\ (A - B)^2 = A^2 - 2AB + B^2\end{array}}$$

Memorize these formulas.

Remember:

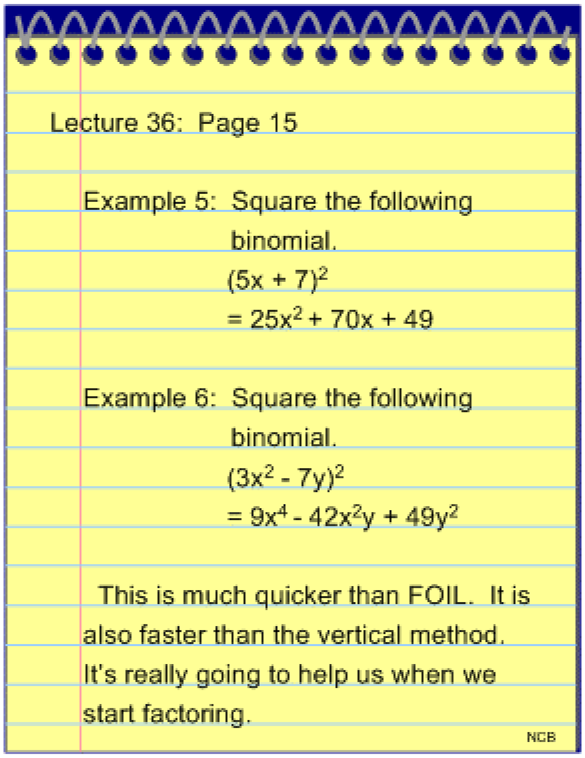
When you have a binomial squared:

Square the first
Twice the product
Square the last

If you memorize this little phrase, it will help you remember these formulas.

NCB

ALG2036-15



Lecture 36: Page 15

Example 5: Square the following binomial.

$(5x + 7)^2$

$= 25x^2 + 70x + 49$

Example 6: Square the following binomial.

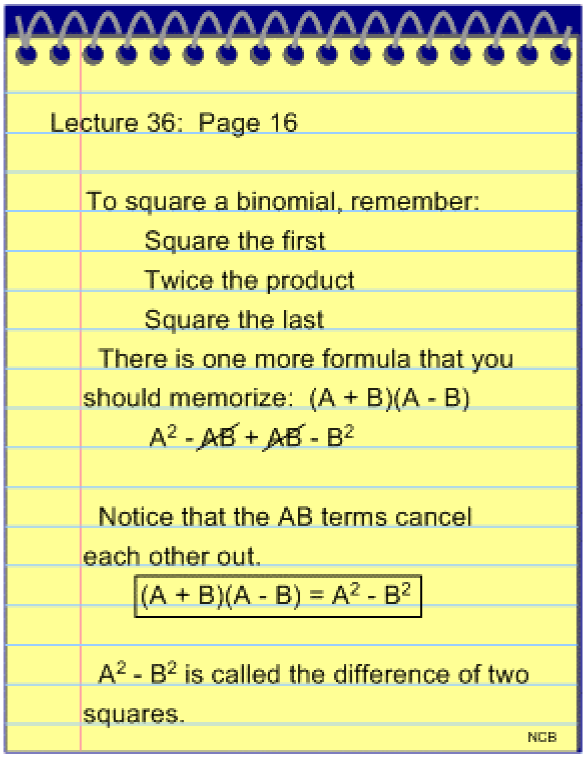
$(3x^2 - 7y)^2$

$= 9x^4 - 42x^2y + 49y^2$

This is much quicker than FOIL. It is also faster than the vertical method. It's really going to help us when we start factoring.

NCB

ALG2036-16



Lecture 36: Page 16

To square a binomial, remember:

Square the first
Twice the product
Square the last

There is one more formula that you should memorize: $(A + B)(A - B)$

$A^2 - \cancel{AB} + \cancel{AB} - B^2$

Notice that the AB terms cancel each other out.

$\boxed{(A + B)(A - B) = A^2 - B^2}$

$A^2 - B^2$ is called the difference of two squares.

NCB

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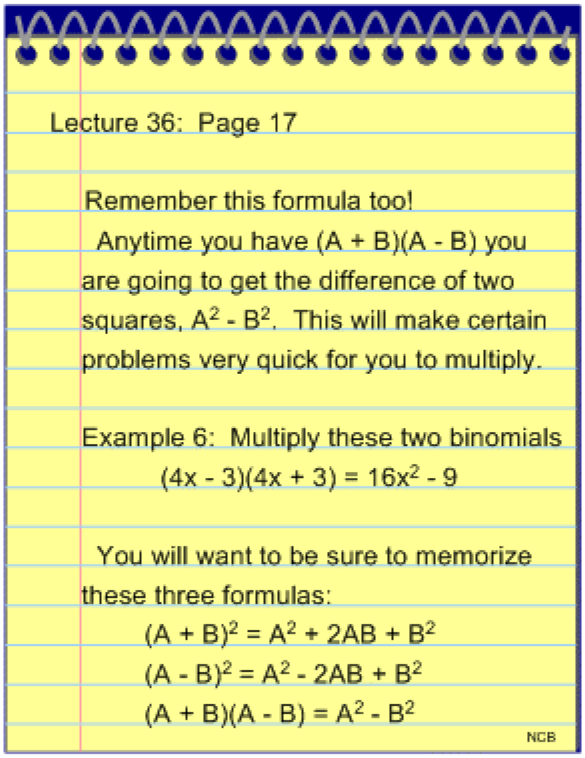

Lecture 36: Page 17

Remember this formula too!

Anytime you have $(A + B)(A - B)$ you are going to get the difference of two squares, $A^2 - B^2$. This will make certain problems very quick for you to multiply.

Example 6: Multiply these two binomials

$$(4x - 3)(4x + 3) = 16x^2 - 9$$

You will want to be sure to memorize these three formulas:

$$(A + B)^2 = A^2 + 2AB + B^2$$

$$(A - B)^2 = A^2 - 2AB + B^2$$

$$(A + B)(A - B) = A^2 - B^2$$

NCB

Lecture 37 Notes

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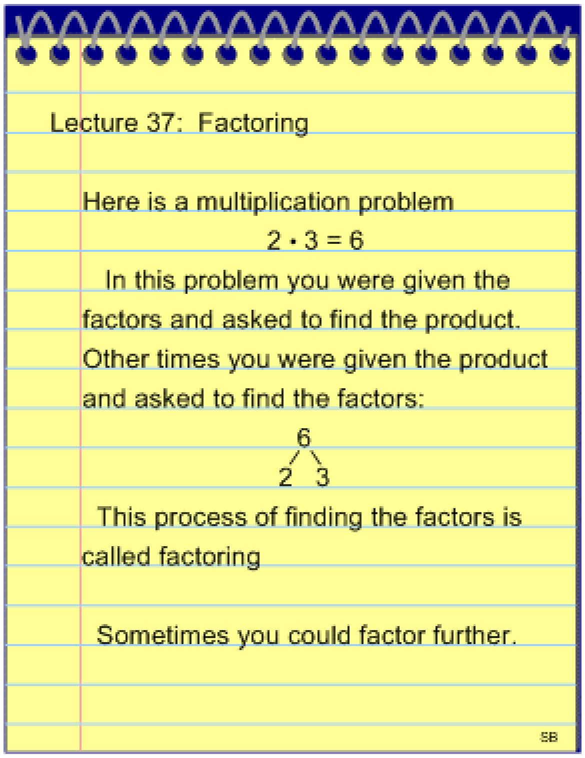


Lecture 37: Factoring

Here is a multiplication problem

$2 \cdot 3 = 6$

In this problem you were given the factors and asked to find the product. Other times you were given the product and asked to find the factors:

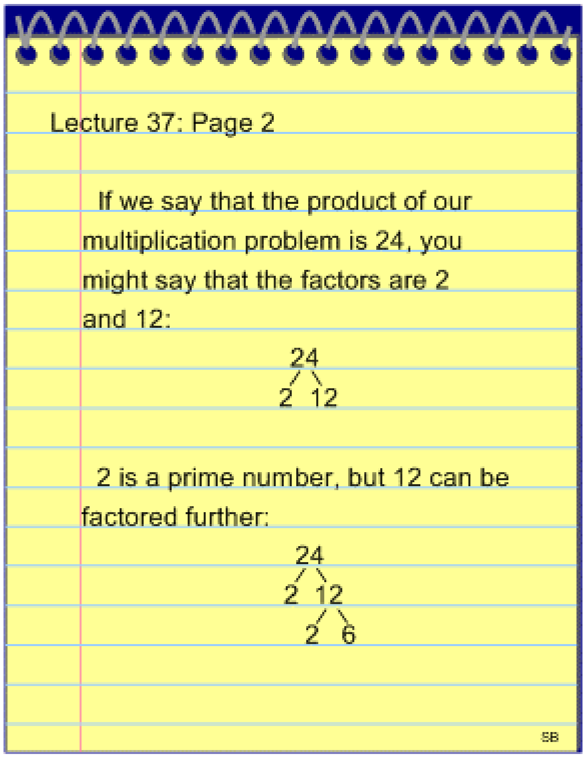
6
/ \
2 3

This process of finding the factors is called factoring

Sometimes you could factor further.

SB

ALG2037-02



Lecture 37: Page 2

If we say that the product of our multiplication problem is 24, you might say that the factors are 2 and 12:

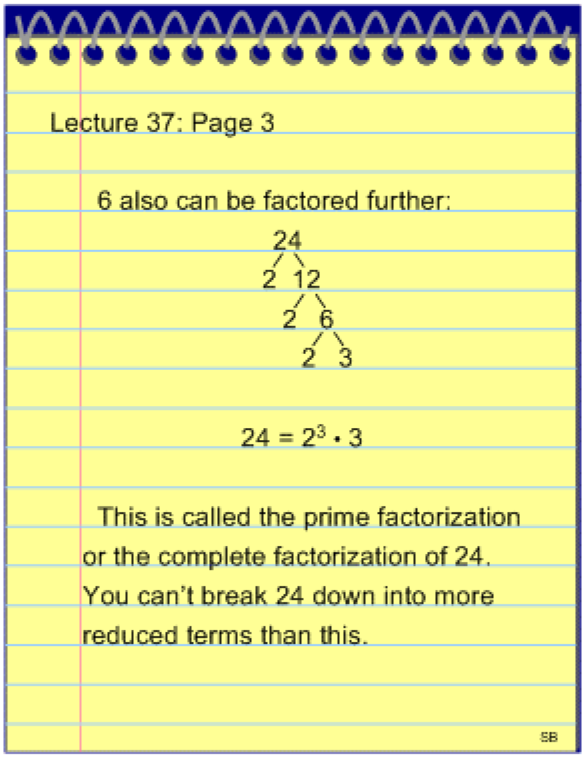
24
/ \
2 12

2 is a prime number, but 12 can be factored further:

24
/ \
2 12
/ \
2 6

SB

ALG2037-03



Lecture 37: Page 3

6 also can be factored further:

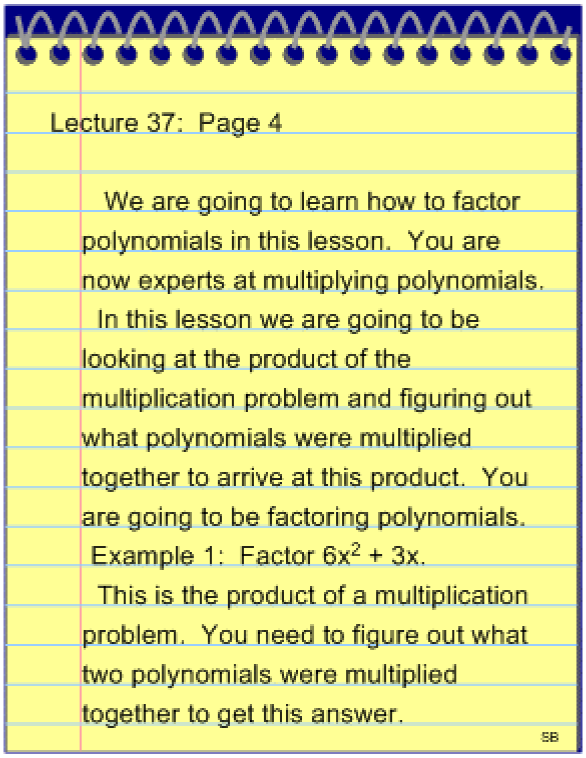
24
/ \
2 12
/ \
2 6
/ \
2 3

$24 = 2^3 \cdot 3$

This is called the prime factorization or the complete factorization of 24. You can't break 24 down into more reduced terms than this.

SB

ALG2037-04



Lecture 37: Page 4

We are going to learn how to factor polynomials in this lesson. You are now experts at multiplying polynomials. In this lesson we are going to be looking at the product of the multiplication problem and figuring out what polynomials were multiplied together to arrive at this product. You are going to be factoring polynomials.

Example 1: Factor $6x^2 + 3x$.

This is the product of a multiplication problem. You need to figure out what two polynomials were multiplied together to get this answer.

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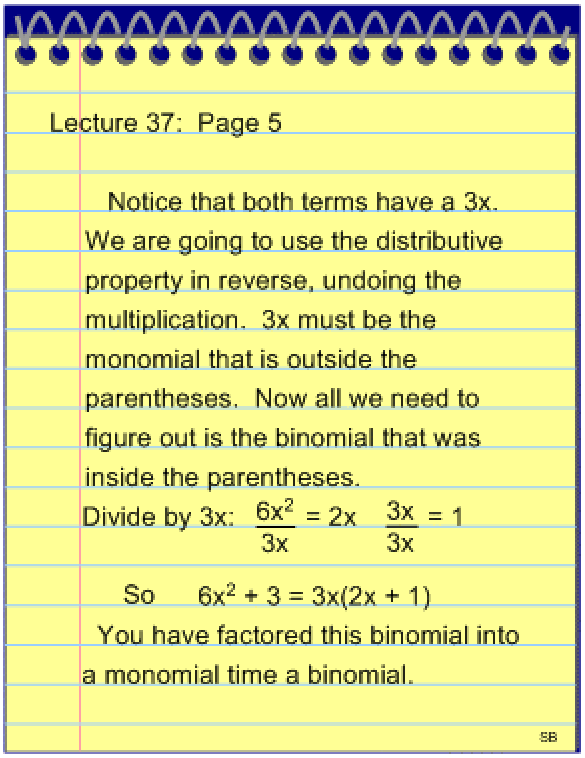
Lecture 37 Notes

ALG2037-05



Lecture 37: Page 5

Notice that both terms have a 3x. We are going to use the distributive property in reverse, undoing the multiplication. 3x must be the monomial that is outside the parentheses. Now all we need to figure out is the binomial that was inside the parentheses.

Divide by 3x: $\frac{6x^2}{3x} = 2x$ $\frac{3x}{3x} = 1$

So $6x^2 + 3 = 3x(2x + 1)$

You have factored this binomial into a monomial time a binomial.

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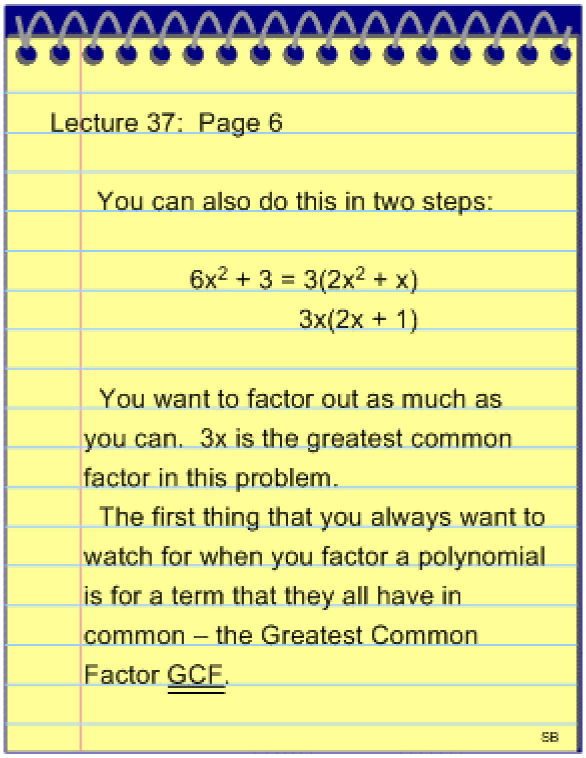

Lecture 37: Page 6

You can also do this in two steps:

$$6x^2 + 3 = 3(2x^2 + x)$$
$$3x(2x + 1)$$

You want to factor out as much as you can. 3x is the greatest common factor in this problem.

The first thing that you always want to watch for when you factor a polynomial is for a term that they all have in common – the Greatest Common Factor GCF.

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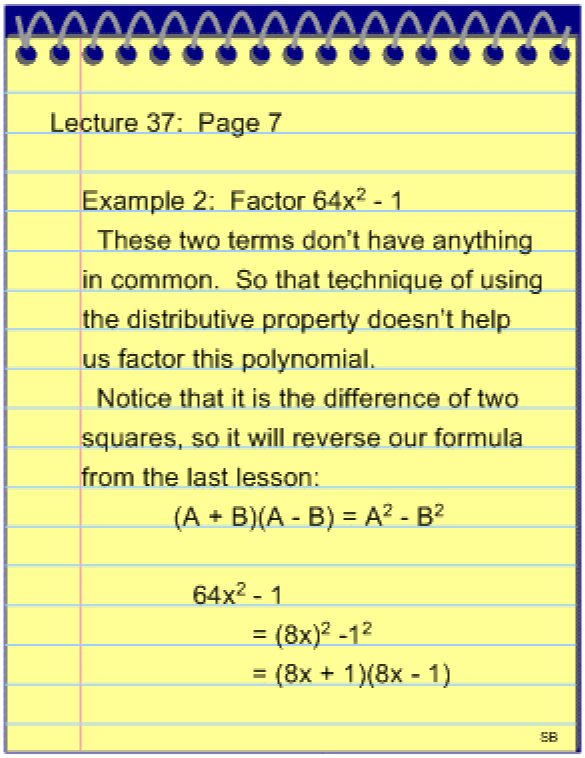

Lecture 37: Page 7

Example 2: Factor $64x^2 - 1$

These two terms don't have anything in common. So that technique of using the distributive property doesn't help us factor this polynomial.

Notice that it is the difference of two squares, so it will reverse our formula from the last lesson:

$$(A + B)(A - B) = A^2 - B^2$$

$$64x^2 - 1$$
$$= (8x)^2 - 1^2$$
$$= (8x + 1)(8x - 1)$$

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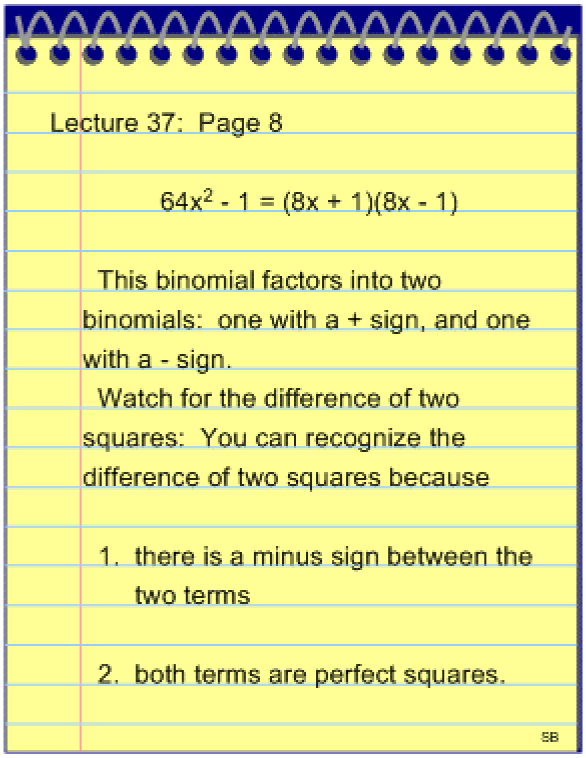

Lecture 37: Page 8

$$64x^2 - 1 = (8x + 1)(8x - 1)$$

This binomial factors into two binomials: one with a + sign, and one with a - sign.

Watch for the difference of two squares: You can recognize the difference of two squares because

1. there is a minus sign between the two terms

2. both terms are perfect squares.

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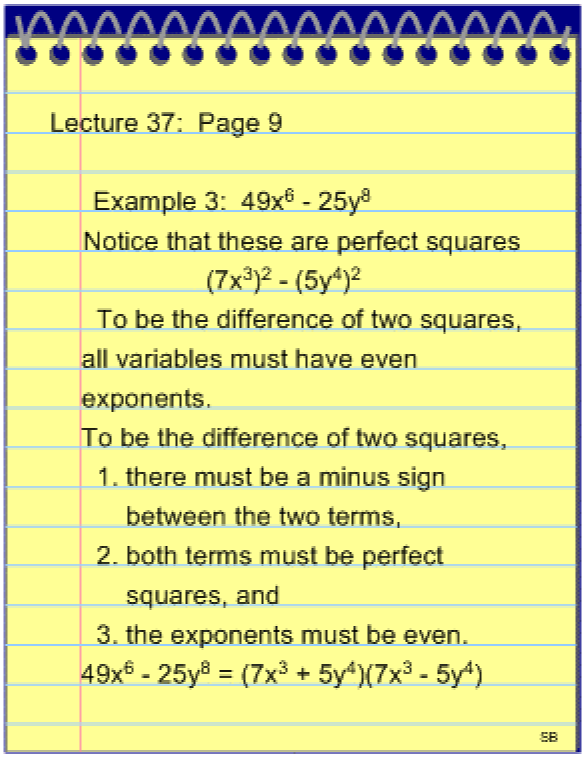
Lecture 37 Notes

ALG2037-09



Lecture 37: Page 9

Example 3: $49x^6 - 25y^8$

Notice that these are perfect squares

$(7x^3)^2 - (5y^4)^2$

To be the difference of two squares, all variables must have even exponents.

To be the difference of two squares,

1. there must be a minus sign between the two terms,
2. both terms must be perfect squares, and
3. the exponents must be even.

$49x^6 - 25y^8 = (7x^3 + 5y^4)(7x^3 - 5y^4)$

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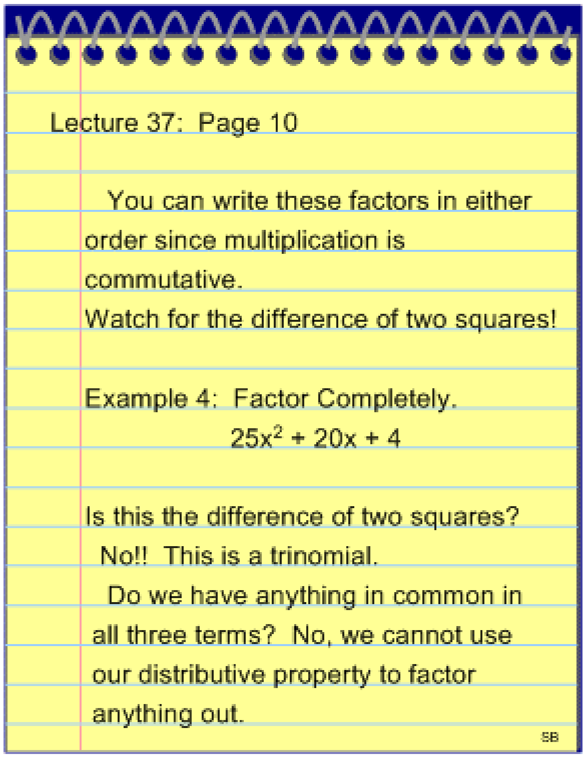
ALG2037-10



Lecture 37: Page 10

You can write these factors in either order since multiplication is commutative.

Watch for the difference of two squares!

Example 4: Factor Completely.

$25x^2 + 20x + 4$

Is this the difference of two squares?

No!! This is a trinomial.

Do we have anything in common in all three terms? No, we cannot use our distributive property to factor anything out.

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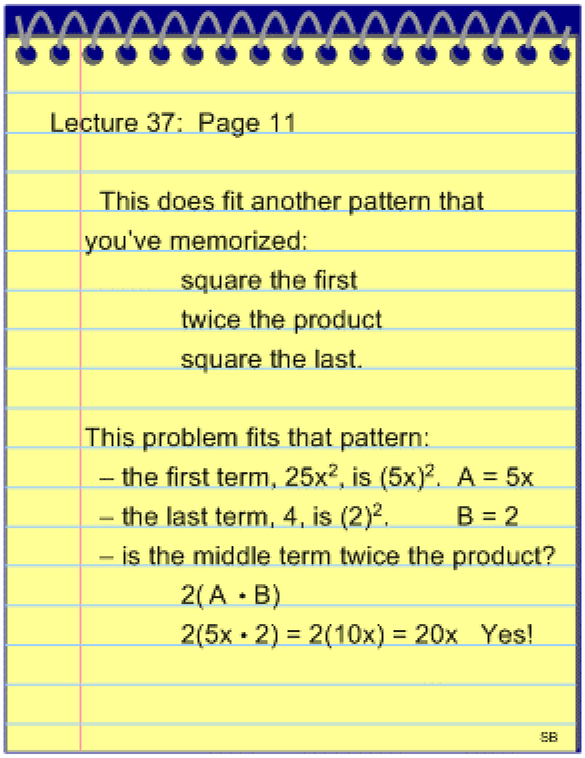
ALG2037-11



Lecture 37: Page 11

This does fit another pattern that you've memorized:

square the first
twice the product
square the last.

This problem fits that pattern:

– the first term, $25x^2$, is $(5x)^2$. $A = 5x$
– the last term, 4, is $(2)^2$. $B = 2$
– is the middle term twice the product?

$2(A \cdot B)$

$2(5x \cdot 2) = 2(10x) = 20x$ Yes!

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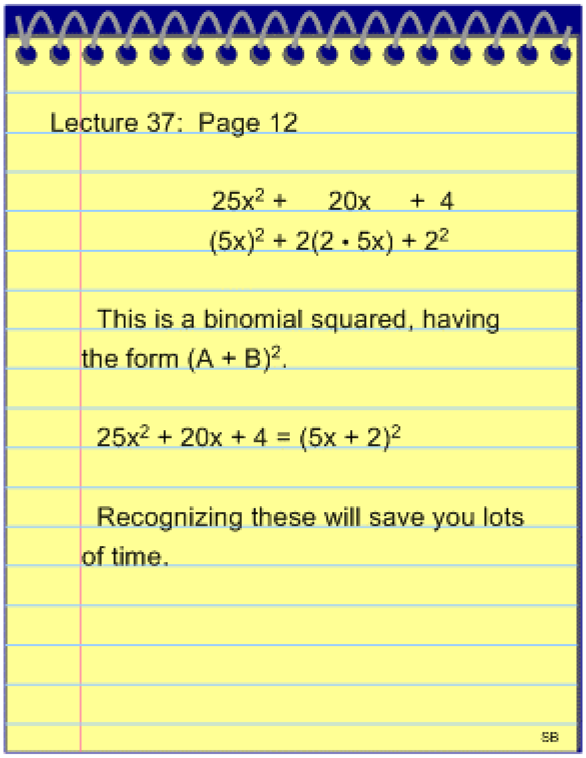
ALG2037-12



Lecture 37: Page 12

$25x^2 + \quad 20x \quad + \ 4$

$(5x)^2 + 2(2 \cdot 5x) + 2^2$

This is a binomial squared, having the form $(A + B)^2$.

$25x^2 + 20x + 4 = (5x + 2)^2$

Recognizing these will save you lots of time.

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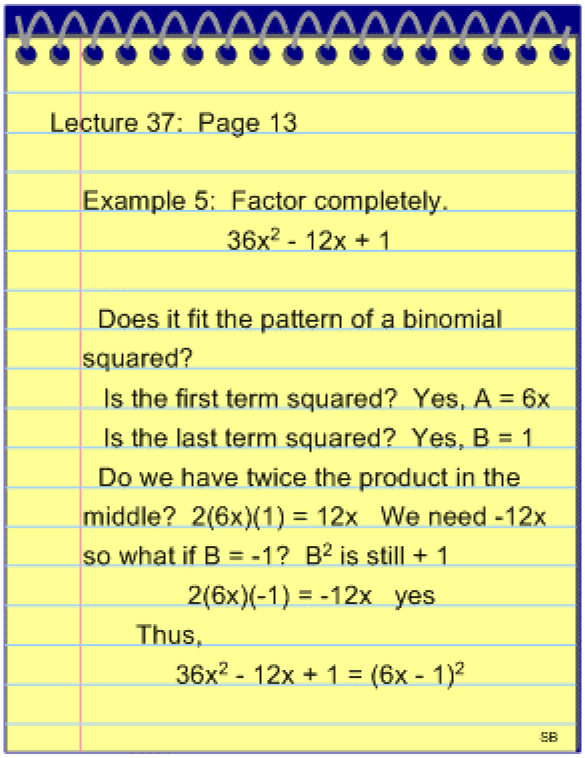
Lecture 37 Notes

ALG2037-13



Lecture 37: Page 13

Example 5: Factor completely.

$36x^2 - 12x + 1$

Does it fit the pattern of a binomial squared?

Is the first term squared? Yes, A = 6x

Is the last term squared? Yes, B = 1

Do we have twice the product in the middle? 2(6x)(1) = 12x We need -12x so what if B = -1? B^2 is still + 1

2(6x)(-1) = -12x yes

Thus,

$36x^2 - 12x + 1 = (6x - 1)^2$

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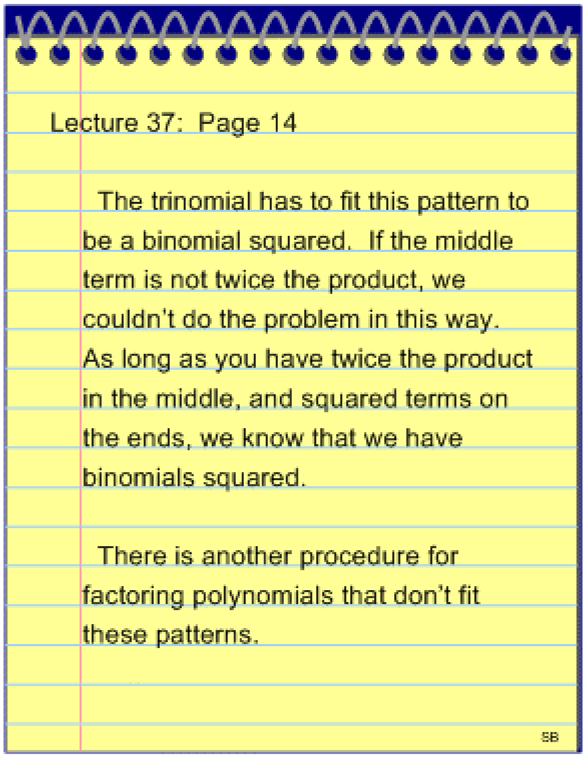
ALG2037-14



Lecture 37: Page 14

The trinomial has to fit this pattern to be a binomial squared. If the middle term is not twice the product, we couldn't do the problem in this way. As long as you have twice the product in the middle, and squared terms on the ends, we know that we have binomials squared.

There is another procedure for factoring polynomials that don't fit these patterns.

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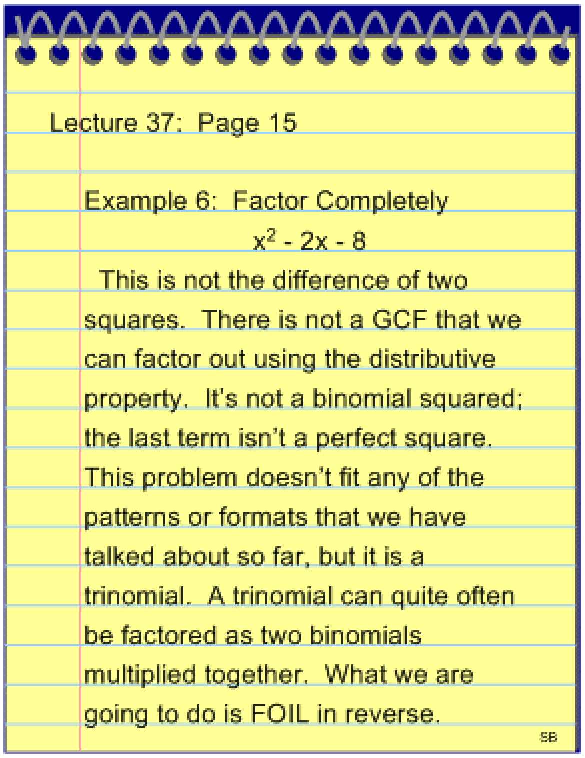
ALG2037-15



Lecture 37: Page 15

Example 6: Factor Completely

$x^2 - 2x - 8$

This is not the difference of two squares. There is not a GCF that we can factor out using the distributive property. It's not a binomial squared; the last term isn't a perfect square. This problem doesn't fit any of the patterns or formats that we have talked about so far, but it is a trinomial. A trinomial can quite often be factored as two binomials multiplied together. What we are going to do is FOIL in reverse.

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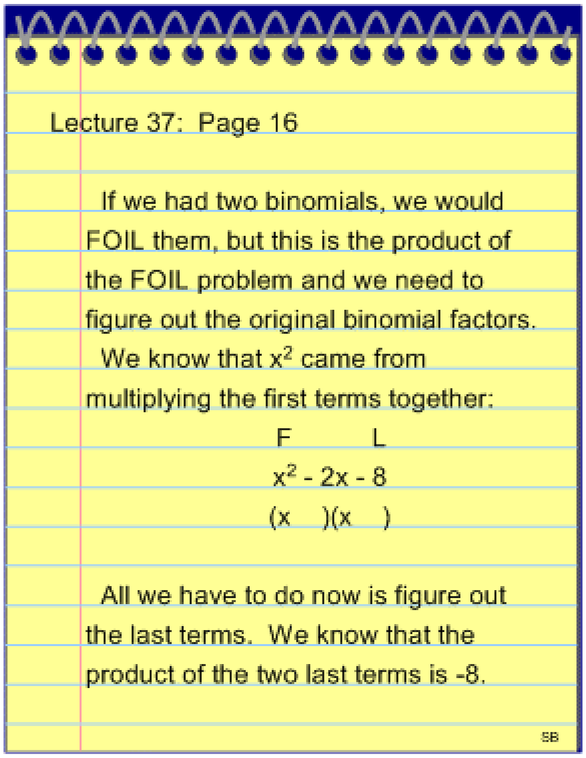
ALG2037-16



Lecture 37: Page 16

If we had two binomials, we would FOIL them, but this is the product of the FOIL problem and we need to figure out the original binomial factors.

We know that x^2 came from multiplying the first terms together:

F L

$x^2 - 2x - 8$

(x)(x)

All we have to do now is figure out the last terms. We know that the product of the two last terms is -8.

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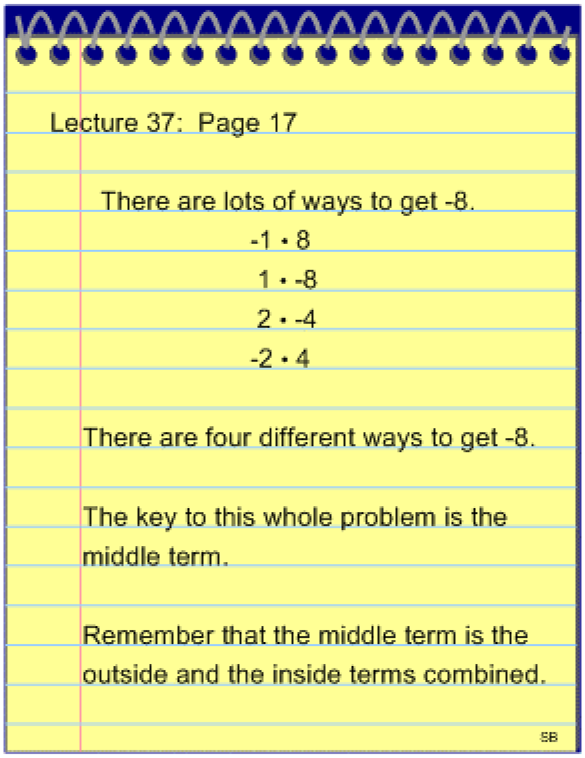
Lecture 37 Notes

ALG2037-17



Lecture 37: Page 17

There are lots of ways to get -8.

-1 • 8

1 • -8

2 • -4

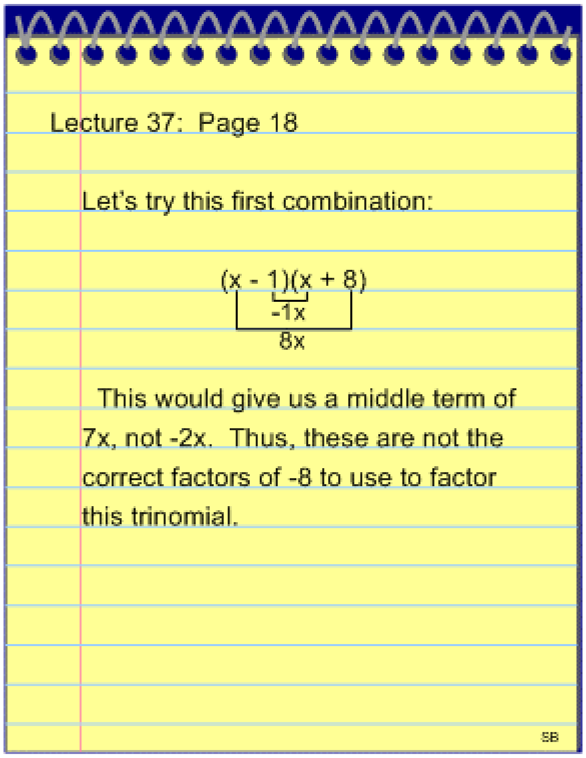
-2 • 4

There are four different ways to get -8.

The key to this whole problem is the middle term.

Remember that the middle term is the outside and the inside terms combined.

ALG2037-18



Lecture 37: Page 18

Let's try this first combination:

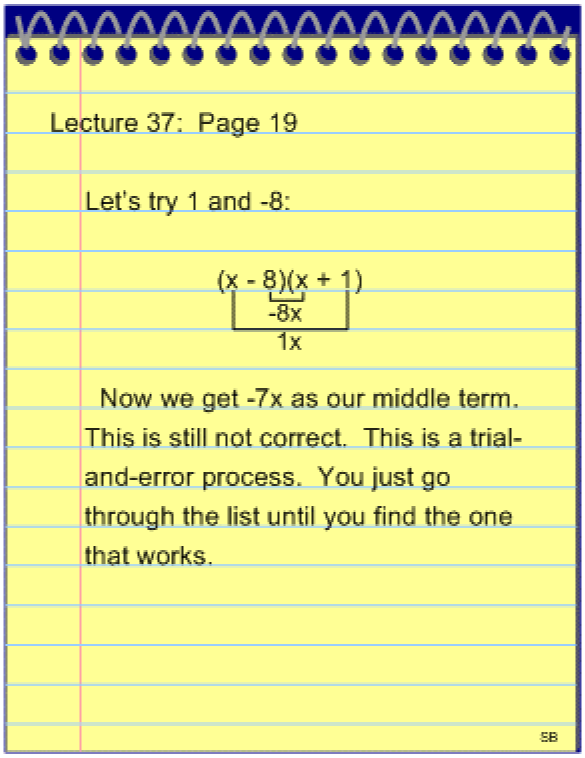
(x - 1)(x + 8)

-1x

8x

This would give us a middle term of 7x, not -2x. Thus, these are not the correct factors of -8 to use to factor this trinomial.

ALG2037-19



Lecture 37: Page 19

Let's try 1 and -8:

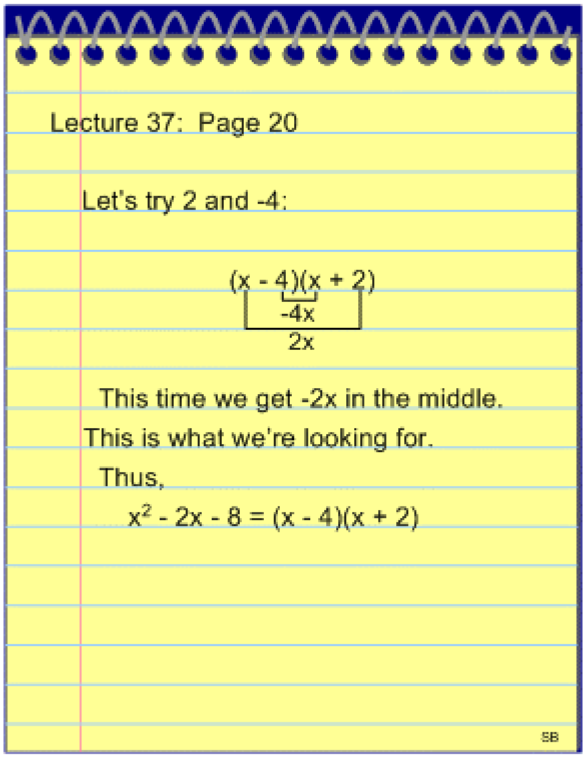
(x - 8)(x + 1)

-8x

1x

Now we get -7x as our middle term. This is still not correct. This is a trial-and-error process. You just go through the list until you find the one that works.

ALG2037-20



Lecture 37: Page 20

Let's try 2 and -4:

(x - 4)(x + 2)

-4x

2x

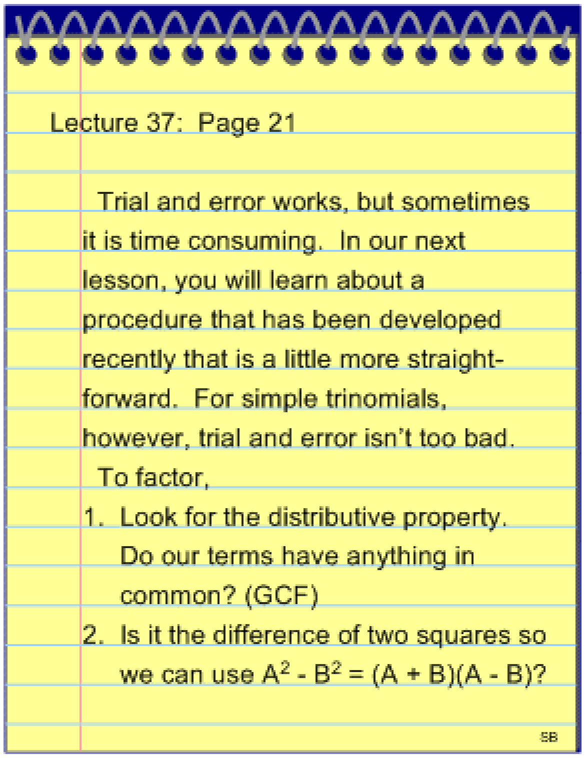
This time we get -2x in the middle. This is what we're looking for.

Thus,

$x^2 - 2x - 8 = (x - 4)(x + 2)$

Lecture 37 Notes

ALG2037-21



Lecture 37: Page 21

Trial and error works, but sometimes it is time consuming. In our next lesson, you will learn about a procedure that has been developed recently that is a little more straightforward. For simple trinomials, however, trial and error isn't too bad.

To factor,

1. Look for the distributive property. Do our terms have anything in common? (GCF)
2. Is it the difference of two squares so we can use $A^2 - B^2 = (A + B)(A - B)$?

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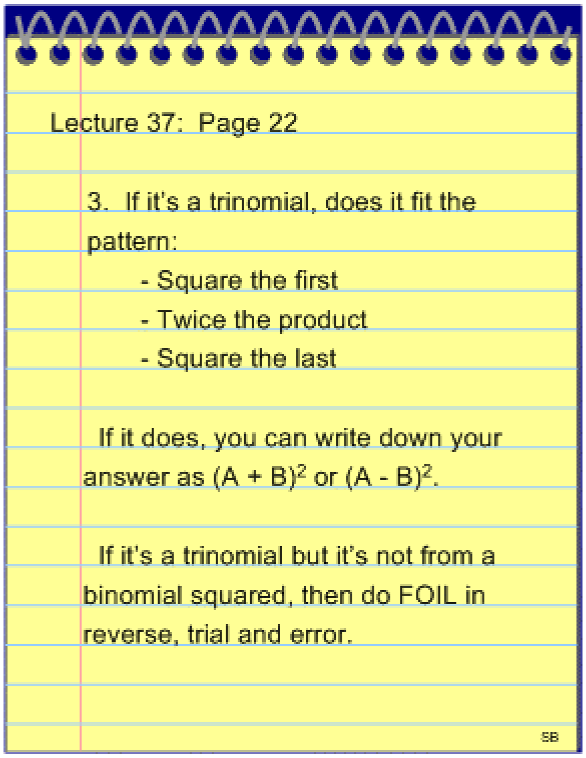
ALG2037-22



Lecture 37: Page 22

3. If it's a trinomial, does it fit the pattern:
    - Square the first
    - Twice the product
    - Square the last

If it does, you can write down your answer as $(A + B)^2$ or $(A - B)^2$.

If it's a trinomial but it's not from a binomial squared, then do FOIL in reverse, trial and error.

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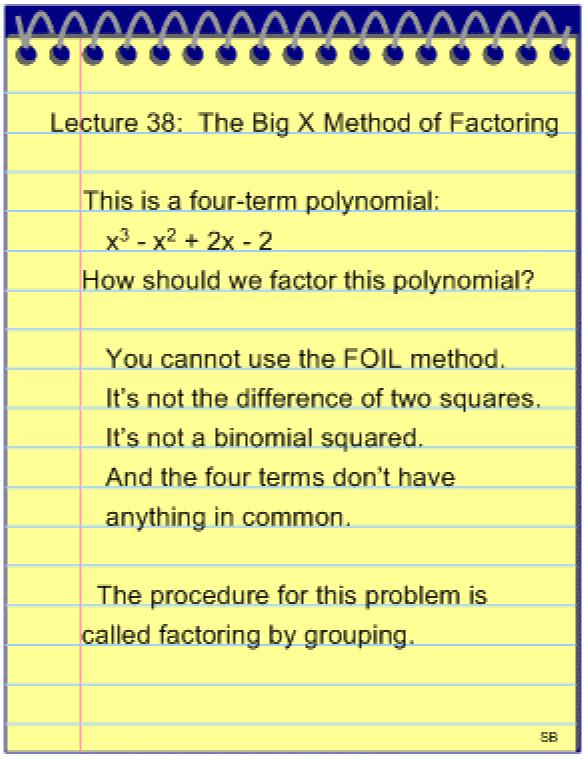
Lecture 38 Notes

ALG2038-01



Lecture 38: The Big X Method of Factoring

This is a four-term polynomial:

$x^3 - x^2 + 2x - 2$

How should we factor this polynomial?

You cannot use the FOIL method.

It's not the difference of two squares.

It's not a binomial squared.

And the four terms don't have anything in common.

The procedure for this problem is called factoring by grouping.

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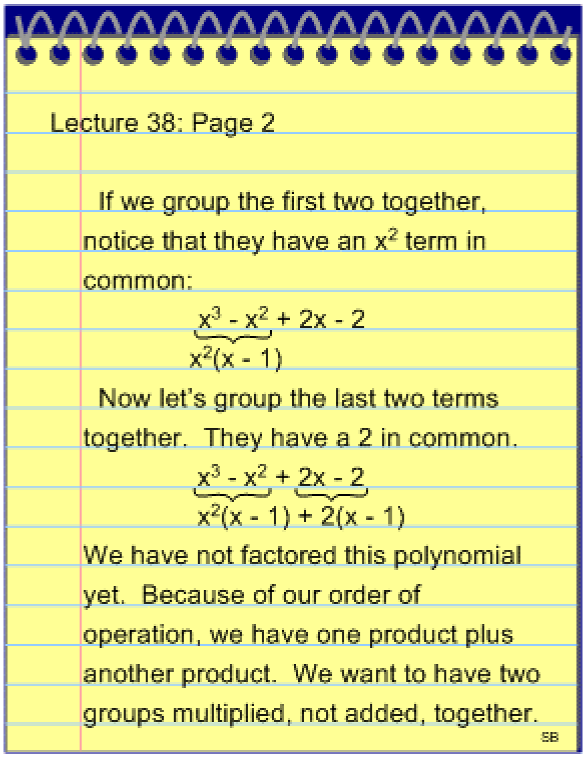
ALG2038-02



Lecture 38: Page 2

If we group the first two together, notice that they have an x^2 term in common:

$$\underbrace{x^3 - x^2}_{x^2(x - 1)} + 2x - 2$$

Now let's group the last two terms together. They have a 2 in common.

$$\underbrace{x^3 - x^2}_{x^2(x - 1)} + \underbrace{2x - 2}_{2(x - 1)}$$

We have not factored this polynomial yet. Because of our order of operation, we have one product plus another product. We want to have two groups multiplied, not added, together.

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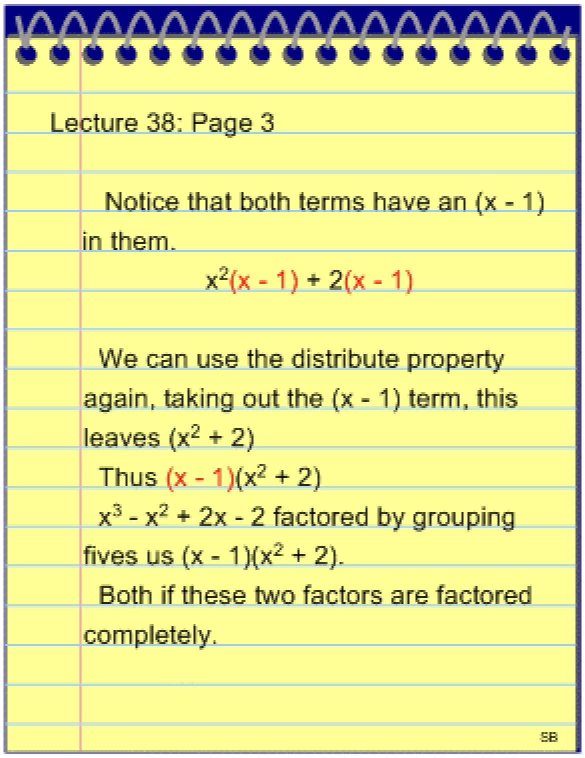

Lecture 38: Page 3

Notice that both terms have an (x - 1) in them.

$$x^2(x - 1) + 2(x - 1)$$

We can use the distribute property again, taking out the (x - 1) term, this leaves $(x^2 + 2)$

Thus $(x - 1)(x^2 + 2)$

$x^3 - x^2 + 2x - 2$ factored by grouping fives us $(x - 1)(x^2 + 2)$.

Both if these two factors are factored completely.

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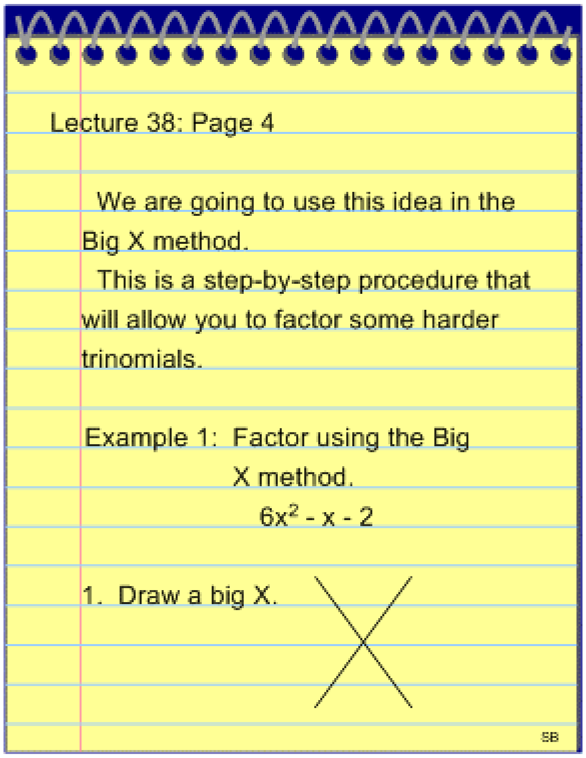

Lecture 38: Page 4

We are going to use this idea in the Big X method.

This is a step-by-step procedure that will allow you to factor some harder trinomials.

Example 1: Factor using the Big X method.

$6x^2 - x - 2$

1. Draw a big X.

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Lecture 38 Notes, Continued

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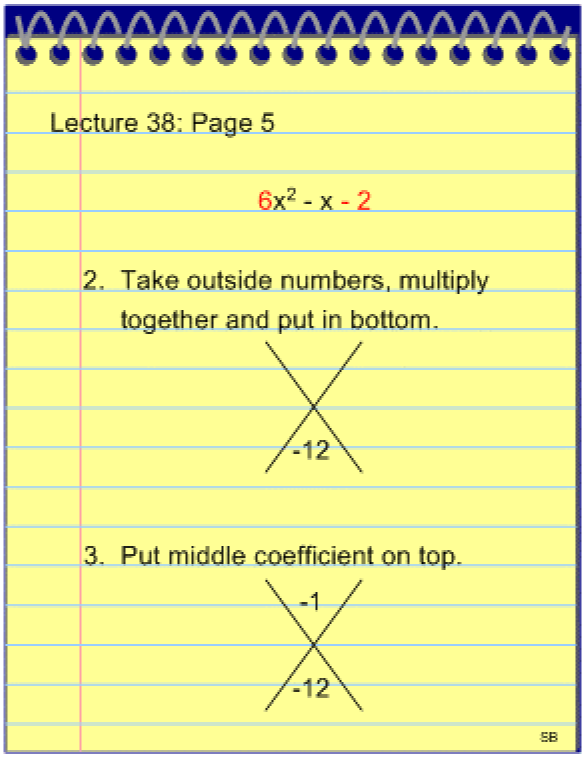

Lecture 38: Page 5

$6x^2 - x - 2$

2. Take outside numbers, multiply together and put in bottom.

3. Put middle coefficient on top.

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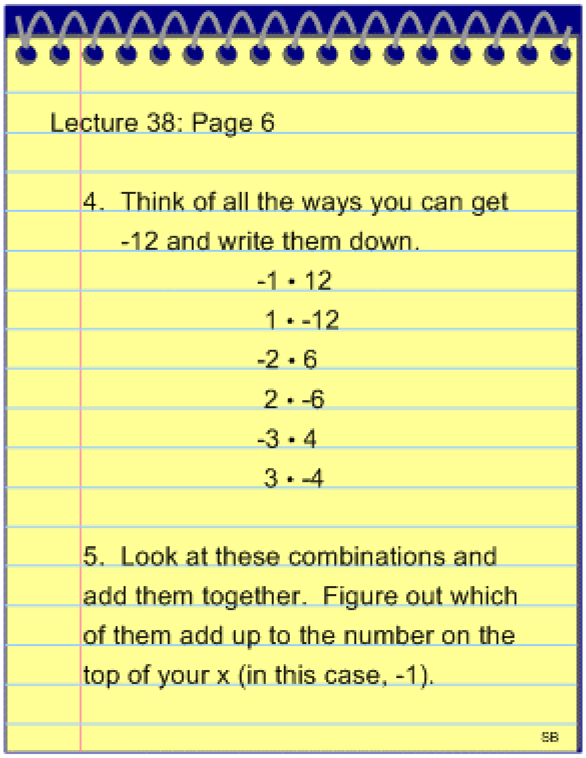
ALG2038-06



Lecture 38: Page 6

4. Think of all the ways you can get -12 and write them down.

$-1 \cdot 12$
$1 \cdot -12$
$-2 \cdot 6$
$2 \cdot -6$
$-3 \cdot 4$
$3 \cdot -4$

5. Look at these combinations and add them together. Figure out which of them add up to the number on the top of your x (in this case, -1).

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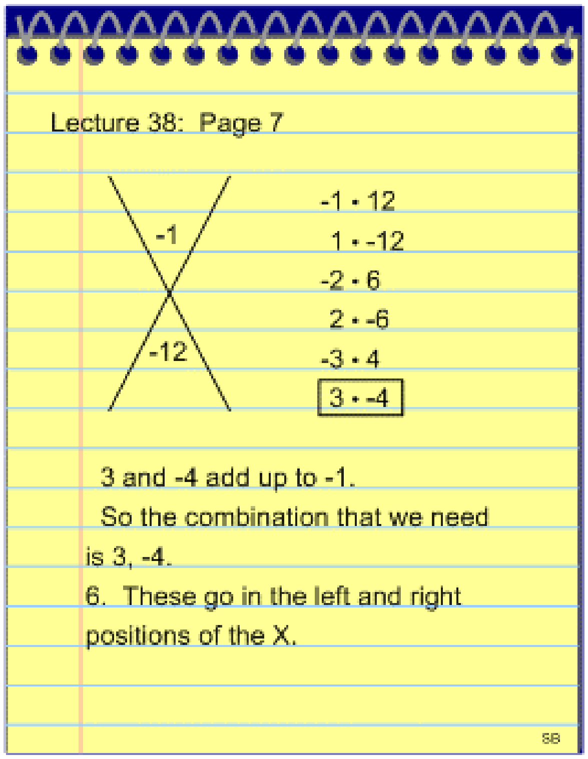

Lecture 38: Page 7

$-1 \cdot 12$
$1 \cdot -12$
$-2 \cdot 6$
$2 \cdot -6$
$-3 \cdot 4$
$\boxed{3 \cdot -4}$

3 and -4 add up to -1.
So the combination that we need is 3, -4.

6. These go in the left and right positions of the X.

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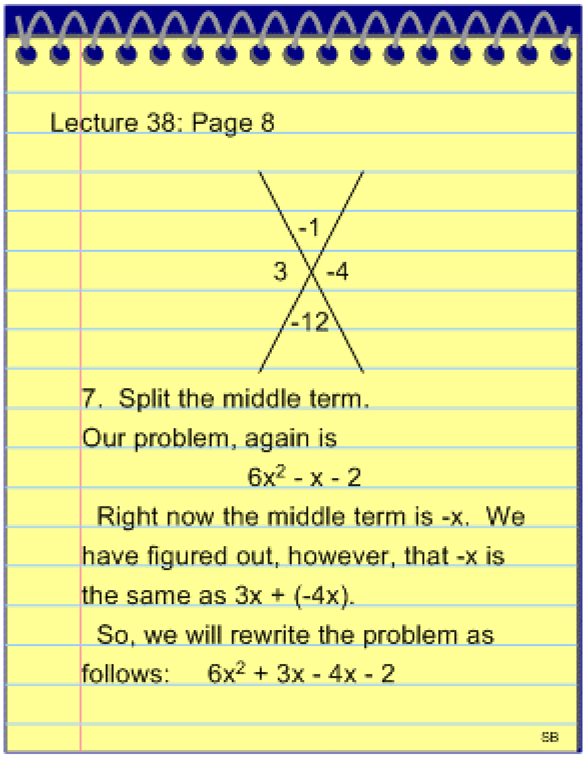

Lecture 38: Page 8

7. Split the middle term.
Our problem, again is

$$6x^2 - x - 2$$

Right now the middle term is -x. We have figured out, however, that -x is the same as 3x + (-4x).

So, we will rewrite the problem as follows: $6x^2 + 3x - 4x - 2$

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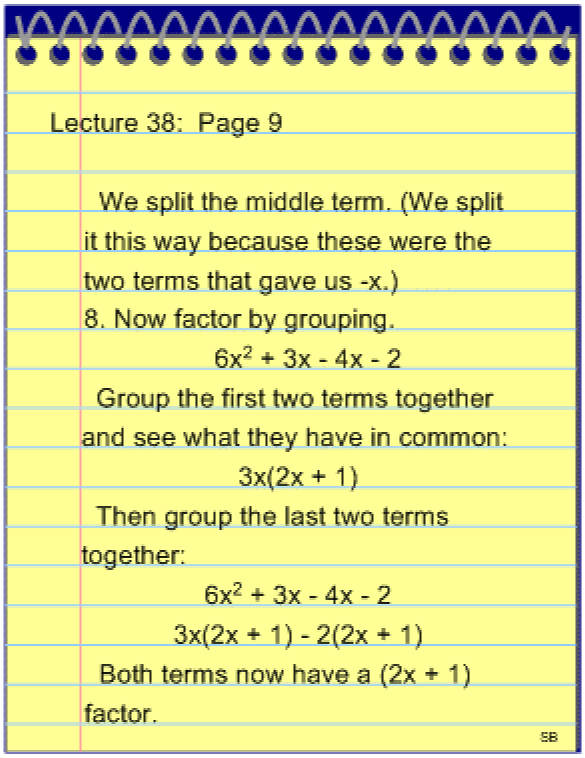
Lecture 38 Notes, Continued

ALG2038-09



Lecture 38: Page 9

We split the middle term. (We split it this way because these were the two terms that gave us -x.)

8. Now factor by grouping.

$6x^2 + 3x - 4x - 2$

Group the first two terms together and see what they have in common:

$3x(2x + 1)$

Then group the last two terms together:

$6x^2 + 3x - 4x - 2$

$3x(2x + 1) - 2(2x + 1)$

Both terms now have a (2x + 1) factor.

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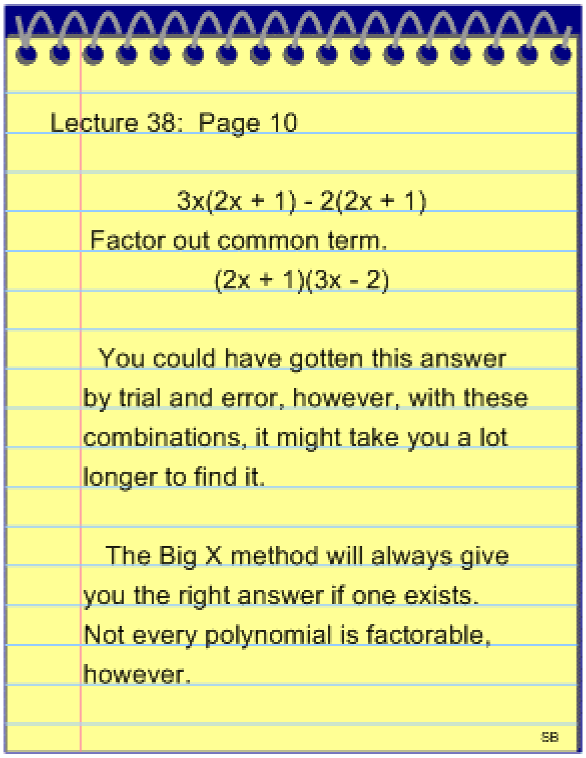
ALG2038-10



Lecture 38: Page 10

$3x(2x + 1) - 2(2x + 1)$

Factor out common term.

$(2x + 1)(3x - 2)$

You could have gotten this answer by trial and error, however, with these combinations, it might take you a lot longer to find it.

The Big X method will always give you the right answer if one exists. Not every polynomial is factorable, however.

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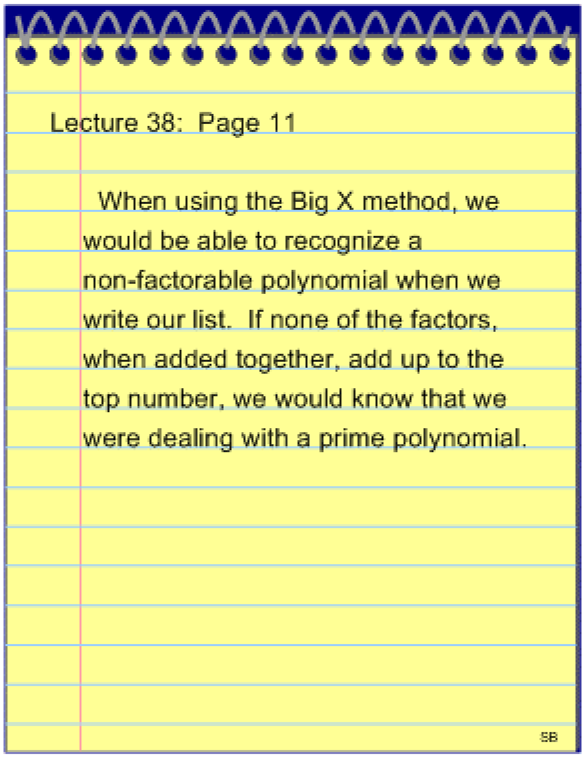
ALG2038-11



Lecture 38: Page 11

When using the Big X method, we would be able to recognize a non-factorable polynomial when we write our list. If none of the factors, when added together, add up to the top number, we would know that we were dealing with a prime polynomial.

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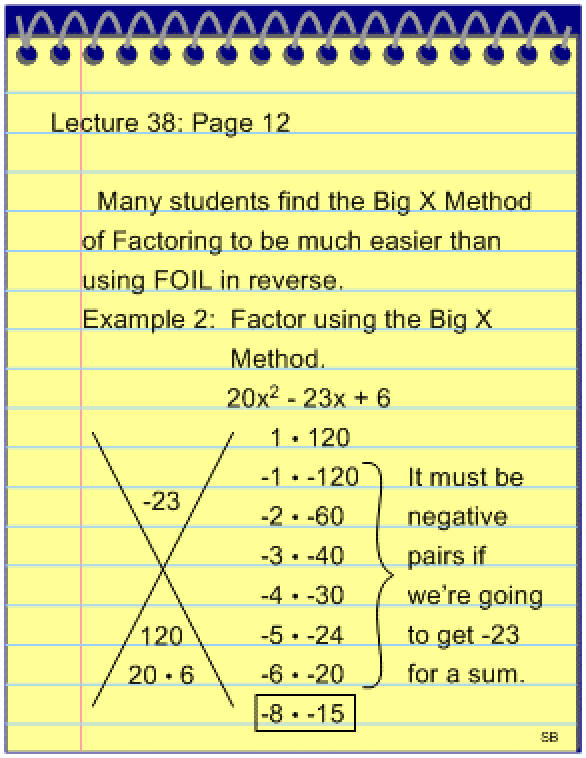

Lecture 38: Page 12

Many students find the Big X Method of Factoring to be much easier than using FOIL in reverse.

Example 2: Factor using the Big X Method.

$20x^2 - 23x + 6$

-23

120

20 • 6

1 • 120
-1 • -120
-2 • -60
-3 • -40
-4 • -30
-5 • -24
-6 • -20
$\boxed{-8 \cdot -15}$

It must be negative pairs if we're going to get -23 for a sum.

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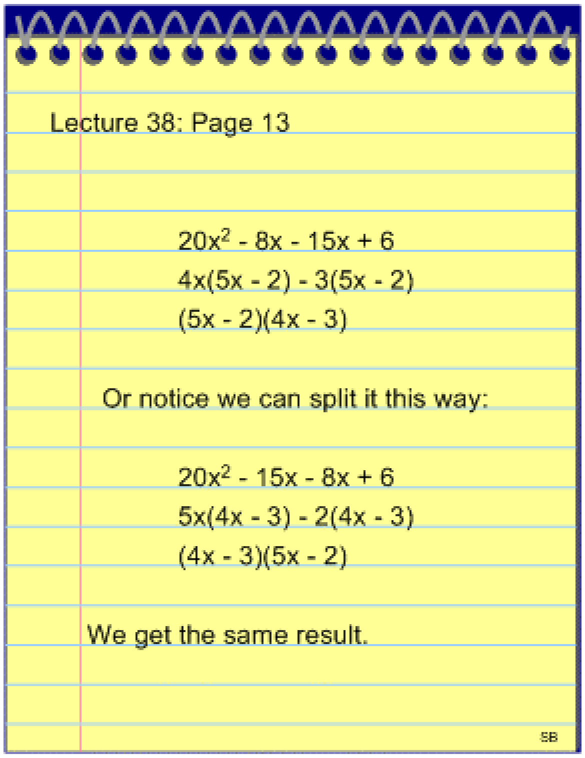
ALG2038-13



Lecture 38: Page 13

$20x^2 - 8x - 15x + 6$

$4x(5x - 2) - 3(5x - 2)$

$(5x - 2)(4x - 3)$

Or notice we can split it this way:

$20x^2 - 15x - 8x + 6$

$5x(4x - 3) - 2(4x - 3)$

$(4x - 3)(5x - 2)$

We get the same result.

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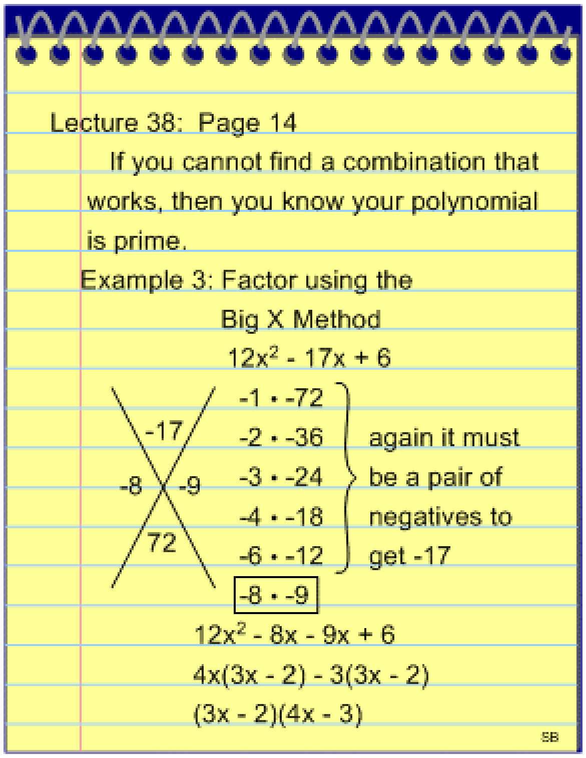
ALG2038-14



Lecture 38: Page 14

If you cannot find a combination that works, then you know your polynomial is prime.

Example 3: Factor using the Big X Method

$12x^2 - 17x + 6$

$-1 \cdot -72$
$-2 \cdot -36$
$-3 \cdot -24$
$-4 \cdot -18$
$-6 \cdot -12$
$\boxed{-8 \cdot -9}$

again it must be a pair of negatives to get -17

$12x^2 - 8x - 9x + 6$

$4x(3x - 2) - 3(3x - 2)$

$(3x - 2)(4x - 3)$

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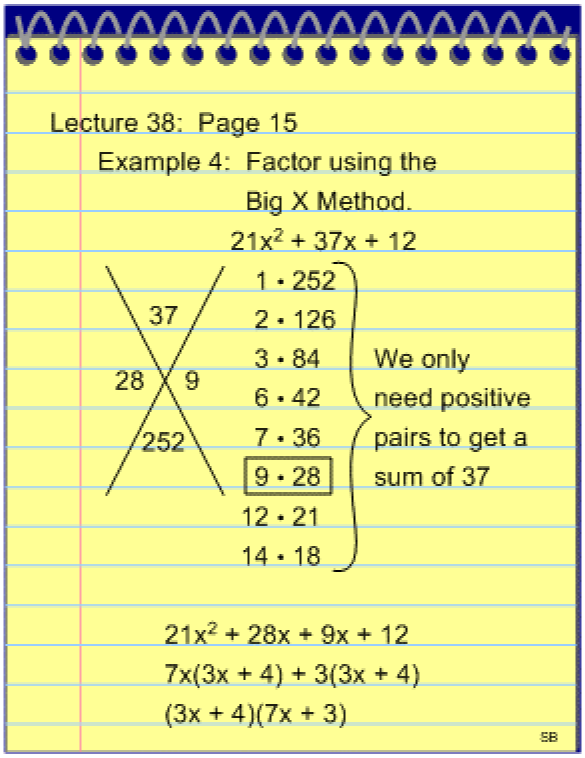
ALG2038-15



Lecture 38: Page 15

Example 4: Factor using the Big X Method.

$21x^2 + 37x + 12$

$1 \cdot 252$
$2 \cdot 126$
$3 \cdot 84$
$6 \cdot 42$
$7 \cdot 36$
$\boxed{9 \cdot 28}$
$12 \cdot 21$
$14 \cdot 18$

We only need positive pairs to get a sum of 37

$21x^2 + 28x + 9x + 12$

$7x(3x + 4) + 3(3x + 4)$

$(3x + 4)(7x + 3)$

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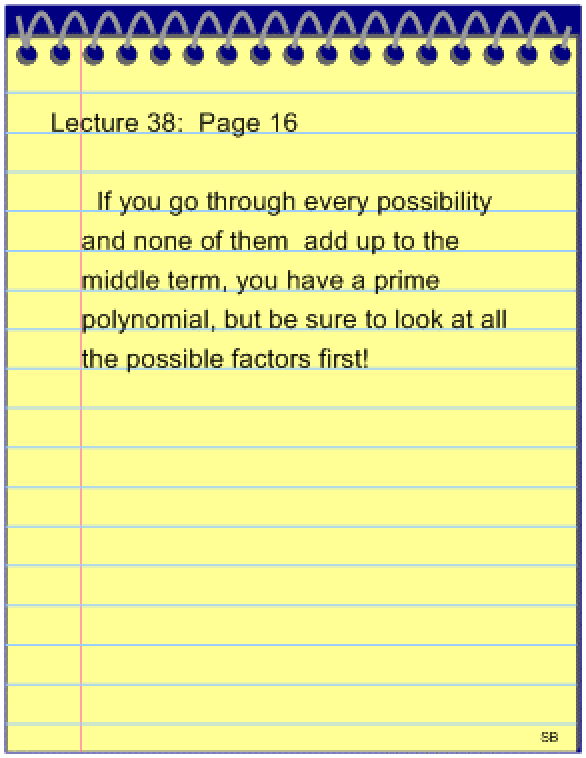
ALG2038-16



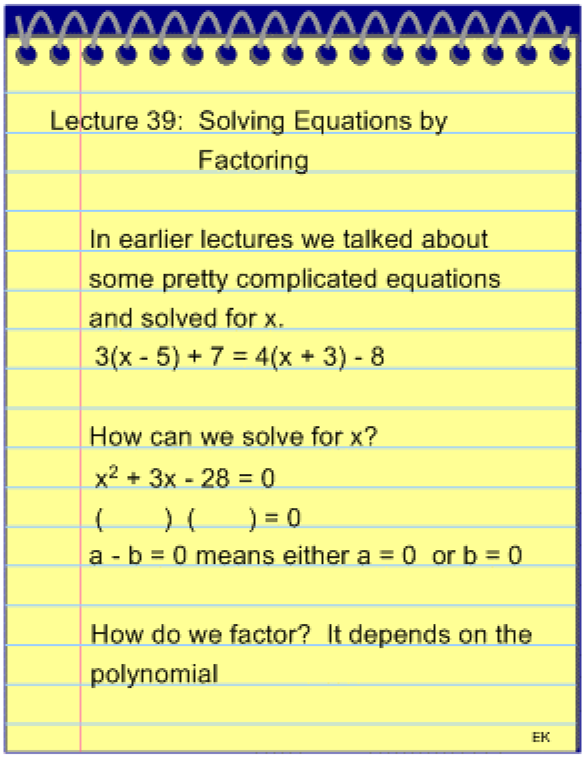
Lecture 38: Page 16

If you go through every possibility and none of them add up to the middle term, you have a prime polynomial, but be sure to look at all the possible factors first!

SB

Lecture 39 Notes

ALG2039-01



Lecture 39: Solving Equations by Factoring

In earlier lectures we talked about some pretty complicated equations and solved for x.

$3(x - 5) + 7 = 4(x + 3) - 8$

How can we solve for x?

$x^2 + 3x - 28 = 0$

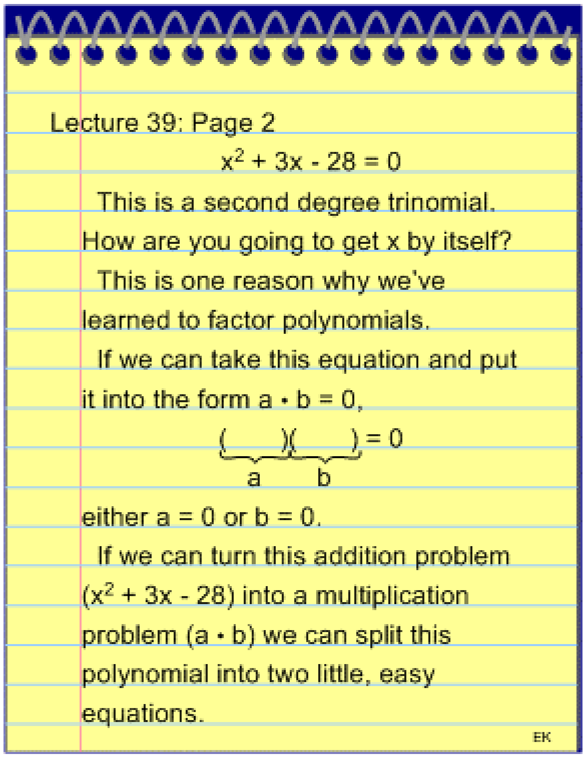
$(\quad)\ (\quad) = 0$

$a - b = 0$ means either $a = 0$ or $b = 0$

How do we factor? It depends on the polynomial

EK

ALG2039-02



Lecture 39: Page 2

$x^2 + 3x - 28 = 0$

This is a second degree trinomial. How are you going to get x by itself?

This is one reason why we've learned to factor polynomials.

If we can take this equation and put it into the form $a \cdot b = 0$,

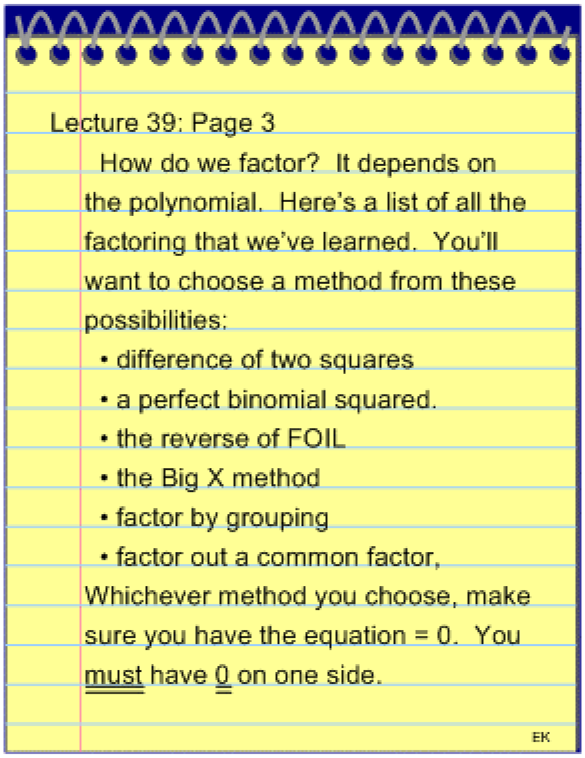
$$\underbrace{(\quad)}_{a}\underbrace{(\quad)}_{b} = 0$$

either $a = 0$ or $b = 0$.

If we can turn this addition problem $(x^2 + 3x - 28)$ into a multiplication problem $(a \cdot b)$ we can split this polynomial into two little, easy equations.

EK

ALG2039-03



Lecture 39: Page 3

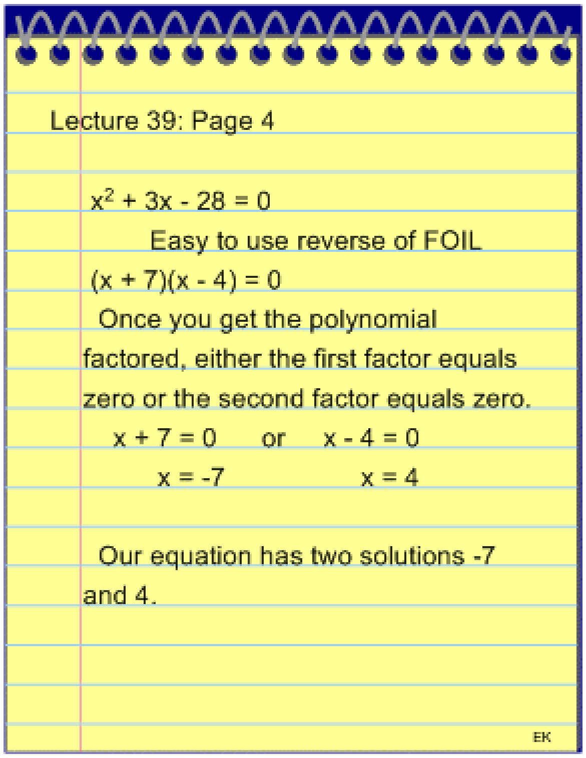
How do we factor? It depends on the polynomial. Here's a list of all the factoring that we've learned. You'll want to choose a method from these possibilities:

- difference of two squares
- a perfect binomial squared.
- the reverse of FOIL
- the Big X method
- factor by grouping
- factor out a common factor,

Whichever method you choose, make sure you have the equation = 0. You must have 0 on one side.

EK

ALG2039-04



Lecture 39: Page 4

$x^2 + 3x - 28 = 0$

Easy to use reverse of FOIL

$(x + 7)(x - 4) = 0$

Once you get the polynomial factored, either the first factor equals zero or the second factor equals zero.

$x + 7 = 0$ or $x - 4 = 0$

$x = -7$ $\quad$ $x = 4$

Our equation has two solutions -7 and 4.

EK

Lecture 39 Notes, Continued

ALG2039-05

Lecture 39: Page 5

Let's test our answers to see if they are correct:

$$(4)^2 + 3(4) - 28$$

$$= 16 + 12 - 28 = 0 \text{ works}$$

$$(-7)^2 + 3(-7) - 28$$

$$= 49 - 21 - 28 = 0 \text{ works}$$

Therefore -7 and 4 are the two solutions to this equation.

Since not every polynomial is factorable, this method isn't going to work all the time. If we have a prime polynomial, we do not yet know how to solve it. As long as we can factor, we will be able to solve these kinds of equations.

EK

ALG2039-06

Lecture 39: Page 6

Example 1: Solve for x

$$3x^2 + 9x = 0$$

Begin by factoring. Notice that both terms have a factor of 3x:

$$3x(x + 3) = 0$$

$$\frac{3x}{3} = \frac{0}{3} \quad \text{or} \quad x + 3 = 0$$

$$x = 0 \qquad\qquad x = -3$$

This equation has two solutions.

Not all polynomials have two different solutions. You might get the same solution twice for one equation.

EK

ALG2039-07

Lecture 39: Page 7

Example 2: Solve for x.

$$x^2 + 4x = -4$$

Notice that this time, the right-hand side of this equation does not equal 0. We need the right-hand side to be zero because this process only works when a • b = 0. Just add 4 to both sides, making the right-hand side be zero.

EK

ALG2039-08

Lecture 39: Page 8

$$\begin{array}{r} x^2 + 4x = -4 \\ \underline{\quad +4 \quad +4} \\ x^2 + 4x + 4 = 0 \end{array}$$

Always make one side of your equation be 0 first! Then factor. This is a binomial squared.

$$(x + 2)^2 = 0$$

$$x + 2 = 0$$

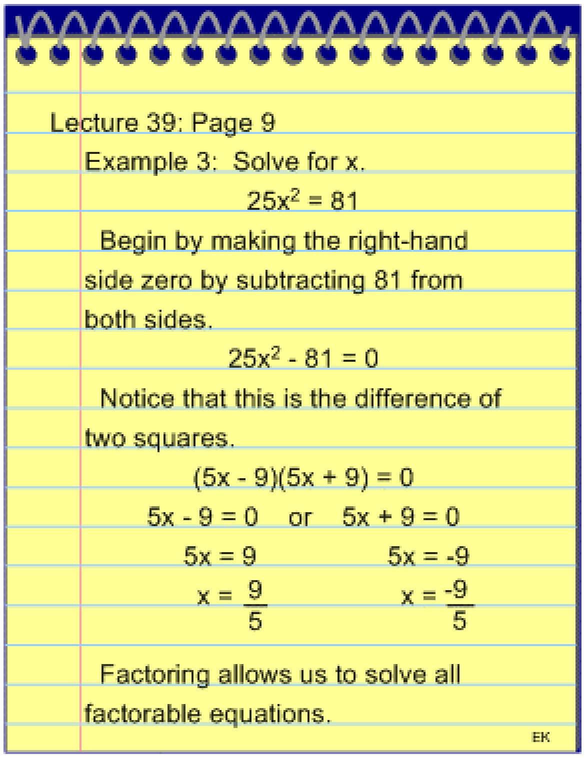
$$x = -2$$

In this case there is only one solution since x + 2 is the factor twice.

EK

Lecture 39 Notes, Continued

ALG2039-09



Lecture 39: Page 9

Example 3: Solve for x.

$$25x^2 = 81$$

Begin by making the right-hand side zero by subtracting 81 from both sides.

$$25x^2 - 81 = 0$$

Notice that this is the difference of two squares.

$$(5x - 9)(5x + 9) = 0$$

$$5x - 9 = 0 \quad \text{or} \quad 5x + 9 = 0$$

$$5x = 9 \qquad 5x = -9$$

$$x = \frac{9}{5} \qquad x = \frac{-9}{5}$$

Factoring allows us to solve all factorable equations.

EK

Lecture 40 Notes

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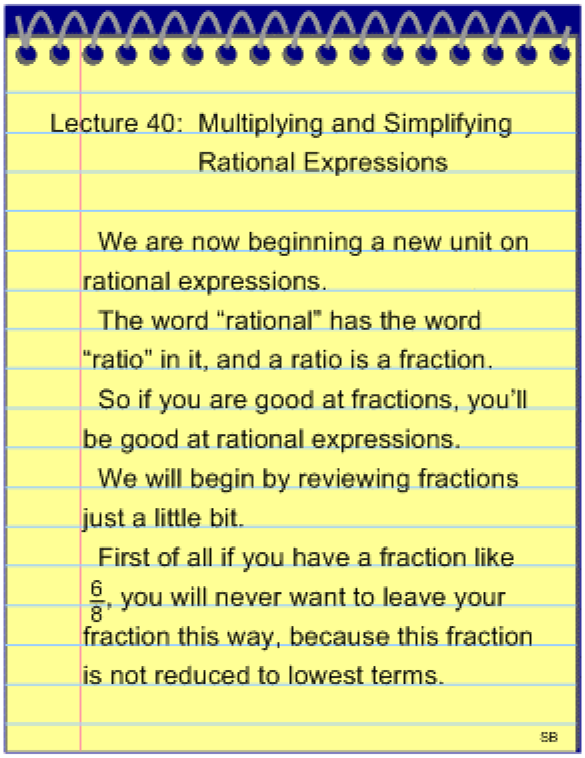

Lecture 40: Multiplying and Simplifying Rational Expressions

We are now beginning a new unit on rational expressions.

The word "rational" has the word "ratio" in it, and a ratio is a fraction.

So if you are good at fractions, you'll be good at rational expressions.

We will begin by reviewing fractions just a little bit.

First of all if you have a fraction like $\frac{6}{8}$, you will never want to leave your fraction this way, because this fraction is not reduced to lowest terms.

SB

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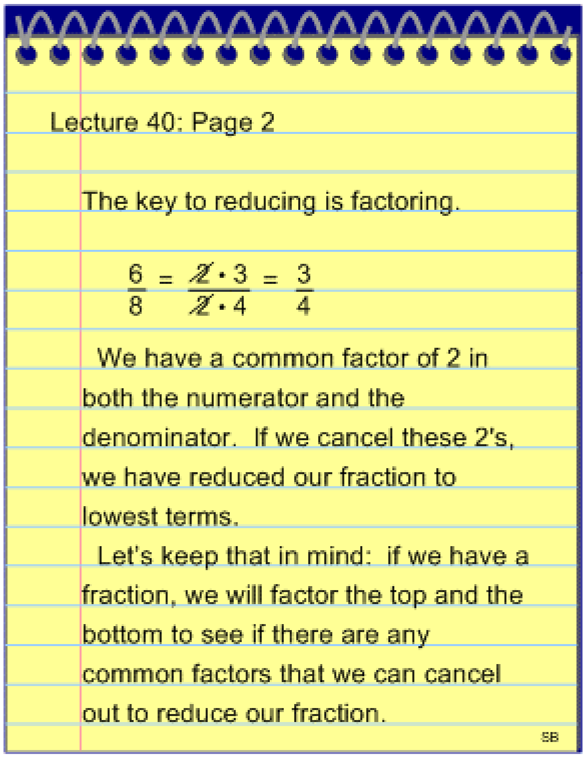


Lecture 40: Page 2

The key to reducing is factoring.

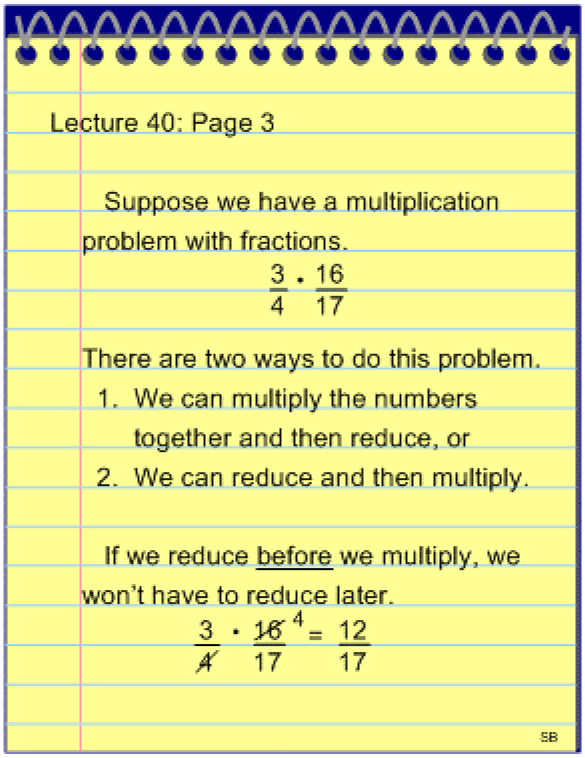
$$\frac{6}{8} = \frac{\cancel{2}\cdot 3}{\cancel{2}\cdot 4} = \frac{3}{4}$$

We have a common factor of 2 in both the numerator and the denominator. If we cancel these 2's, we have reduced our fraction to lowest terms.

Let's keep that in mind: if we have a fraction, we will factor the top and the bottom to see if there are any common factors that we can cancel out to reduce our fraction.

SB

ALG2040-03



Lecture 40: Page 3

Suppose we have a multiplication problem with fractions.

$$\frac{3}{4}\cdot\frac{16}{17}$$

There are two ways to do this problem.

1. We can multiply the numbers together and then reduce, or
2. We can reduce and then multiply.

If we reduce before we multiply, we won't have to reduce later.

$$\frac{3}{\cancel{4}}\cdot\frac{\overset{4}{\cancel{16}}}{17} = \frac{12}{17}$$

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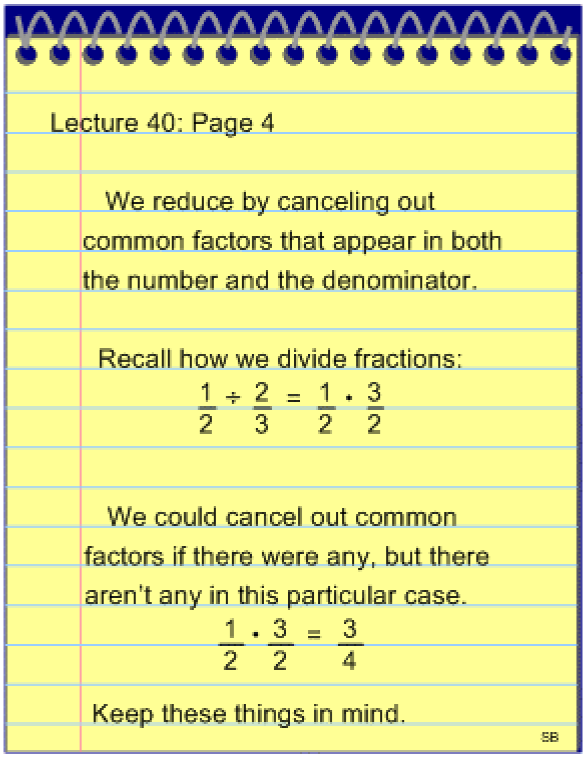


Lecture 40: Page 4

We reduce by canceling out common factors that appear in both the number and the denominator.

Recall how we divide fractions:

$$\frac{1}{2}\div\frac{2}{3} = \frac{1}{2}\cdot\frac{3}{2}$$

We could cancel out common factors if there were any, but there aren't any in this particular case.

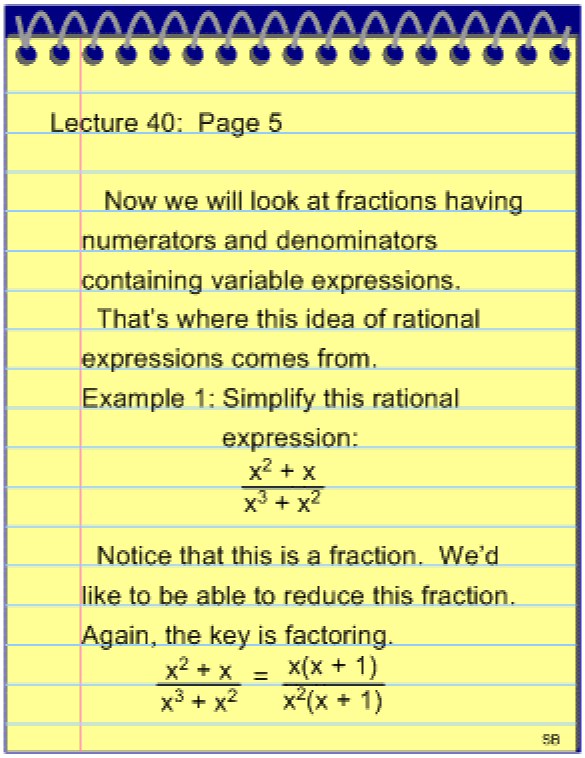
$$\frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4}$$

Keep these things in mind.

SB

Lecture 40 Notes, Continued

ALG2040-05



Lecture 40: Page 5

Now we will look at fractions having numerators and denominators containing variable expressions.

That's where this idea of rational expressions comes from.

Example 1: Simplify this rational expression:

$$\frac{x^2 + x}{x^3 + x^2}$$

Notice that this is a fraction. We'd like to be able to reduce this fraction. Again, the key is factoring.

$$\frac{x^2 + x}{x^3 + x^2} = \frac{x(x + 1)}{x^2(x + 1)}$$

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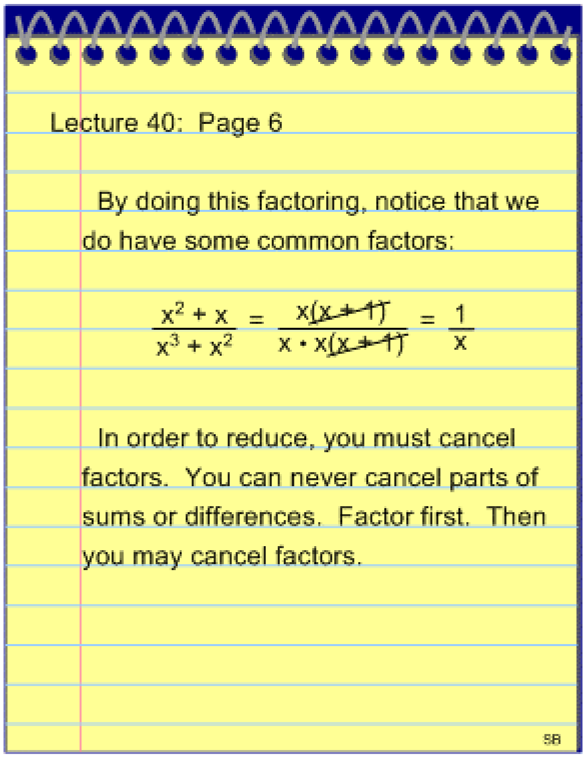
ALG2040-06



Lecture 40: Page 6

By doing this factoring, notice that we do have some common factors:

$$\frac{x^2 + x}{x^3 + x^2} = \frac{x\cancel{(x + 1)}}{x \cdot x\cancel{(x + 1)}} = \frac{1}{x}$$

In order to reduce, you must cancel factors. You can never cancel parts of sums or differences. Factor first. Then you may cancel factors.

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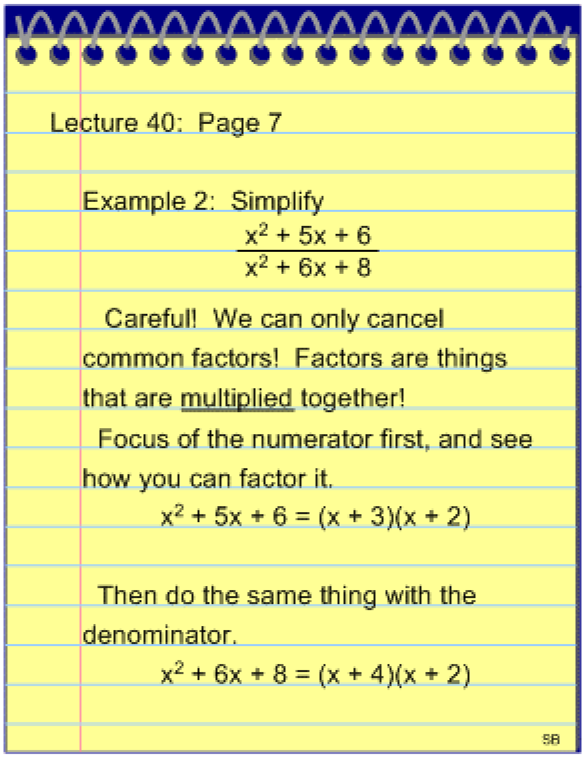
ALG2040-07



Lecture 40: Page 7

Example 2: Simplify

$$\frac{x^2 + 5x + 6}{x^2 + 6x + 8}$$

Careful! We can only cancel common factors! Factors are things that are multiplied together!

Focus of the numerator first, and see how you can factor it.

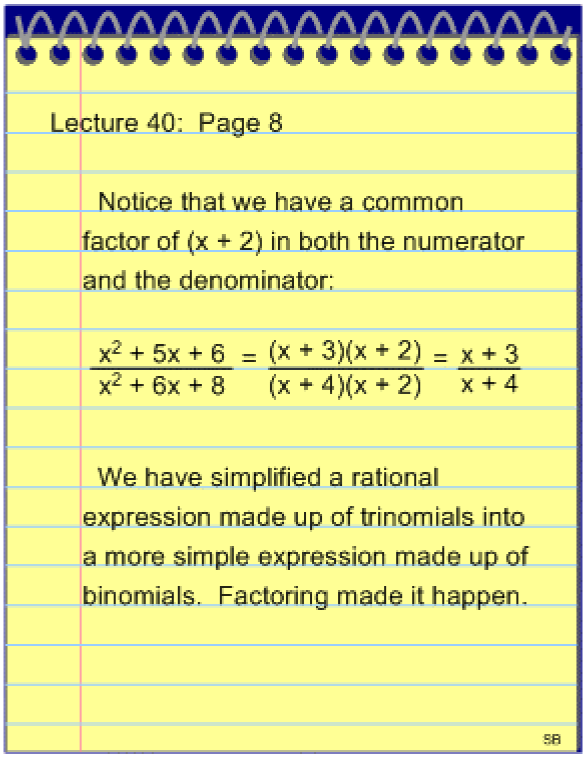
$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

Then do the same thing with the denominator.

$$x^2 + 6x + 8 = (x + 4)(x + 2)$$

SB

ALG2040-08



Lecture 40: Page 8

Notice that we have a common factor of (x + 2) in both the numerator and the denominator:

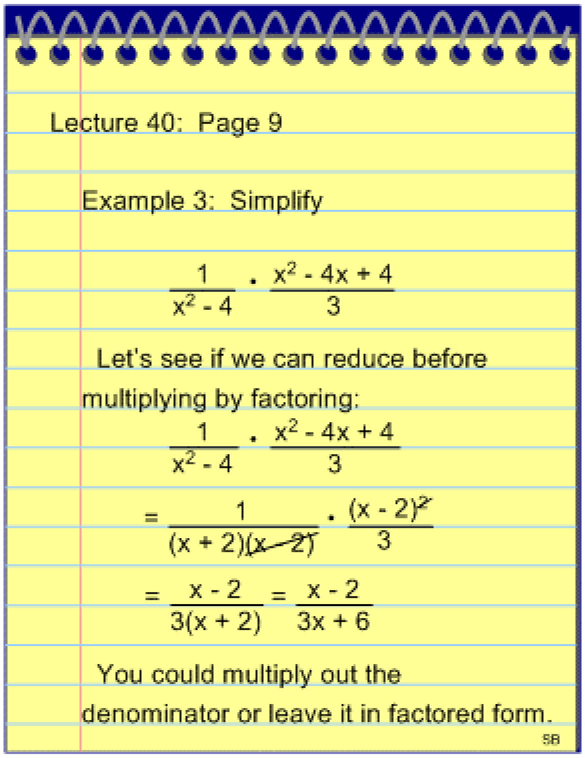
$$\frac{x^2 + 5x + 6}{x^2 + 6x + 8} = \frac{(x + 3)(x + 2)}{(x + 4)(x + 2)} = \frac{x + 3}{x + 4}$$

We have simplified a rational expression made up of trinomials into a more simple expression made up of binomials. Factoring made it happen.

SB

Lecture 40 Notes, Continued

ALG2040-09



Lecture 40: Page 9

Example 3: Simplify

$$\frac{1}{x^2 - 4} \cdot \frac{x^2 - 4x + 4}{3}$$

Let's see if we can reduce before multiplying by factoring:

$$\frac{1}{x^2 - 4} \cdot \frac{x^2 - 4x + 4}{3}$$

$$= \frac{1}{(x + 2)\cancel{(x - 2)}} \cdot \frac{(x - 2)^{\cancel{2}}}{3}$$

$$= \frac{x - 2}{3(x + 2)} = \frac{x - 2}{3x + 6}$$

You could multiply out the denominator or leave it in factored form.

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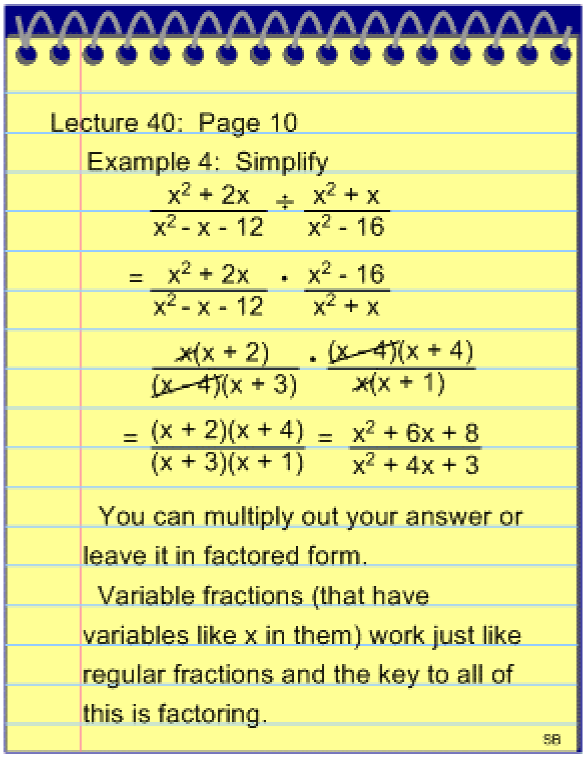
ALG2040-10



Lecture 40: Page 10

Example 4: Simplify

$$\frac{x^2 + 2x}{x^2 - x - 12} \div \frac{x^2 + x}{x^2 - 16}$$

$$= \frac{x^2 + 2x}{x^2 - x - 12} \cdot \frac{x^2 - 16}{x^2 + x}$$

$$\frac{\cancel{x}(x + 2)}{\cancel{(x - 4)}(x + 3)} \cdot \frac{\cancel{(x - 4)}(x + 4)}{\cancel{x}(x + 1)}$$

$$= \frac{(x + 2)(x + 4)}{(x + 3)(x + 1)} = \frac{x^2 + 6x + 8}{x^2 + 4x + 3}$$

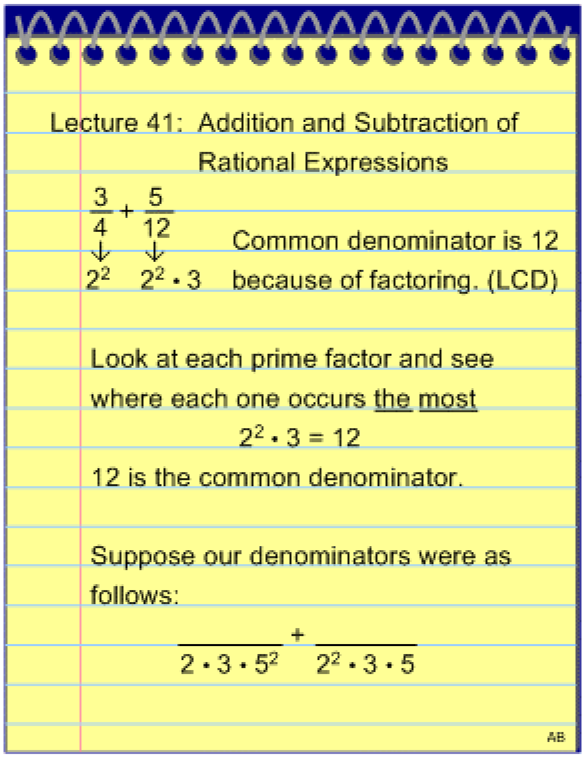
You can multiply out your answer or leave it in factored form.

Variable fractions (that have variables like x in them) work just like regular fractions and the key to all of this is factoring.

SB

Lecture 41 Notes

ALG2041-01



Lecture 41: Addition and Subtraction of Rational Expressions

$$\begin{array}{ccc} \frac{3}{4} & + & \frac{5}{12} \\ \downarrow & & \downarrow \\ 2^2 & & 2^2 \cdot 3 \end{array}$$

Common denominator is 12 because of factoring. (LCD)

Look at each prime factor and see where each one occurs the most

$$2^2 \cdot 3 = 12$$

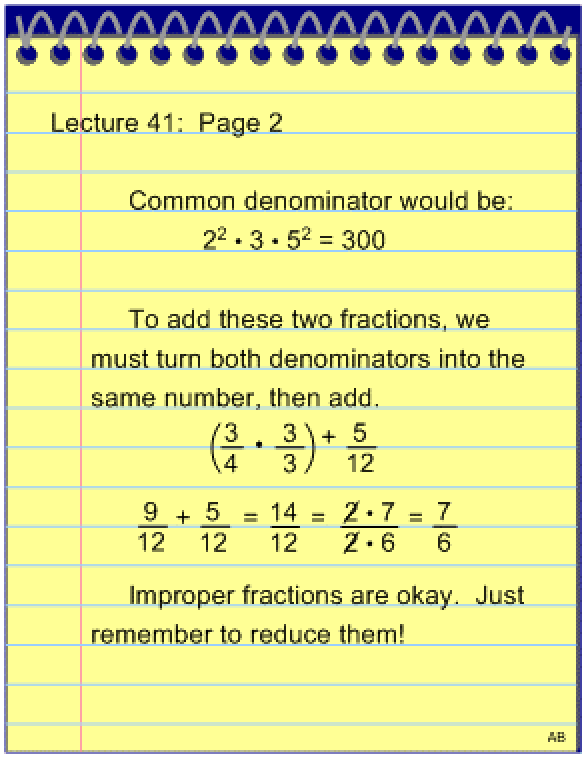
12 is the common denominator.

Suppose our denominators were as follows:

$$\frac{\quad}{2 \cdot 3 \cdot 5^2} + \frac{\quad}{2^2 \cdot 3 \cdot 5}$$

AB

ALG2041-02



Lecture 41: Page 2

Common denominator would be:

$$2^2 \cdot 3 \cdot 5^2 = 300$$

To add these two fractions, we must turn both denominators into the same number, then add.

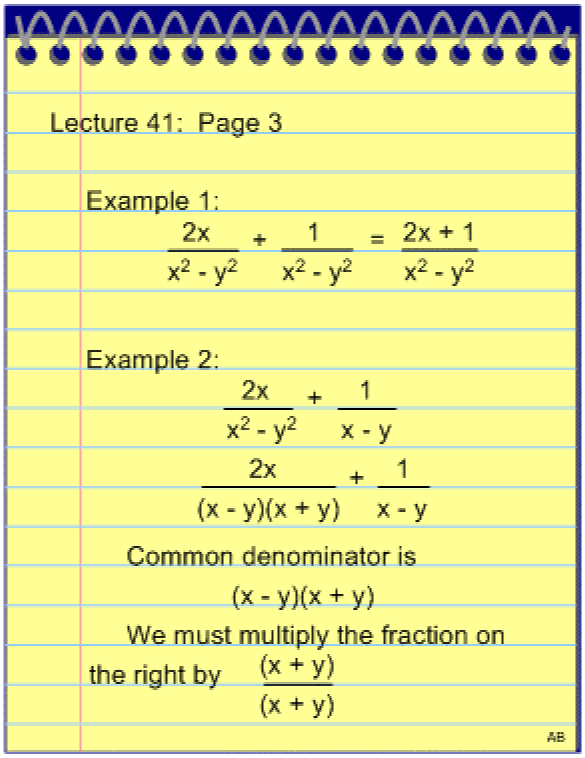
$$\left(\frac{3}{4} \cdot \frac{3}{3}\right) + \frac{5}{12}$$

$$\frac{9}{12} + \frac{5}{12} = \frac{14}{12} = \frac{\cancel{2} \cdot 7}{\cancel{2} \cdot 6} = \frac{7}{6}$$

Improper fractions are okay. Just remember to reduce them!

AB

ALG2041-03



Lecture 41: Page 3

Example 1:

$$\frac{2x}{x^2 - y^2} + \frac{1}{x^2 - y^2} = \frac{2x + 1}{x^2 - y^2}$$

Example 2:

$$\frac{2x}{x^2 - y^2} + \frac{1}{x - y}$$

$$\frac{2x}{(x - y)(x + y)} + \frac{1}{x - y}$$

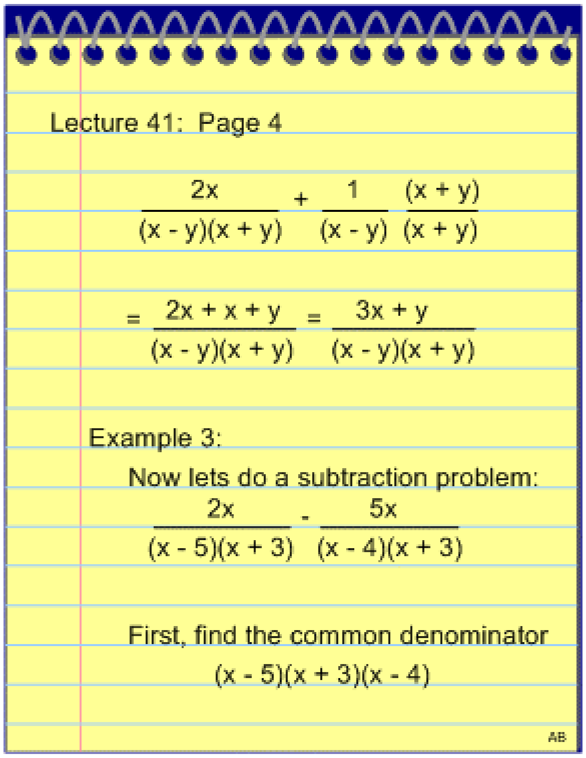
Common denominator is

$$(x - y)(x + y)$$

We must multiply the fraction on the right by $\frac{(x + y)}{(x + y)}$

AB

ALG2041-04



Lecture 41: Page 4

$$\frac{2x}{(x - y)(x + y)} + \frac{1}{(x - y)} \frac{(x + y)}{(x + y)}$$

$$= \frac{2x + x + y}{(x - y)(x + y)} = \frac{3x + y}{(x - y)(x + y)}$$

Example 3:

Now lets do a subtraction problem:

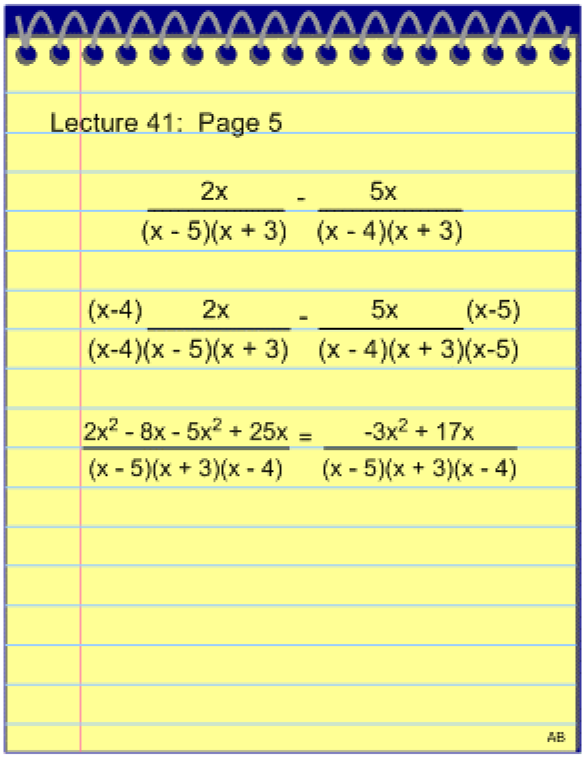
$$\frac{2x}{(x - 5)(x + 3)} - \frac{5x}{(x - 4)(x + 3)}$$

First, find the common denominator

$$(x - 5)(x + 3)(x - 4)$$

AB

ALG2041-05



Lecture 41: Page 5

$$\frac{2x}{(x - 5)(x + 3)} - \frac{5x}{(x - 4)(x + 3)}$$

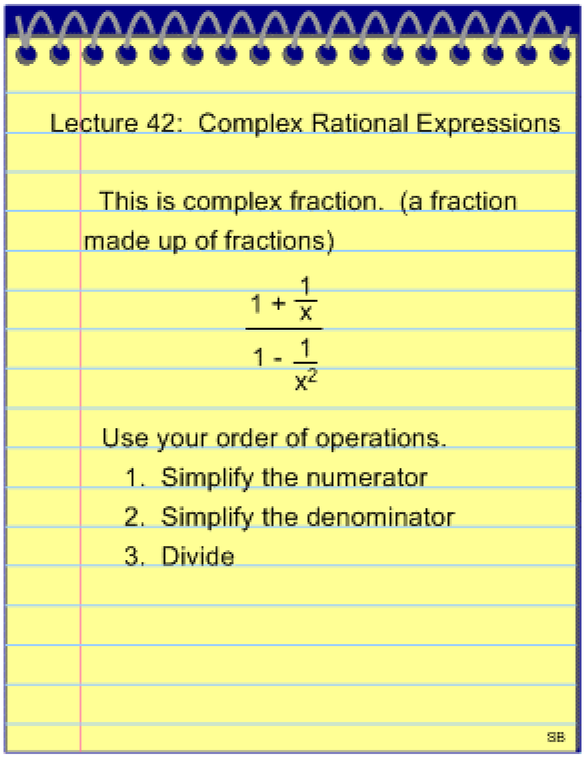
$$(x-4)\frac{2x}{(x-4)(x - 5)(x + 3)} - \frac{5x}{(x - 4)(x + 3)(x-5)}(x-5)$$

$$\frac{2x^2 - 8x - 5x^2 + 25x}{(x - 5)(x + 3)(x - 4)} = \frac{-3x^2 + 17x}{(x - 5)(x + 3)(x - 4)}$$

AB

Lecture 42 Notes

ALG2042-01



Lecture 42: Complex Rational Expressions

This is complex fraction. (a fraction made up of fractions)

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x^2}}$$

Use your order of operations.

1. Simplify the numerator
2. Simplify the denominator
3. Divide

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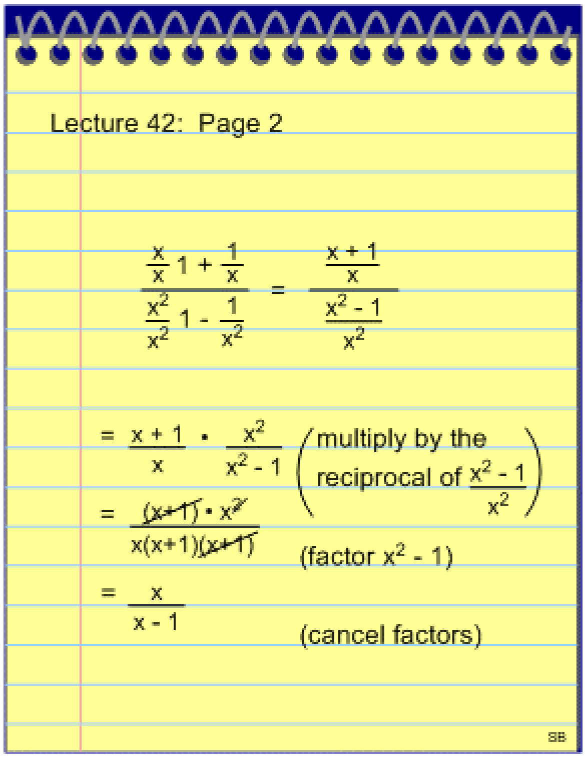

Lecture 42: Page 2

$$\frac{\frac{x}{x}1+\frac{1}{x}}{\frac{x^2}{x^2}1-\frac{1}{x^2}} = \frac{\frac{x+1}{x}}{\frac{x^2-1}{x^2}}$$

$$= \frac{x+1}{x}\cdot\frac{x^2}{x^2-1}$$ (multiply by the reciprocal of $\frac{x^2-1}{x^2}$)

$$= \frac{\cancel{(x+1)}\cdot x^{\cancel{2}}}{x(x+1)\cancel{(x+1)}}$$ (factor $x^2 - 1$)

$$= \frac{x}{x-1}$$ (cancel factors)

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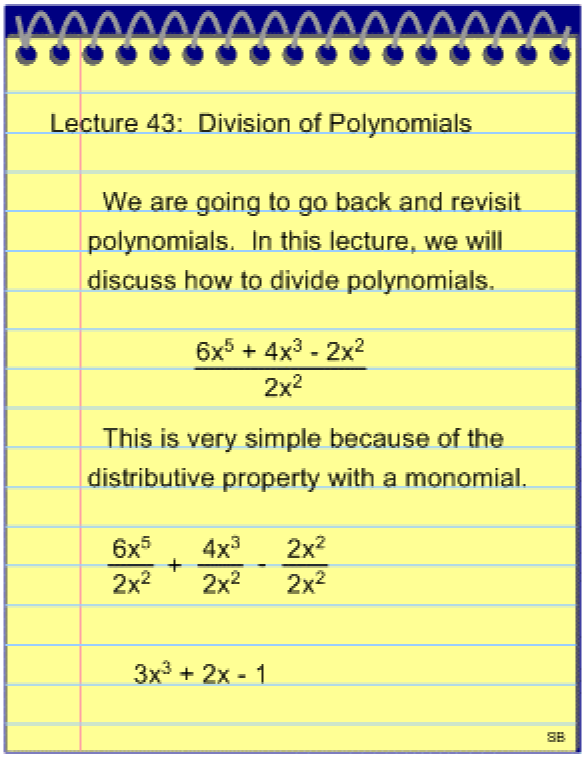
Lecture 43 Notes

ALG2043-01



Lecture 43: Division of Polynomials

We are going to go back and revisit polynomials. In this lecture, we will discuss how to divide polynomials.

$$\frac{6x^5 + 4x^3 - 2x^2}{2x^2}$$

This is very simple because of the distributive property with a monomial.

$$\frac{6x^5}{2x^2} + \frac{4x^3}{2x^2} - \frac{2x^2}{2x^2}$$

$$3x^3 + 2x - 1$$

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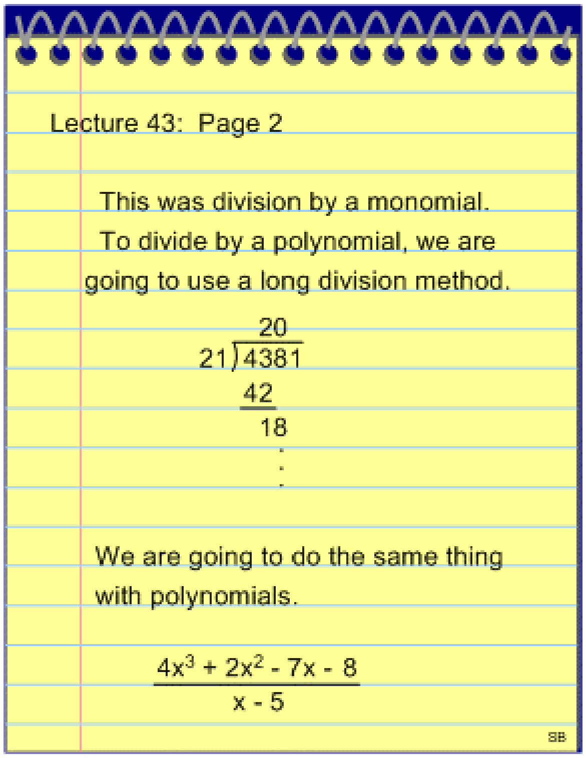
ALG2043-02



Lecture 43: Page 2

This was division by a monomial. To divide by a polynomial, we are going to use a long division method.

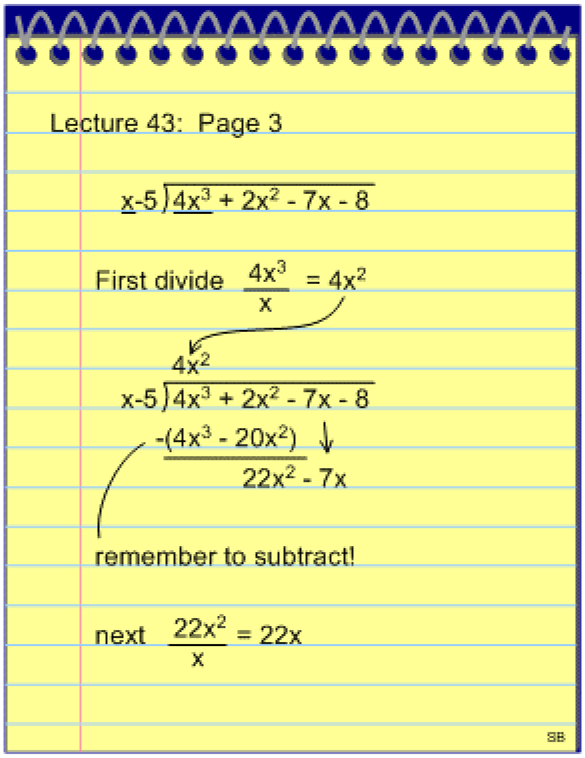
$$\begin{array}{r} 20 \\ 21\overline{)4381} \\ \underline{42} \\ 18 \\ \vdots \end{array}$$

We are going to do the same thing with polynomials.

$$\frac{4x^3 + 2x^2 - 7x - 8}{x - 5}$$

SB

ALG2043-03



Lecture 43: Page 3

$$x-5\overline{)4x^3 + 2x^2 - 7x - 8}$$

First divide $\frac{4x^3}{x} = 4x^2$

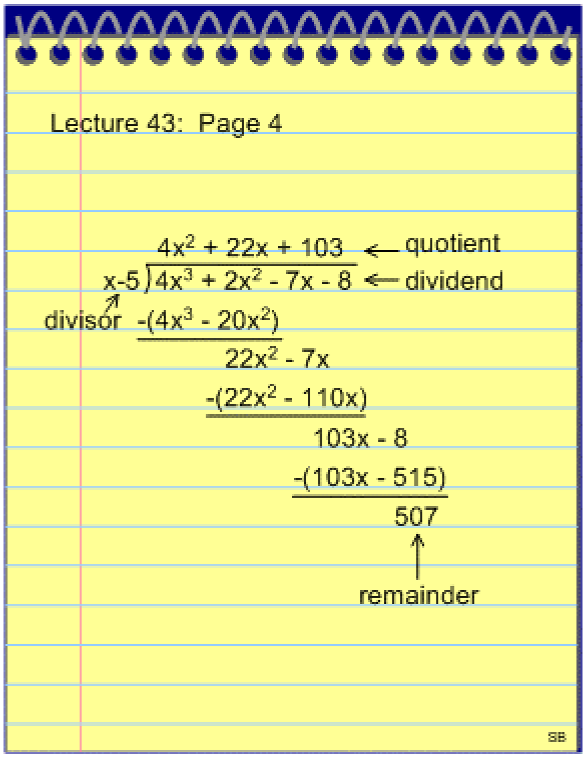
$$\begin{array}{r} 4x^2 \\ x-5\overline{)4x^3 + 2x^2 - 7x - 8} \\ \underline{-(4x^3 - 20x^2)} \downarrow \\ 22x^2 - 7x \end{array}$$

remember to subtract!

next $\frac{22x^2}{x} = 22x$

SB

ALG2043-04



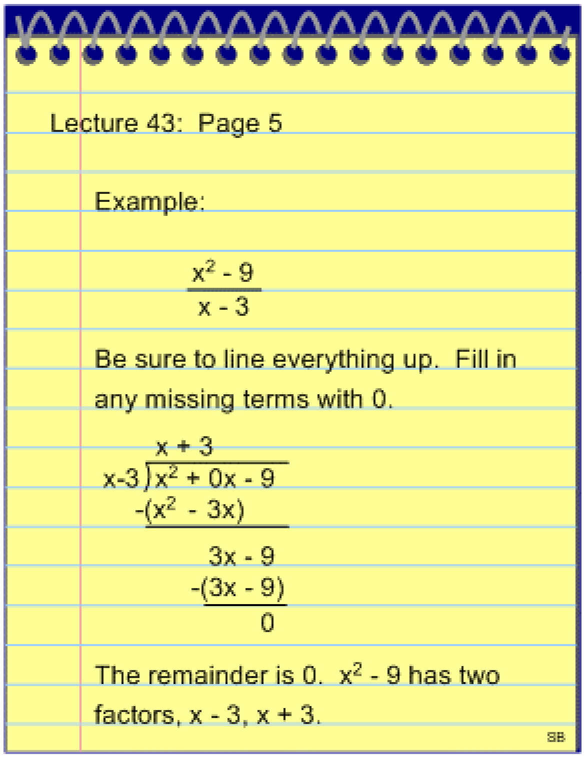
Lecture 43: Page 4

$$\begin{array}{lr} & 4x^2 + 22x + 103 \leftarrow \text{quotient} \\ & x-5\overline{)4x^3 + 2x^2 - 7x - 8} \leftarrow \text{dividend} \\ \text{divisor} & \underline{-(4x^3 - 20x^2)} \\ & 22x^2 - 7x \\ & \underline{-(22x^2 - 110x)} \\ & 103x - 8 \\ & \underline{-(103x - 515)} \\ & 507 \\ & \uparrow \\ & \text{remainder} \end{array}$$

SB

Lecture 43 Notes, Continued

ALG2043-05



Lecture 43: Page 5

Example:

$$\frac{x^2 - 9}{x - 3}$$

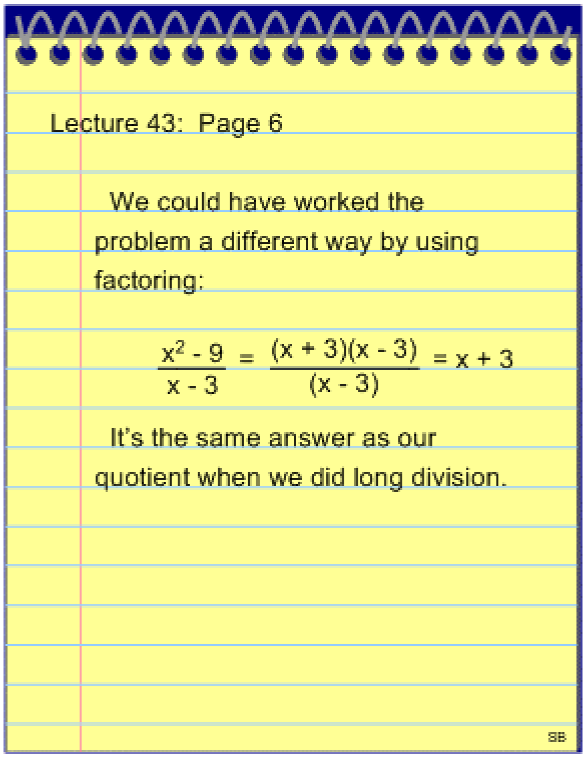
Be sure to line everything up. Fill in any missing terms with 0.

$$\begin{array}{r}
x + 3 \\
x-3 \,\overline{)\, x^2 + 0x - 9} \\
\underline{-(x^2 - 3x) } \\
3x - 9 \\
\underline{-(3x - 9)} \\
0
\end{array}$$

The remainder is 0. $x^2 - 9$ has two factors, $x - 3$, $x + 3$.

SB

ALG2043-06



Lecture 43: Page 6

We could have worked the problem a different way by using factoring:

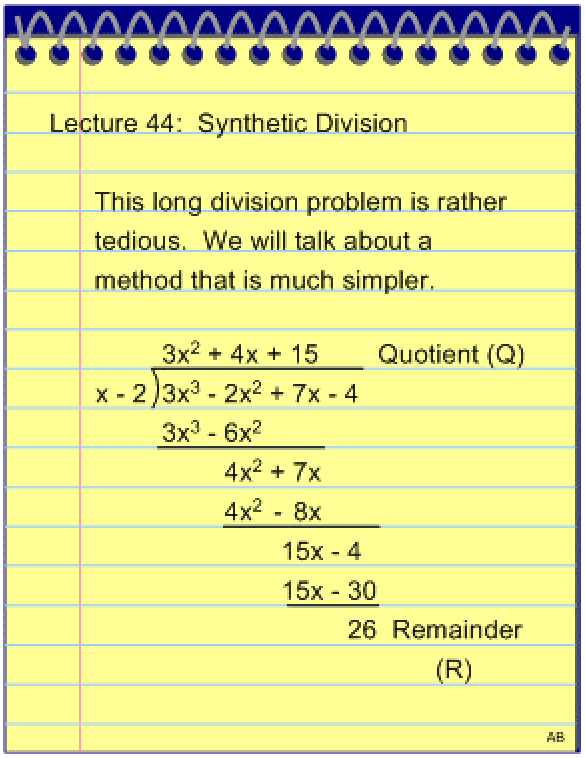
$$\frac{x^2 - 9}{x - 3} = \frac{(x + 3)(x - 3)}{(x - 3)} = x + 3$$

It's the same answer as our quotient when we did long division.

SB

Lecture 44 Notes

ALG2044-01



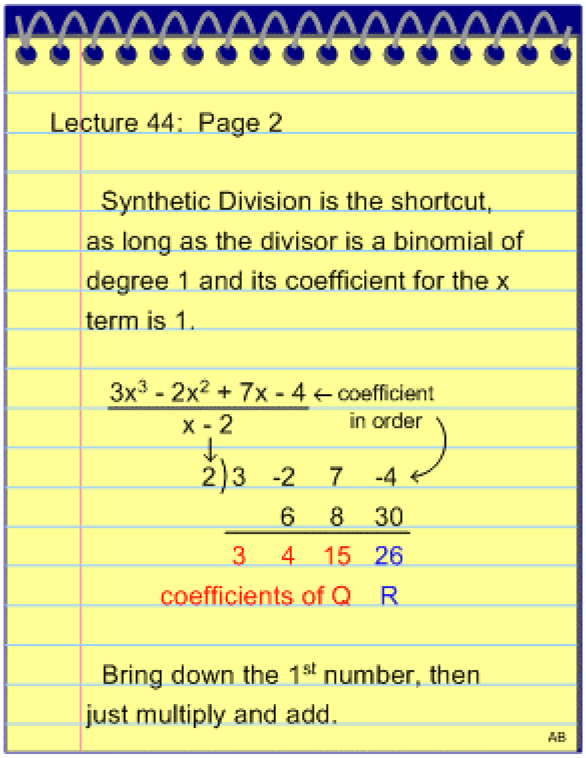
Lecture 44: Synthetic Division

This long division problem is rather tedious. We will talk about a method that is much simpler.

$$\begin{array}{r l} & \underline{3x^2 + 4x + 15} \quad \text{Quotient (Q)} \\ x - 2 \,\big) & 3x^3 - 2x^2 + 7x - 4 \\ & \underline{3x^3 - 6x^2} \\ & \quad 4x^2 + 7x \\ & \quad \underline{4x^2 - 8x} \\ & \qquad 15x - 4 \\ & \qquad \underline{15x - 30} \\ & \qquad\quad 26 \ \text{Remainder} \\ & \qquad\qquad \text{(R)} \end{array}$$

AB

ALG2044-02



Lecture 44: Page 2

Synthetic Division is the shortcut, as long as the divisor is a binomial of degree 1 and its coefficient for the x term is 1.

$$\frac{3x^3 - 2x^2 + 7x - 4}{x - 2} \leftarrow \text{coefficient in order}$$

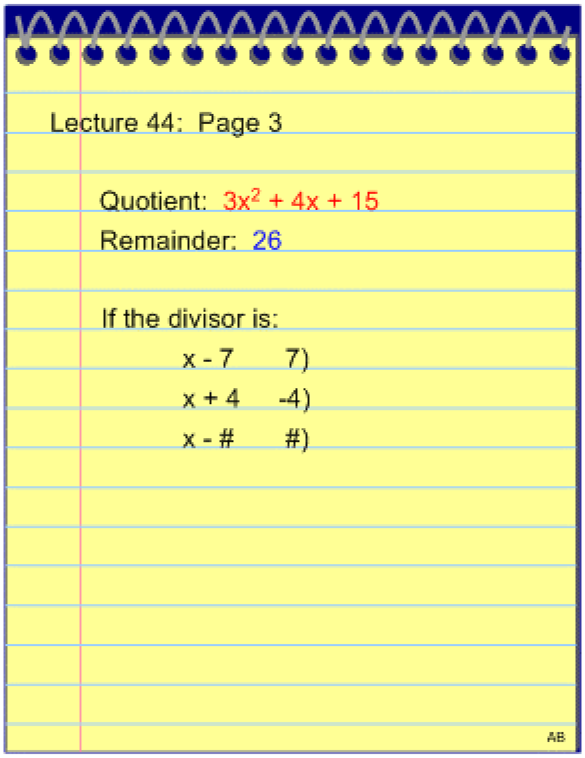
$$\begin{array}{r|rrrr} 2 & 3 & -2 & 7 & -4 \\ & & 6 & 8 & 30 \\ \hline & 3 & 4 & 15 & 26 \end{array}$$

coefficients of Q R

Bring down the 1st number, then just multiply and add.

AB

ALG2044-03



Lecture 44: Page 3

Quotient: $3x^2 + 4x + 15$

Remainder: 26

If the divisor is:

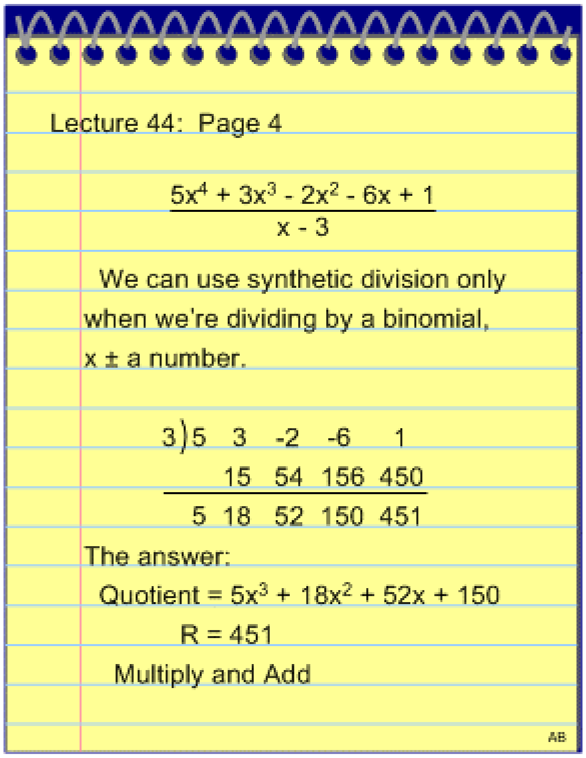
$x - 7$ 7)

$x + 4$ -4)

$x - \#$ #)

AB

ALG2044-04



Lecture 44: Page 4

$$\frac{5x^4 + 3x^3 - 2x^2 - 6x + 1}{x - 3}$$

We can use synthetic division only when we're dividing by a binomial, x ± a number.

$$\begin{array}{r|rrrrr} 3 & 5 & 3 & -2 & -6 & 1 \\ & & 15 & 54 & 156 & 450 \\ \hline & 5 & 18 & 52 & 150 & 451 \end{array}$$

The answer:

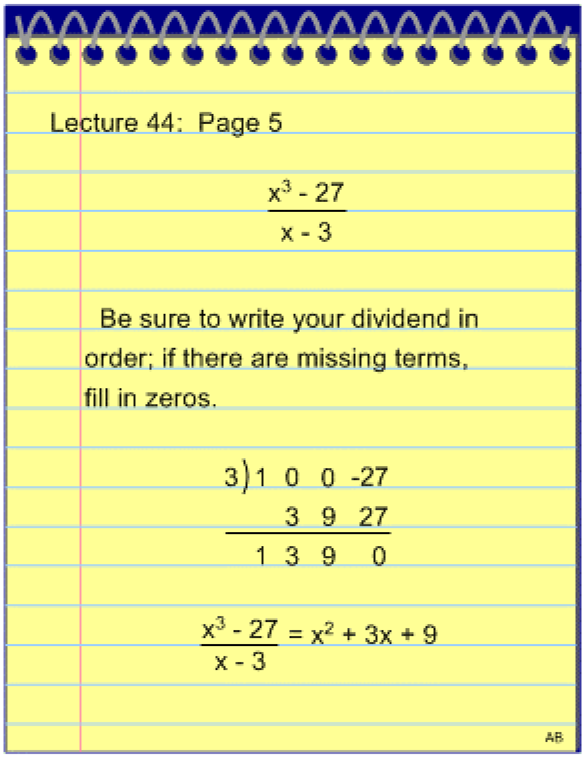
Quotient = $5x^3 + 18x^2 + 52x + 150$

R = 451

Multiply and Add

AB

ALG2044-05



Lecture 44: Page 5

$$\frac{x^3 - 27}{x - 3}$$

Be sure to write your dividend in order; if there are missing terms, fill in zeros.

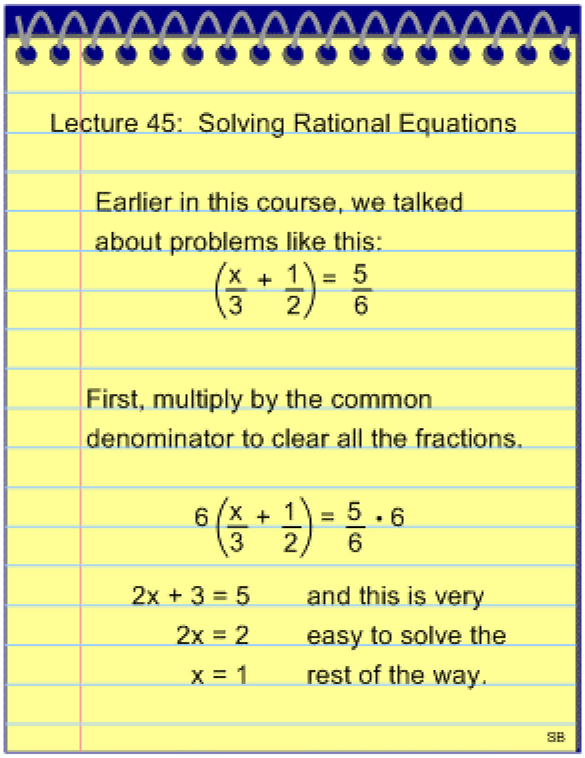
$$\begin{array}{r|rrrr} 3 & 1 & 0 & 0 & -27 \\ & & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & 0 \end{array}$$

$$\frac{x^3 - 27}{x - 3} = x^2 + 3x + 9$$

AB

Lecture 45 Notes

ALG2045-01



Lecture 45: Solving Rational Equations

Earlier in this course, we talked about problems like this:

$$\left(\frac{x}{3} + \frac{1}{2}\right) = \frac{5}{6}$$

First, multiply by the common denominator to clear all the fractions.

$$6\left(\frac{x}{3} + \frac{1}{2}\right) = \frac{5}{6} \cdot 6$$

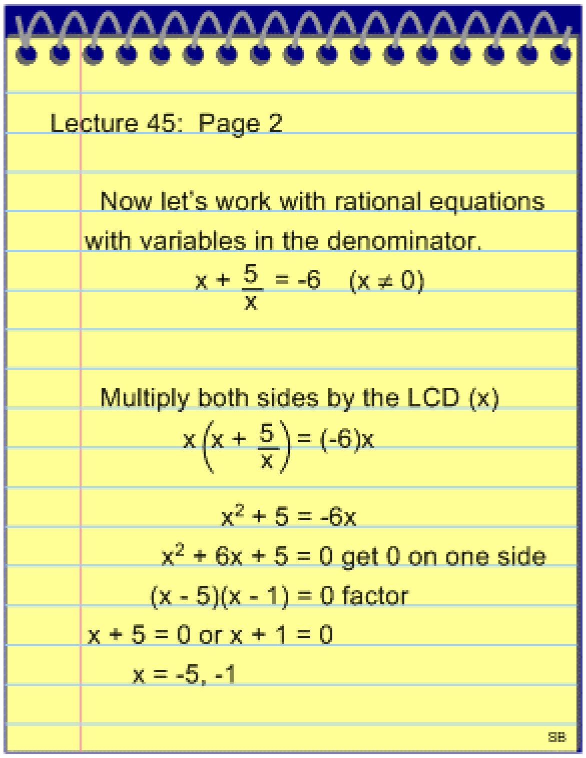
$2x + 3 = 5$ and this is very

$2x = 2$ easy to solve the

$x = 1$ rest of the way.

SB

ALG2045-02



Lecture 45: Page 2

Now let's work with rational equations with variables in the denominator.

$$x + \frac{5}{x} = -6 \quad (x \neq 0)$$

Multiply both sides by the LCD (x)

$$x\left(x + \frac{5}{x}\right) = (-6)x$$

$x^2 + 5 = -6x$

$x^2 + 6x + 5 = 0$ get 0 on one side

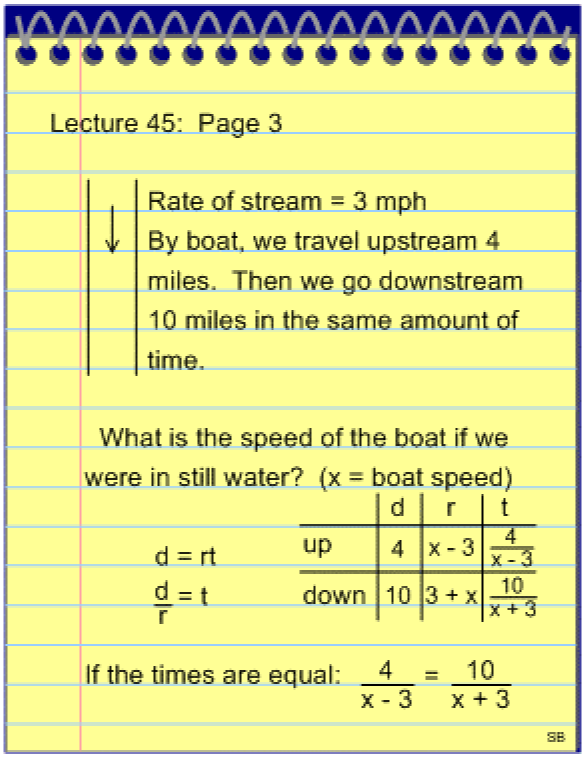
$(x - 5)(x - 1) = 0$ factor

$x + 5 = 0$ or $x + 1 = 0$

$x = -5, -1$

SB

ALG2045-03



Lecture 45: Page 3

↓ Rate of stream = 3 mph
By boat, we travel upstream 4 miles. Then we go downstream 10 miles in the same amount of time.

What is the speed of the boat if we were in still water? (x = boat speed)

$d = rt$

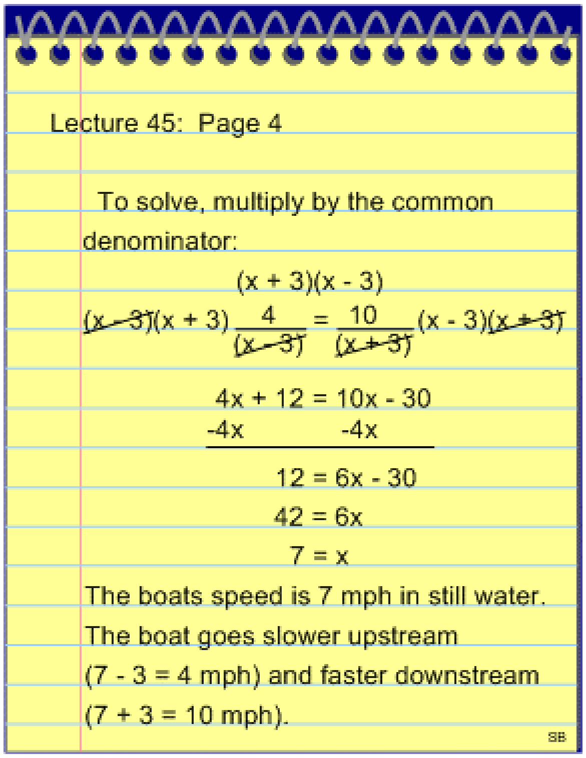
$\frac{d}{r} = t$

	d	r	t
up	4	$x - 3$	$\frac{4}{x-3}$
down	10	$3 + x$	$\frac{10}{x+3}$

If the times are equal: $\frac{4}{x-3} = \frac{10}{x+3}$

SB

ALG2045-04



Lecture 45: Page 4

To solve, multiply by the common denominator:

$(x + 3)(x - 3)$

$$\cancel{(x-3)}(x+3)\frac{4}{\cancel{(x-3)}} = \frac{10}{\cancel{(x+3)}}(x-3)\cancel{(x+3)}$$

$$\begin{aligned} 4x + 12 &= 10x - 30 \\ -4x \quad &\quad\; -4x \end{aligned}$$

$12 = 6x - 30$

$42 = 6x$

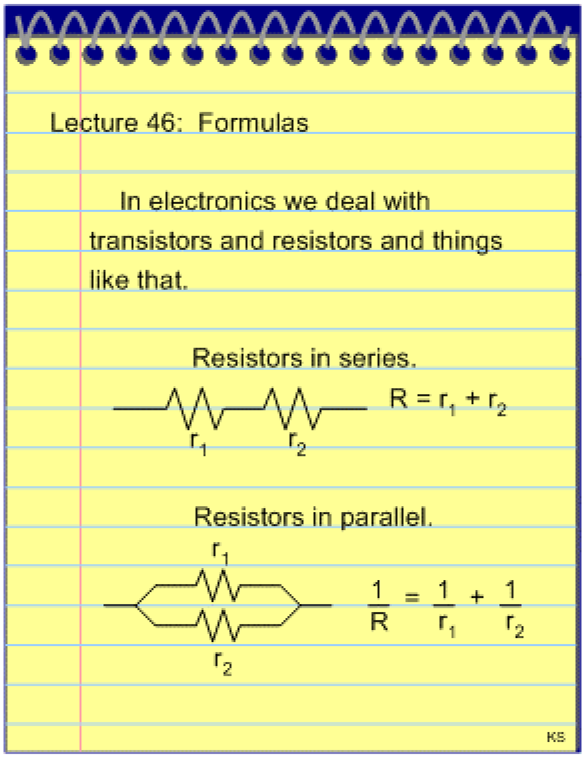
$7 = x$

The boats speed is 7 mph in still water. The boat goes slower upstream (7 - 3 = 4 mph) and faster downstream (7 + 3 = 10 mph).

SB

Lecture 46 Notes

ALG2046-01



Lecture 46: Formulas

In electronics we deal with transistors and resistors and things like that.

Resistors in series.

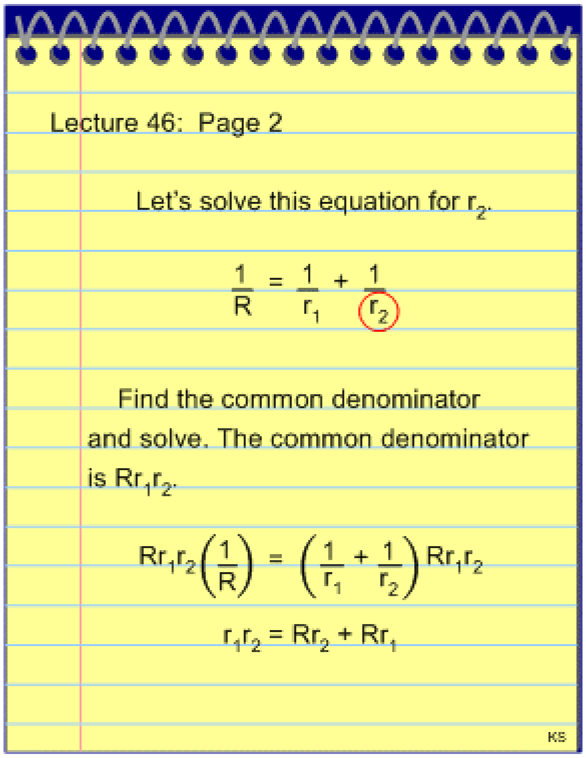
$R = r_1 + r_2$

Resistors in parallel.

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$$

KS

ALG2046-02



Lecture 46: Page 2

Let's solve this equation for r_2.

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{\boxed{r_2}}$$

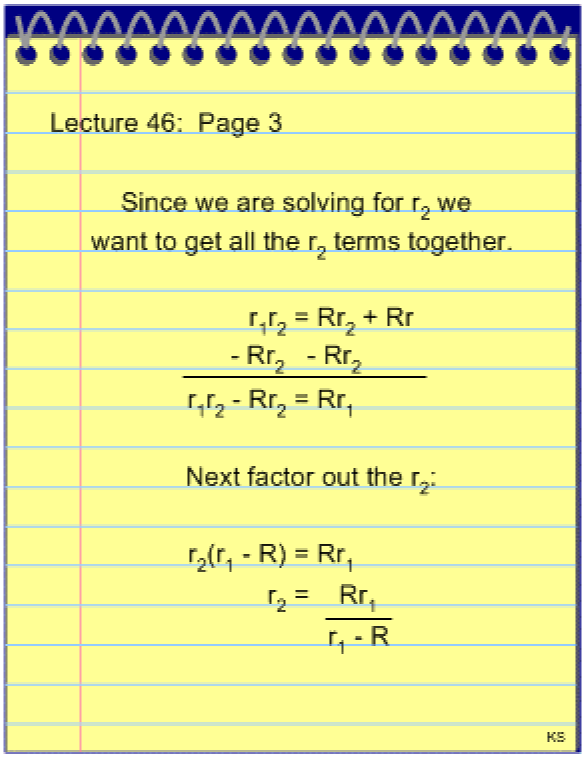
Find the common denominator and solve. The common denominator is Rr_1r_2.

$$Rr_1r_2\left(\frac{1}{R}\right) = \left(\frac{1}{r_1} + \frac{1}{r_2}\right)Rr_1r_2$$

$$r_1r_2 = Rr_2 + Rr_1$$

KS

ALG2046-03



Lecture 46: Page 3

Since we are solving for r_2 we want to get all the r_2 terms together.

$$\begin{array}{r} r_1r_2 = Rr_2 + Rr \\ - Rr_2 \quad - Rr_2 \\ \hline r_1r_2 - Rr_2 = Rr_1 \end{array}$$

Next factor out the r_2:

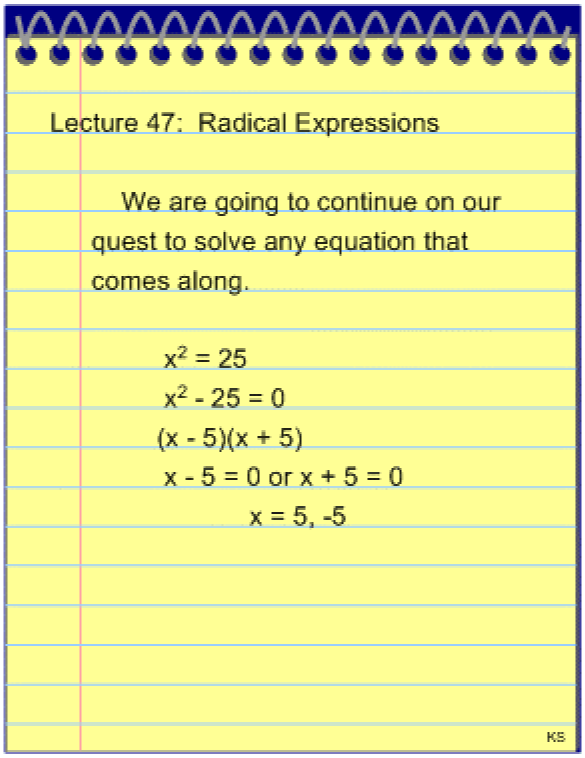
$$r_2(r_1 - R) = Rr_1$$

$$r_2 = \frac{Rr_1}{r_1 - R}$$

KS

Lecture 47 Notes

ALG2047-01



Lecture 47: Radical Expressions

We are going to continue on our quest to solve any equation that comes along.

$x^2 = 25$

$x^2 - 25 = 0$

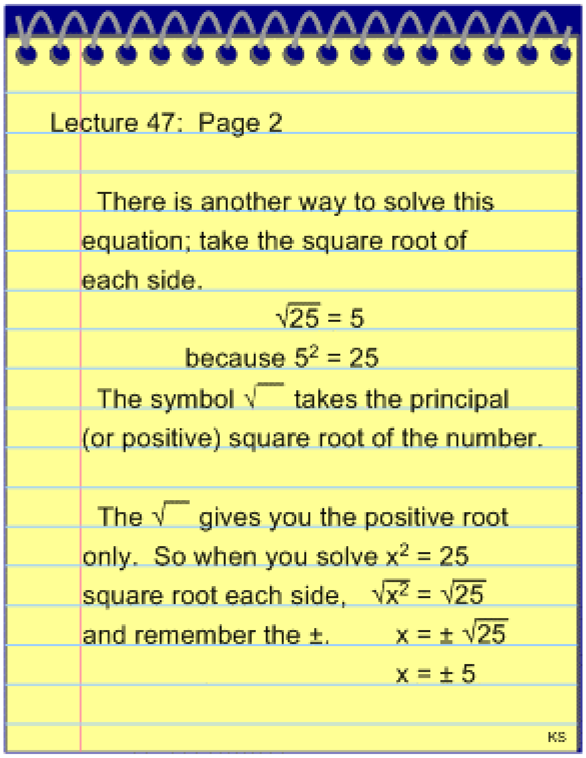
$(x - 5)(x + 5)$

$x - 5 = 0$ or $x + 5 = 0$

$x = 5, -5$

KS

ALG2047-02



Lecture 47: Page 2

There is another way to solve this equation; take the square root of each side.

$\sqrt{25} = 5$

because $5^2 = 25$

The symbol $\sqrt{\ }$ takes the principal (or positive) square root of the number.

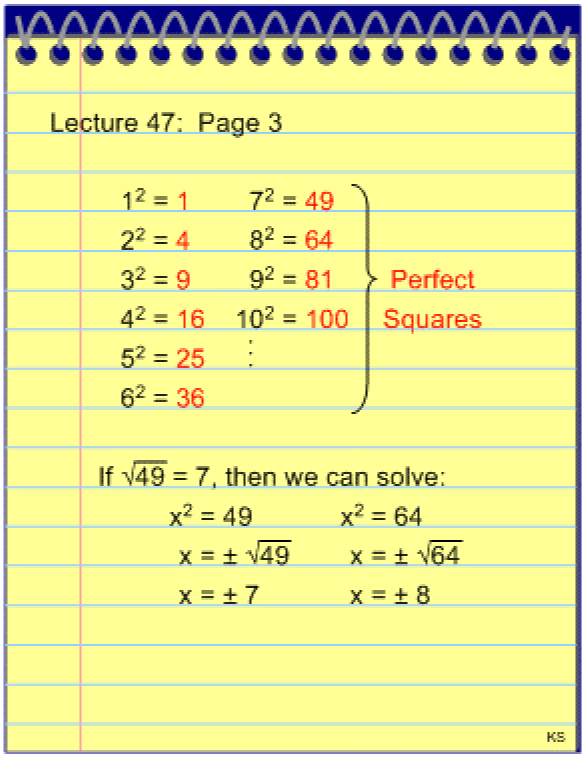
The $\sqrt{\ }$ gives you the positive root only. So when you solve $x^2 = 25$ square root each side, $\sqrt{x^2} = \sqrt{25}$

and remember the ±. $x = \pm\sqrt{25}$

$x = \pm 5$

KS

ALG2047-03



Lecture 47: Page 3

$1^2 = 1$ $7^2 = 49$

$2^2 = 4$ $8^2 = 64$

$3^2 = 9$ $9^2 = 81$ Perfect

$4^2 = 16$ $10^2 = 100$ Squares

$5^2 = 25$ ⋮

$6^2 = 36$

If $\sqrt{49} = 7$, then we can solve:

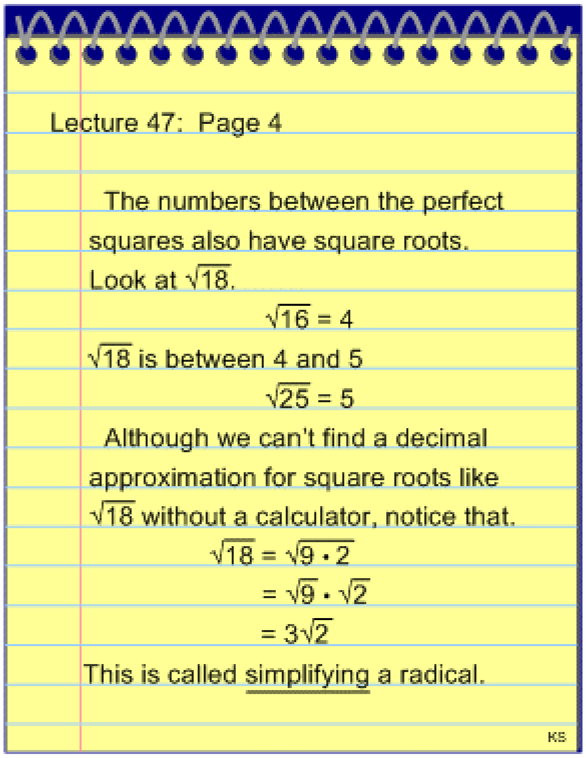
$x^2 = 49$ $x^2 = 64$

$x = \pm\sqrt{49}$ $x = \pm\sqrt{64}$

$x = \pm 7$ $x = \pm 8$

KS

ALG2047-04



Lecture 47: Page 4

The numbers between the perfect squares also have square roots. Look at $\sqrt{18}$.

$\sqrt{16} = 4$

$\sqrt{18}$ is between 4 and 5

$\sqrt{25} = 5$

Although we can't find a decimal approximation for square roots like $\sqrt{18}$ without a calculator, notice that.

$\sqrt{18} = \sqrt{9 \cdot 2}$

$= \sqrt{9} \cdot \sqrt{2}$

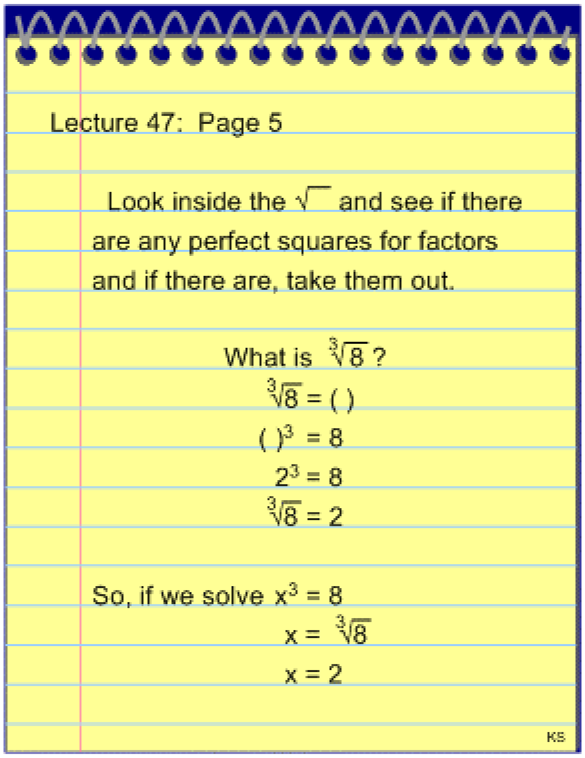
$= 3\sqrt{2}$

This is called simplifying a radical.

KS

Lecture 47 Notes, Continued

ALG2047-05



Lecture 47: Page 5

Look inside the $\sqrt{\ }$ and see if there are any perfect squares for factors and if there are, take them out.

What is $\sqrt[3]{8}$?

$\sqrt[3]{8} = (\ )$

$(\ )^3 = 8$

$2^3 = 8$

$\sqrt[3]{8} = 2$

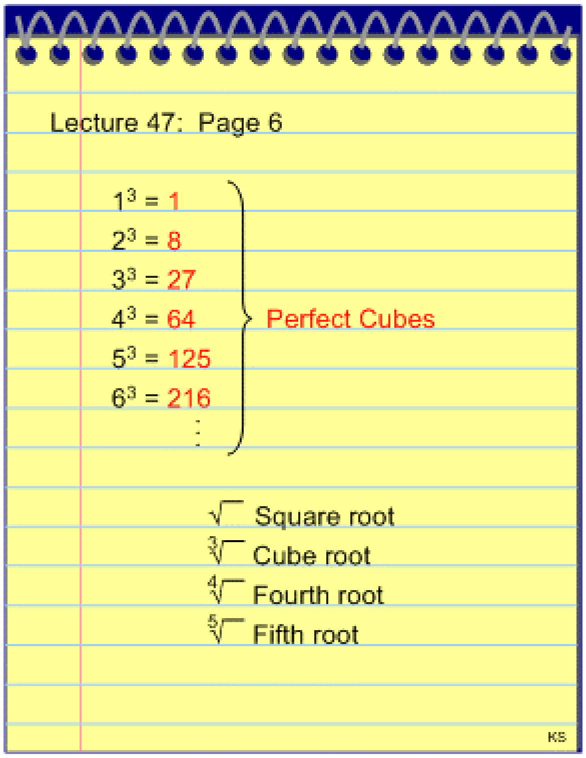
So, if we solve $x^3 = 8$

$x = \sqrt[3]{8}$

$x = 2$

KS

ALG2047-06



Lecture 47: Page 6

$$\left.\begin{array}{l} 1^3 = 1 \\ 2^3 = 8 \\ 3^3 = 27 \\ 4^3 = 64 \\ 5^3 = 125 \\ 6^3 = 216 \\ \quad\vdots \end{array}\right\} \text{Perfect Cubes}$$

$\sqrt{\ }$ Square root

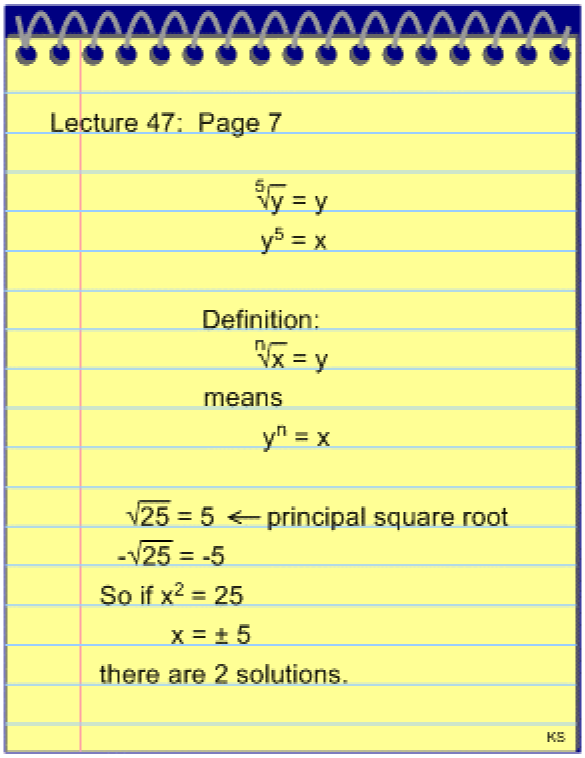
$\sqrt[3]{\ }$ Cube root

$\sqrt[4]{\ }$ Fourth root

$\sqrt[5]{\ }$ Fifth root

KS

ALG2047-07



Lecture 47: Page 7

$\sqrt[5]{y} = y$

$y^5 = x$

Definition:

$\sqrt[n]{x} = y$

means

$y^n = x$

$\sqrt{25} = 5 \leftarrow$ principal square root

$-\sqrt{25} = -5$

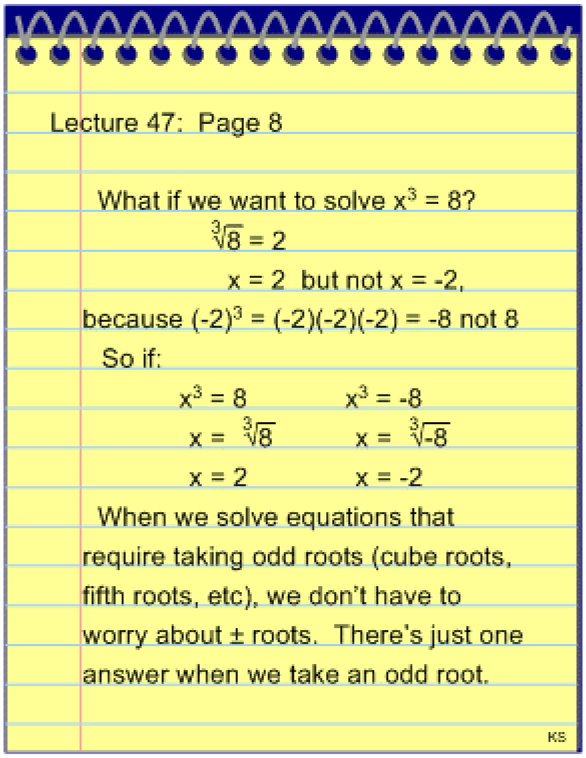
So if $x^2 = 25$

$x = \pm 5$

there are 2 solutions.

KS

ALG2047-08



Lecture 47: Page 8

What if we want to solve $x^3 = 8$?

$\sqrt[3]{8} = 2$

$x = 2$ but not $x = -2$,

because $(-2)^3 = (-2)(-2)(-2) = -8$ not 8

So if:

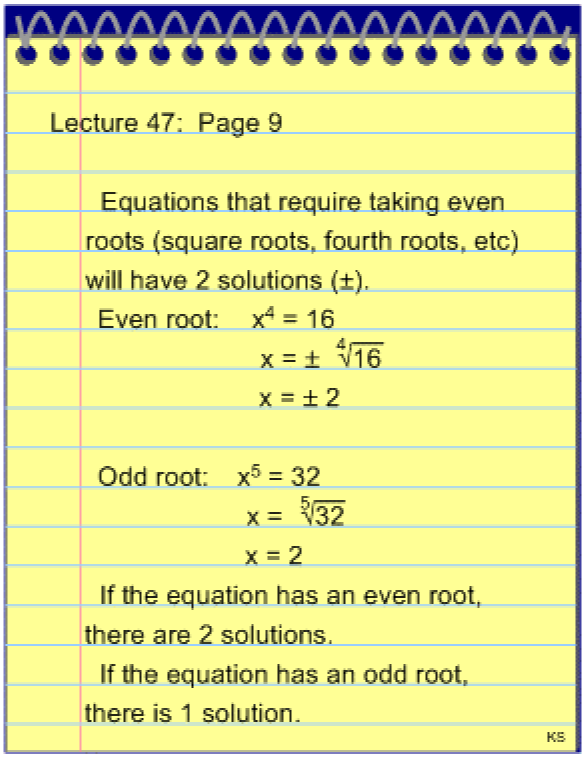
$$\begin{array}{ll} x^3 = 8 & x^3 = -8 \\ x = \sqrt[3]{8} & x = \sqrt[3]{-8} \\ x = 2 & x = -2 \end{array}$$

When we solve equations that require taking odd roots (cube roots, fifth roots, etc), we don't have to worry about ± roots. There's just one answer when we take an odd root.

KS

Lecture 47 Notes, Continued

ALG2047-09



Lecture 47: Page 9

Equations that require taking even roots (square roots, fourth roots, etc) will have 2 solutions (±).

Even root: $x^4 = 16$

$x = \pm \sqrt[4]{16}$

$x = \pm 2$

Odd root: $x^5 = 32$

$x = \sqrt[5]{32}$

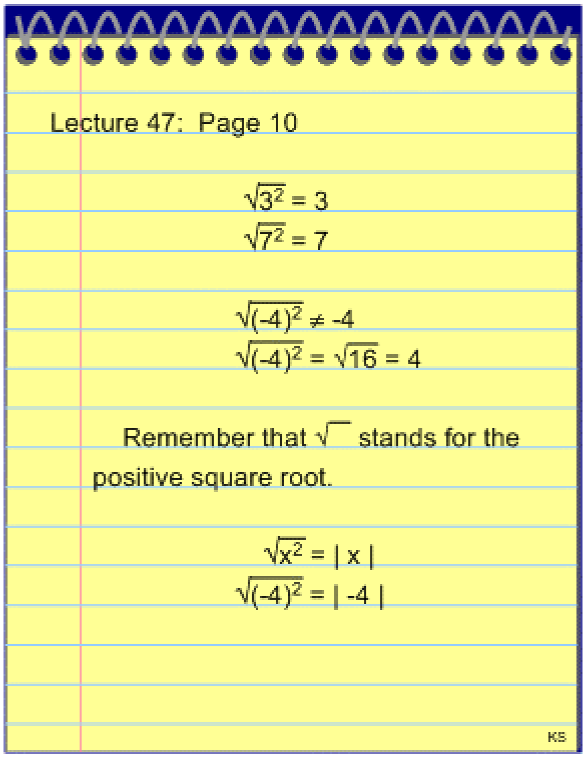
$x = 2$

If the equation has an even root, there are 2 solutions.

If the equation has an odd root, there is 1 solution.

KS

ALG2047-10



Lecture 47: Page 10

$\sqrt{3^2} = 3$

$\sqrt{7^2} = 7$

$\sqrt{(-4)^2} \neq -4$

$\sqrt{(-4)^2} = \sqrt{16} = 4$

Remember that $\sqrt{\ }$ stands for the positive square root.

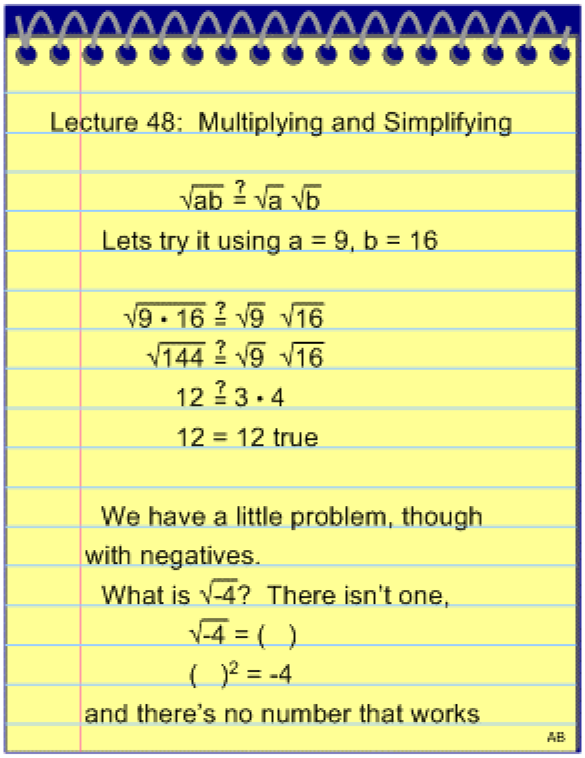
$\sqrt{x^2} = | x |$

$\sqrt{(-4)^2} = | -4 |$

KS

Lecture 48 Notes

ALG2048-01



Lecture 48: Multiplying and Simplifying

$\sqrt{ab} \overset{?}{=} \sqrt{a}\ \sqrt{b}$

Lets try it using a = 9, b = 16

$\sqrt{9 \cdot 16} \overset{?}{=} \sqrt{9}\ \ \sqrt{16}$

$\sqrt{144} \overset{?}{=} \sqrt{9}\ \ \sqrt{16}$

$12 \overset{?}{=} 3 \cdot 4$

12 = 12 true

We have a little problem, though with negatives.

What is $\sqrt{-4}$? There isn't one.

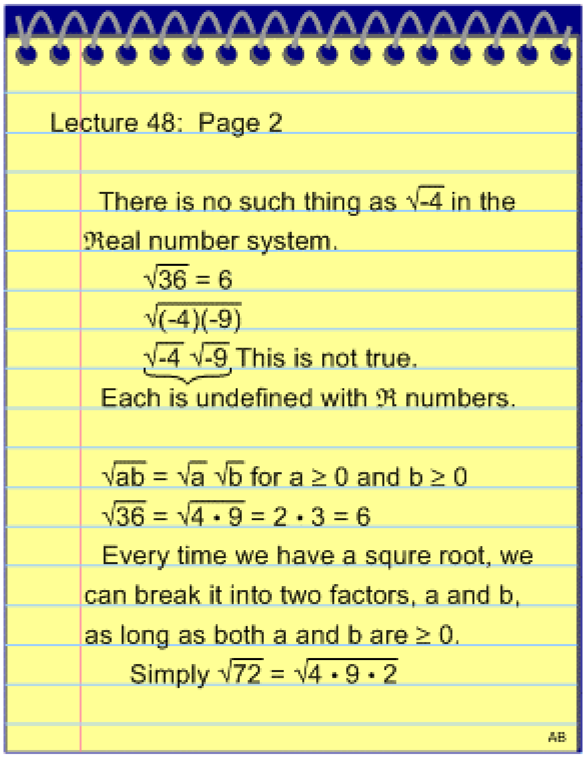
$\sqrt{-4} = (\ \ \ )$

$(\ \ \ )^2 = -4$

and there's no number that works

AB

ALG2048-02



Lecture 48: Page 2

There is no such thing as $\sqrt{-4}$ in the ℜeal number system.

$\sqrt{36} = 6$

$\sqrt{(-4)(-9)}$

$\underbrace{\sqrt{-4}\ \sqrt{-9}}$ This is not true.

Each is undefined with ℜ numbers.

$\sqrt{ab} = \sqrt{a}\ \sqrt{b}$ for $a \geq 0$ and $b \geq 0$

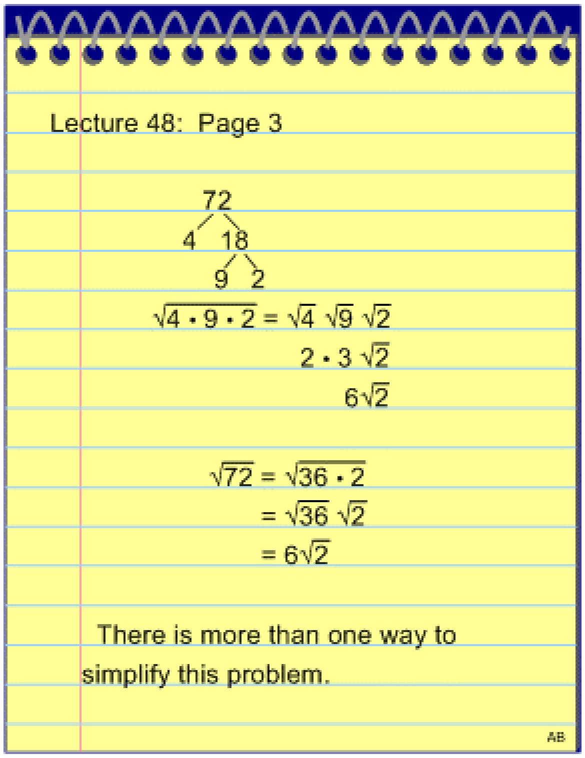
$\sqrt{36} = \sqrt{4 \cdot 9} = 2 \cdot 3 = 6$

Every time we have a squre root, we can break it into two factors, a and b, as long as both a and b are ≥ 0.

Simply $\sqrt{72} = \sqrt{4 \cdot 9 \cdot 2}$

AB

ALG2048-03



Lecture 48: Page 3

72

4 18

9 2

$\sqrt{4 \cdot 9 \cdot 2} = \sqrt{4}\ \sqrt{9}\ \sqrt{2}$

$2 \cdot 3\ \sqrt{2}$

$6\sqrt{2}$

$\sqrt{72} = \sqrt{36 \cdot 2}$

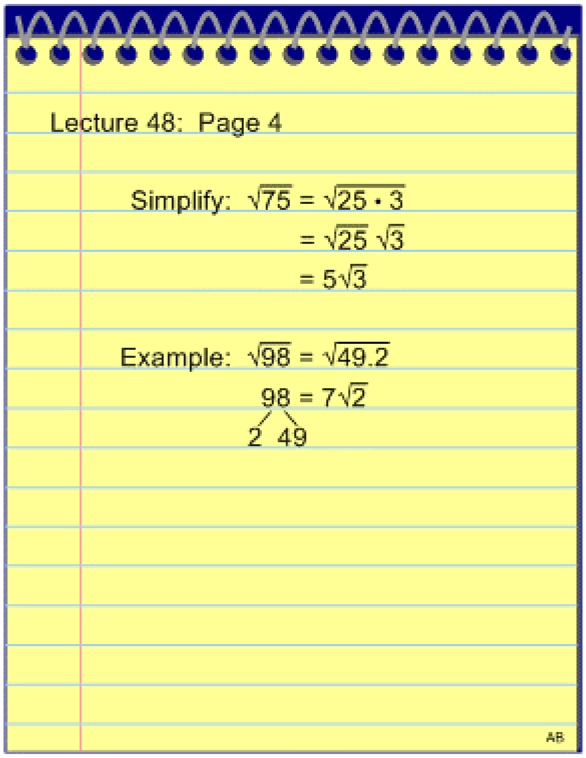
$= \sqrt{36}\ \sqrt{2}$

$= 6\sqrt{2}$

There is more than one way to simplify this problem.

AB

ALG2048-04



Lecture 48: Page 4

Simplify: $\sqrt{75} = \sqrt{25 \cdot 3}$

$= \sqrt{25}\ \sqrt{3}$

$= 5\sqrt{3}$

Example: $\sqrt{98} = \sqrt{49.2}$

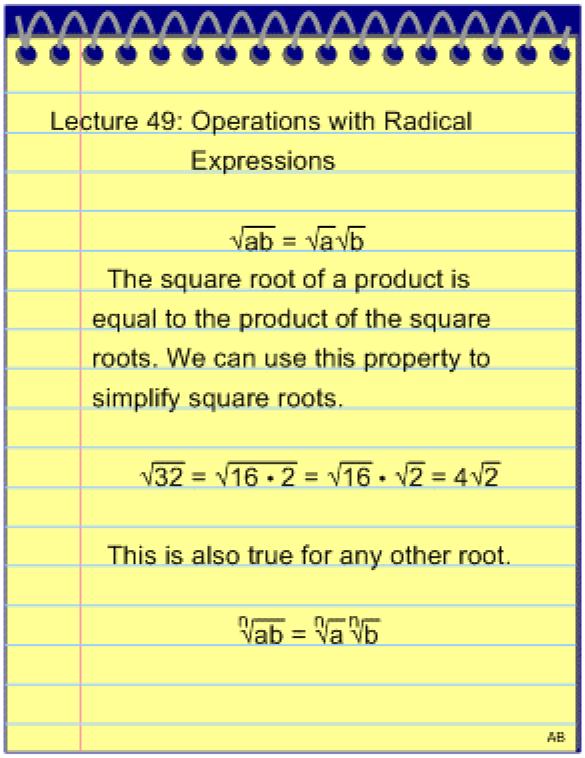
$98 = 7\sqrt{2}$

2 49

AB

Lecture 49 Notes

ALG2049-01



Lecture 49: Operations with Radical Expressions

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

The square root of a product is equal to the product of the square roots. We can use this property to simplify square roots.

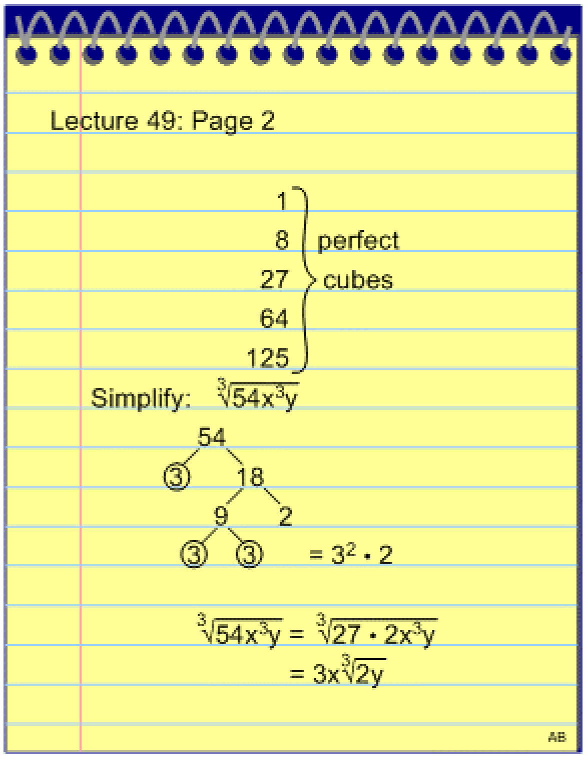
$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

This is also true for any other root.

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

AB

ALG2049-02



Lecture 49: Page 2

$$\left.\begin{matrix}1\\8\\27\\64\\125\end{matrix}\right\}\text{perfect cubes}$$

Simplify: $\sqrt[3]{54x^3y}$

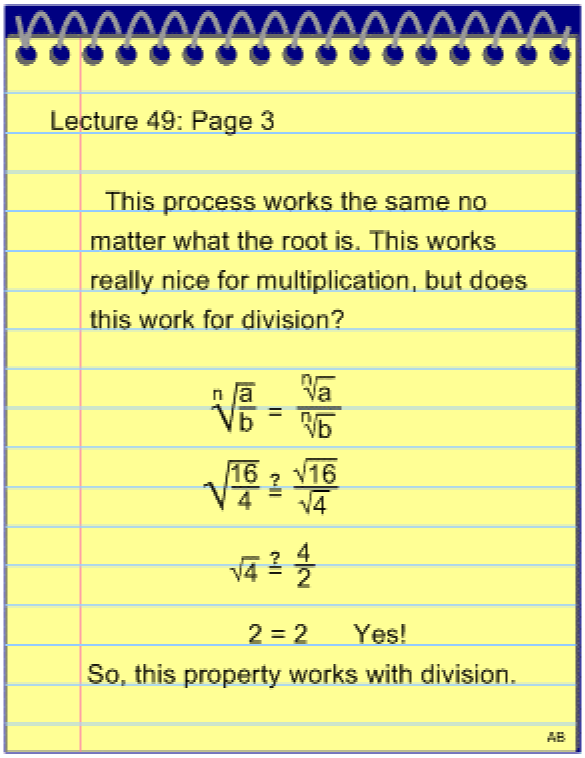
54
③ 18
9 2
③ ③ $= 3^2 \cdot 2$

$\sqrt[3]{54x^3y} = \sqrt[3]{27 \cdot 2x^3y}$

$= 3x\sqrt[3]{2y}$

AB

ALG2049-03



Lecture 49: Page 3

This process works the same no matter what the root is. This works really nice for multiplication, but does this work for division?

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt{\frac{16}{4}} \overset{?}{=} \frac{\sqrt{16}}{\sqrt{4}}$

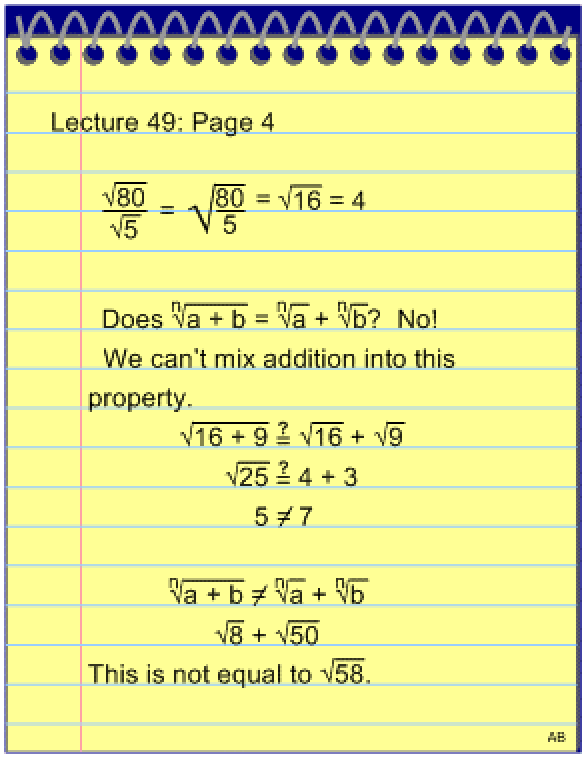
$\sqrt{4} \overset{?}{=} \frac{4}{2}$

$2 = 2$ Yes!

So, this property works with division.

AB

ALG2049-04



Lecture 49: Page 4

$\frac{\sqrt{80}}{\sqrt{5}} = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$

Does $\sqrt[n]{a+b} = \sqrt[n]{a} + \sqrt[n]{b}$? No!
We can't mix addition into this property.

$\sqrt{16+9} \overset{?}{=} \sqrt{16} + \sqrt{9}$

$\sqrt{25} \overset{?}{=} 4 + 3$

$5 \neq 7$

$\sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}$

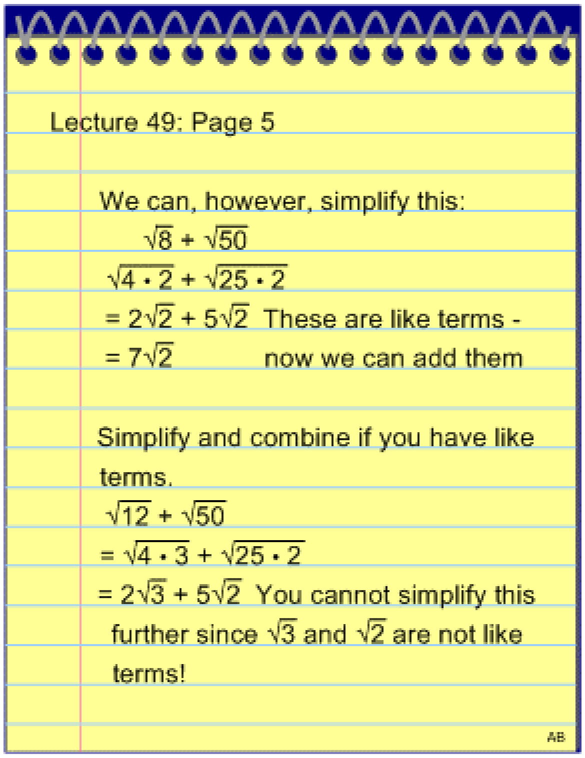
$\sqrt{8} + \sqrt{50}$

This is not equal to $\sqrt{58}$.

AB

Lecture 49 Notes, Continued

ALG2049-05



Lecture 49: Page 5

We can, however, simplify this:

$\sqrt{8} + \sqrt{50}$

$\sqrt{4 \cdot 2} + \sqrt{25 \cdot 2}$

$= 2\sqrt{2} + 5\sqrt{2}$ These are like terms -

$= 7\sqrt{2}$ now we can add them

Simplify and combine if you have like terms.

$\sqrt{12} + \sqrt{50}$

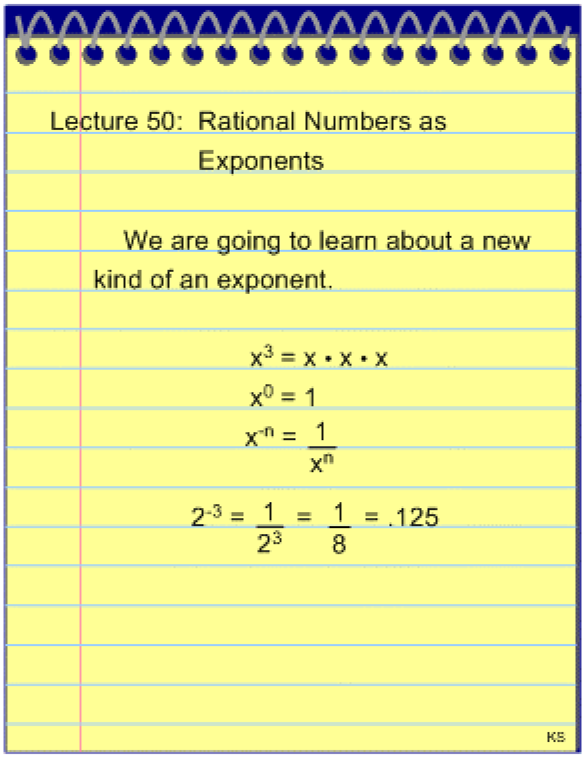
$= \sqrt{4 \cdot 3} + \sqrt{25 \cdot 2}$

$= 2\sqrt{3} + 5\sqrt{2}$ You cannot simplify this further since $\sqrt{3}$ and $\sqrt{2}$ are not like terms!

AB

Lecture 50 Notes

ALG2050-01



Lecture 50: Rational Numbers as Exponents

We are going to learn about a new kind of an exponent.

$x^3 = x \cdot x \cdot x$

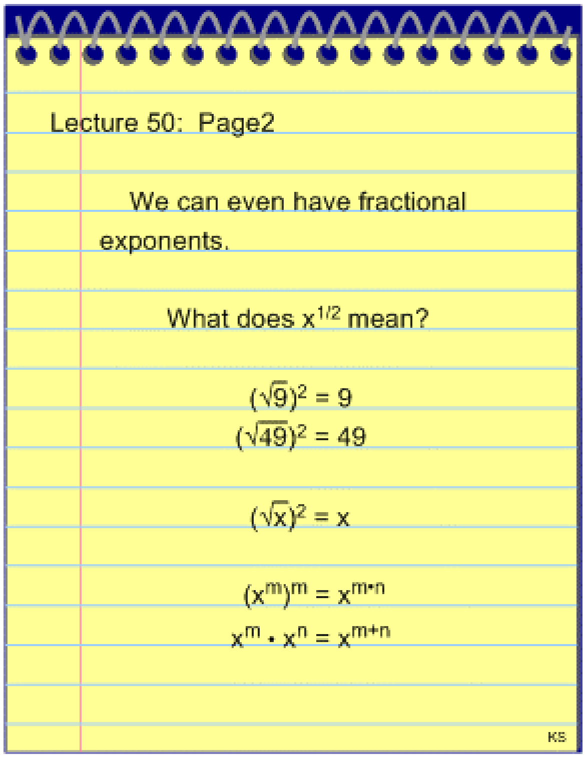
$x^0 = 1$

$x^{-n} = \frac{1}{x^n}$

$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = .125$

KS

ALG2050-02



Lecture 50: Page2

We can even have fractional exponents.

What does $x^{1/2}$ mean?

$(\sqrt{9})^2 = 9$

$(\sqrt{49})^2 = 49$

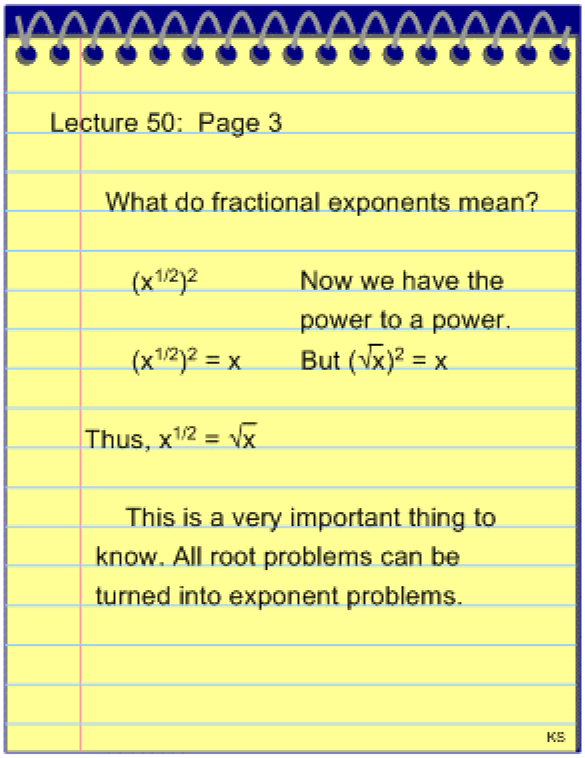
$(\sqrt{x})^2 = x$

$(x^m)^m = x^{m \cdot n}$

$x^m \cdot x^n = x^{m+n}$

KS

ALG2050-03



Lecture 50: Page 3

What do fractional exponents mean?

$(x^{1/2})^2$ Now we have the power to a power.

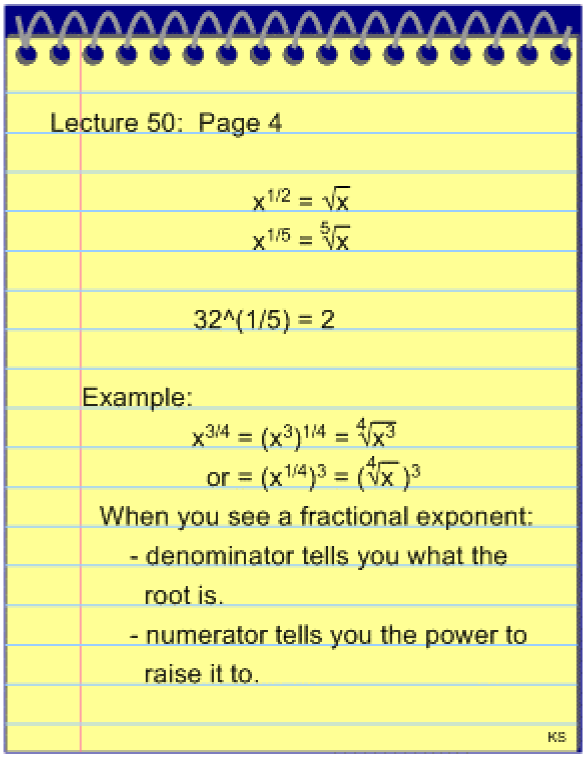
$(x^{1/2})^2 = x$ But $(\sqrt{x})^2 = x$

Thus, $x^{1/2} = \sqrt{x}$

This is a very important thing to know. All root problems can be turned into exponent problems.

KS

ALG2050-04



Lecture 50: Page 4

$x^{1/2} = \sqrt{x}$

$x^{1/5} = \sqrt[5]{x}$

32^(1/5) = 2

Example:

$x^{3/4} = (x^3)^{1/4} = \sqrt[4]{x^3}$

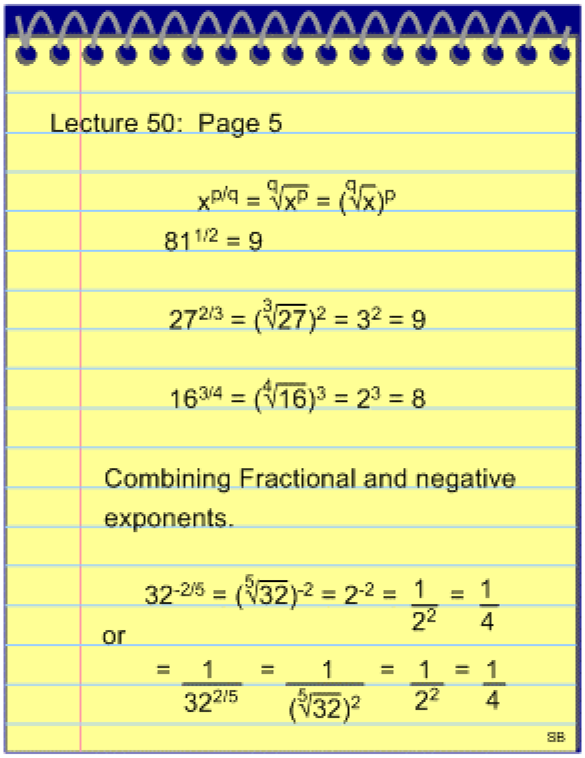
or $= (x^{1/4})^3 = (\sqrt[4]{x}\ )^3$

When you see a fractional exponent:

- denominator tells you what the root is.
- numerator tells you the power to raise it to.

KS

ALG2050-05



Lecture 50: Page 5

$x^{p/q} = \sqrt[q]{x^p} = (\sqrt[q]{x})^p$

$81^{1/2} = 9$

$27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$

$16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$

Combining Fractional and negative exponents.

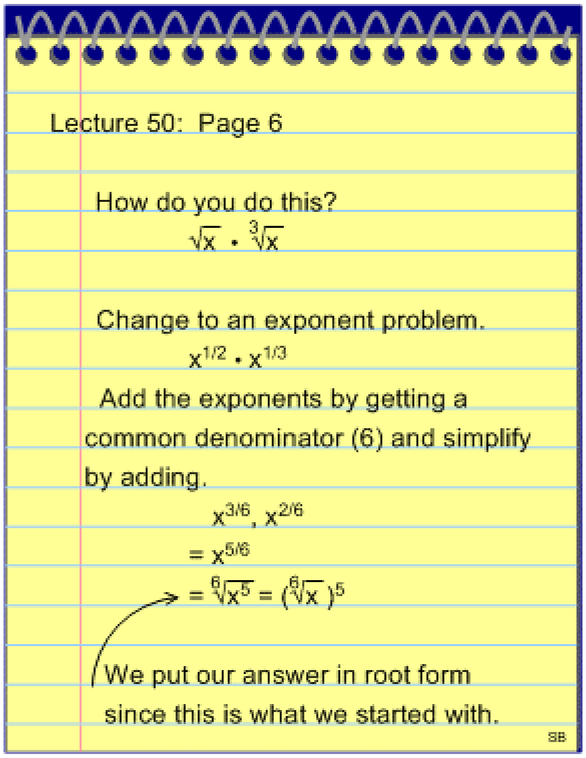
$32^{-2/5} = (\sqrt[5]{32})^{-2} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

or

$= \frac{1}{32^{2/5}} = \frac{1}{(\sqrt[5]{32})^2} = \frac{1}{2^2} = \frac{1}{4}$

SB

ALG2050-06



Lecture 50: Page 6

How do you do this?

$\sqrt{x} \cdot \sqrt[3]{x}$

Change to an exponent problem.

$x^{1/2} \cdot x^{1/3}$

Add the exponents by getting a common denominator (6) and simplify by adding.

$x^{3/6}, x^{2/6}$

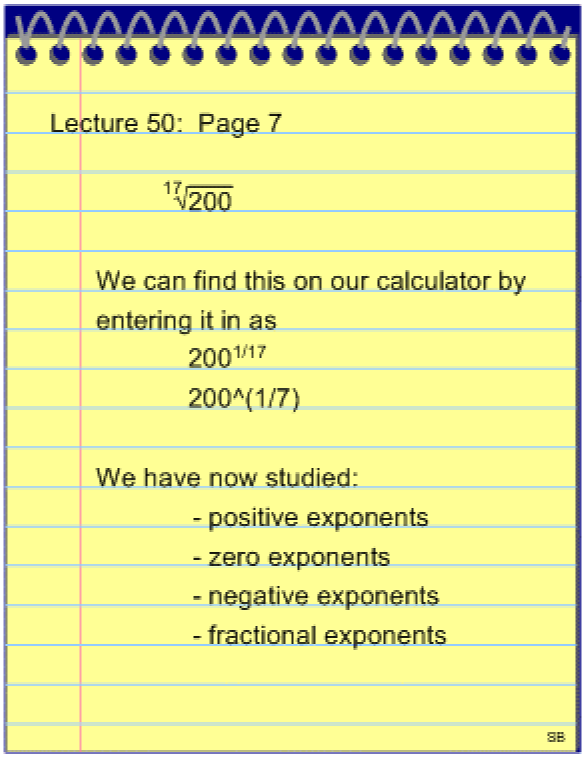
$= x^{5/6}$

$= \sqrt[6]{x^5} = (\sqrt[6]{x}\,)^5$

We put our answer in root form since this is what we started with.

SB

ALG2050-07



Lecture 50: Page 7

$\sqrt[17]{200}$

We can find this on our calculator by entering it in as

$200^{1/17}$

200^(1/7)

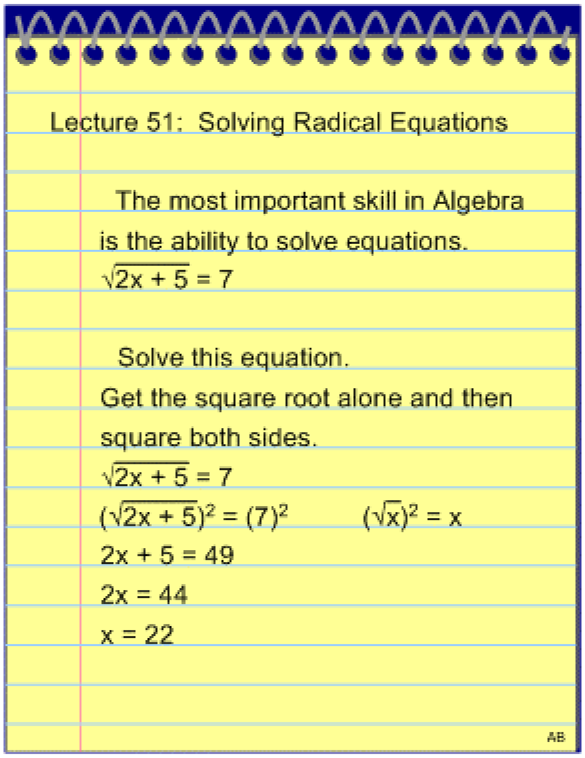
We have now studied:

- positive exponents
- zero exponents
- negative exponents
- fractional exponents

SB

Lecture 51 Notes

ALG2051-01



Lecture 51: Solving Radical Equations

The most important skill in Algebra is the ability to solve equations.

$\sqrt{2x + 5} = 7$

Solve this equation.

Get the square root alone and then square both sides.

$\sqrt{2x + 5} = 7$

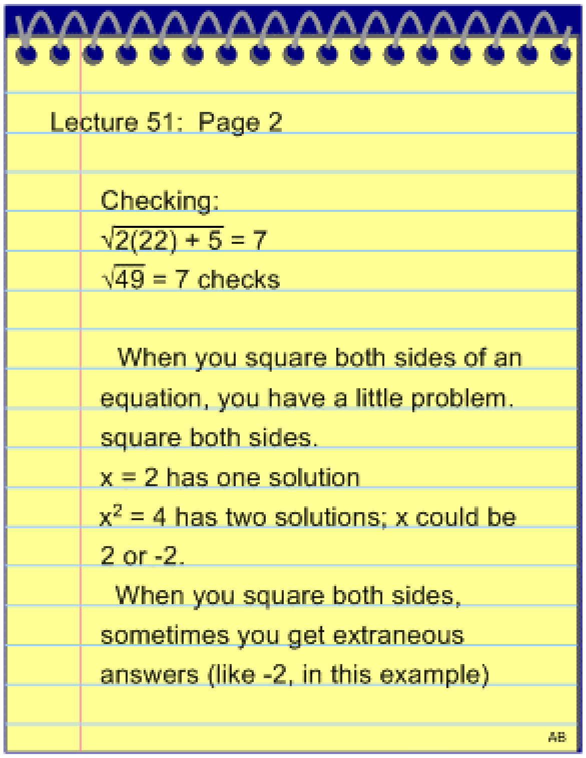
$(\sqrt{2x + 5})^2 = (7)^2 \qquad (\sqrt{x})^2 = x$

$2x + 5 = 49$

$2x = 44$

$x = 22$

ALG2051-02



Lecture 51: Page 2

Checking:

$\sqrt{2(22) + 5} = 7$

$\sqrt{49} = 7$ checks

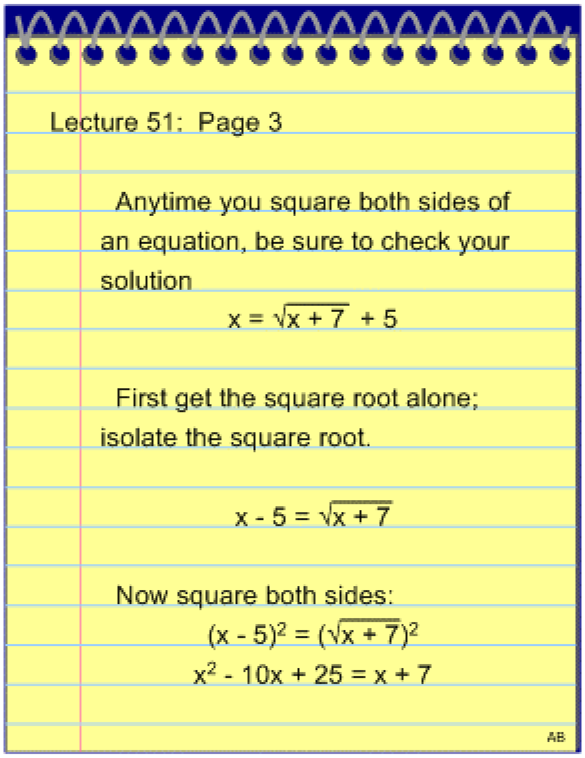
When you square both sides of an equation, you have a little problem. square both sides.

$x = 2$ has one solution

$x^2 = 4$ has two solutions; x could be 2 or -2.

When you square both sides, sometimes you get extraneous answers (like -2, in this example)

ALG2051-03



Lecture 51: Page 3

Anytime you square both sides of an equation, be sure to check your solution

$x = \sqrt{x + 7} + 5$

First get the square root alone; isolate the square root.

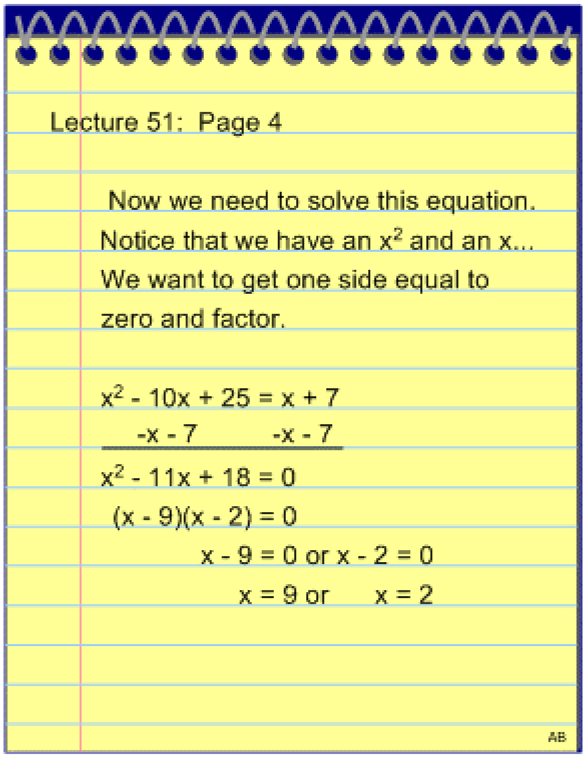
$x - 5 = \sqrt{x + 7}$

Now square both sides:

$(x - 5)^2 = (\sqrt{x + 7})^2$

$x^2 - 10x + 25 = x + 7$

ALG2051-04



Lecture 51: Page 4

Now we need to solve this equation. Notice that we have an x^2 and an x... We want to get one side equal to zero and factor.

$x^2 - 10x + 25 = x + 7$

$\underline{\quad -x - 7 \qquad\quad -x - 7}$

$x^2 - 11x + 18 = 0$

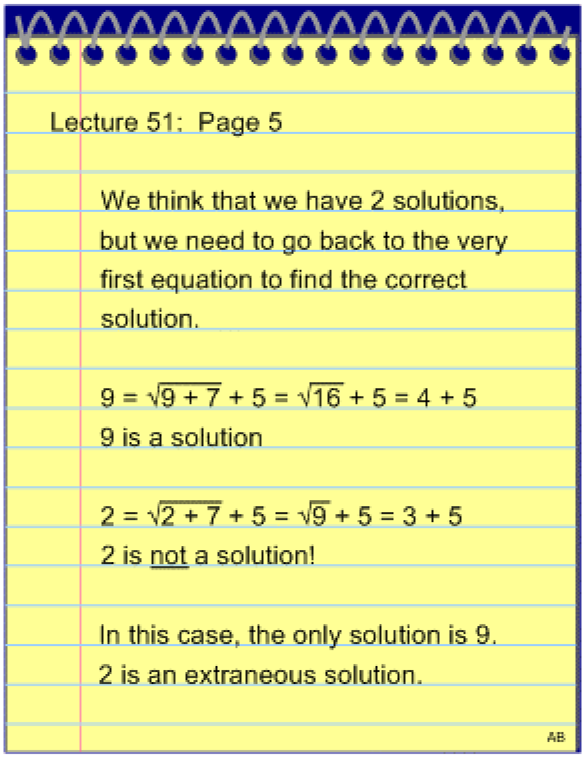
$(x - 9)(x - 2) = 0$

$x - 9 = 0$ or $x - 2 = 0$

$x = 9$ or $\quad x = 2$

Lecture 51 Notes, Continued

ALG2051-05



Lecture 51: Page 5

We think that we have 2 solutions, but we need to go back to the very first equation to find the correct solution.

$9 = \sqrt{9 + 7} + 5 = \sqrt{16} + 5 = 4 + 5$

9 is a solution

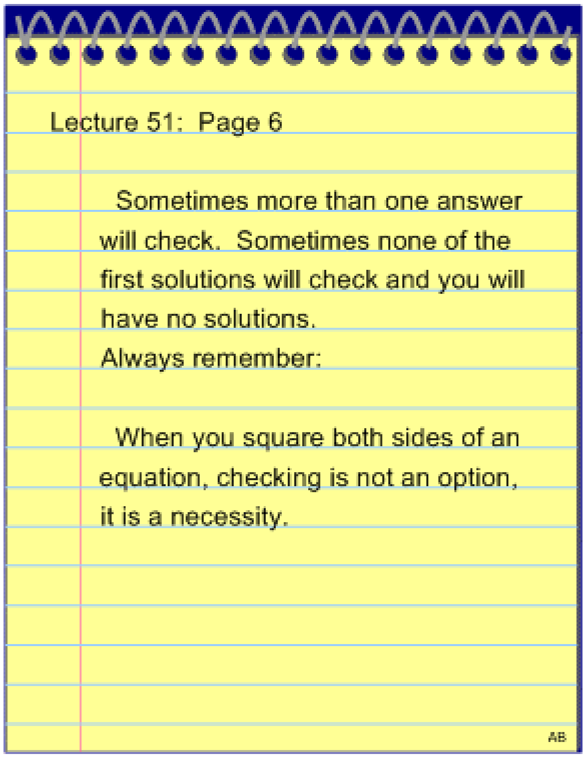
$2 = \sqrt{2 + 7} + 5 = \sqrt{9} + 5 = 3 + 5$

2 is not a solution!

In this case, the only solution is 9.
2 is an extraneous solution.

AB

ALG2051-06



Lecture 51: Page 6

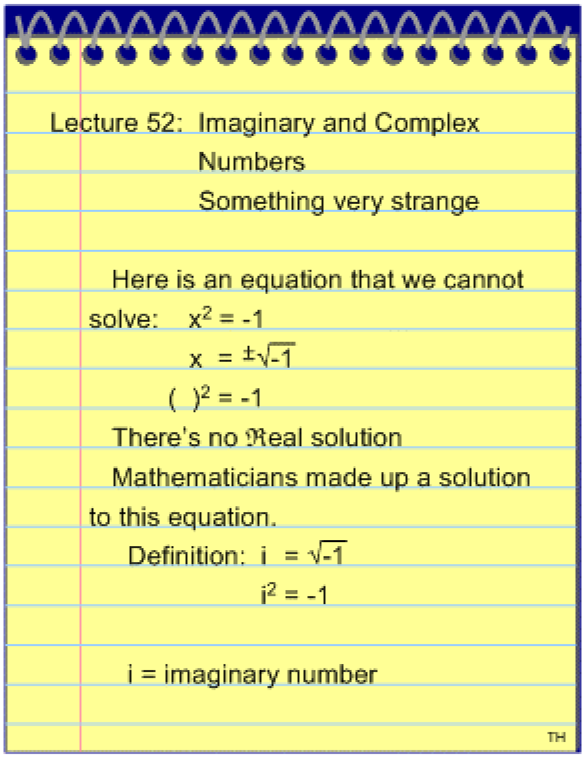
Sometimes more than one answer will check. Sometimes none of the first solutions will check and you will have no solutions.
Always remember:

When you square both sides of an equation, checking is not an option, it is a necessity.

AB

Lecture 52 Notes

ALG2052-01



Lecture 52: Imaginary and Complex Numbers

Something very strange

Here is an equation that we cannot solve: $x^2 = -1$

$x = \pm\sqrt{-1}$

$(\ \ )^2 = -1$

There's no ℜeal solution

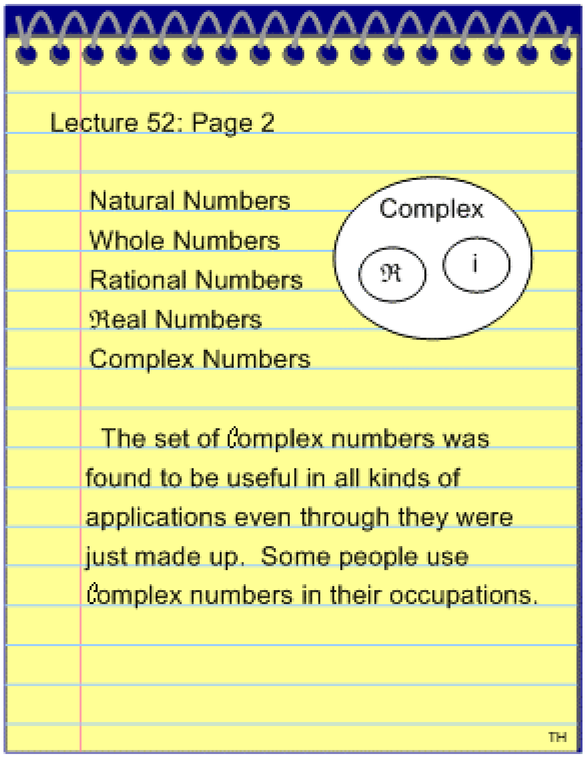
Mathematicians made up a solution to this equation.

Definition: $i = \sqrt{-1}$

$i^2 = -1$

i = imaginary number

ALG2052-02

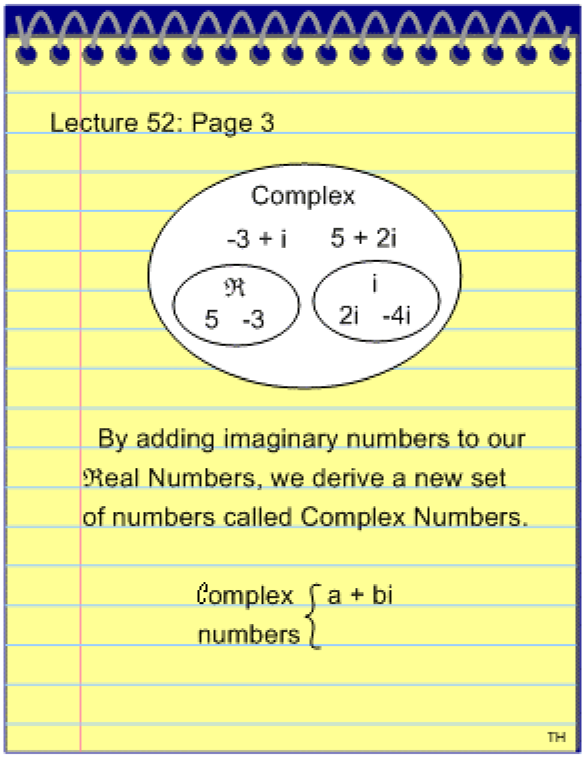


Lecture 52: Page 2

Natural Numbers
Whole Numbers
Rational Numbers
ℜeal Numbers
Complex Numbers

The set of 𝒞omplex numbers was found to be useful in all kinds of applications even through they were just made up. Some people use 𝒞omplex numbers in their occupations.

ALG2052-03

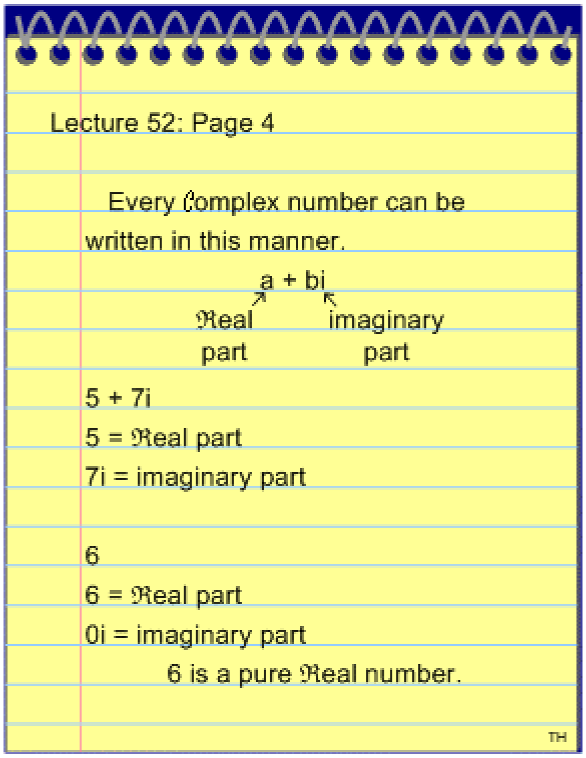


Lecture 52: Page 3

By adding imaginary numbers to our ℜeal Numbers, we derive a new set of numbers called Complex Numbers.

𝒞omplex numbers $\Big\{\ a + bi$

ALG2052-04



Lecture 52: Page 4

Every 𝒞omplex number can be written in this manner.

$a + bi$

ℜeal part imaginary part

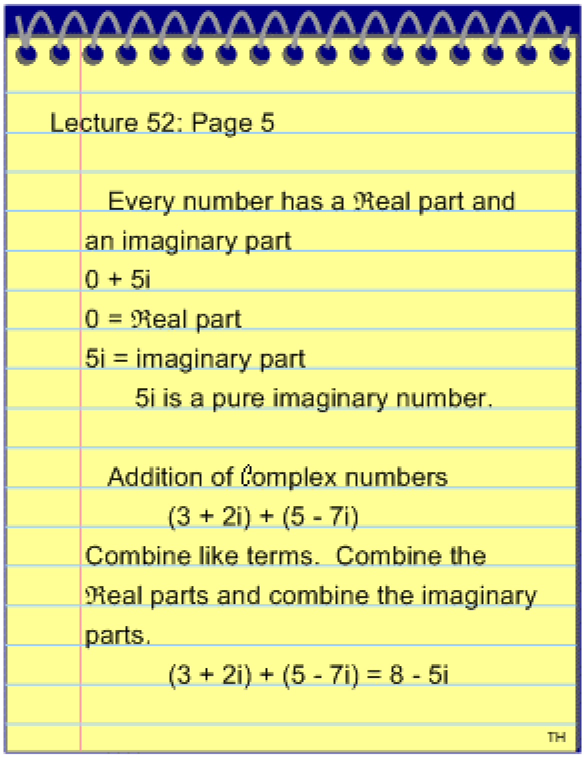
5 + 7i
5 = ℜeal part
7i = imaginary part

6
6 = ℜeal part
0i = imaginary part

6 is a pure ℜeal number.

Lecture 52 Notes, Continued

ALG2052-05



Lecture 52: Page 5

Every number has a ℜeal part and an imaginary part

$0 + 5i$

0 = ℜeal part

$5i$ = imaginary part

$5i$ is a pure imaginary number.

Addition of 𝒞omplex numbers

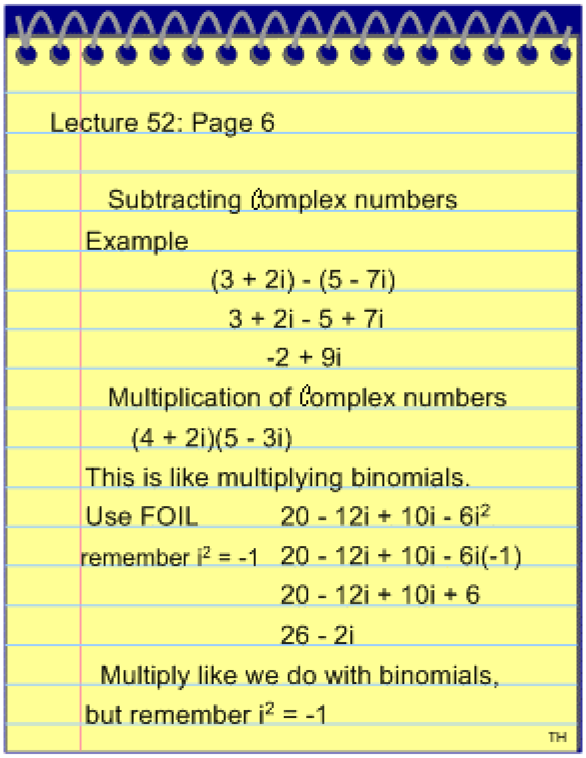
$(3 + 2i) + (5 - 7i)$

Combine like terms. Combine the ℜeal parts and combine the imaginary parts.

$(3 + 2i) + (5 - 7i) = 8 - 5i$

TH

ALG2052-06



Lecture 52: Page 6

Subtracting 𝒞omplex numbers

Example

$(3 + 2i) - (5 - 7i)$

$3 + 2i - 5 + 7i$

$-2 + 9i$

Multiplication of 𝒞omplex numbers

$(4 + 2i)(5 - 3i)$

This is like multiplying binomials.

Use FOIL $20 - 12i + 10i - 6i^2$

remember $i^2 = -1$ $20 - 12i + 10i - 6i(-1)$

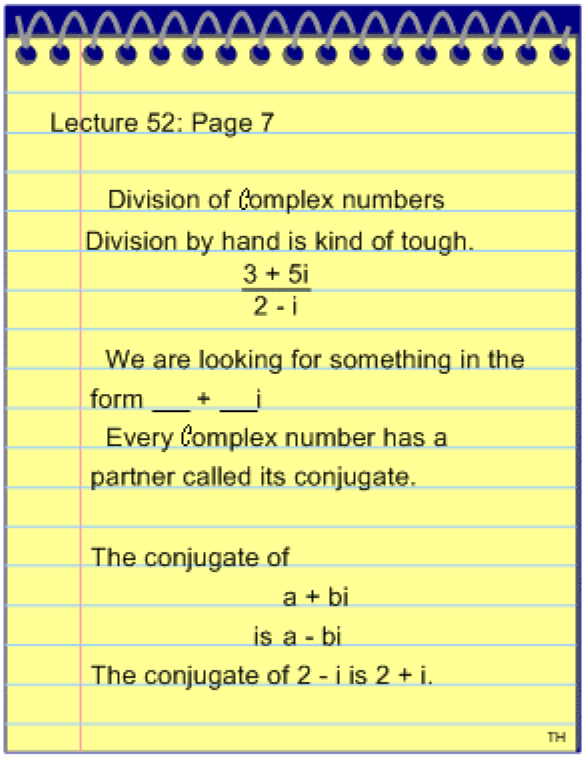
$20 - 12i + 10i + 6$

$26 - 2i$

Multiply like we do with binomials, but remember $i^2 = -1$

TH

ALG2052-07



Lecture 52: Page 7

Division of 𝒞omplex numbers

Division by hand is kind of tough.

$$\frac{3 + 5i}{2 - i}$$

We are looking for something in the form ___ + ___i

Every 𝒞omplex number has a partner called its conjugate.

The conjugate of

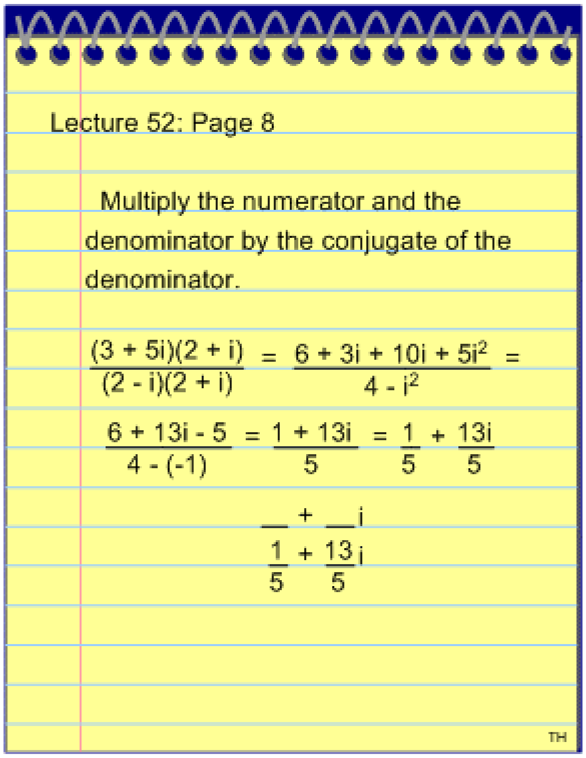
$a + bi$

is $a - bi$

The conjugate of $2 - i$ is $2 + i$.

TH

ALG2052-08



Lecture 52: Page 8

Multiply the numerator and the denominator by the conjugate of the denominator.

$$\frac{(3 + 5i)(2 + i)}{(2 - i)(2 + i)} = \frac{6 + 3i + 10i + 5i^2}{4 - i^2} =$$

$$\frac{6 + 13i - 5}{4 - (-1)} = \frac{1 + 13i}{5} = \frac{1}{5} + \frac{13i}{5}$$

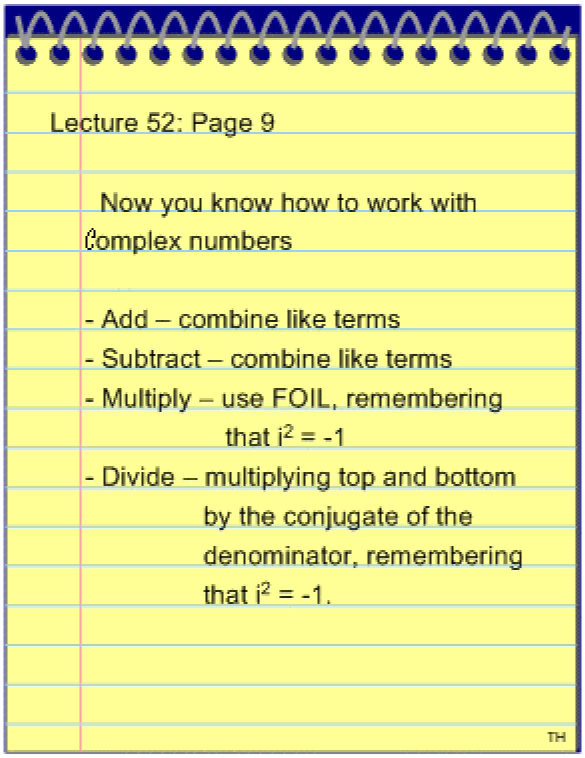
__ + __i

$$\frac{1}{5} + \frac{13}{5}i$$

TH

Lecture 52 Notes, Continued

ALG2052-09



Lecture 52: Page 9

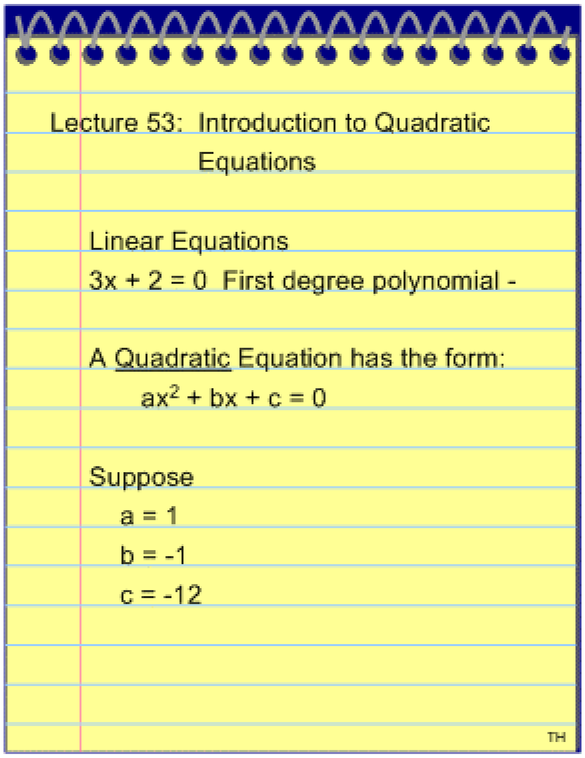
Now you know how to work with Complex numbers

- Add – combine like terms
- Subtract – combine like terms
- Multiply – use FOIL, remembering that $i^2 = -1$
- Divide – multiplying top and bottom by the conjugate of the denominator, remembering that $i^2 = -1$.

TH

Lecture 53 Notes

ALG2053-01



Lecture 53: Introduction to Quadratic Equations

Linear Equations

$3x + 2 = 0$ First degree polynomial -

A Quadratic Equation has the form:

$ax^2 + bx + c = 0$

Suppose

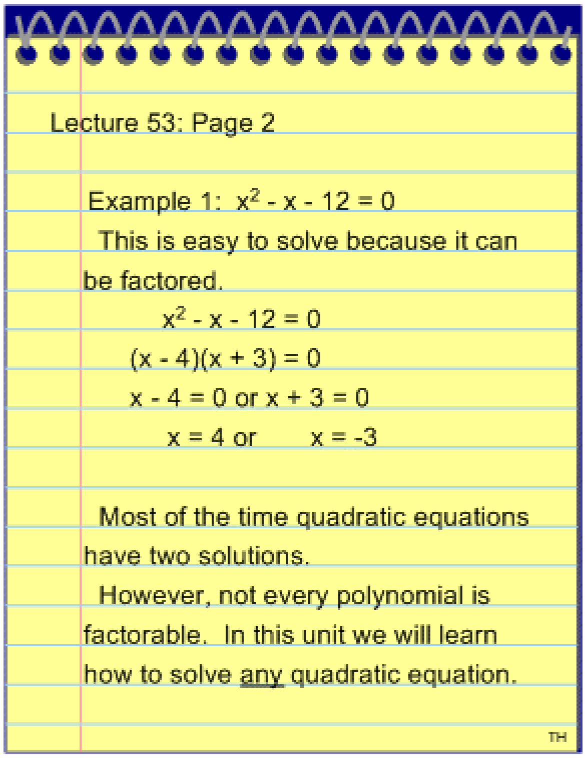
$a = 1$

$b = -1$

$c = -12$

TH

ALG2053-02



Lecture 53: Page 2

Example 1: $x^2 - x - 12 = 0$

This is easy to solve because it can be factored.

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x - 4 = 0$ or $x + 3 = 0$

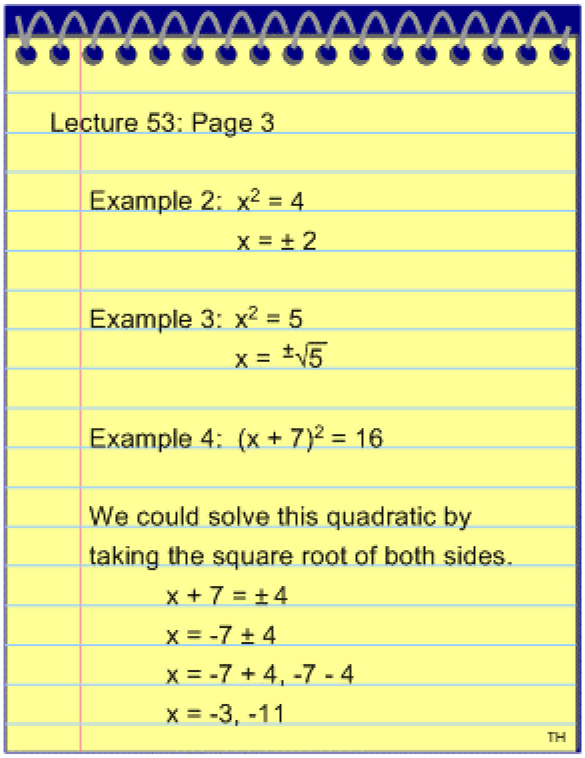
$x = 4$ or $x = -3$

Most of the time quadratic equations have two solutions.

However, not every polynomial is factorable. In this unit we will learn how to solve any quadratic equation.

TH

ALG2053-03



Lecture 53: Page 3

Example 2: $x^2 = 4$

$x = \pm 2$

Example 3: $x^2 = 5$

$x = \pm\sqrt{5}$

Example 4: $(x + 7)^2 = 16$

We could solve this quadratic by taking the square root of both sides.

$x + 7 = \pm 4$

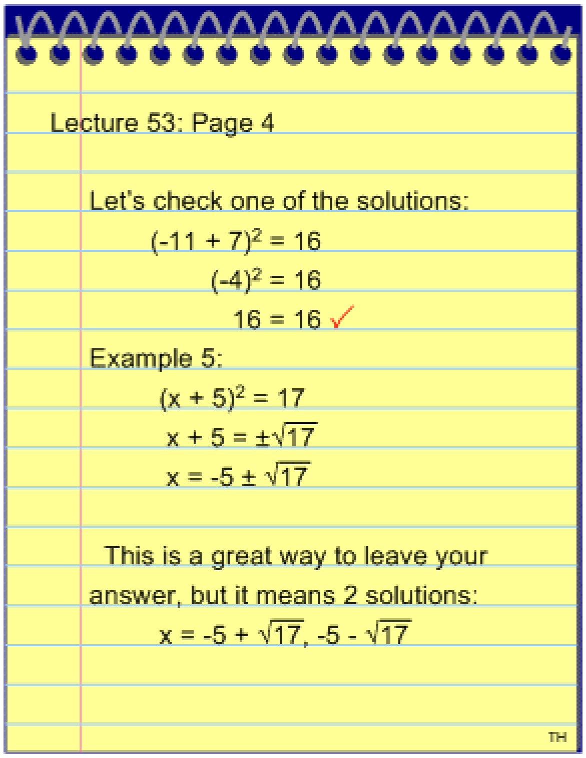
$x = -7 \pm 4$

$x = -7 + 4, -7 - 4$

$x = -3, -11$

TH

ALG2053-04



Lecture 53: Page 4

Let's check one of the solutions:

$(-11 + 7)^2 = 16$

$(-4)^2 = 16$

$16 = 16$ ✓

Example 5:

$(x + 5)^2 = 17$

$x + 5 = \pm\sqrt{17}$

$x = -5 \pm \sqrt{17}$

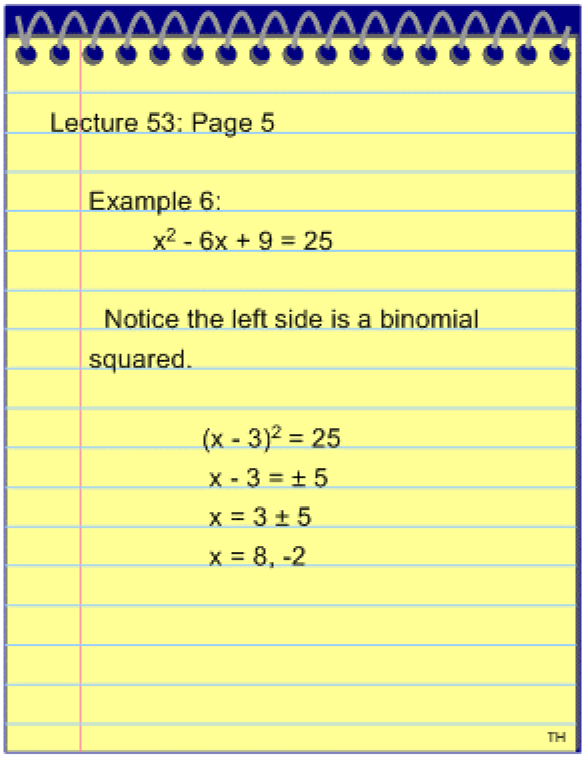
This is a great way to leave your answer, but it means 2 solutions:

$x = -5 + \sqrt{17}, -5 - \sqrt{17}$

TH

Lecture 53 Notes, Continued

ALG2053-05



Lecture 53: Page 5

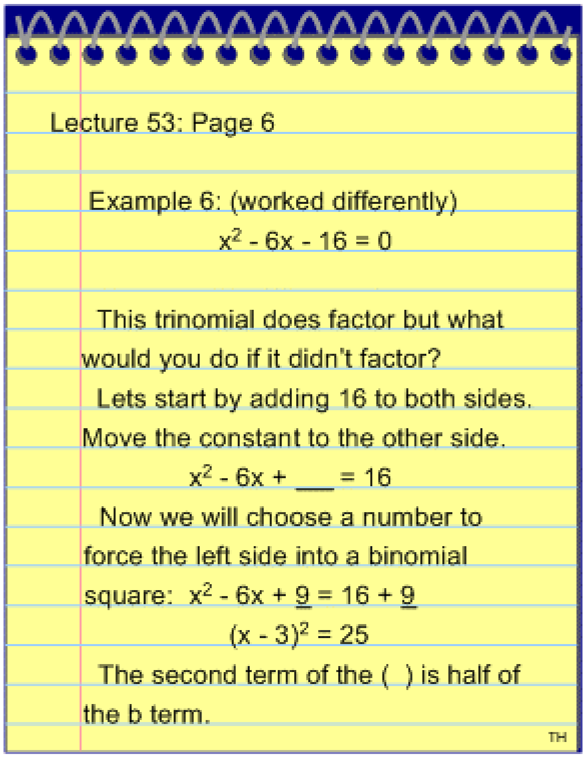
Example 6:
$x^2 - 6x + 9 = 25$

Notice the left side is a binomial squared.

$(x - 3)^2 = 25$
$x - 3 = \pm 5$
$x = 3 \pm 5$
$x = 8, -2$

TH

ALG2053-06



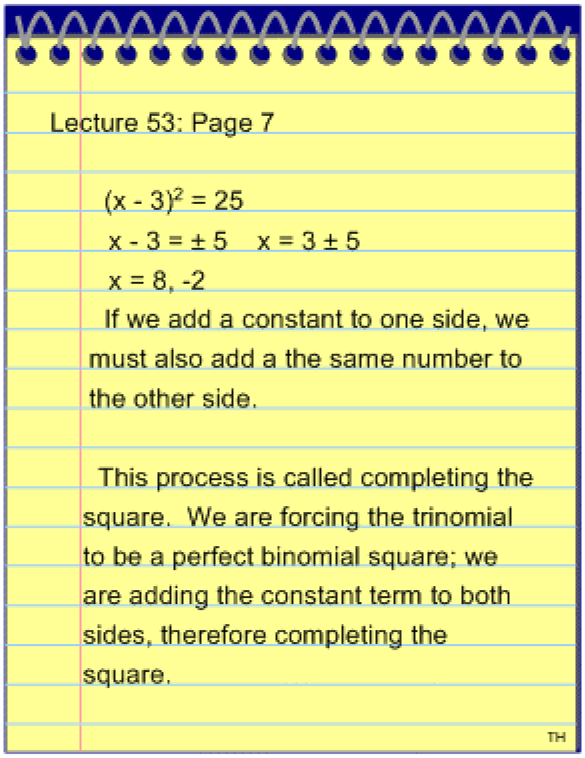
Lecture 53: Page 6

Example 6: (worked differently)
$x^2 - 6x - 16 = 0$

This trinomial does factor but what would you do if it didn't factor?
Lets start by adding 16 to both sides. Move the constant to the other side.
$x^2 - 6x + \underline{\quad} = 16$
Now we will choose a number to force the left side into a binomial square: $x^2 - 6x + \underline{9} = 16 + \underline{9}$
$(x - 3)^2 = 25$
The second term of the ( ) is half of the b term.

TH

ALG2053-07



Lecture 53: Page 7

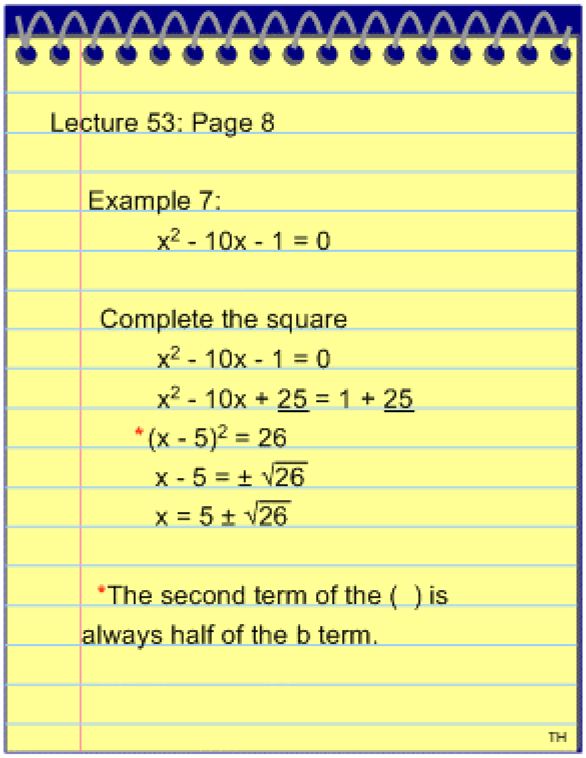
$(x - 3)^2 = 25$
$x - 3 = \pm 5 \quad x = 3 \pm 5$
$x = 8, -2$

If we add a constant to one side, we must also add a the same number to the other side.

This process is called completing the square. We are forcing the trinomial to be a perfect binomial square; we are adding the constant term to both sides, therefore completing the square.

TH

ALG2053-08



Lecture 53: Page 8

Example 7:
$x^2 - 10x - 1 = 0$

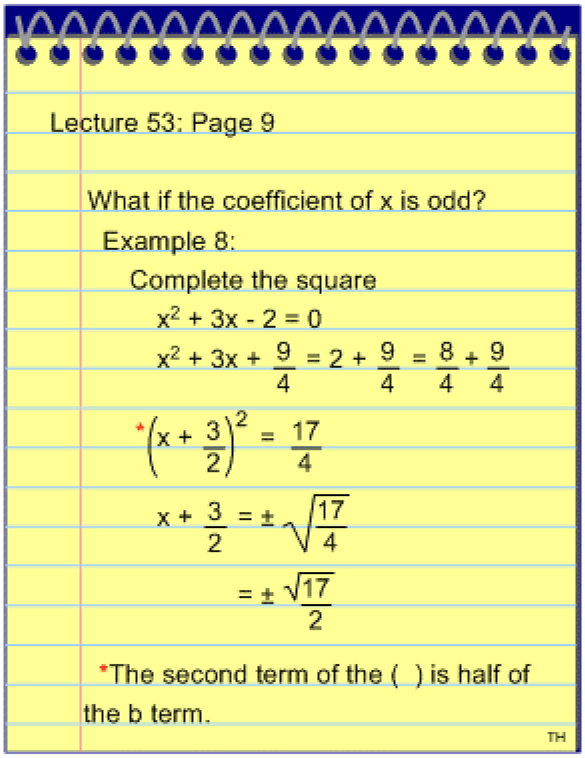
Complete the square
$x^2 - 10x - 1 = 0$
$x^2 - 10x + \underline{25} = 1 + \underline{25}$
$^*(x - 5)^2 = 26$
$x - 5 = \pm \sqrt{26}$
$x = 5 \pm \sqrt{26}$

*The second term of the ( ) is always half of the b term.

TH

Lecture 53 Notes, Continued

ALG2053-09



Lecture 53: Page 9

What if the coefficient of x is odd?

Example 8:

Complete the square

$$x^2 + 3x - 2 = 0$$

$$x^2 + 3x + \frac{9}{4} = 2 + \frac{9}{4} = \frac{8}{4} + \frac{9}{4}$$

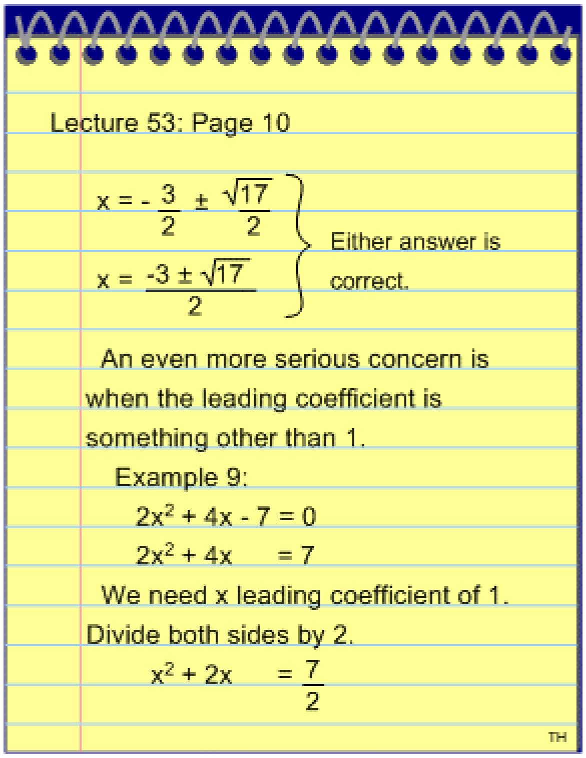
$${}^*\left(x + \frac{3}{2}\right)^2 = \frac{17}{4}$$

$$x + \frac{3}{2} = \pm\sqrt{\frac{17}{4}}$$

$$= \pm\frac{\sqrt{17}}{2}$$

*The second term of the ( ) is half of the b term.

ALG2053-10



Lecture 53: Page 10

$$\left.\begin{array}{l} x = -\frac{3}{2} \pm \frac{\sqrt{17}}{2} \\ x = \frac{-3 \pm \sqrt{17}}{2} \end{array}\right\}$$ Either answer is correct.

An even more serious concern is when the leading coefficient is something other than 1.

Example 9:

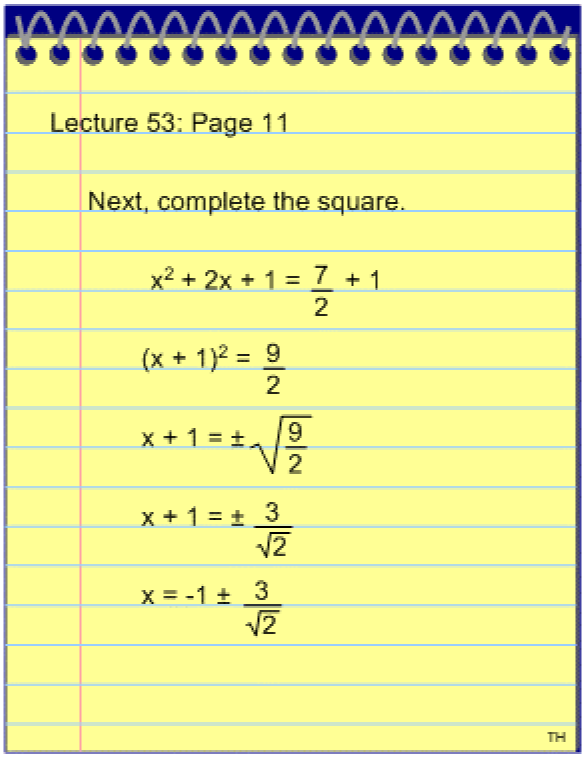
$$2x^2 + 4x - 7 = 0$$

$$2x^2 + 4x \quad = 7$$

We need x leading coefficient of 1. Divide both sides by 2.

$$x^2 + 2x \quad = \frac{7}{2}$$

ALG2053-11



Lecture 53: Page 11

Next, complete the square.

$$x^2 + 2x + 1 = \frac{7}{2} + 1$$

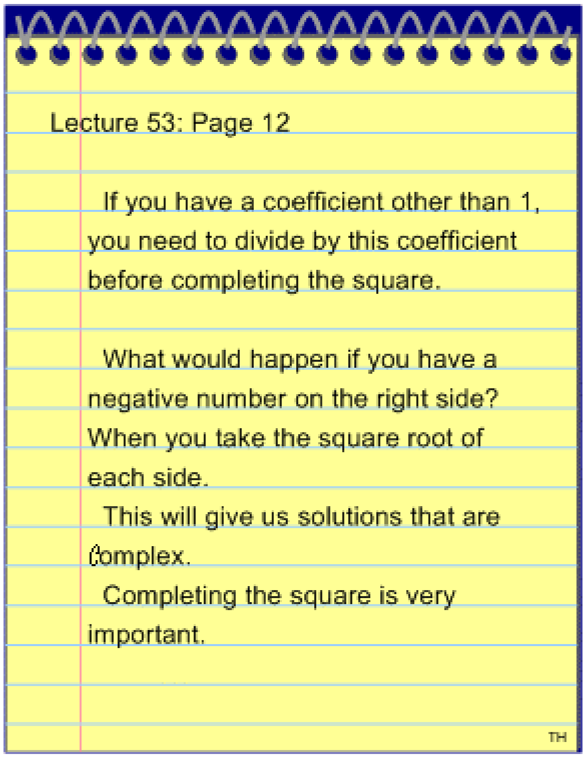
$$(x + 1)^2 = \frac{9}{2}$$

$$x + 1 = \pm\sqrt{\frac{9}{2}}$$

$$x + 1 = \pm\frac{3}{\sqrt{2}}$$

$$x = -1 \pm \frac{3}{\sqrt{2}}$$

ALG2053-12



Lecture 53: Page 12

If you have a coefficient other than 1, you need to divide by this coefficient before completing the square.

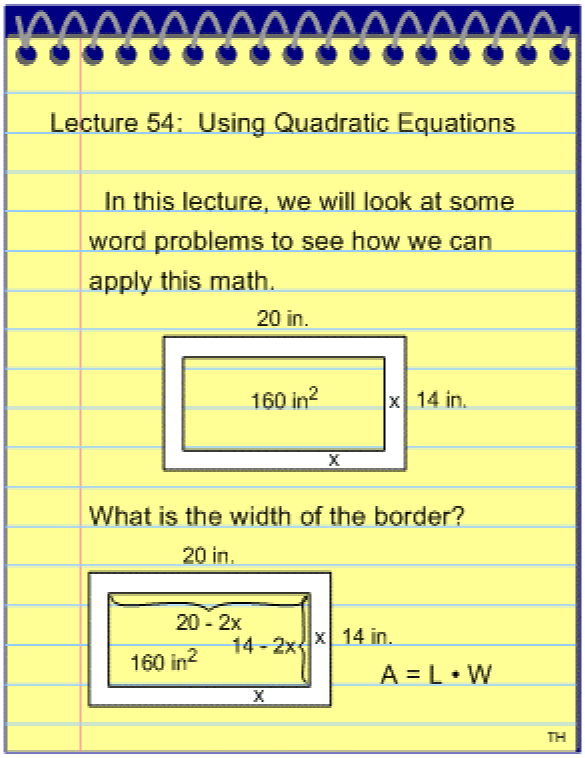
What would happen if you have a negative number on the right side? When you take the square root of each side.

This will give us solutions that are complex.

Completing the square is very important.

Lecture 54 Notes

ALG2054-01



Lecture 54: Using Quadratic Equations

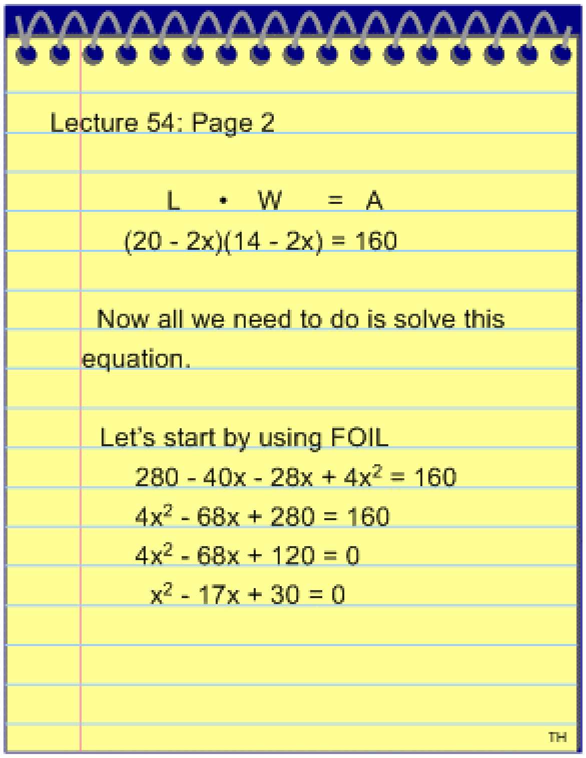
In this lecture, we will look at some word problems to see how we can apply this math.

What is the width of the border?

A = L • W

TH

ALG2054-02



Lecture 54: Page 2

L • W = A

$(20 - 2x)(14 - 2x) = 160$

Now all we need to do is solve this equation.

Let's start by using FOIL

$280 - 40x - 28x + 4x^2 = 160$

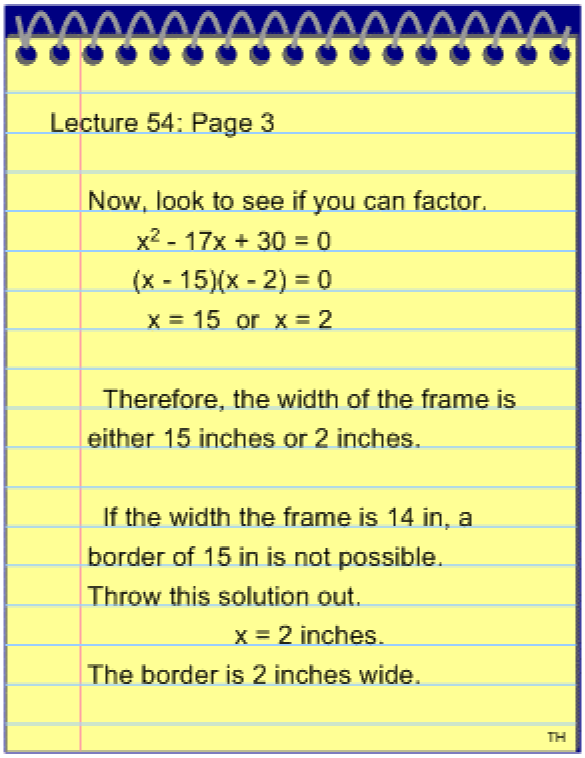
$4x^2 - 68x + 280 = 160$

$4x^2 - 68x + 120 = 0$

$x^2 - 17x + 30 = 0$

TH

ALG2054-03



Lecture 54: Page 3

Now, look to see if you can factor.

$x^2 - 17x + 30 = 0$

$(x - 15)(x - 2) = 0$

$x = 15$ or $x = 2$

Therefore, the width of the frame is either 15 inches or 2 inches.

If the width the frame is 14 in, a border of 15 in is not possible. Throw this solution out.

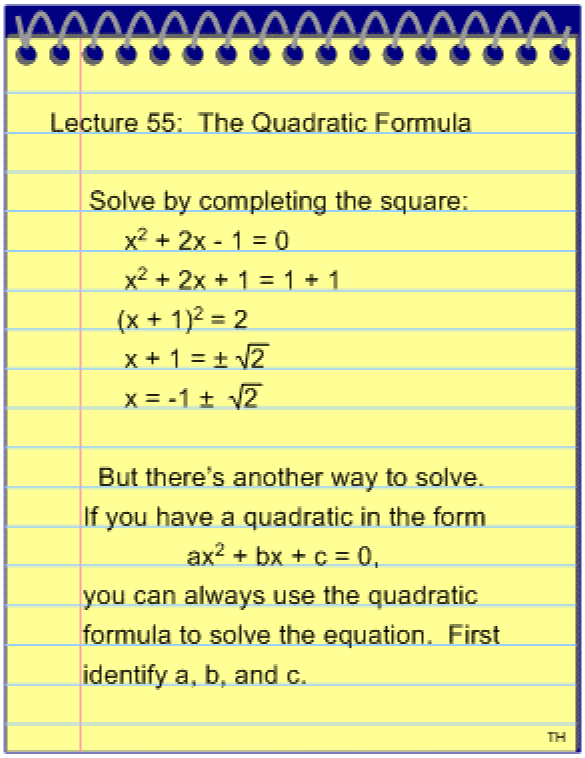
x = 2 inches.

The border is 2 inches wide.

TH

Lecture 55 Notes

ALG2055-01



Lecture 55: The Quadratic Formula

Solve by completing the square:

$x^2 + 2x - 1 = 0$

$x^2 + 2x + 1 = 1 + 1$

$(x + 1)^2 = 2$

$x + 1 = \pm \sqrt{2}$

$x = -1 \pm \sqrt{2}$

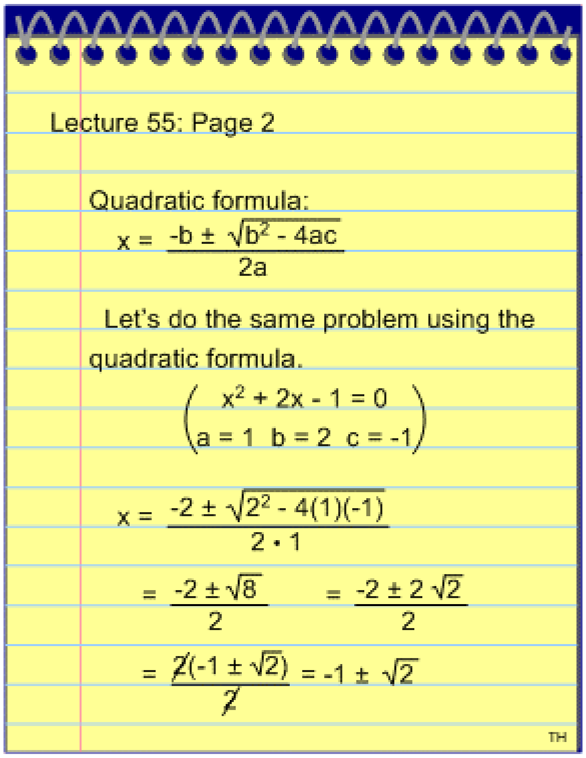
But there's another way to solve. If you have a quadratic in the form

$ax^2 + bx + c = 0,$

you can always use the quadratic formula to solve the equation. First identify a, b, and c.

TH

ALG2055-02



Lecture 55: Page 2

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let's do the same problem using the quadratic formula.

$$\left(\begin{array}{c} x^2 + 2x - 1 = 0 \\ a = 1 \quad b = 2 \quad c = -1 \end{array}\right)$$

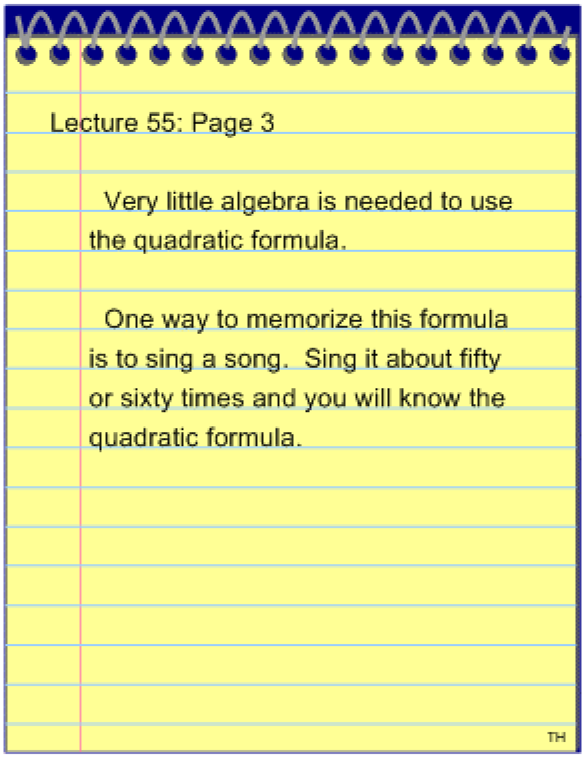
$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{8}}{2} \qquad = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$= \frac{\cancel{2}(-1 \pm \sqrt{2})}{\cancel{2}} = -1 \pm \sqrt{2}$$

TH

ALG2055-03



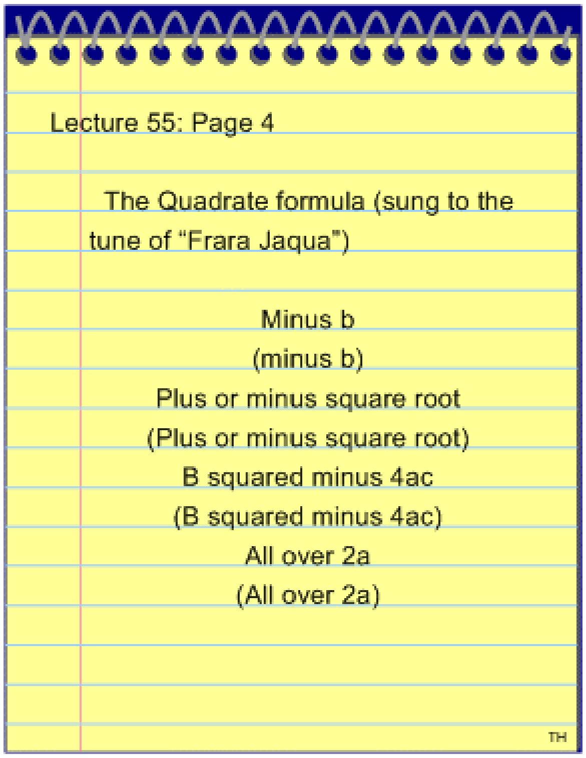
Lecture 55: Page 3

Very little algebra is needed to use the quadratic formula.

One way to memorize this formula is to sing a song. Sing it about fifty or sixty times and you will know the quadratic formula.

TH

ALG2055-04



Lecture 55: Page 4

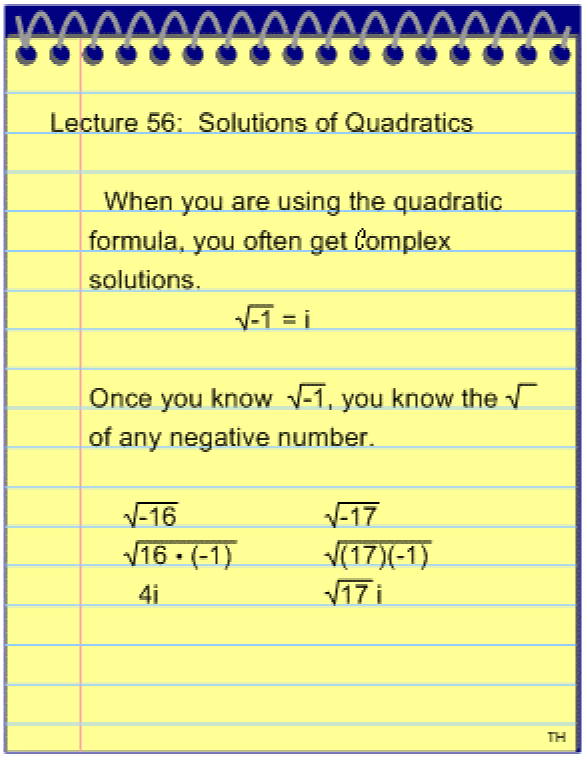
The Quadrate formula (sung to the tune of "Frara Jaqua")

Minus b
(minus b)
Plus or minus square root
(Plus or minus square root)
B squared minus 4ac
(B squared minus 4ac)
All over 2a
(All over 2a)

TH

Lecture 56 Notes

ALG2056-01



Lecture 56: Solutions of Quadratics

When you are using the quadratic formula, you often get 𝒞omplex solutions.

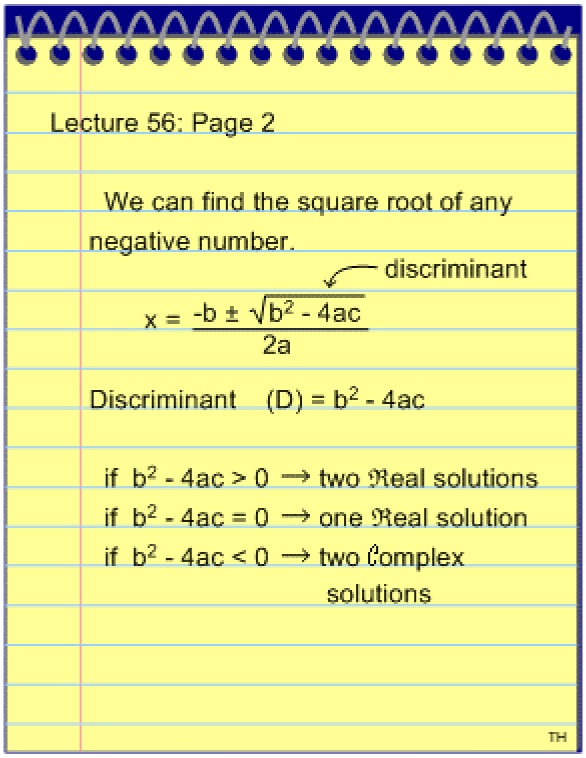
$\sqrt{-1} = i$

Once you know $\sqrt{-1}$, you know the $\sqrt{\ }$ of any negative number.

$\sqrt{-16}$	$\sqrt{-17}$
$\sqrt{16 \cdot (-1)}$	$\sqrt{(17)(-1)}$
$4i$	$\sqrt{17}\, i$

TH

ALG2056-02



Lecture 56: Page 2

We can find the square root of any negative number.

discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant (D) = $b^2 - 4ac$

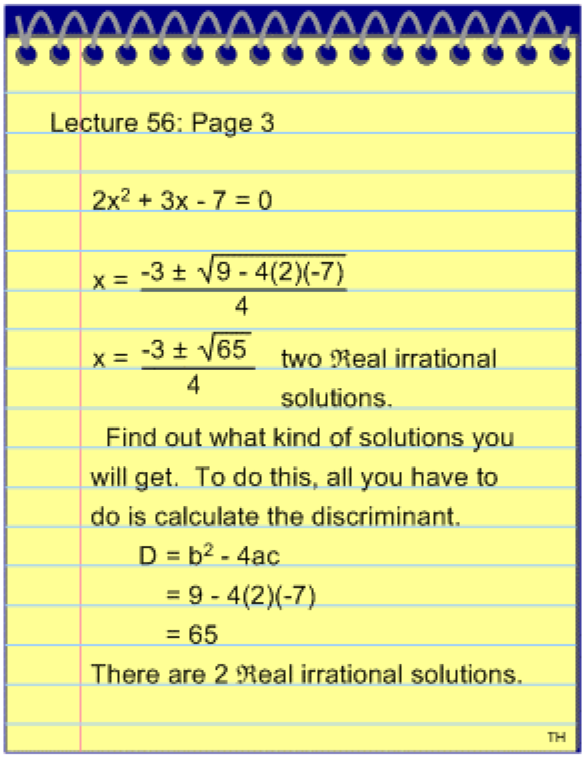
if $b^2 - 4ac > 0$ → two ℜeal solutions

if $b^2 - 4ac = 0$ → one ℜeal solution

if $b^2 - 4ac < 0$ → two 𝒞omplex solutions

TH

ALG2056-03



Lecture 56: Page 3

$2x^2 + 3x - 7 = 0$

$$x = \frac{-3 \pm \sqrt{9 - 4(2)(-7)}}{4}$$

$$x = \frac{-3 \pm \sqrt{65}}{4}$$ two ℜeal irrational solutions.

Find out what kind of solutions you will get. To do this, all you have to do is calculate the discriminant.

$D = b^2 - 4ac$

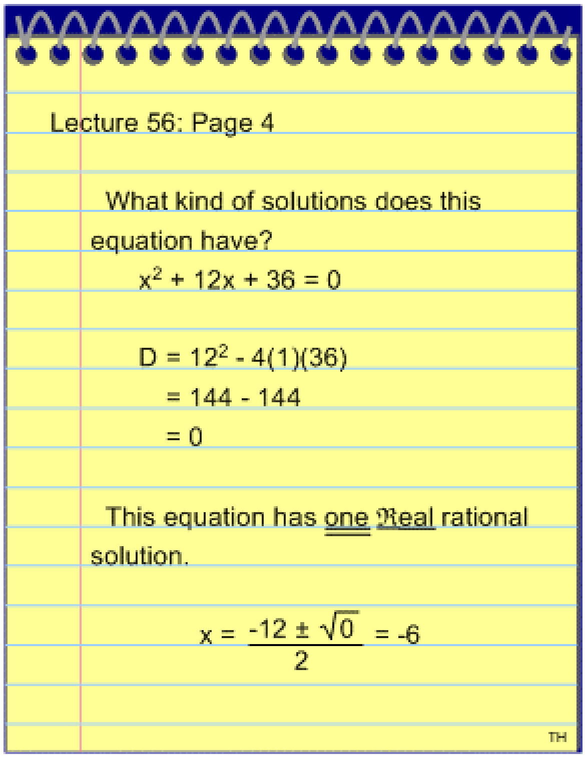
$= 9 - 4(2)(-7)$

$= 65$

There are 2 ℜeal irrational solutions.

TH

ALG2056-04



Lecture 56: Page 4

What kind of solutions does this equation have?

$x^2 + 12x + 36 = 0$

$D = 12^2 - 4(1)(36)$

$= 144 - 144$

$= 0$

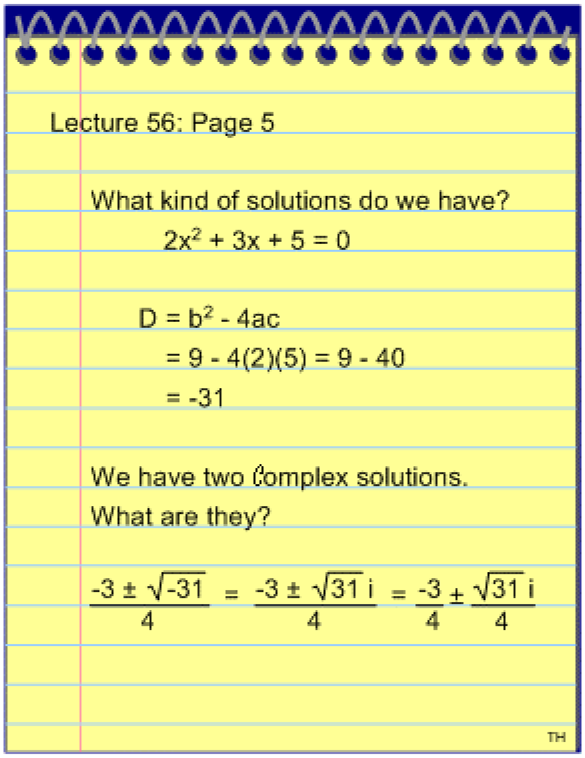
This equation has one ℜeal rational solution.

$$x = \frac{-12 \pm \sqrt{0}}{2} = -6$$

TH

Lecture 56 Notes, Continued

ALG2056-05



Lecture 56: Page 5

What kind of solutions do we have?

$$2x^2 + 3x + 5 = 0$$

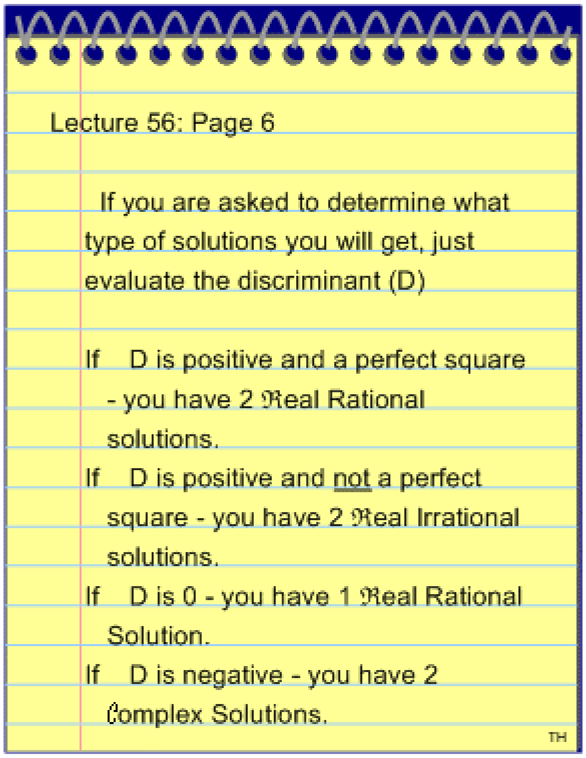
$$\begin{aligned} D &= b^2 - 4ac \\ &= 9 - 4(2)(5) = 9 - 40 \\ &= -31 \end{aligned}$$

We have two 𝒞omplex solutions.
What are they?

$$\frac{-3 \pm \sqrt{-31}}{4} = \frac{-3 \pm \sqrt{31}\,i}{4} = \frac{-3}{4} \pm \frac{\sqrt{31}\,i}{4}$$

TH

ALG2056-06



Lecture 56: Page 6

If you are asked to determine what type of solutions you will get, just evaluate the discriminant (D)

If D is positive and a perfect square - you have 2 ℜeal Rational solutions.

If D is positive and <u>not</u> a perfect square - you have 2 ℜeal Irrational solutions.

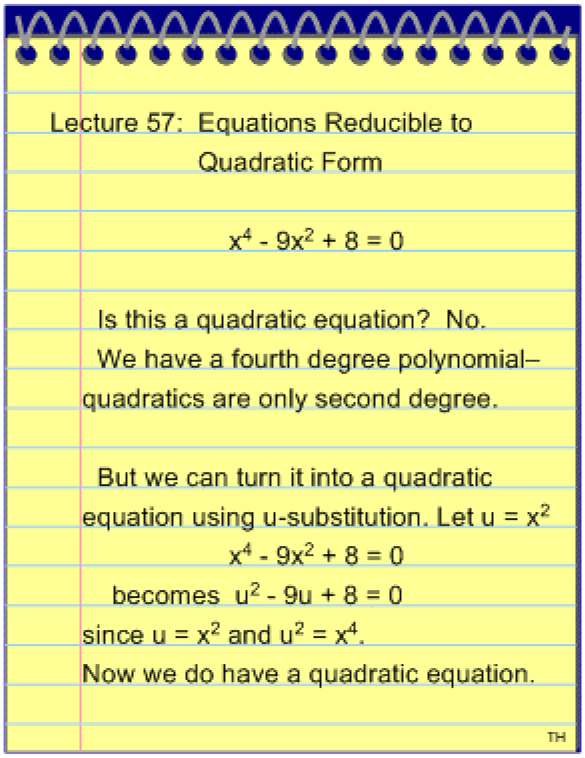
If D is 0 - you have 1 ℜeal Rational Solution.

If D is negative - you have 2 𝒞omplex Solutions.

TH

Lecture 57 Notes

ALG2057-01



Lecture 57: Equations Reducible to Quadratic Form

$x^4 - 9x^2 + 8 = 0$

Is this a quadratic equation? No. We have a fourth degree polynomial– quadratics are only second degree.

But we can turn it into a quadratic equation using u-substitution. Let $u = x^2$

$x^4 - 9x^2 + 8 = 0$

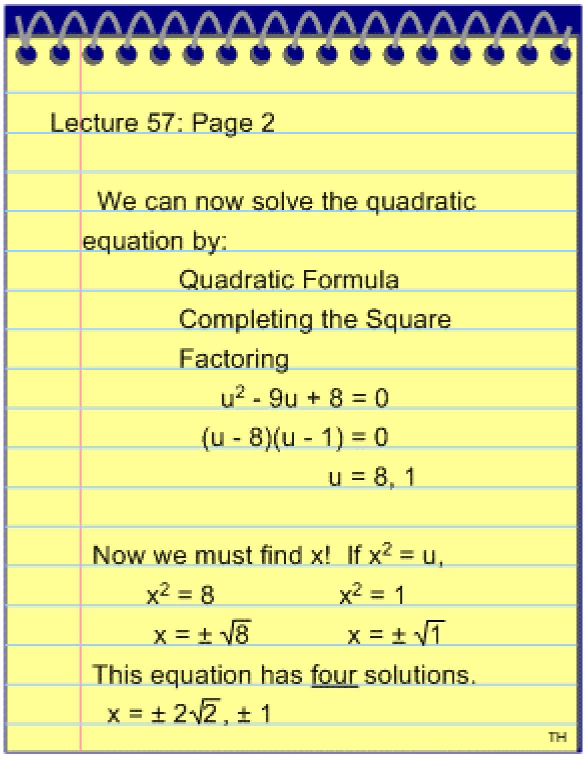
becomes $u^2 - 9u + 8 = 0$

since $u = x^2$ and $u^2 = x^4$.

Now we do have a quadratic equation.

TH

ALG2057-02



Lecture 57: Page 2

We can now solve the quadratic equation by:

Quadratic Formula

Completing the Square

Factoring

$u^2 - 9u + 8 = 0$

$(u - 8)(u - 1) = 0$

$u = 8, 1$

Now we must find x! If $x^2 = u$,

$x^2 = 8 \qquad x^2 = 1$

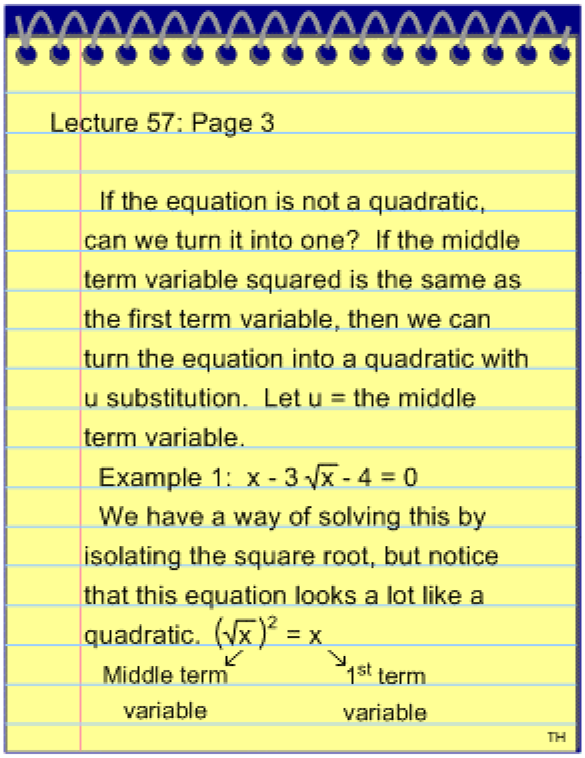
$x = \pm\sqrt{8} \qquad x = \pm\sqrt{1}$

This equation has <u>four</u> solutions.

$x = \pm 2\sqrt{2}, \pm 1$

TH

ALG2057-03



Lecture 57: Page 3

If the equation is not a quadratic, can we turn it into one? If the middle term variable squared is the same as the first term variable, then we can turn the equation into a quadratic with u substitution. Let u = the middle term variable.

Example 1: $x - 3\sqrt{x} - 4 = 0$

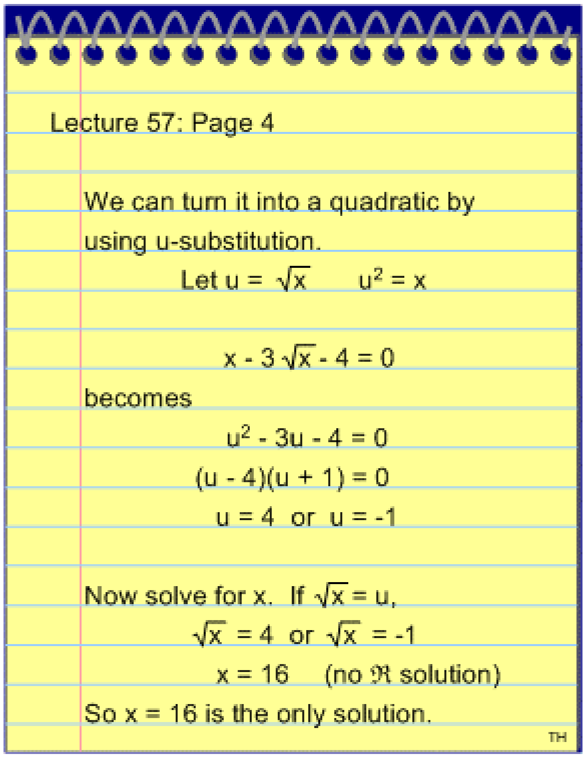
We have a way of solving this by isolating the square root, but notice that this equation looks a lot like a quadratic. $(\sqrt{x})^2 = x$

Middle term variable

1st term variable

TH

ALG2057-04



Lecture 57: Page 4

We can turn it into a quadratic by using u-substitution.

Let $u = \sqrt{x} \qquad u^2 = x$

$x - 3\sqrt{x} - 4 = 0$

becomes

$u^2 - 3u - 4 = 0$

$(u - 4)(u + 1) = 0$

$u = 4$ or $u = -1$

Now solve for x. If $\sqrt{x} = u$,

$\sqrt{x} = 4$ or $\sqrt{x} = -1$

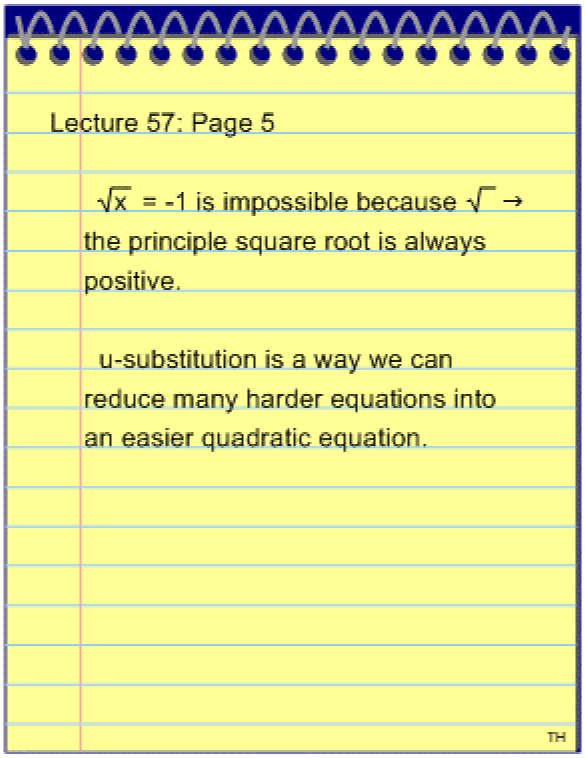
$x = 16$ (no $\Re$ solution)

So $x = 16$ is the only solution.

TH

Lecture 57 Notes, Continued

ALG2057-05



Lecture 57: Page 5

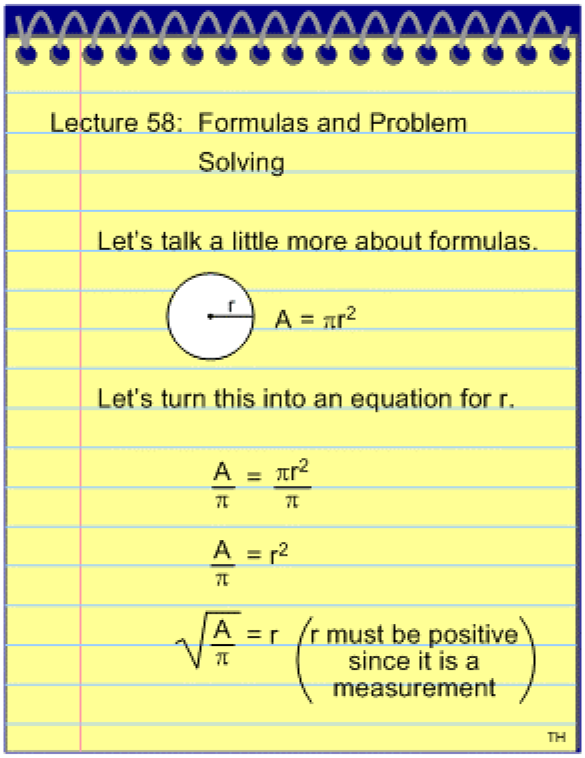
$\sqrt{x}$ = -1 is impossible because $\sqrt{\ }$ → the principle square root is always positive.

u-substitution is a way we can reduce many harder equations into an easier quadratic equation.

TH

Lecture 58 Notes

ALG2058-01



Lecture 58: Formulas and Problem Solving

Let's talk a little more about formulas.

$A = \pi r^2$

Let's turn this into an equation for r.

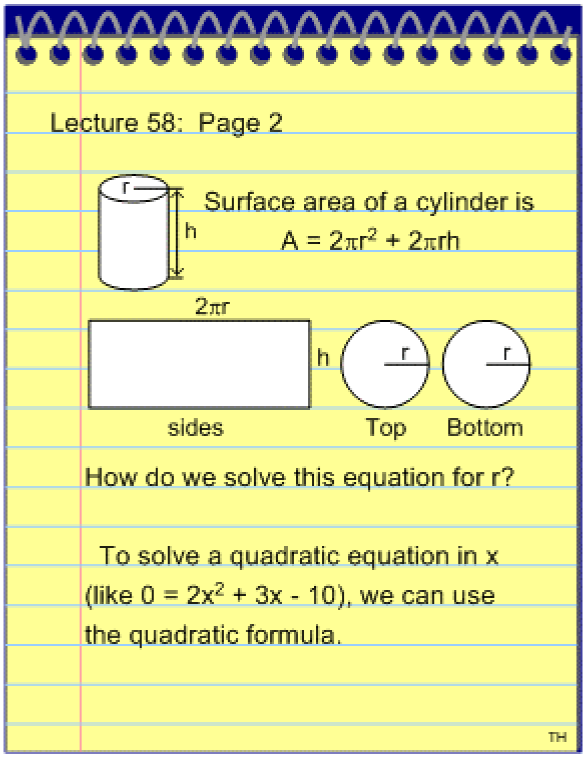
$$\frac{A}{\pi} = \frac{\pi r^2}{\pi}$$

$$\frac{A}{\pi} = r^2$$

$$\sqrt{\frac{A}{\pi}} = r \quad \left(\text{r must be positive since it is a measurement}\right)$$

TH

ALG2058-02



Lecture 58: Page 2

Surface area of a cylinder is

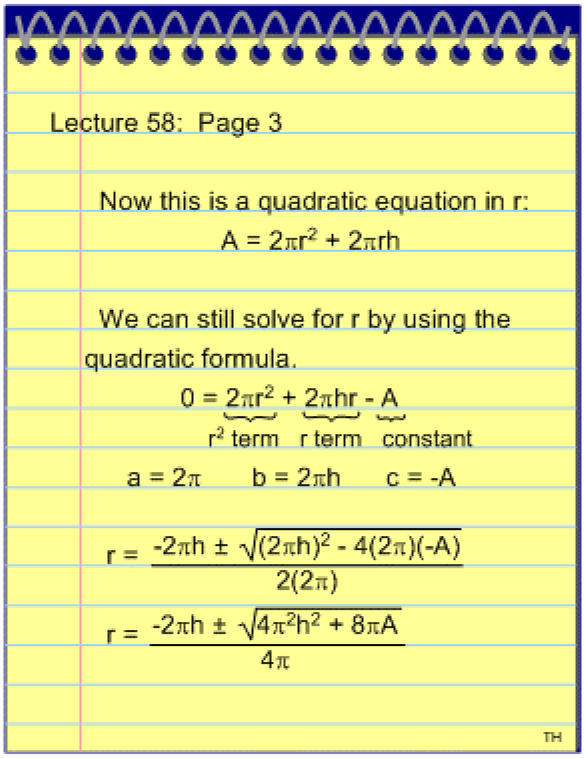
$A = 2\pi r^2 + 2\pi rh$

How do we solve this equation for r?

To solve a quadratic equation in x (like $0 = 2x^2 + 3x - 10$), we can use the quadratic formula.

TH

ALG2058-03



Lecture 58: Page 3

Now this is a quadratic equation in r:

$A = 2\pi r^2 + 2\pi rh$

We can still solve for r by using the quadratic formula.

$$0 = \underbrace{2\pi r^2}_{r^2\text{ term}} + \underbrace{2\pi hr}_{r\text{ term}} - \underbrace{A}_{\text{constant}}$$

$a = 2\pi \qquad b = 2\pi h \qquad c = -A$

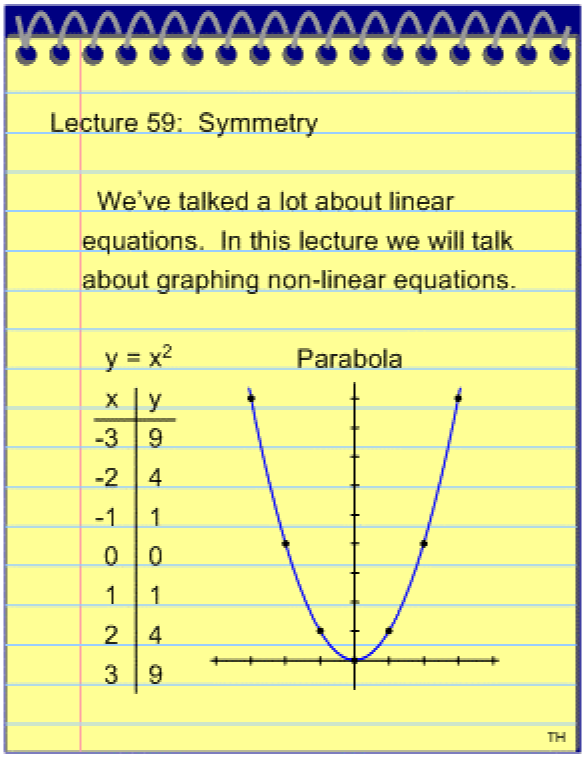
$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-A)}}{2(2\pi)}$$

$$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi A}}{4\pi}$$

TH

Lecture 59 Notes

ALG2059-01



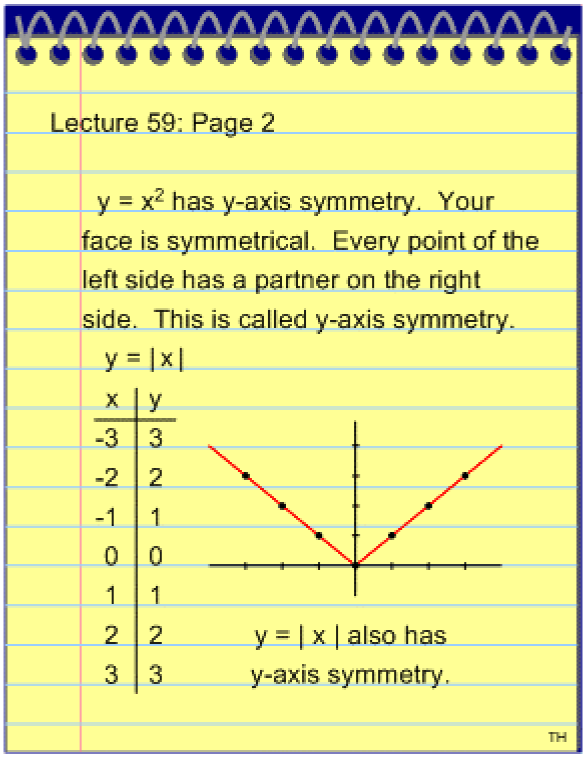
Lecture 59: Symmetry

We've talked a lot about linear equations. In this lecture we will talk about graphing non-linear equations.

$y = x^2$ Parabola

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

ALG2059-02



Lecture 59: Page 2

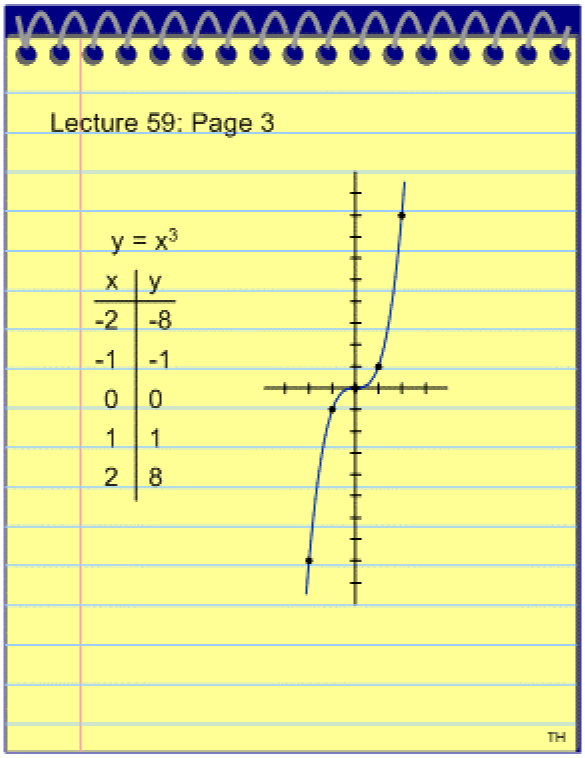
$y = x^2$ has y-axis symmetry. Your face is symmetrical. Every point of the left side has a partner on the right side. This is called y-axis symmetry.

$y = |x|$

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

$y = |x|$ also has y-axis symmetry.

ALG2059-03

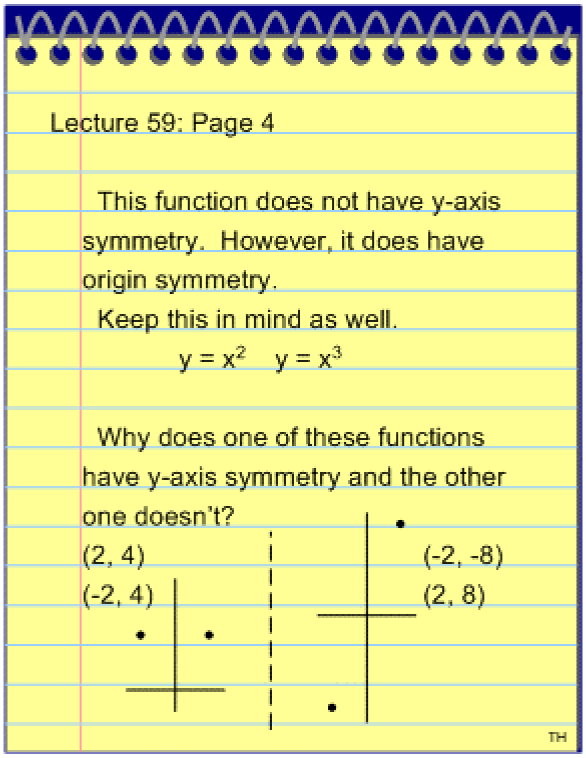


Lecture 59: Page 3

$y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

ALG2059-04



Lecture 59: Page 4

This function does not have y-axis symmetry. However, it does have origin symmetry.

Keep this in mind as well.

$y = x^2$ $y = x^3$

Why does one of these functions have y-axis symmetry and the other one doesn't?

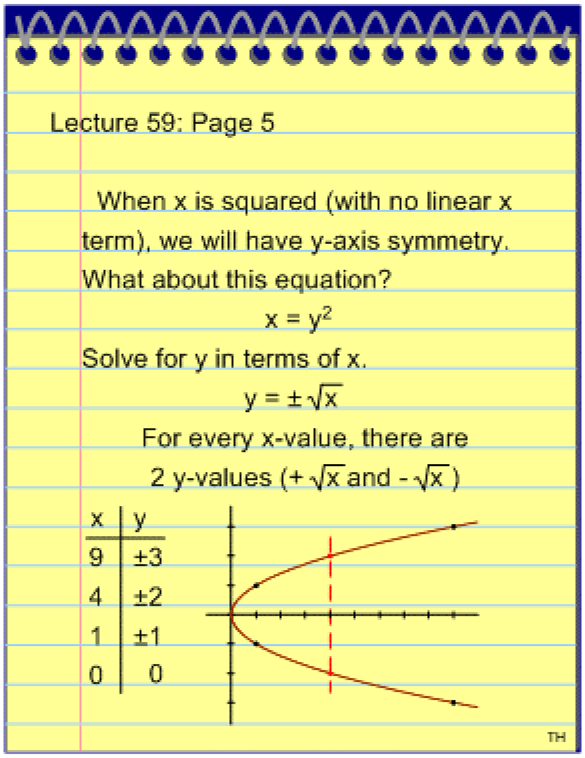
(2, 4)

(-2, 4)

(-2, -8)

(2, 8)

ALG2059-05



Lecture 59: Page 5

When x is squared (with no linear x term), we will have y-axis symmetry. What about this equation?

$$x = y^2$$

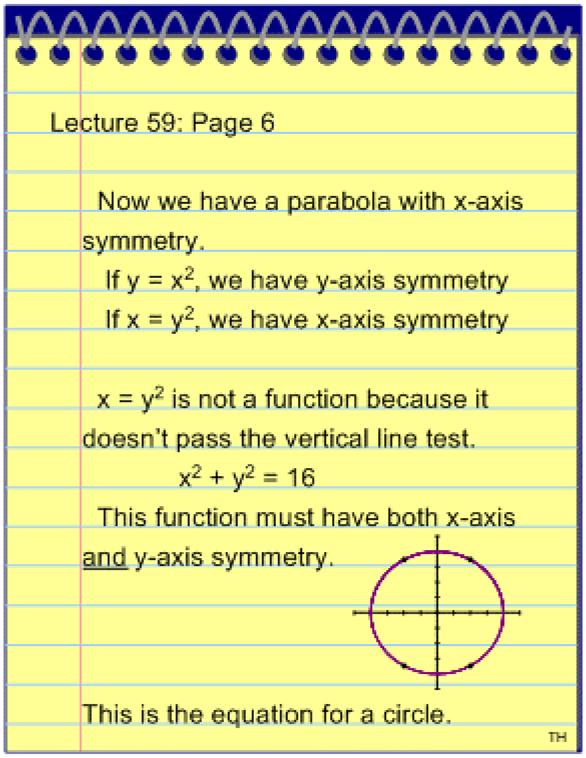
Solve for y in terms of x.

$$y = \pm\sqrt{x}$$

For every x-value, there are 2 y-values ($+\sqrt{x}$ and $-\sqrt{x}$)

x	y
9	±3
4	±2
1	±1
0	0

ALG2059-06



Lecture 59: Page 6

Now we have a parabola with x-axis symmetry.

If $y = x^2$, we have y-axis symmetry

If $x = y^2$, we have x-axis symmetry

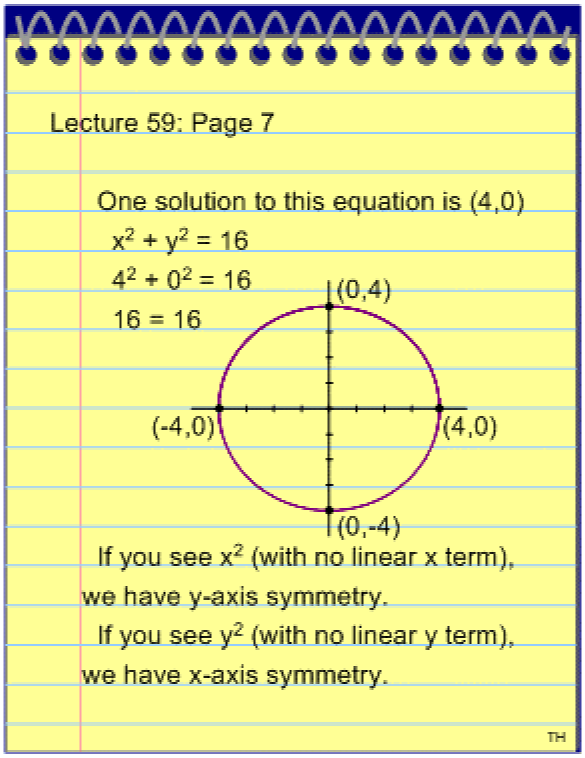
$x = y^2$ is not a function because it doesn't pass the vertical line test.

$$x^2 + y^2 = 16$$

This function must have both x-axis and y-axis symmetry.

This is the equation for a circle.

ALG2059-07



Lecture 59: Page 7

One solution to this equation is (4,0)

$x^2 + y^2 = 16$

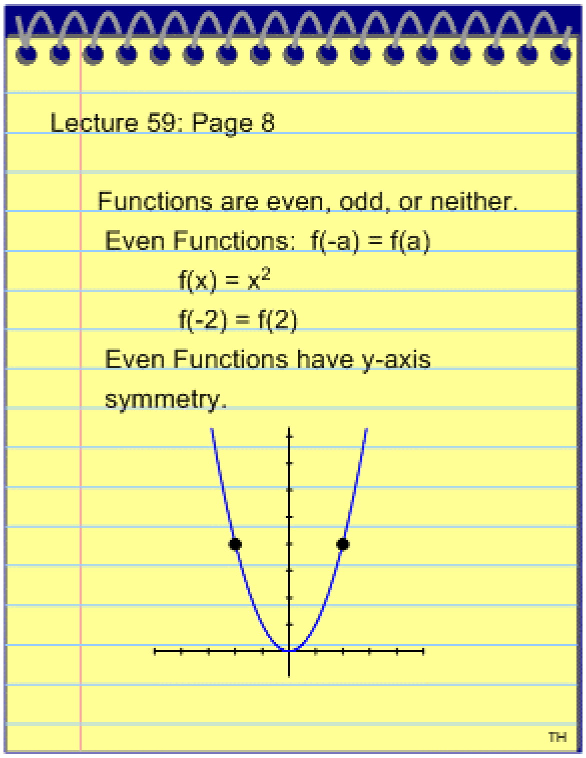
$4^2 + 0^2 = 16$

$16 = 16$

If you see x^2 (with no linear x term), we have y-axis symmetry.

If you see y^2 (with no linear y term), we have x-axis symmetry.

ALG2059-08



Lecture 59: Page 8

Functions are even, odd, or neither.

Even Functions: $f(-a) = f(a)$

$f(x) = x^2$

$f(-2) = f(2)$

Even Functions have y-axis symmetry.

Lecture 59 Notes, Continued

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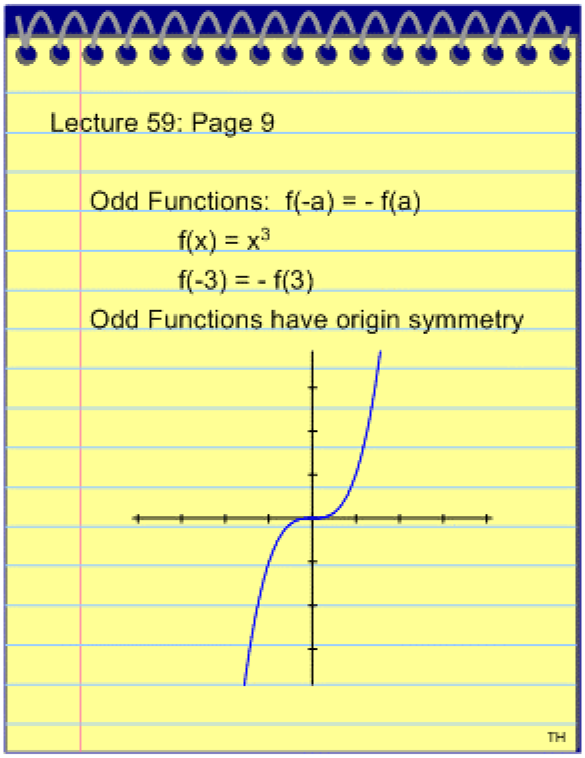


Lecture 59: Page 9

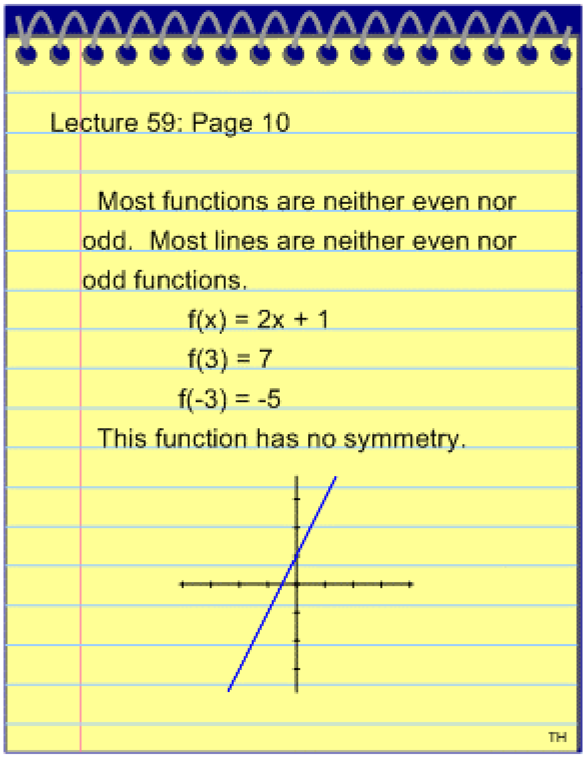
Odd Functions: f(-a) = - f(a)

$f(x) = x^3$

$f(-3) = - f(3)$

Odd Functions have origin symmetry

ALG2059-10



Lecture 59: Page 10

Most functions are neither even nor odd. Most lines are neither even nor odd functions.

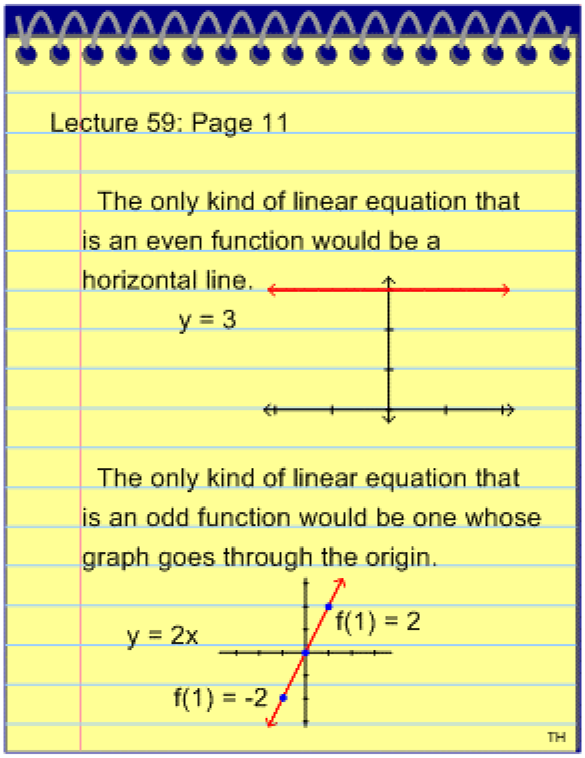
$f(x) = 2x + 1$

$f(3) = 7$

$f(-3) = -5$

This function has no symmetry.

ALG2059-11



Lecture 59: Page 11

The only kind of linear equation that is an even function would be a horizontal line.

$y = 3$

The only kind of linear equation that is an odd function would be one whose graph goes through the origin.

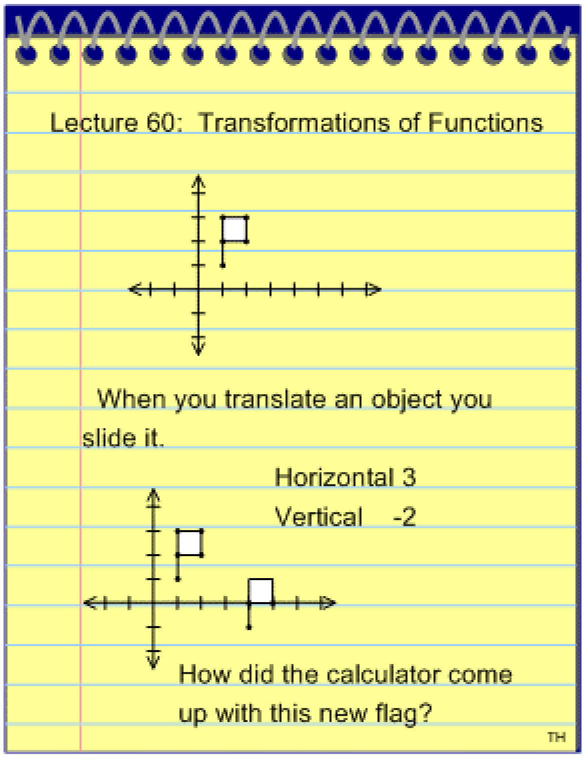
$y = 2x$

$f(1) = 2$

$f(1) = -2$

Lecture 60 Notes

ALG2060-01



Lecture 60: Transformations of Functions

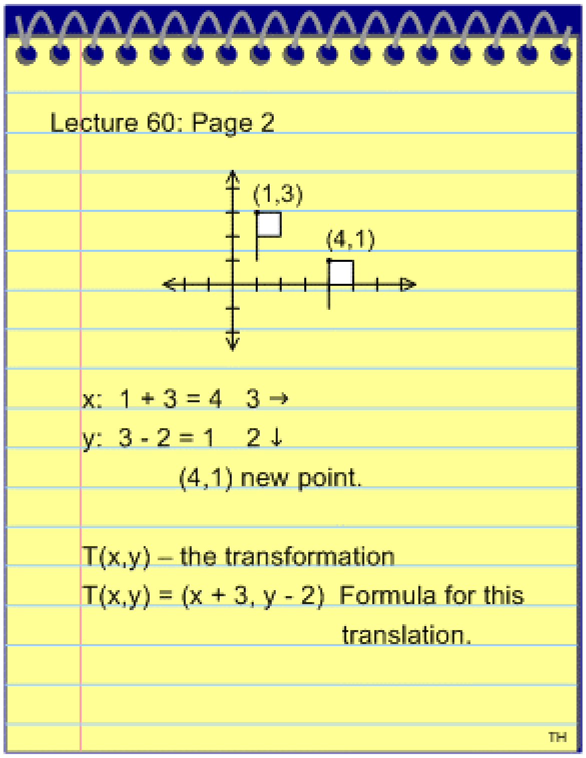
When you translate an object you slide it.

Horizontal 3

Vertical -2

How did the calculator come up with this new flag?

ALG2060-02



Lecture 60: Page 2

x: 1 + 3 = 4 3 →

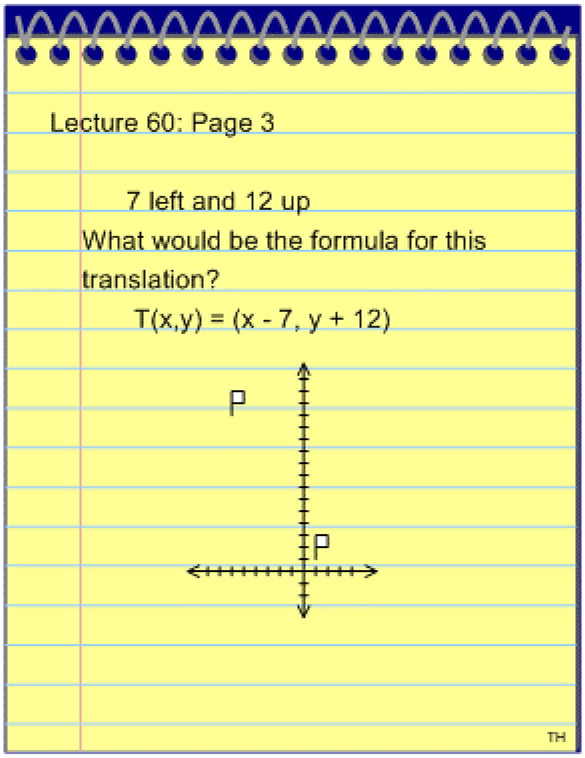
y: 3 - 2 = 1 2 ↓

(4,1) new point.

T(x,y) – the transformation

T(x,y) = (x + 3, y - 2) Formula for this translation.

ALG2060-03



Lecture 60: Page 3

7 left and 12 up

What would be the formula for this translation?

$T(x,y) = (x - 7, y + 12)$

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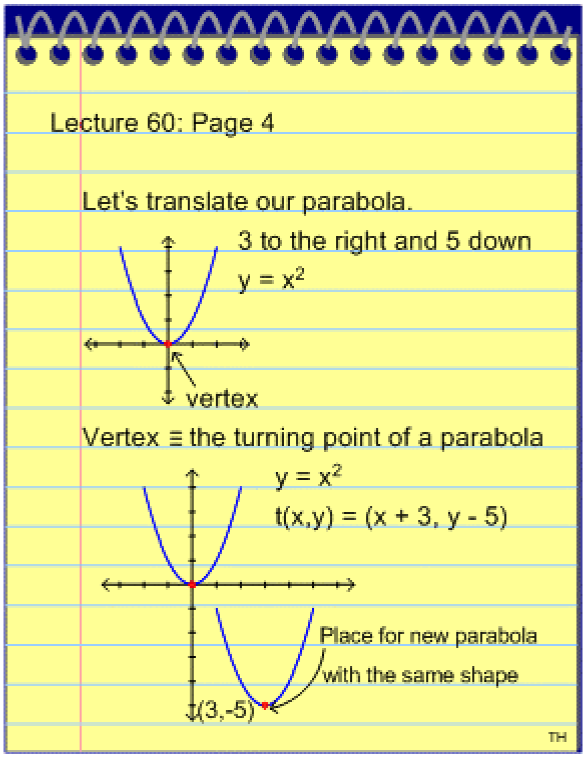


Lecture 60: Page 4

Let's translate our parabola.

3 to the right and 5 down

$y = x^2$

Vertex ≡ the turning point of a parabola

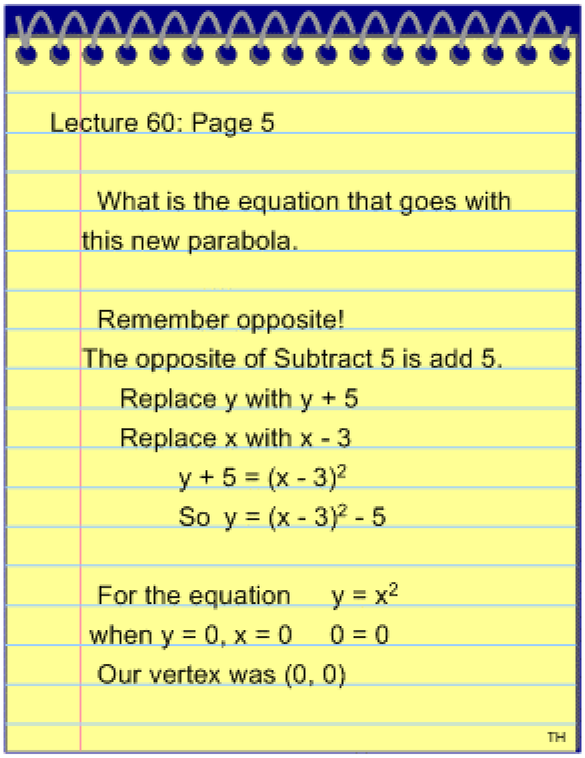
$y = x^2$

$t(x,y) = (x + 3, y - 5)$

Place for new parabola with the same shape

Lecture 60 Notes, Continued

ALG2060-05



Lecture 60: Page 5

What is the equation that goes with this new parabola.

Remember opposite!

The opposite of Subtract 5 is add 5.

Replace y with y + 5

Replace x with x - 3

$y + 5 = (x - 3)^2$

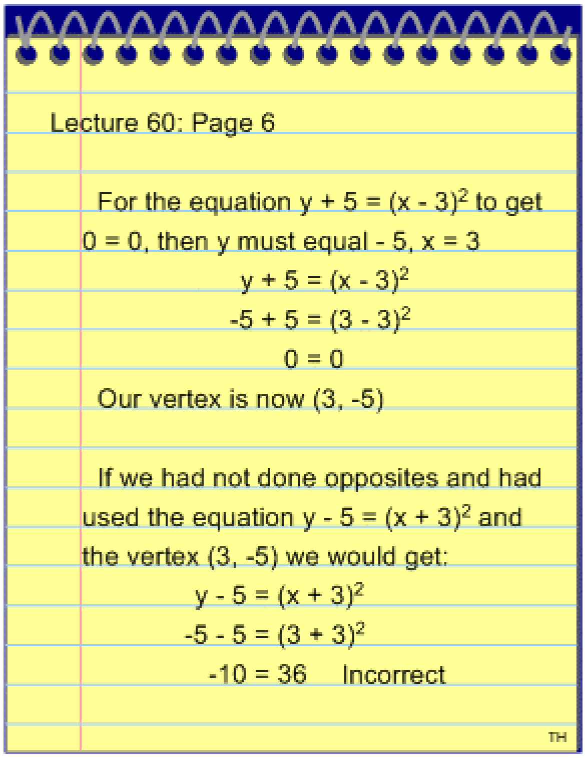
So $y = (x - 3)^2 - 5$

For the equation $y = x^2$

when y = 0, x = 0 $0 = 0$

Our vertex was (0, 0)

ALG2060-06



Lecture 60: Page 6

For the equation $y + 5 = (x - 3)^2$ to get 0 = 0, then y must equal - 5, x = 3

$y + 5 = (x - 3)^2$

$-5 + 5 = (3 - 3)^2$

$0 = 0$

Our vertex is now (3, -5)

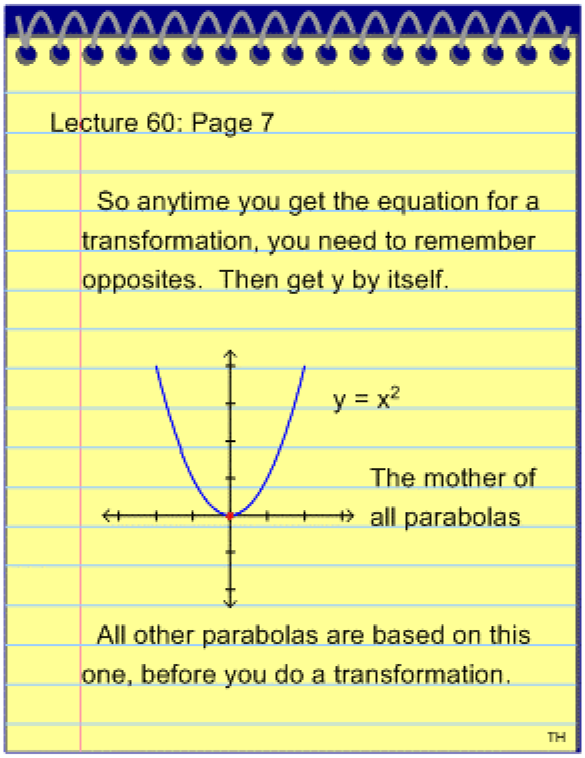
If we had not done opposites and had used the equation $y - 5 = (x + 3)^2$ and the vertex (3, -5) we would get:

$y - 5 = (x + 3)^2$

$-5 - 5 = (3 + 3)^2$

$-10 = 36$ Incorrect

ALG2060-07



Lecture 60: Page 7

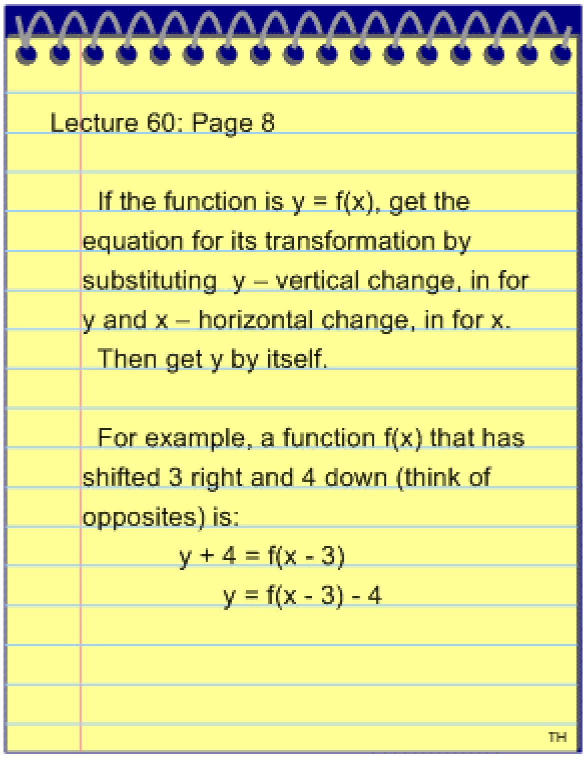
So anytime you get the equation for a transformation, you need to remember opposites. Then get y by itself.

$y = x^2$

The mother of all parabolas

All other parabolas are based on this one, before you do a transformation.

ALG2060-08



Lecture 60: Page 8

If the function is y = f(x), get the equation for its transformation by substituting y – vertical change, in for y and x – horizontal change, in for x. Then get y by itself.

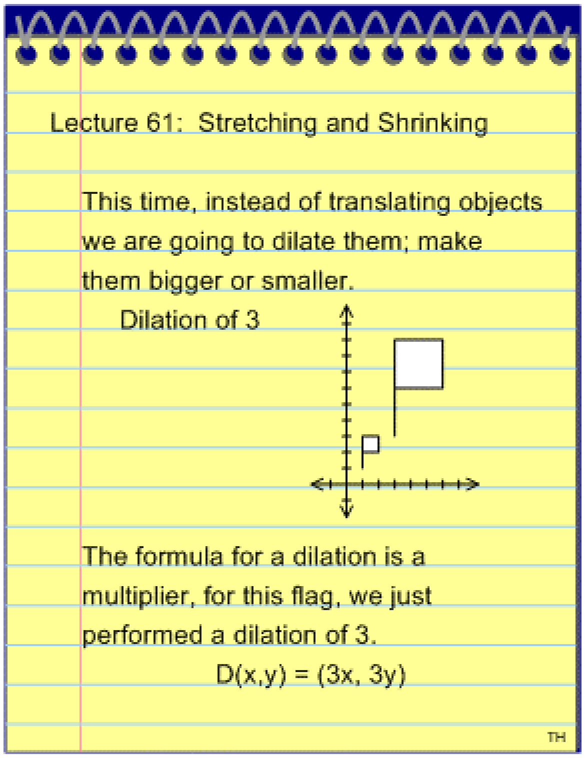
For example, a function f(x) that has shifted 3 right and 4 down (think of opposites) is:

$y + 4 = f(x - 3)$

$y = f(x - 3) - 4$

Lecture 61 Notes

ALG2061-01



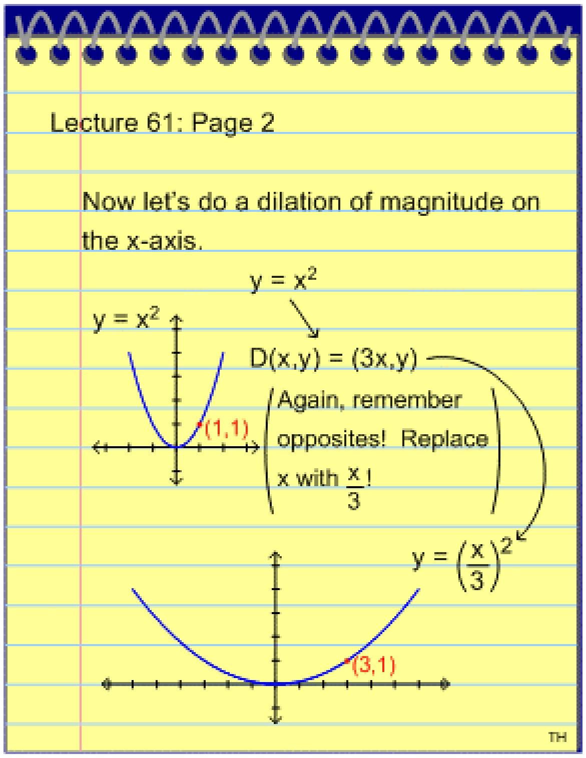
Lecture 61: Stretching and Shrinking

This time, instead of translating objects we are going to dilate them; make them bigger or smaller.

The formula for a dilation is a multiplier, for this flag, we just performed a dilation of 3.

$D(x,y) = (3x, 3y)$

ALG2061-02



Lecture 61: Page 2

Now let's do a dilation of magnitude on the x-axis.

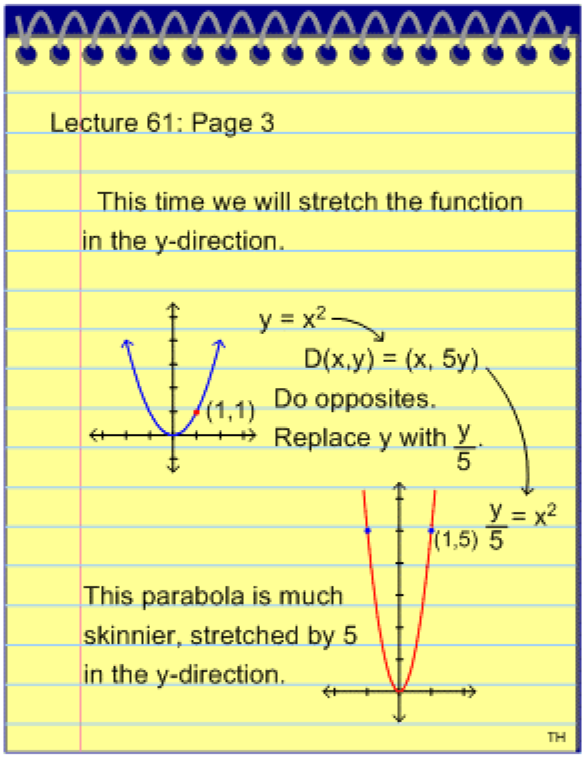
$y = x^2$

$D(x,y) = (3x,y)$

Again, remember opposites! Replace x with $\frac{x}{3}$!

$y = \left(\frac{x}{3}\right)^2$

ALG2061-03



Lecture 61: Page 3

This time we will stretch the function in the y-direction.

$y = x^2$

$D(x,y) = (x, 5y)$

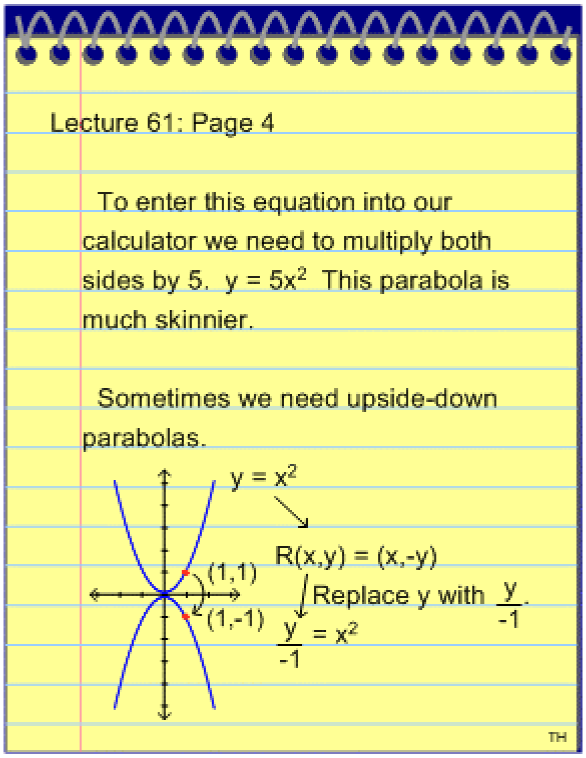
Do opposites.

Replace y with $\frac{y}{5}$.

$\frac{y}{5} = x^2$

This parabola is much skinnier, stretched by 5 in the y-direction.

ALG2061-04



Lecture 61: Page 4

To enter this equation into our calculator we need to multiply both sides by 5. $y = 5x^2$ This parabola is much skinnier.

Sometimes we need upside-down parabolas.

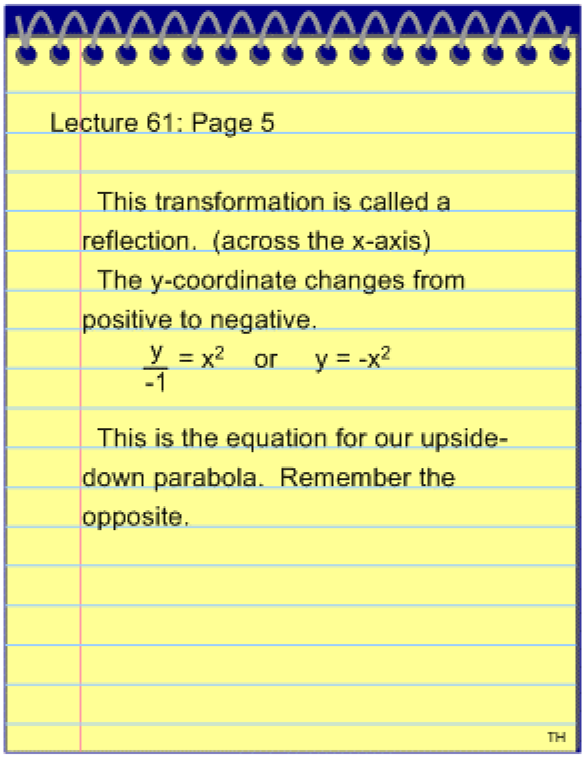
$y = x^2$

$R(x,y) = (x,-y)$

Replace y with $\frac{y}{-1}$.

$\frac{y}{-1} = x^2$

ALG2061-05



Lecture 61: Page 5

This transformation is called a reflection. (across the x-axis)

The y-coordinate changes from positive to negative.

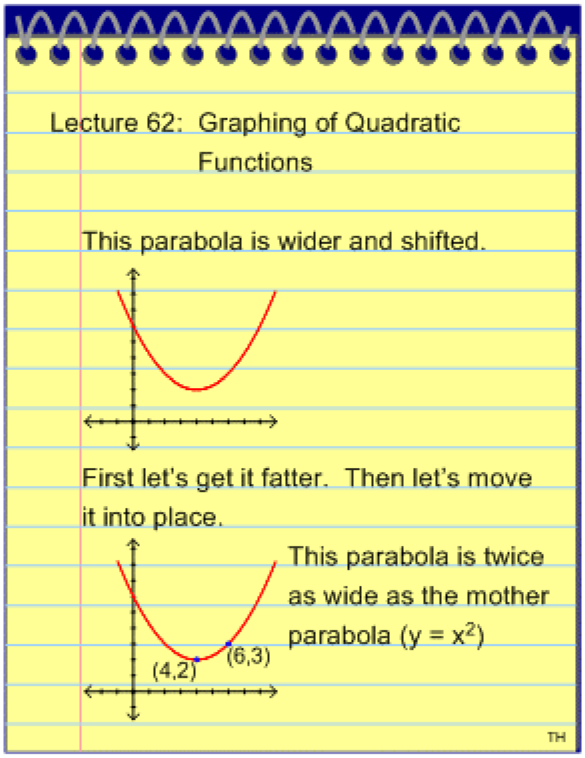
$\frac{y}{-1} = x^2$ or $y = -x^2$

This is the equation for our upside-down parabola. Remember the opposite.

TH

Lecture 62 Notes

ALG2062-01



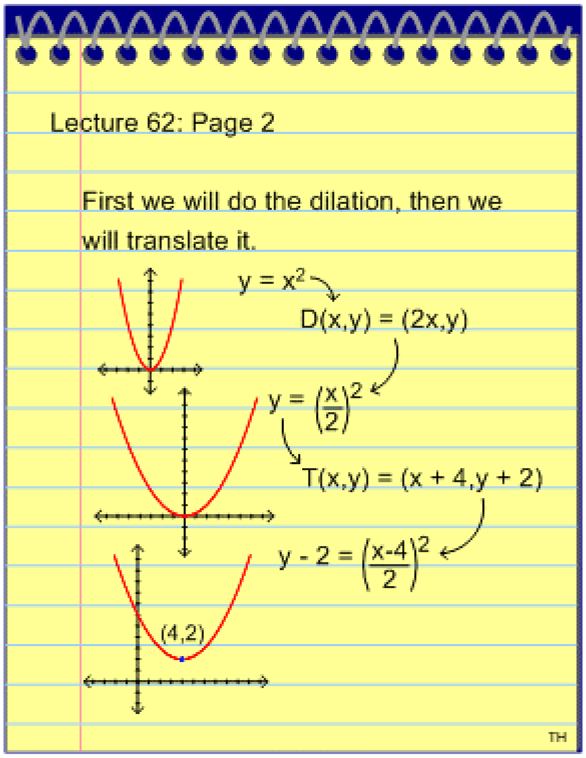
Lecture 62: Graphing of Quadratic Functions

This parabola is wider and shifted.

First let's get it fatter. Then let's move it into place.

This parabola is twice as wide as the mother parabola ($y = x^2$)

ALG2062-02



Lecture 62: Page 2

First we will do the dilation, then we will translate it.

$y = x^2$

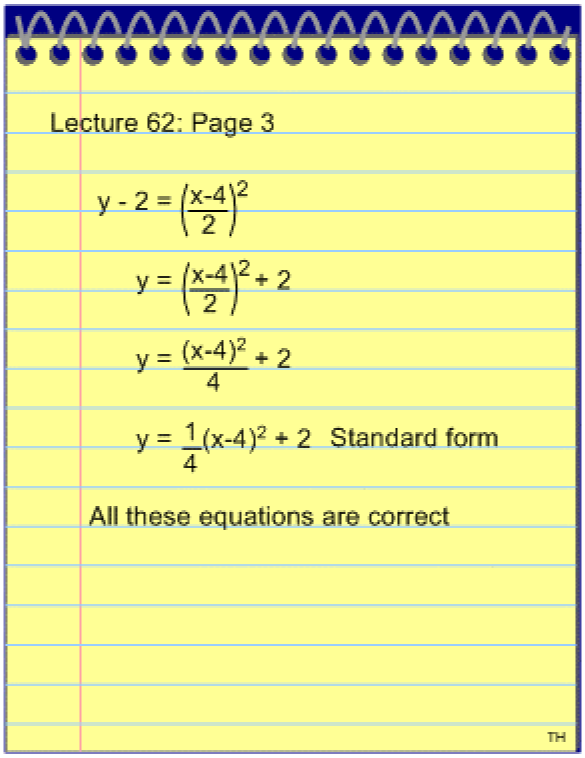
$D(x,y) = (2x,y)$

$y = \left(\frac{x}{2}\right)^2$

$T(x,y) = (x + 4,y + 2)$

$y - 2 = \left(\frac{x-4}{2}\right)^2$

ALG2062-03



Lecture 62: Page 3

$y - 2 = \left(\frac{x-4}{2}\right)^2$

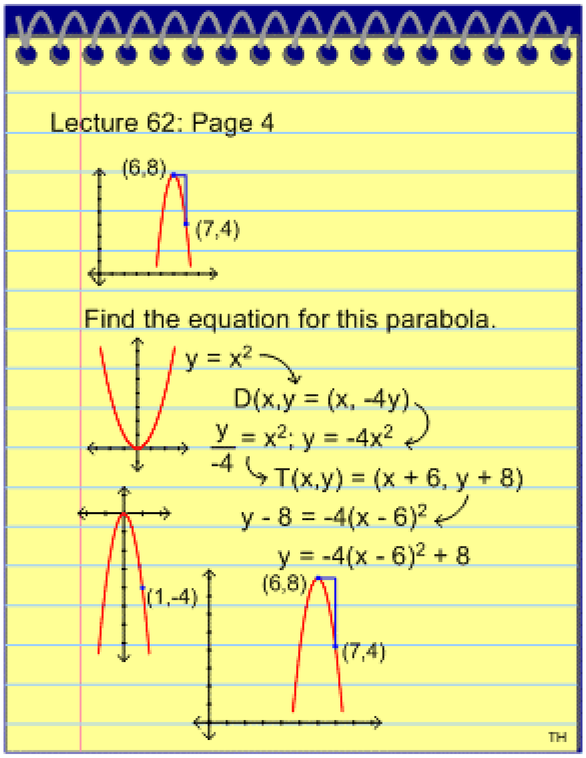
$y = \left(\frac{x-4}{2}\right)^2 + 2$

$y = \frac{(x-4)^2}{4} + 2$

$y = \frac{1}{4}(x-4)^2 + 2$ Standard form

All these equations are correct

ALG2062-04



Lecture 62: Page 4

Find the equation for this parabola.

$y = x^2$

$D(x,y = (x, -4y)$

$\frac{y}{-4} = x^2$; $y = -4x^2$

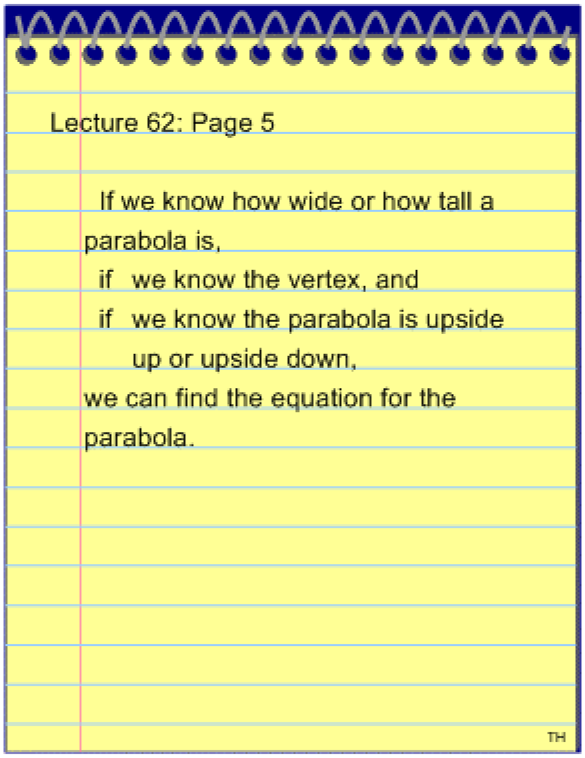
$T(x,y) = (x + 6, y + 8)$

$y - 8 = -4(x - 6)^2$

$y = -4(x - 6)^2 + 8$

Lecture 62 Notes, Continued

ALG2062-05



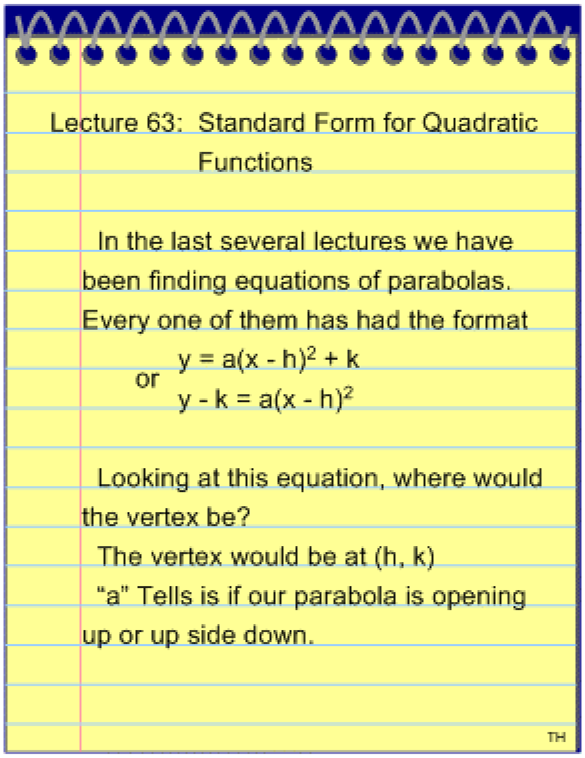
Lecture 62: Page 5

If we know how wide or how tall a parabola is,
if we know the vertex, and
if we know the parabola is upside up or upside down,
we can find the equation for the parabola.

TH

Lecture 63 Notes

ALG2063-01



Lecture 63: Standard Form for Quadratic Functions

In the last several lectures we have been finding equations of parabolas. Every one of them has had the format

$y = a(x - h)^2 + k$

or

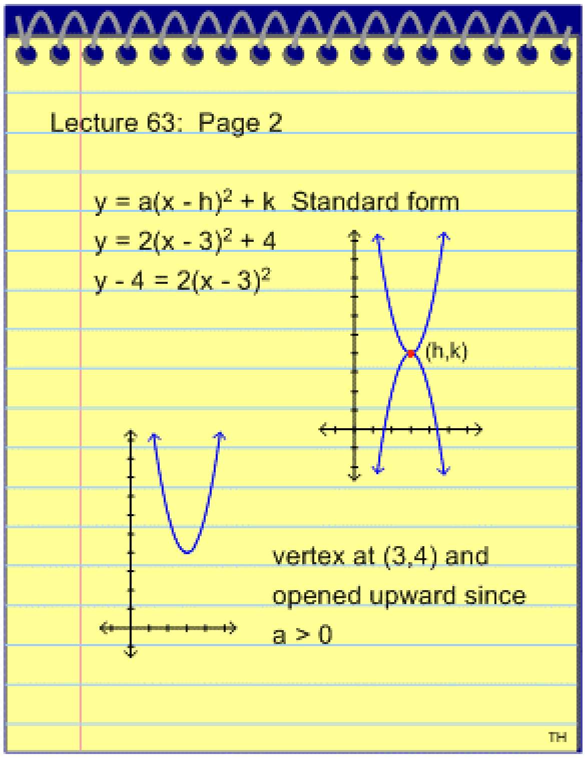
$y - k = a(x - h)^2$

Looking at this equation, where would the vertex be?

The vertex would be at (h, k)

"a" Tells is if our parabola is opening up or up side down.

ALG2063-02



Lecture 63: Page 2

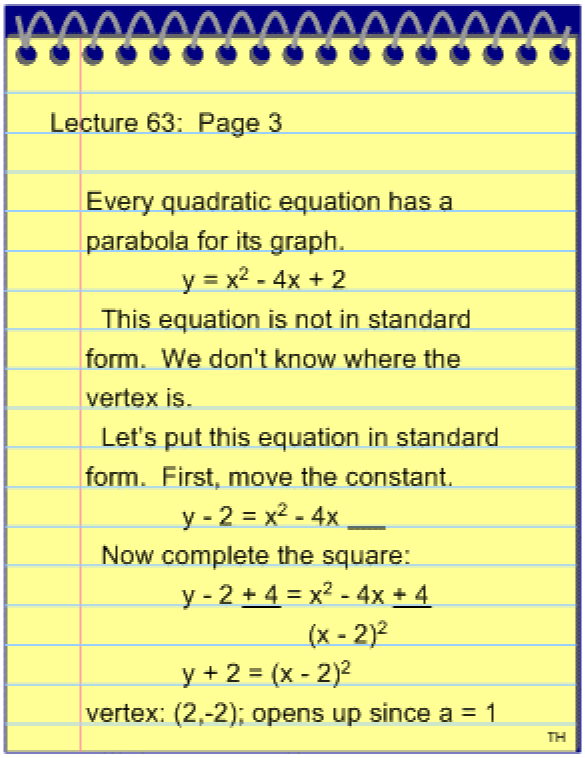
$y = a(x - h)^2 + k$ Standard form

$y = 2(x - 3)^2 + 4$

$y - 4 = 2(x - 3)^2$

vertex at (3,4) and opened upward since $a > 0$

ALG2063-03



Lecture 63: Page 3

Every quadratic equation has a parabola for its graph.

$y = x^2 - 4x + 2$

This equation is not in standard form. We don't know where the vertex is.

Let's put this equation in standard form. First, move the constant.

$y - 2 = x^2 - 4x$ ___

Now complete the square:

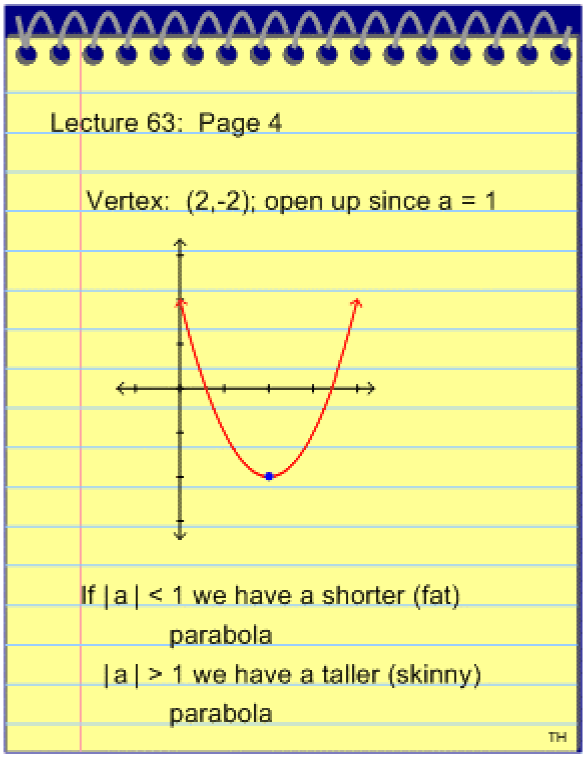
$y - 2 \underline{+ 4} = x^2 - 4x \underline{+ 4}$

$(x - 2)^2$

$y + 2 = (x - 2)^2$

vertex: (2,-2); opens up since a = 1

ALG2063-04



Lecture 63: Page 4

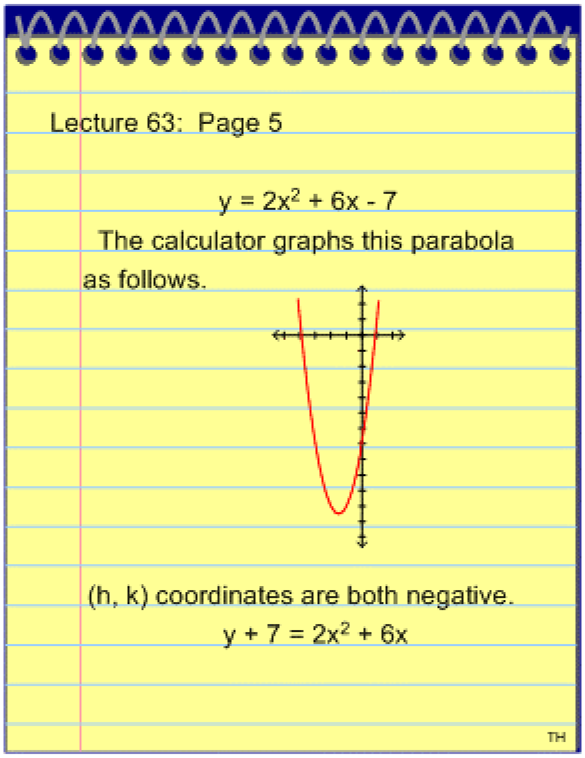
Vertex: (2,-2); open up since a = 1

If $|a| < 1$ we have a shorter (fat) parabola

$|a| > 1$ we have a taller (skinny) parabola

Lecture 63 Notes, Continued

ALG2063-05



Lecture 63: Page 5

$y = 2x^2 + 6x - 7$

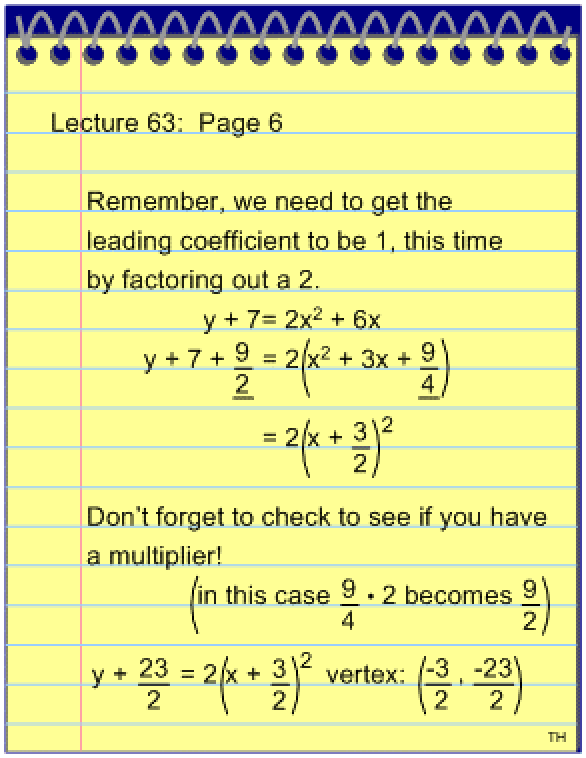
The calculator graphs this parabola as follows.

(h, k) coordinates are both negative.

$y + 7 = 2x^2 + 6x$

TH

ALG2063-06



Lecture 63: Page 6

Remember, we need to get the leading coefficient to be 1, this time by factoring out a 2.

$$y + 7 = 2x^2 + 6x$$

$$y + 7 + \frac{9}{2} = 2\left(x^2 + 3x + \frac{9}{4}\right)$$

$$= 2\left(x + \frac{3}{2}\right)^2$$

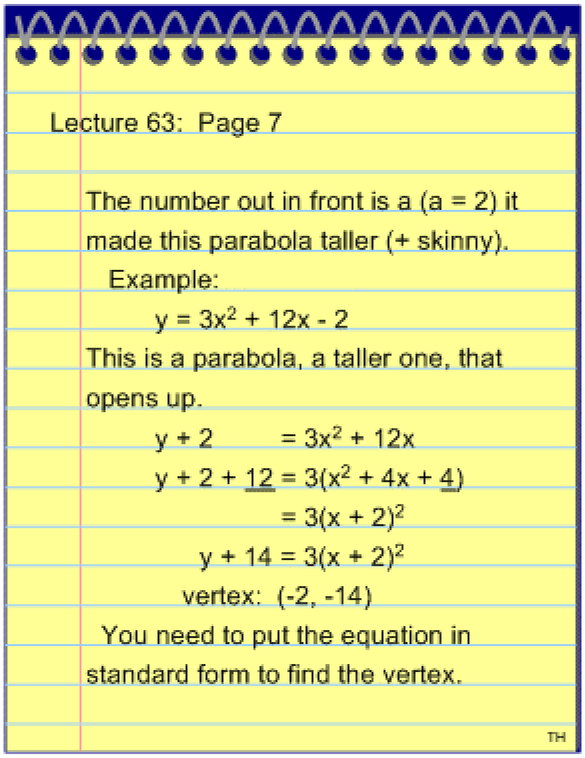
Don't forget to check to see if you have a multiplier!

$\left(\text{in this case } \frac{9}{4} \cdot 2 \text{ becomes } \frac{9}{2}\right)$

$y + \frac{23}{2} = 2\left(x + \frac{3}{2}\right)^2$ vertex: $\left(\frac{-3}{2}, \frac{-23}{2}\right)$

TH

ALG2063-07



Lecture 63: Page 7

The number out in front is a (a = 2) it made this parabola taller (+ skinny).

Example:

$y = 3x^2 + 12x - 2$

This is a parabola, a taller one, that opens up.

$$y + 2 = 3x^2 + 12x$$

$$y + 2 + \underline{12} = 3(x^2 + 4x + \underline{4})$$

$$= 3(x + 2)^2$$

$$y + 14 = 3(x + 2)^2$$

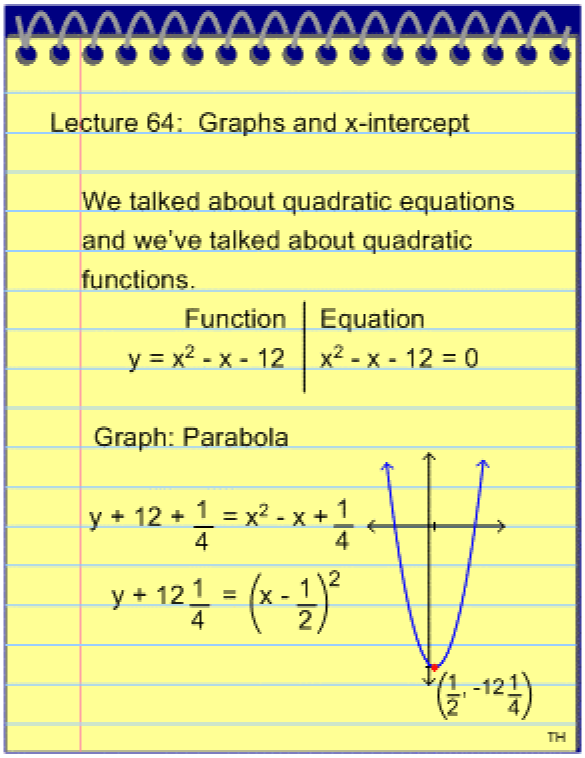
vertex: (-2, -14)

You need to put the equation in standard form to find the vertex.

TH

Lecture 64 Notes

ALG2064-01



Lecture 64: Graphs and x-intercept

We talked about quadratic equations and we've talked about quadratic functions.

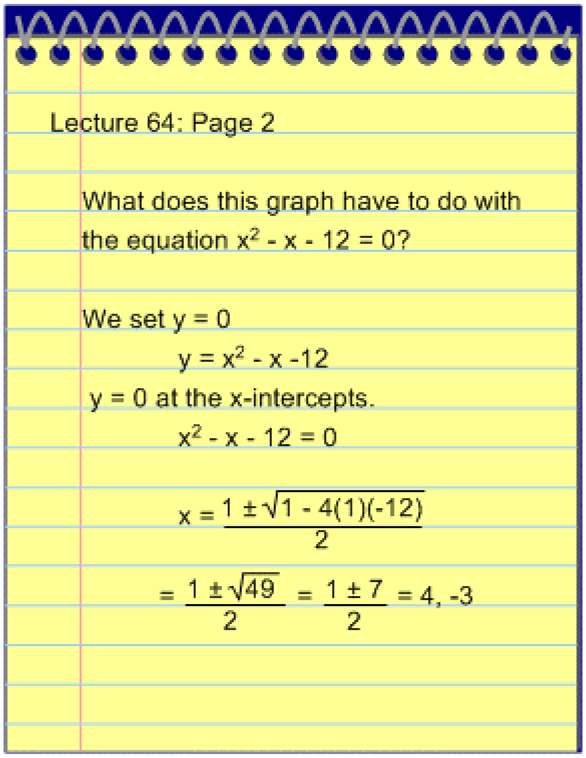
Function	Equation
$y = x^2 - x - 12$	$x^2 - x - 12 = 0$

Graph: Parabola

$$y + 12 + \frac{1}{4} = x^2 - x + \frac{1}{4}$$

$$y + 12\frac{1}{4} = \left(x - \frac{1}{2}\right)^2$$

ALG2064-02



Lecture 64: Page 2

What does this graph have to do with the equation $x^2 - x - 12 = 0$?

We set y = 0

$y = x^2 - x -12$

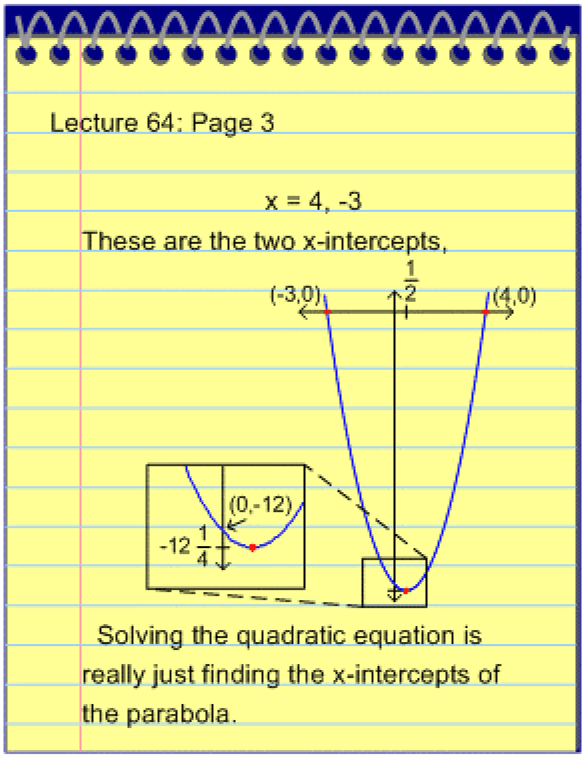
y = 0 at the x-intercepts.

$x^2 - x - 12 = 0$

$$x = \frac{1 \pm \sqrt{1 - 4(1)(-12)}}{2}$$

$$= \frac{1 \pm \sqrt{49}}{2} = \frac{1 \pm 7}{2} = 4, -3$$

ALG2064-03



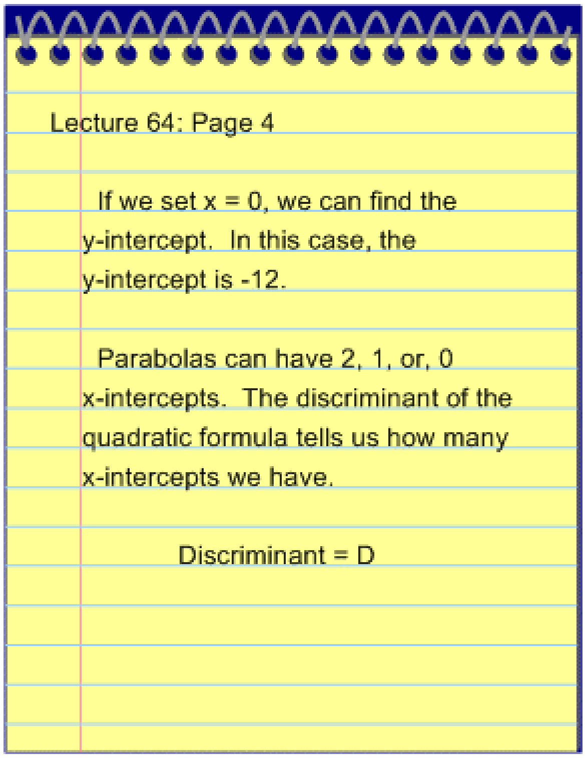
Lecture 64: Page 3

x = 4, -3

These are the two x-intercepts,

Solving the quadratic equation is really just finding the x-intercepts of the parabola.

ALG2064-04



Lecture 64: Page 4

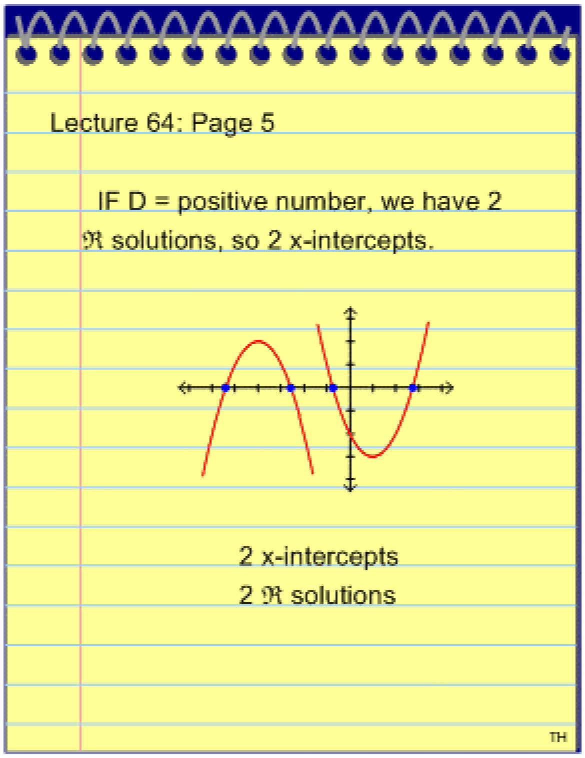
If we set x = 0, we can find the y-intercept. In this case, the y-intercept is -12.

Parabolas can have 2, 1, or, 0 x-intercepts. The discriminant of the quadratic formula tells us how many x-intercepts we have.

Discriminant = D

Lecture 64 Notes, Continued

ALG2064-05

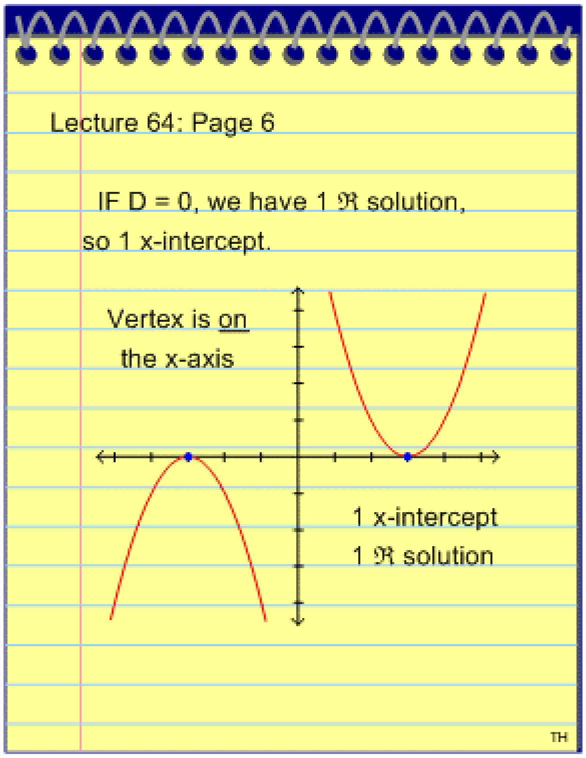


Lecture 64: Page 5

IF D = positive number, we have 2 ℜ solutions, so 2 x-intercepts.

2 x-intercepts
2 ℜ solutions

ALG2064-06



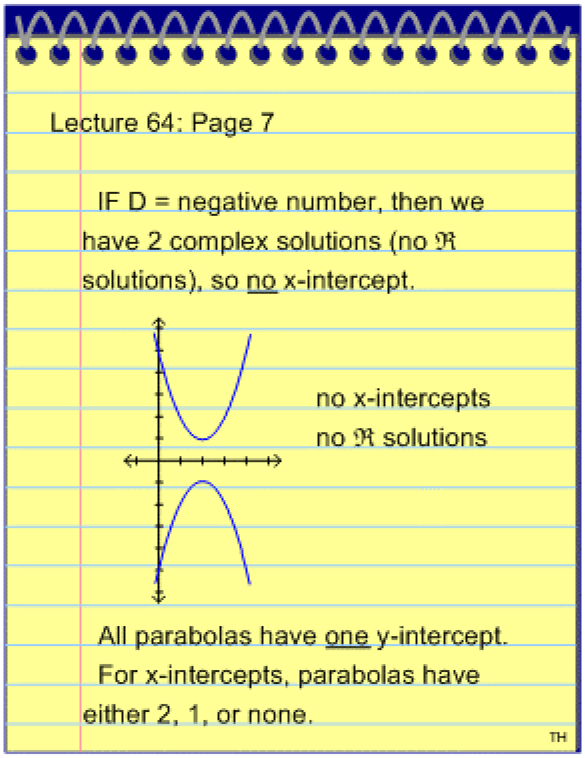
Lecture 64: Page 6

IF D = 0, we have 1 ℜ solution, so 1 x-intercept.

Vertex is on
the x-axis

1 x-intercept
1 ℜ solution

ALG2064-07



Lecture 64: Page 7

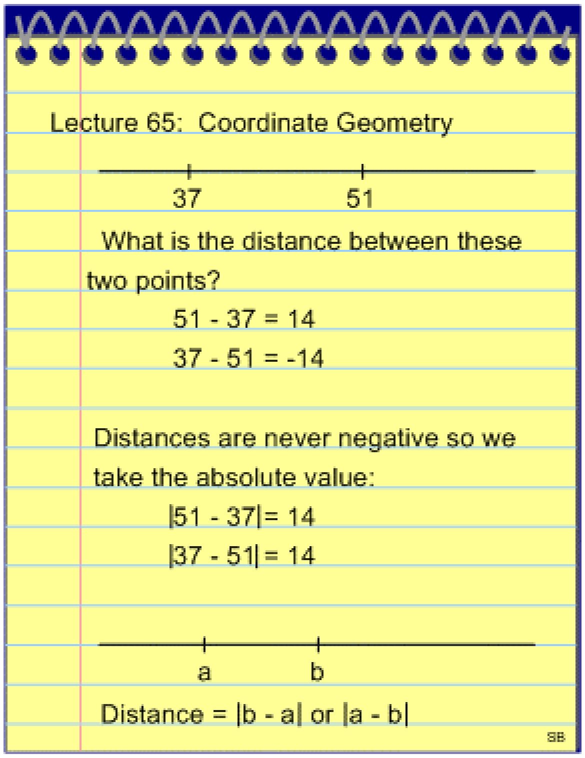
IF D = negative number, then we have 2 complex solutions (no ℜ solutions), so no x-intercept.

no x-intercepts
no ℜ solutions

All parabolas have one y-intercept.
For x-intercepts, parabolas have either 2, 1, or none.

Lecture 65 Notes

ALG2065-01



Lecture 65: Coordinate Geometry

37 51

What is the distance between these two points?

51 - 37 = 14

37 - 51 = -14

Distances are never negative so we take the absolute value:

$|51 - 37| = 14$

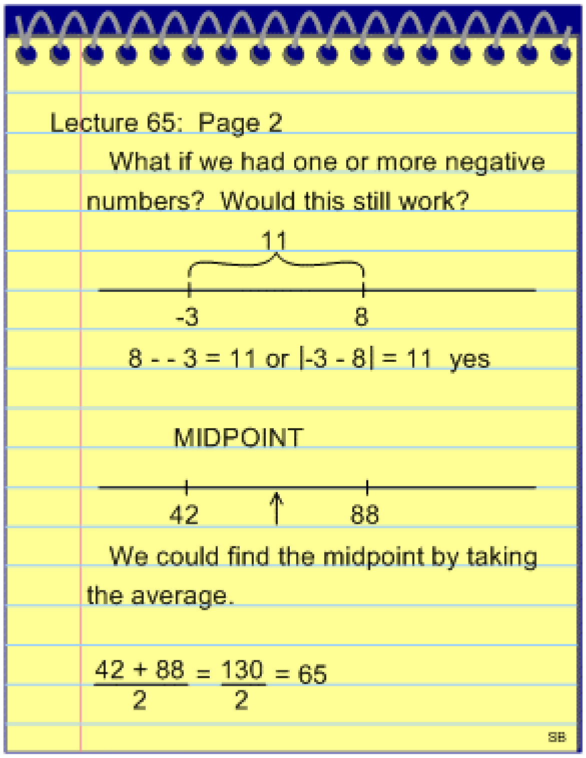
$|37 - 51| = 14$

a b

Distance = $|b - a|$ or $|a - b|$

SB

ALG2065-02



Lecture 65: Page 2

What if we had one or more negative numbers? Would this still work?

11

-3 8

$8 - -3 = 11$ or $|-3 - 8| = 11$ yes

MIDPOINT

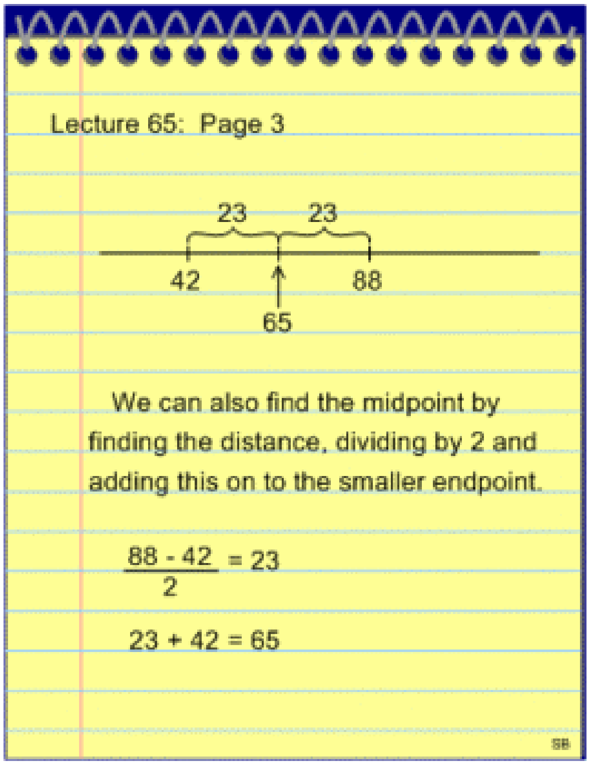
42 88

We could find the midpoint by taking the average.

$$\frac{42 + 88}{2} = \frac{130}{2} = 65$$

SB

ALG2065-03



Lecture 65: Page 3

23 23

42 88

65

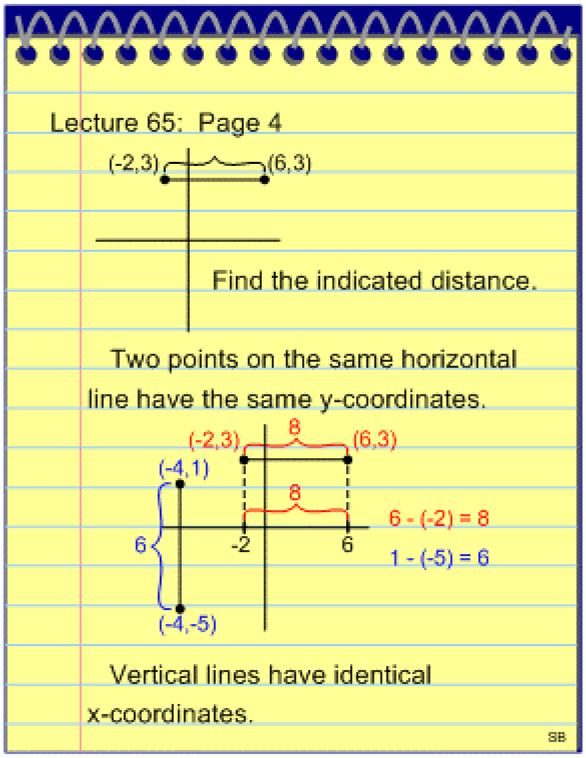
We can also find the midpoint by finding the distance, dividing by 2 and adding this on to the smaller endpoint.

$$\frac{88 - 42}{2} = 23$$

$23 + 42 = 65$

SB

ALG2065-04



Lecture 65: Page 4

(-2,3) (6,3)

Find the indicated distance.

Two points on the same horizontal line have the same y-coordinates.

$6 - (-2) = 8$

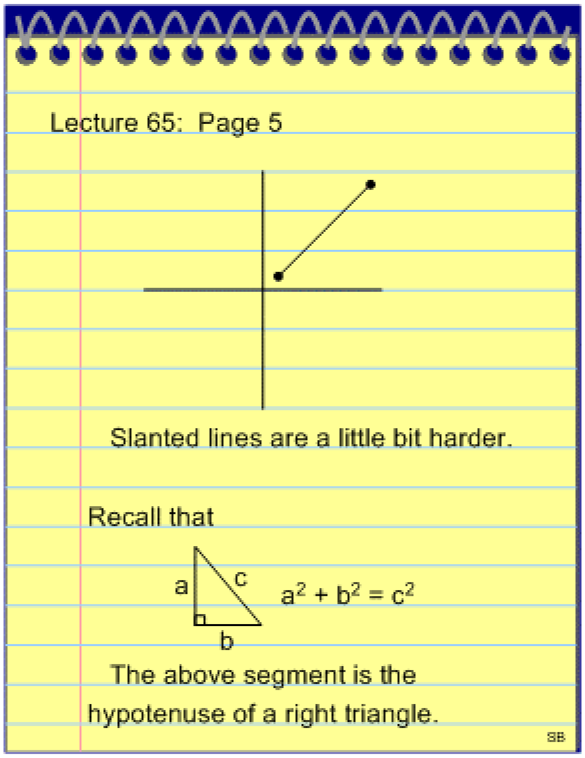
$1 - (-5) = 6$

Vertical lines have identical x-coordinates.

SB

Lecture 65 Notes, Continued

ALG2065-05



Lecture 65: Page 5

Slanted lines are a little bit harder.

Recall that

$a^2 + b^2 = c^2$

The above segment is the hypotenuse of a right triangle.

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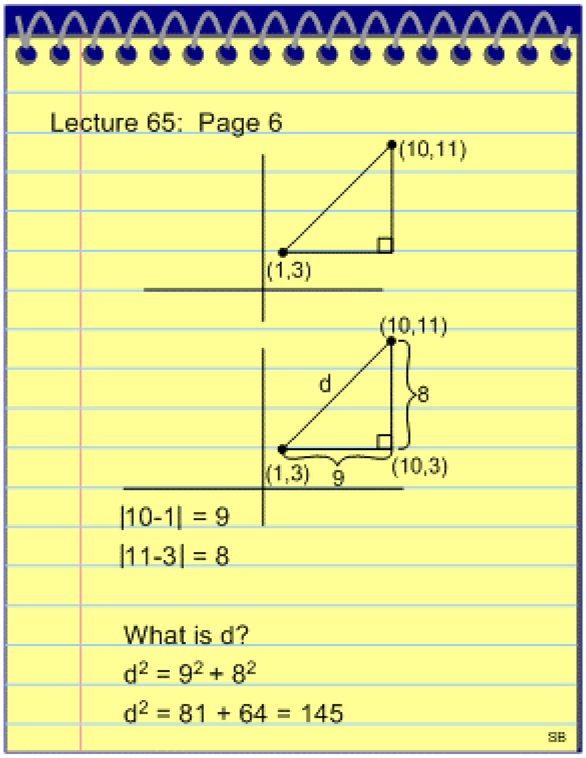


Lecture 65: Page 6

$|10-1| = 9$

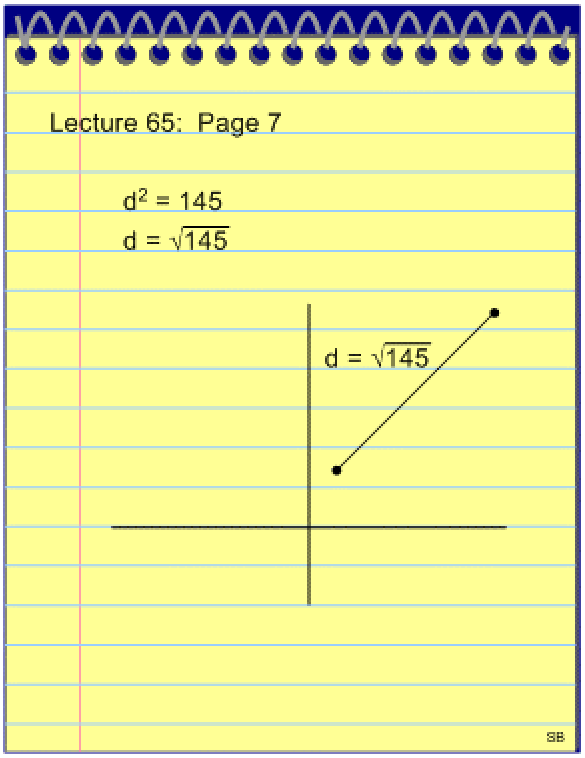
$|11-3| = 8$

What is d?

$d^2 = 9^2 + 8^2$

$d^2 = 81 + 64 = 145$

ALG2065-07

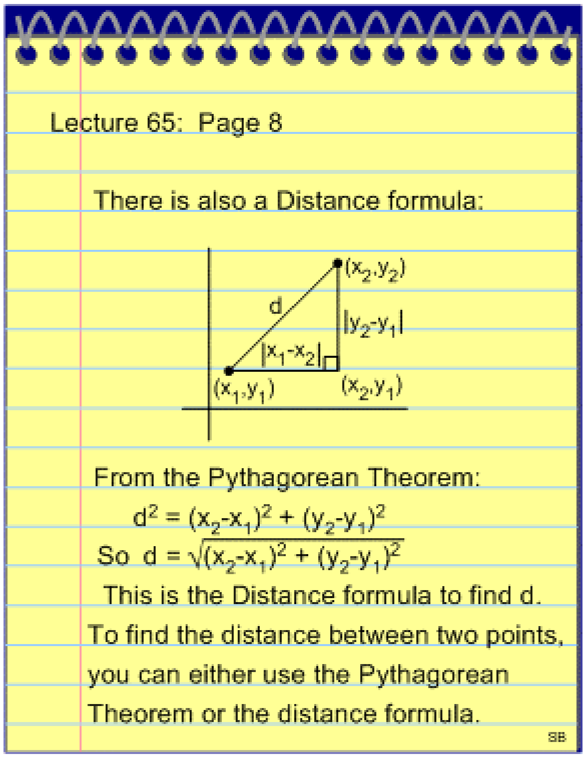


Lecture 65: Page 7

$d^2 = 145$

$d = \sqrt{145}$

ALG2065-08



Lecture 65: Page 8

There is also a Distance formula:

From the Pythagorean Theorem:

$d^2 = (x_2-x_1)^2 + (y_2-y_1)^2$

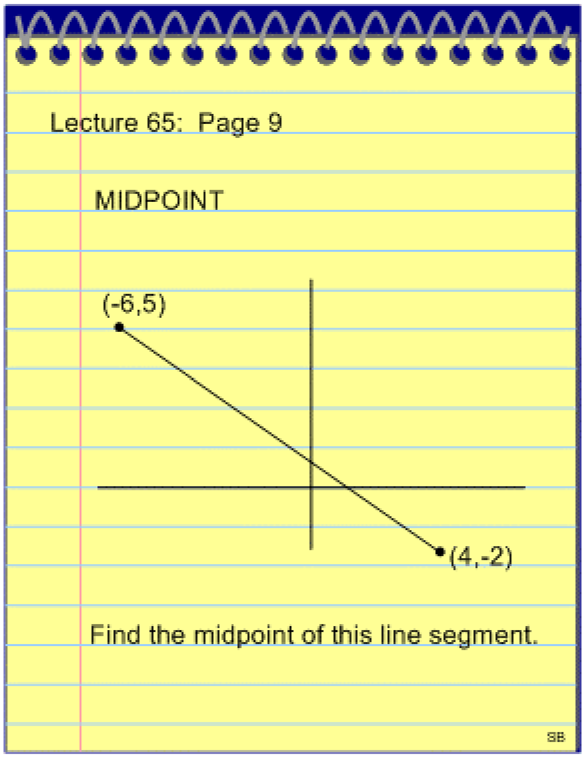
So $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

This is the Distance formula to find d.

To find the distance between two points, you can either use the Pythagorean Theorem or the distance formula.

Lecture 65 Notes, Continued

ALG2065-09



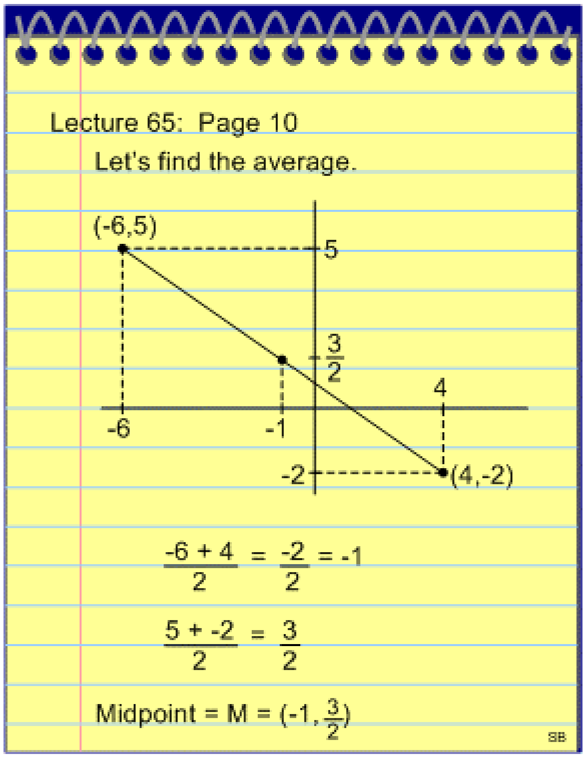
Lecture 65: Page 9

MIDPOINT

Find the midpoint of this line segment.

SB

ALG2065-10



Lecture 65: Page 10

Let's find the average.

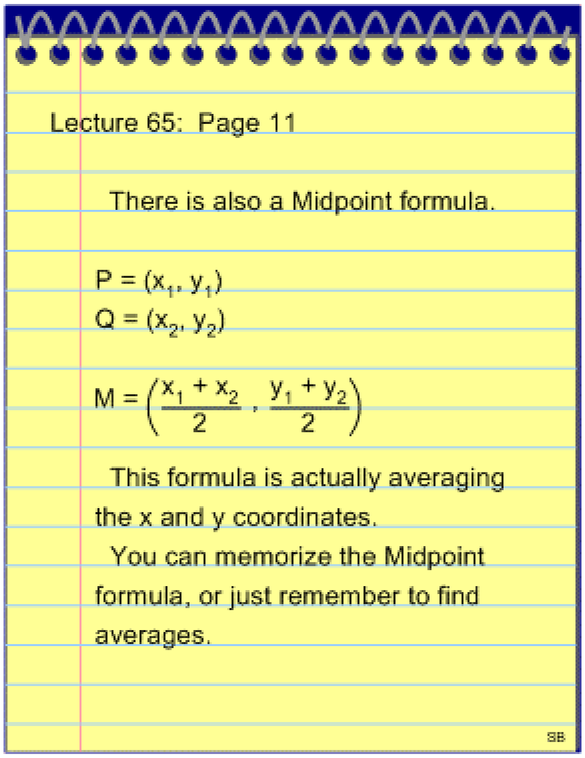
$$\frac{-6 + 4}{2} = \frac{-2}{2} = -1$$

$$\frac{5 + -2}{2} = \frac{3}{2}$$

Midpoint = M = $(-1, \frac{3}{2})$

SB

ALG2065-011



Lecture 65: Page 11

There is also a Midpoint formula.

$P = (x_1, y_1)$
$Q = (x_2, y_2)$

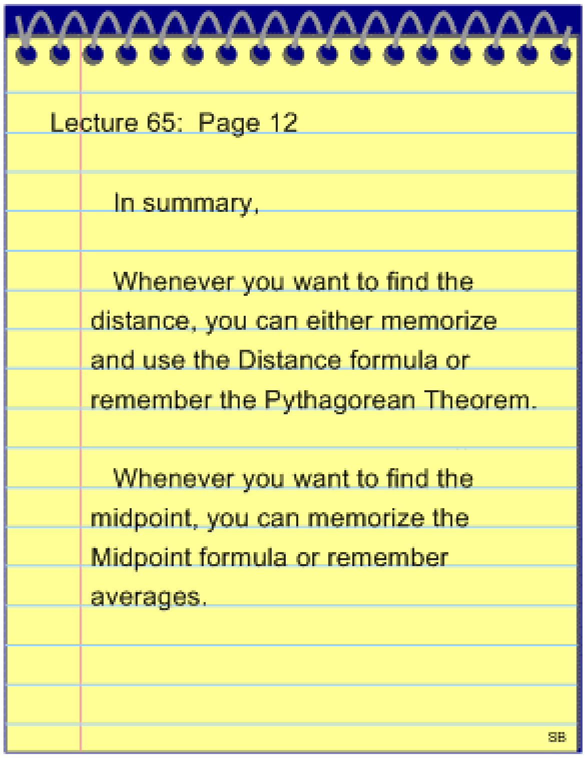
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

This formula is actually averaging the x and y coordinates.

You can memorize the Midpoint formula, or just remember to find averages.

SB

ALG2065-12



Lecture 65: Page 12

In summary,

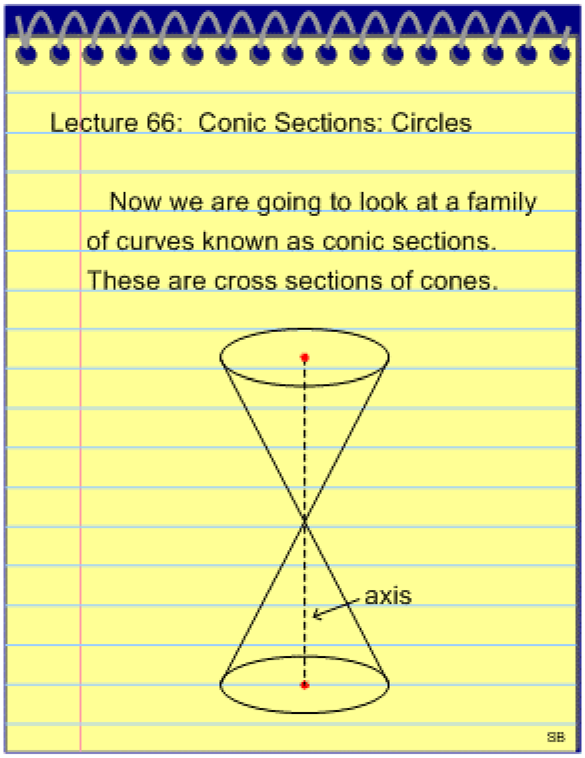
Whenever you want to find the distance, you can either memorize and use the Distance formula or remember the Pythagorean Theorem.

Whenever you want to find the midpoint, you can memorize the Midpoint formula or remember averages.

SB

Lecture 66 Notes

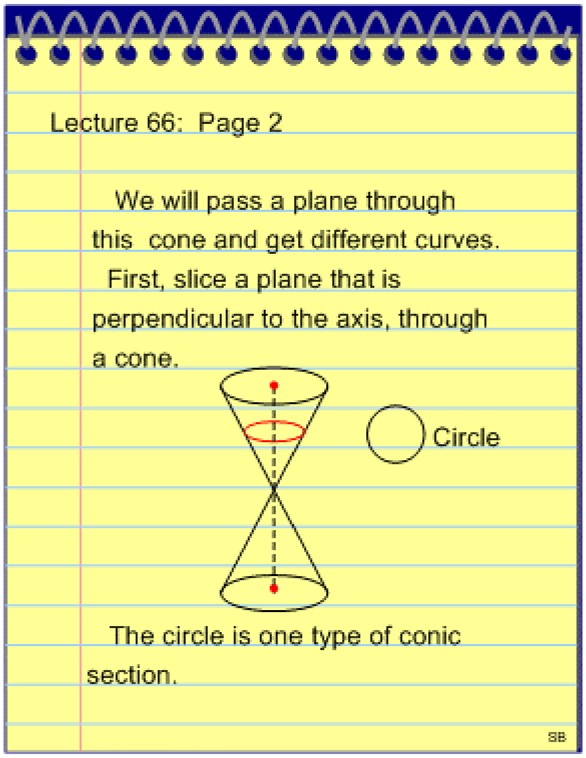
ALG2066-01



Lecture 66: Conic Sections: Circles

Now we are going to look at a family of curves known as conic sections. These are cross sections of cones.

ALG2066-02

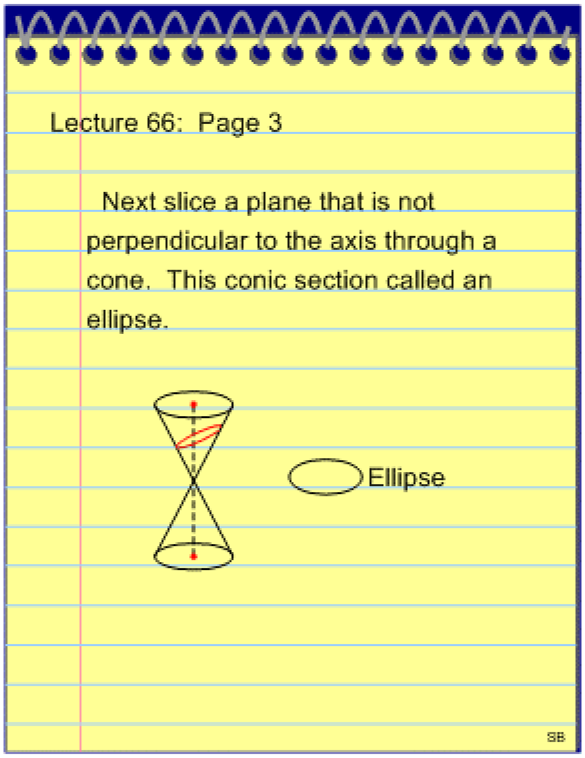


Lecture 66: Page 2

We will pass a plane through this cone and get different curves. First, slice a plane that is perpendicular to the axis, through a cone.

The circle is one type of conic section.

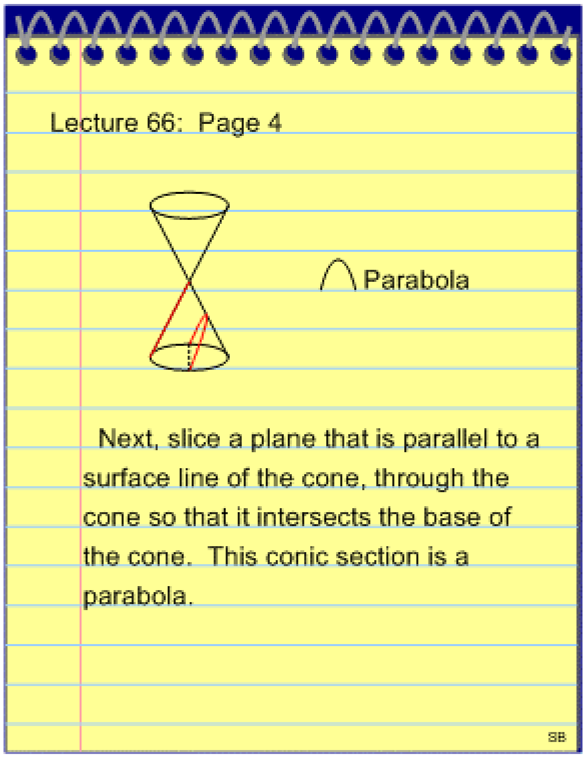
ALG2066-03



Lecture 66: Page 3

Next slice a plane that is not perpendicular to the axis through a cone. This conic section called an ellipse.

ALG2066-04

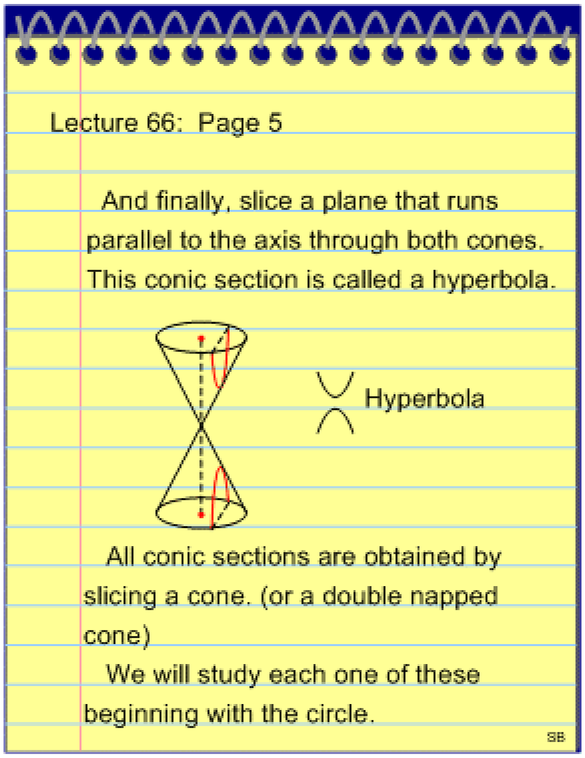


Lecture 66: Page 4

Next, slice a plane that is parallel to a surface line of the cone, through the cone so that it intersects the base of the cone. This conic section is a parabola.

Lecture 66 Notes, Continued

ALG2066-05



Lecture 66: Page 5

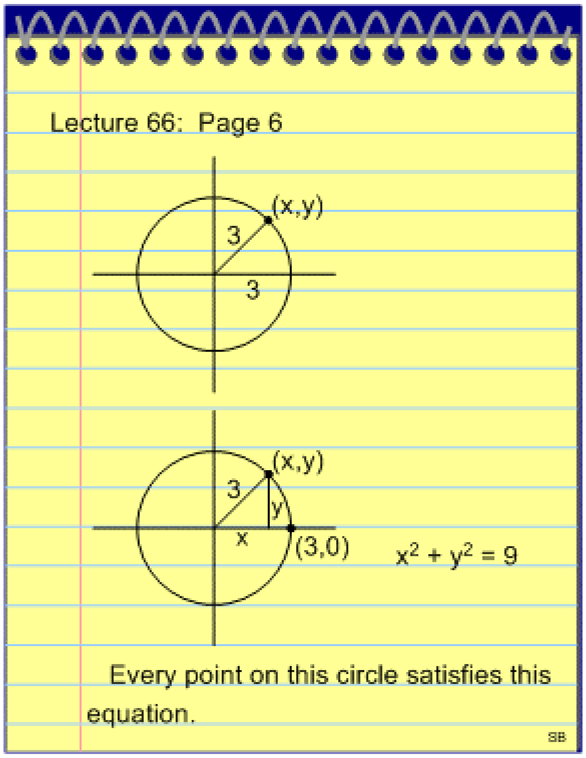
And finally, slice a plane that runs parallel to the axis through both cones. This conic section is called a hyperbola.

All conic sections are obtained by slicing a cone. (or a double napped cone)

We will study each one of these beginning with the circle.

SB

ALG2066-06



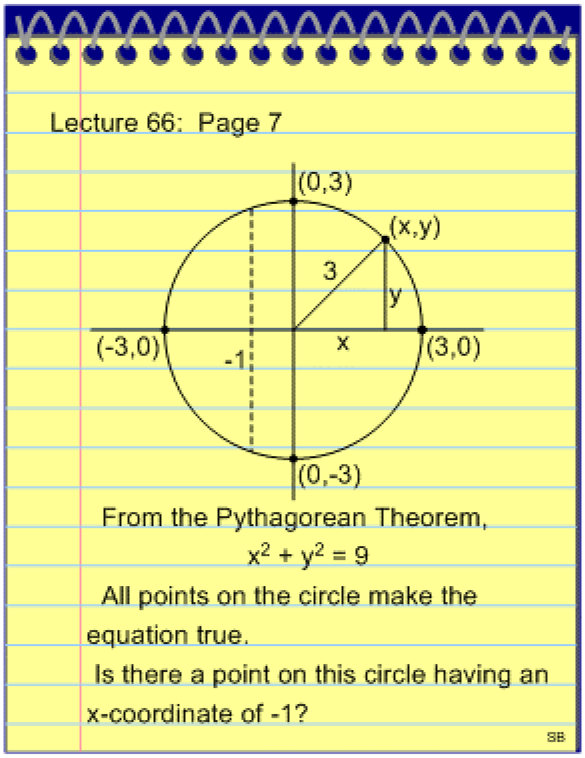
Lecture 66: Page 6

$x^2 + y^2 = 9$

Every point on this circle satisfies this equation.

SB

ALG2066-07



Lecture 66: Page 7

From the Pythagorean Theorem,

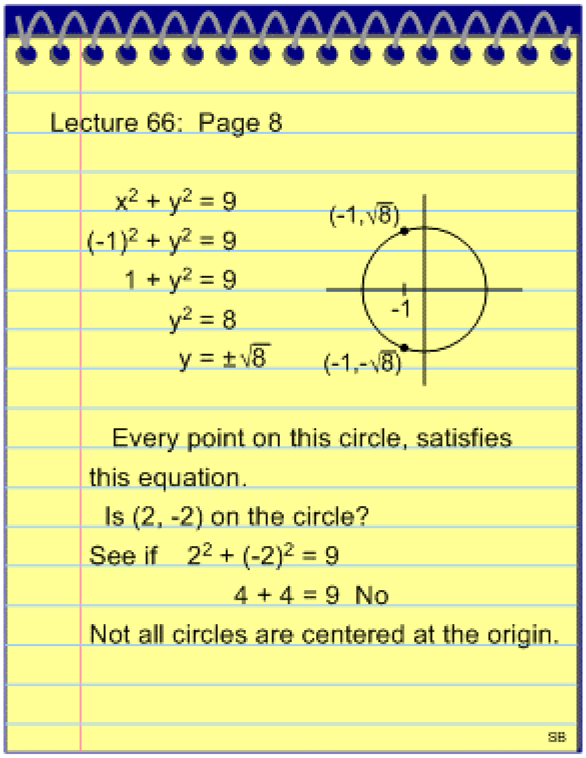
$x^2 + y^2 = 9$

All points on the circle make the equation true.

Is there a point on this circle having an x-coordinate of -1?

SB

ALG2066-08



Lecture 66: Page 8

$x^2 + y^2 = 9$

$(-1)^2 + y^2 = 9$

$1 + y^2 = 9$

$y^2 = 8$

$y = \pm\sqrt{8}$

Every point on this circle, satisfies this equation.

Is (2, -2) on the circle?

See if $2^2 + (-2)^2 = 9$

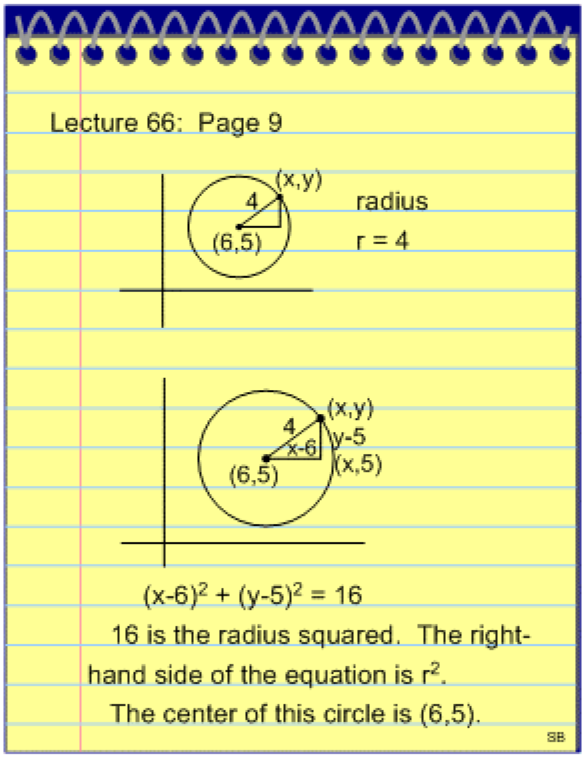
$4 + 4 = 9$ No

Not all circles are centered at the origin.

SB

Lecture 66 Notes, Continued

ALG2066-09



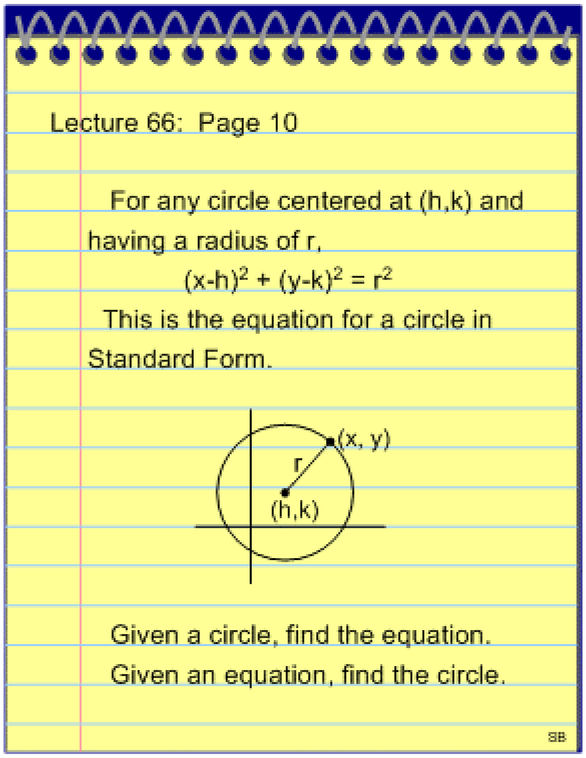
Lecture 66: Page 9

$(x-6)^2 + (y-5)^2 = 16$

16 is the radius squared. The right-hand side of the equation is r^2.

The center of this circle is (6,5).

ALG2066-10



Lecture 66: Page 10

For any circle centered at (h,k) and having a radius of r,

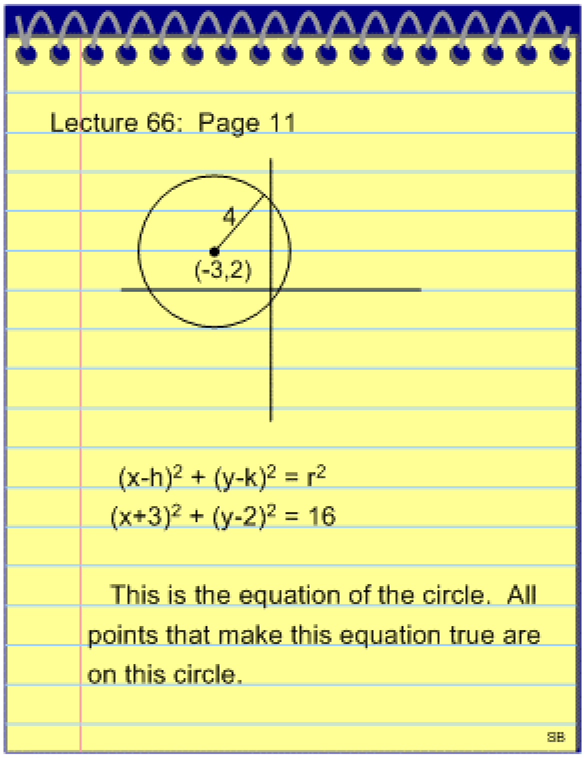
$(x-h)^2 + (y-k)^2 = r^2$

This is the equation for a circle in Standard Form.

Given a circle, find the equation.

Given an equation, find the circle.

ALG2066-11



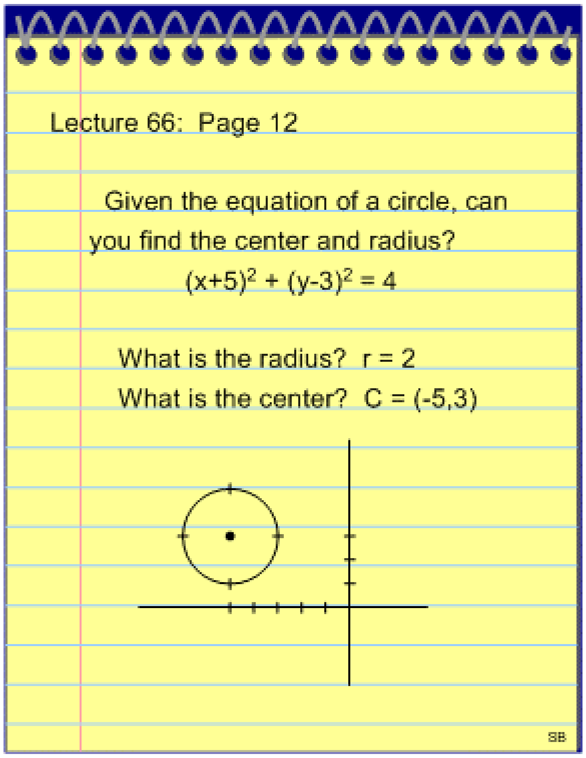
Lecture 66: Page 11

$(x-h)^2 + (y-k)^2 = r^2$

$(x+3)^2 + (y-2)^2 = 16$

This is the equation of the circle. All points that make this equation true are on this circle.

ALG2066-12



Lecture 66: Page 12

Given the equation of a circle, can you find the center and radius?

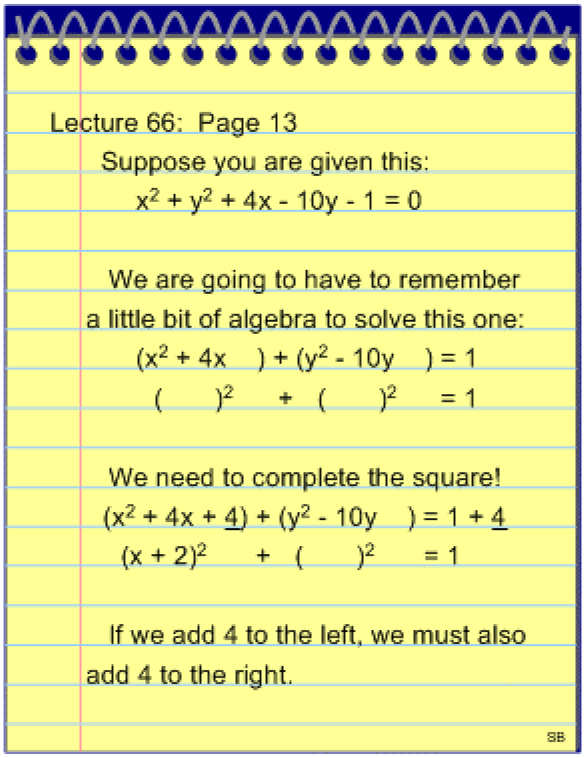
$(x+5)^2 + (y-3)^2 = 4$

What is the radius? r = 2

What is the center? C = (-5,3)

Lecture 66 Notes, Continued

ALG2066-13



Lecture 66: Page 13

Suppose you are given this:

$x^2 + y^2 + 4x - 10y - 1 = 0$

We are going to have to remember a little bit of algebra to solve this one:

$(x^2 + 4x \quad) + (y^2 - 10y \quad) = 1$

$(\quad)^2 + (\quad)^2 = 1$

We need to complete the square!

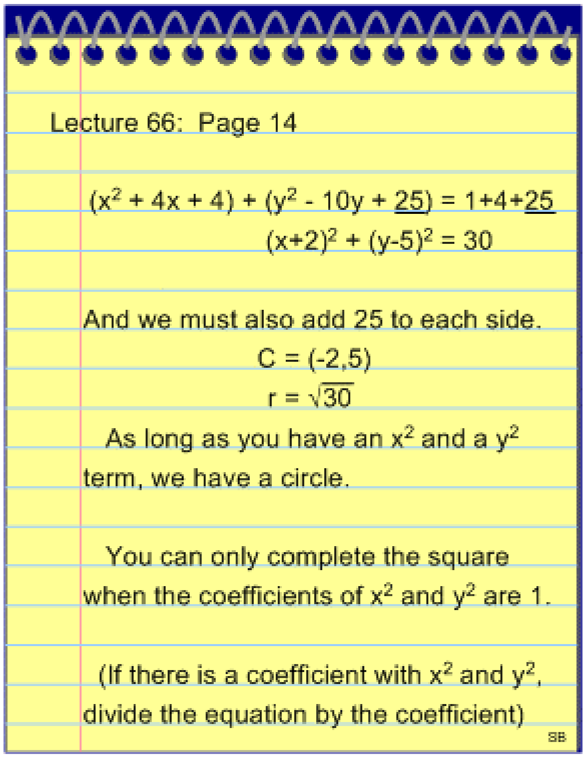
$(x^2 + 4x + \underline{4}) + (y^2 - 10y \quad) = 1 + \underline{4}$

$(x + 2)^2 + (\quad)^2 = 1$

If we add 4 to the left, we must also add 4 to the right.

SB

ALG2066-14



Lecture 66: Page 14

$(x^2 + 4x + 4) + (y^2 - 10y + \underline{25}) = 1+4+\underline{25}$

$(x+2)^2 + (y-5)^2 = 30$

And we must also add 25 to each side.

$C = (-2,5)$

$r = \sqrt{30}$

As long as you have an x^2 and a y^2 term, we have a circle.

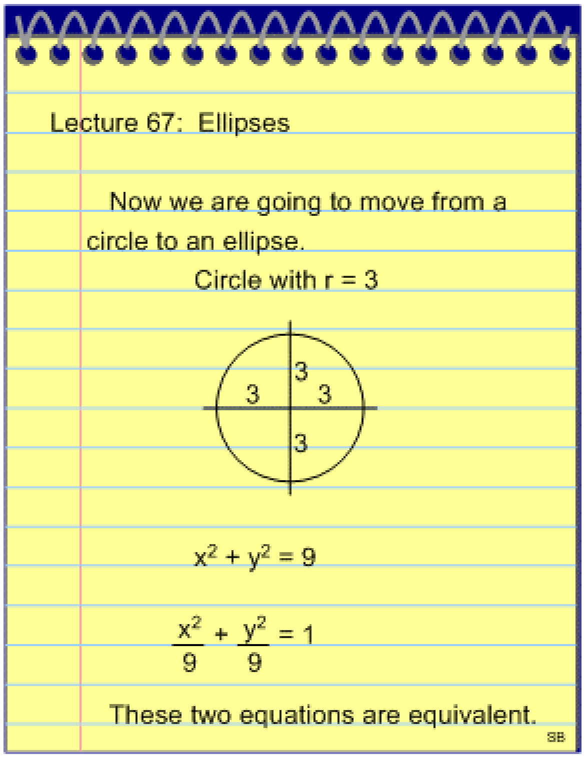
You can only complete the square when the coefficients of x^2 and y^2 are 1.

(If there is a coefficient with x^2 and y^2, divide the equation by the coefficient)

SB

Lecture 67 Notes

ALG2067-01



Lecture 67: Ellipses

Now we are going to move from a circle to an ellipse.

Circle with r = 3

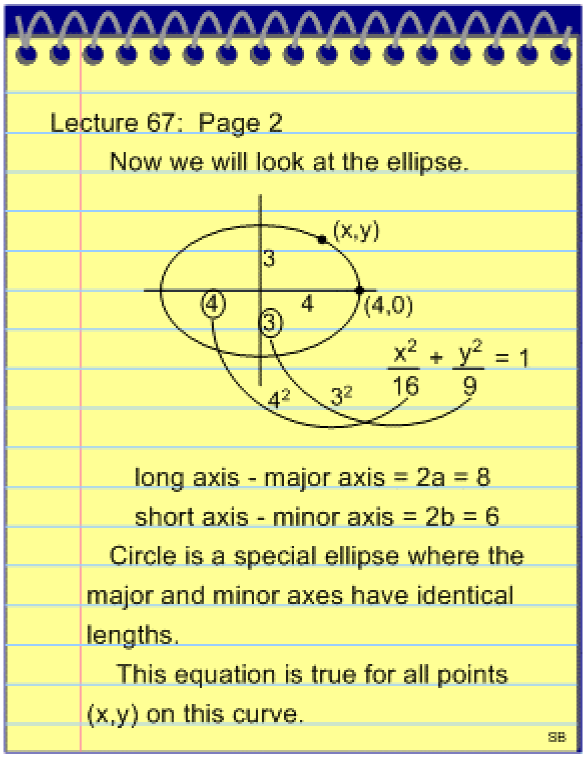
$x^2 + y^2 = 9$

$\frac{x^2}{9} + \frac{y^2}{9} = 1$

These two equations are equivalent.

SB

ALG2067-02



Lecture 67: Page 2

Now we will look at the ellipse.

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

long axis - major axis = 2a = 8

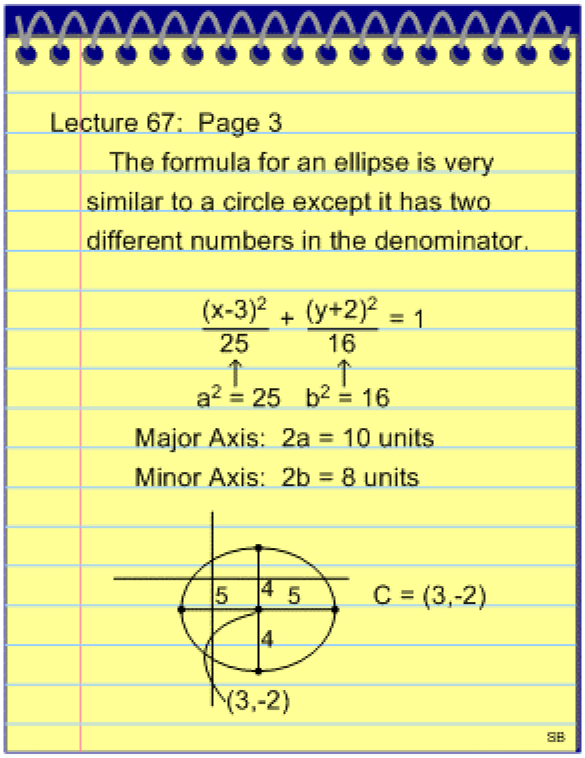
short axis - minor axis = 2b = 6

Circle is a special ellipse where the major and minor axes have identical lengths.

This equation is true for all points (x,y) on this curve.

SB

ALG2067-03



Lecture 67: Page 3

The formula for an ellipse is very similar to a circle except it has two different numbers in the denominator.

$$\underset{\substack{\uparrow \\ a^2 = 25}}{\frac{(x-3)^2}{25}} + \underset{\substack{\uparrow \\ b^2 = 16}}{\frac{(y+2)^2}{16}} = 1$$

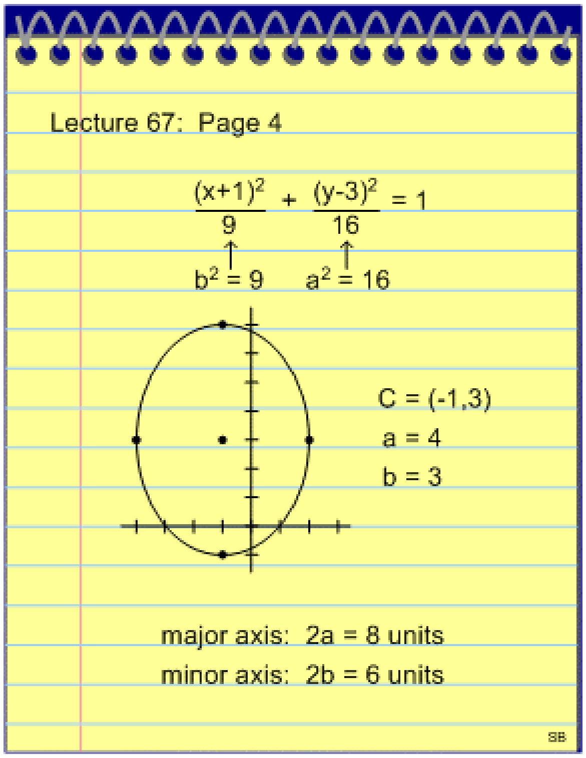
Major Axis: 2a = 10 units

Minor Axis: 2b = 8 units

C = (3,-2)

SB

ALG2067-04



Lecture 67: Page 4

$$\underset{\substack{\uparrow \\ b^2 = 9}}{\frac{(x+1)^2}{9}} + \underset{\substack{\uparrow \\ a^2 = 16}}{\frac{(y-3)^2}{16}} = 1$$

C = (-1,3)

a = 4

b = 3

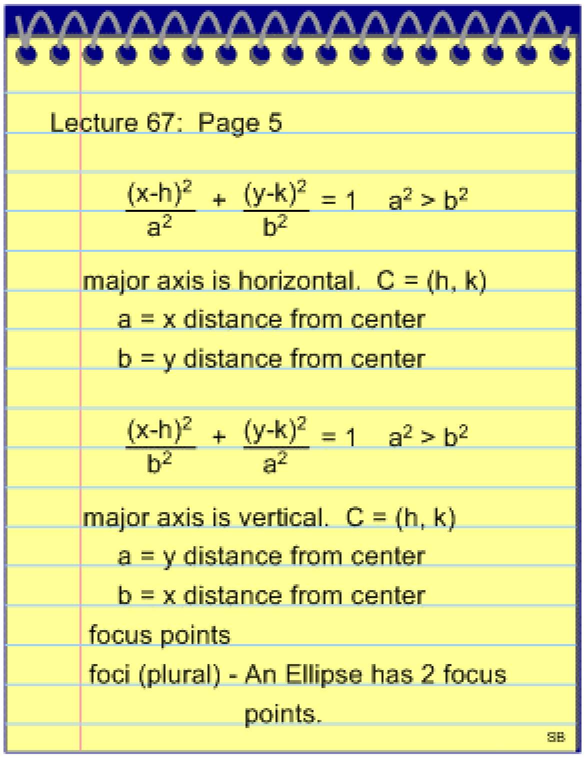
major axis: 2a = 8 units

minor axis: 2b = 6 units

SB

Lecture 67 Notes, Continued

ALG2067-05



Lecture 67: Page 5

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad a^2 > b^2$$

major axis is horizontal. C = (h, k)

a = x distance from center

b = y distance from center

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad a^2 > b^2$$

major axis is vertical. C = (h, k)

a = y distance from center

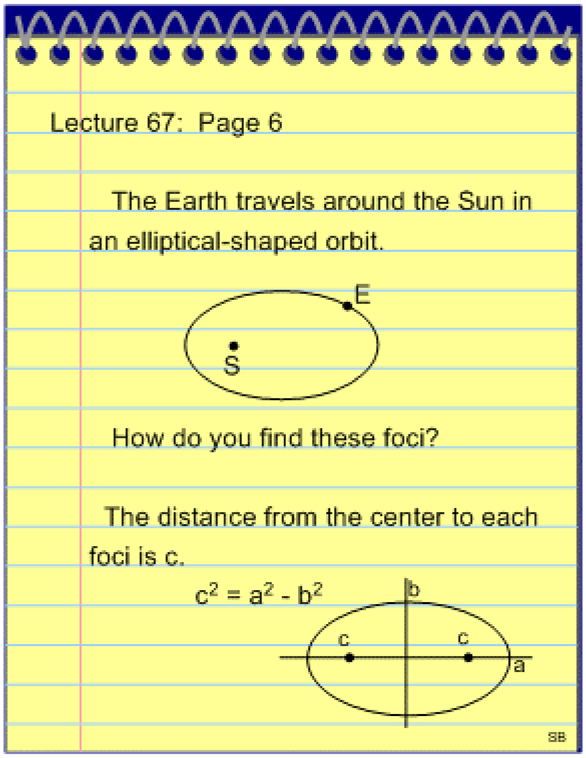
b = x distance from center

focus points

foci (plural) - An Ellipse has 2 focus points.

SB

ALG2067-06



Lecture 67: Page 6

The Earth travels around the Sun in an elliptical-shaped orbit.

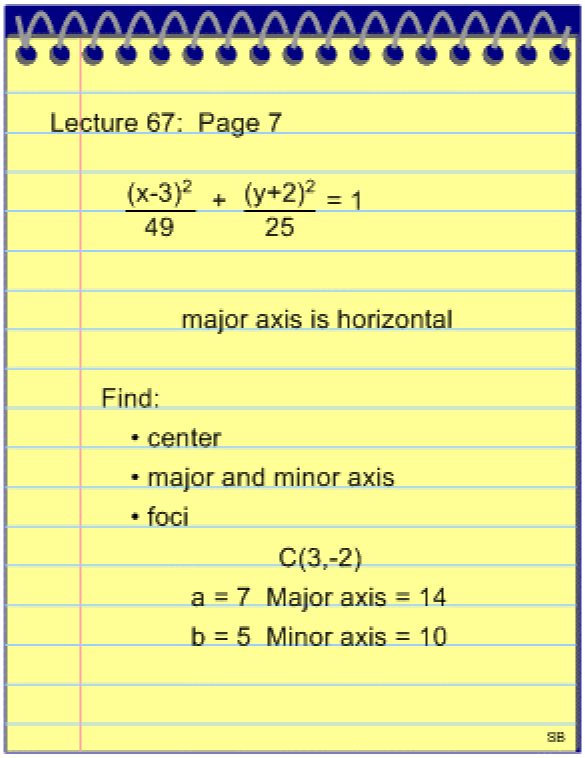
How do you find these foci?

The distance from the center to each foci is c.

$c^2 = a^2 - b^2$

SB

ALG2067-07



Lecture 67: Page 7

$$\frac{(x-3)^2}{49} + \frac{(y+2)^2}{25} = 1$$

major axis is horizontal

Find:

- center
- major and minor axis
- foci

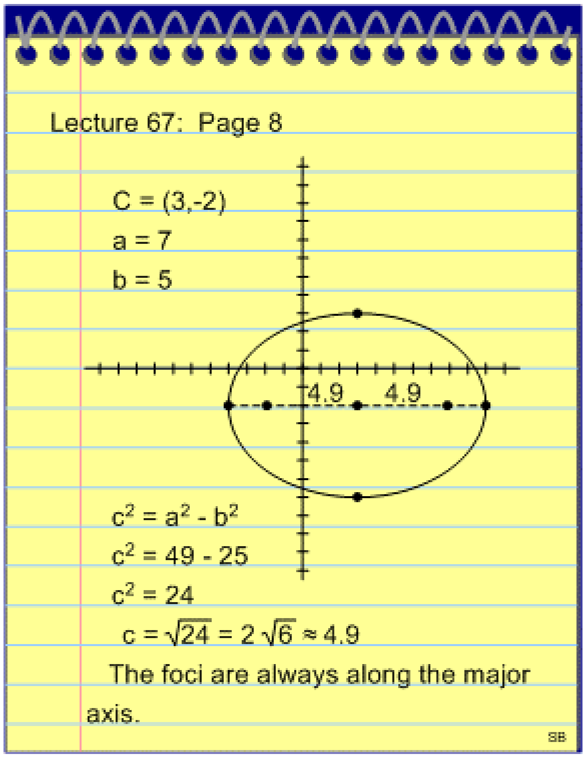
C(3,-2)

a = 7 Major axis = 14

b = 5 Minor axis = 10

SB

ALG2067-08



Lecture 67: Page 8

C = (3,-2)

a = 7

b = 5

$c^2 = a^2 - b^2$

$c^2 = 49 - 25$

$c^2 = 24$

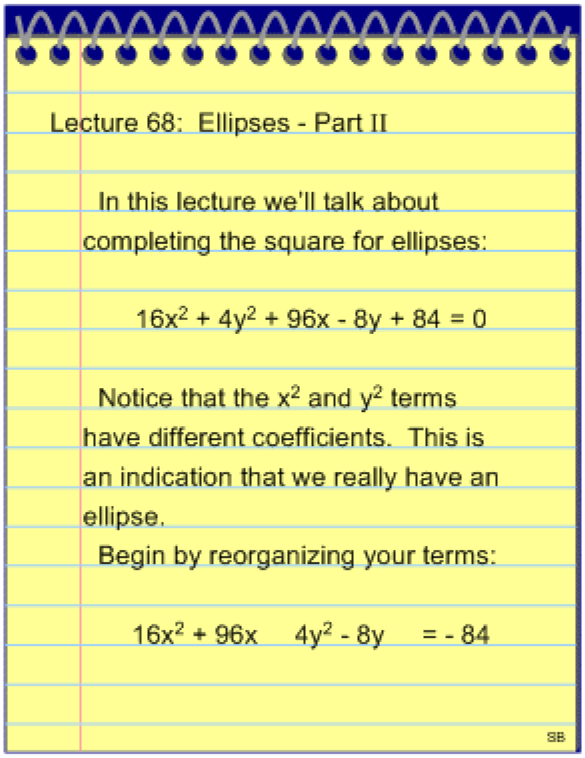
$c = \sqrt{24} = 2\sqrt{6} \approx 4.9$

The foci are always along the major axis.

SB

Lecture 68 Notes

ALG2068-01



Lecture 68: Ellipses - Part II

In this lecture we'll talk about completing the square for ellipses:

$$16x^2 + 4y^2 + 96x - 8y + 84 = 0$$

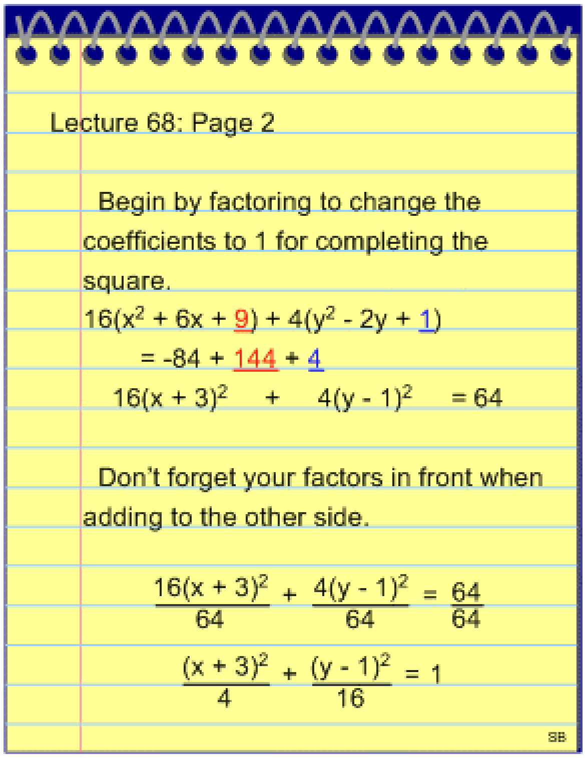
Notice that the x^2 and y^2 terms have different coefficients. This is an indication that we really have an ellipse.

Begin by reorganizing your terms:

$$16x^2 + 96x \quad 4y^2 - 8y \quad = -84$$

SB

ALG2068-02



Lecture 68: Page 2

Begin by factoring to change the coefficients to 1 for completing the square.

$$16(x^2 + 6x + \underline{9}) + 4(y^2 - 2y + \underline{1})$$

$$= -84 + \underline{144} + \underline{4}$$

$$16(x + 3)^2 \quad + \quad 4(y - 1)^2 \quad = 64$$

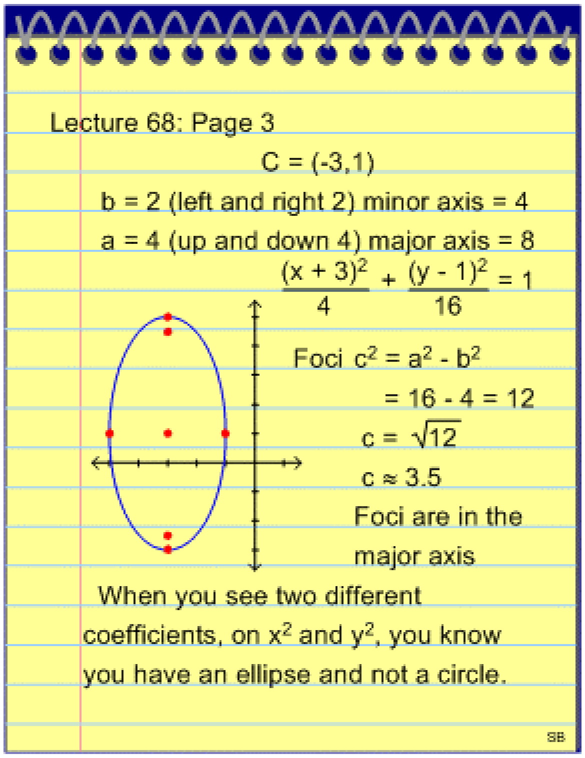
Don't forget your factors in front when adding to the other side.

$$\frac{16(x + 3)^2}{64} + \frac{4(y - 1)^2}{64} = \frac{64}{64}$$

$$\frac{(x + 3)^2}{4} + \frac{(y - 1)^2}{16} = 1$$

SB

ALG2068-03



Lecture 68: Page 3

C = (-3,1)

b = 2 (left and right 2) minor axis = 4

a = 4 (up and down 4) major axis = 8

$$\frac{(x + 3)^2}{4} + \frac{(y - 1)^2}{16} = 1$$

Foci $c^2 = a^2 - b^2$

$= 16 - 4 = 12$

$c = \sqrt{12}$

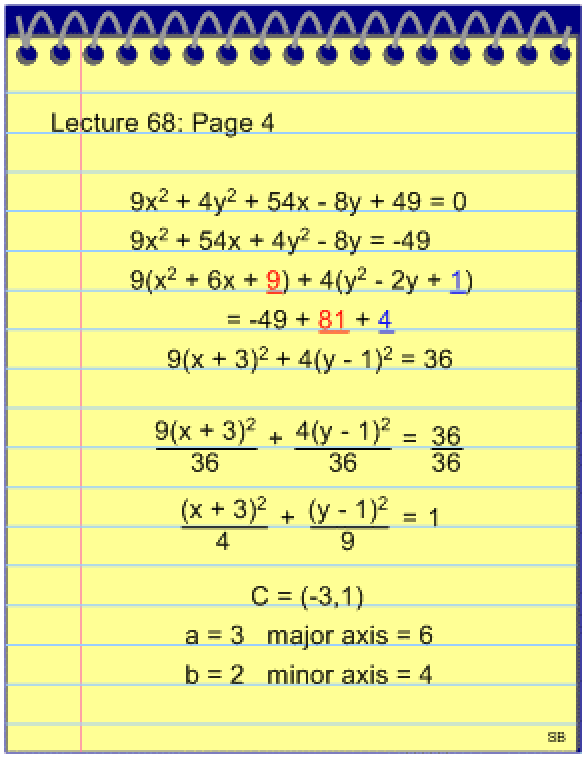
$c \approx 3.5$

Foci are in the major axis

When you see two different coefficients, on x^2 and y^2, you know you have an ellipse and not a circle.

SB

ALG2068-04



Lecture 68: Page 4

$$9x^2 + 4y^2 + 54x - 8y + 49 = 0$$

$$9x^2 + 54x + 4y^2 - 8y = -49$$

$$9(x^2 + 6x + \underline{9}) + 4(y^2 - 2y + \underline{1})$$

$$= -49 + \underline{81} + \underline{4}$$

$$9(x + 3)^2 + 4(y - 1)^2 = 36$$

$$\frac{9(x + 3)^2}{36} + \frac{4(y - 1)^2}{36} = \frac{36}{36}$$

$$\frac{(x + 3)^2}{4} + \frac{(y - 1)^2}{9} = 1$$

C = (-3,1)

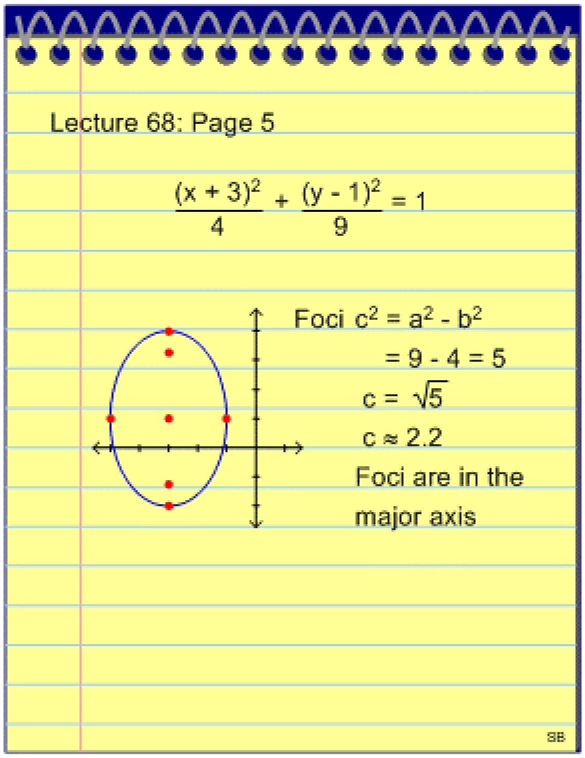
a = 3 major axis = 6

b = 2 minor axis = 4

SB

Lecture 68 Notes, Continued

ALG2068-05



Lecture 68: Page 5

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{9} = 1$$

Foci $c^2 = a^2 - b^2$

$= 9 - 4 = 5$

$c = \sqrt{5}$

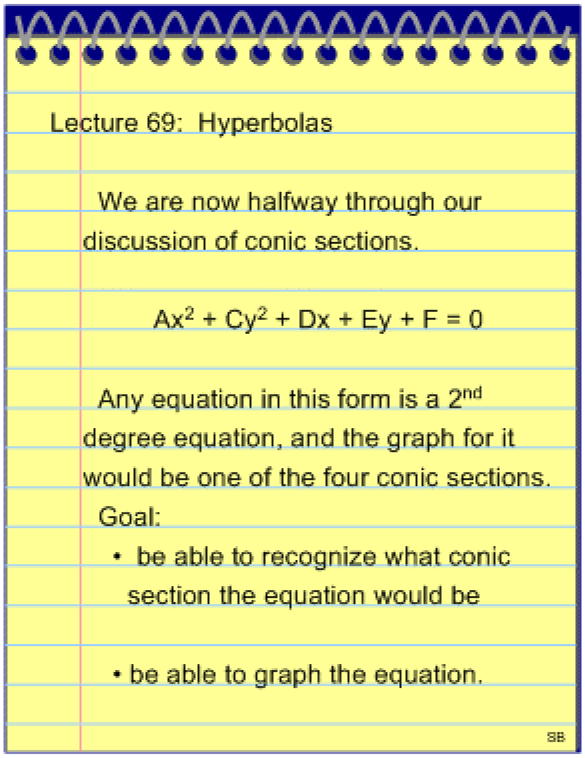
$c \approx 2.2$

Foci are in the major axis

SB

Lecture 69 Notes

ALG2069-01



Lecture 69: Hyperbolas

We are now halfway through our discussion of conic sections.

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Any equation in this form is a 2nd degree equation, and the graph for it would be one of the four conic sections.

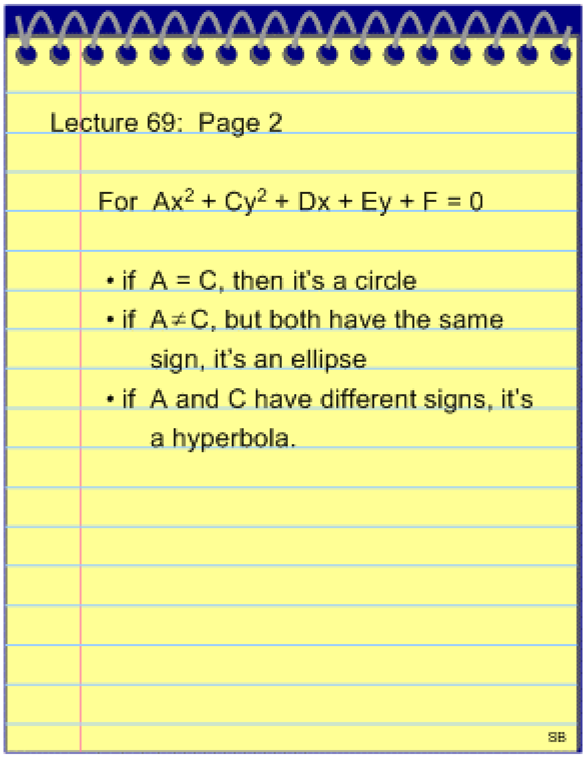
Goal:

- be able to recognize what conic section the equation would be

- be able to graph the equation.

SB

ALG2069-02



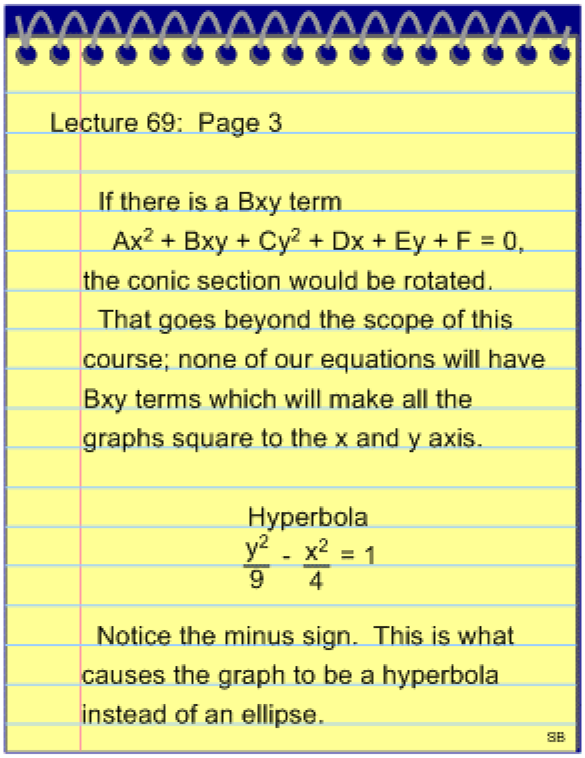
Lecture 69: Page 2

For $Ax^2 + Cy^2 + Dx + Ey + F = 0$

- if $A = C$, then it's a circle
- if $A \neq C$, but both have the same sign, it's an ellipse
- if A and C have different signs, it's a hyperbola.

SB

ALG2069-03



Lecture 69: Page 3

If there is a Bxy term

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

the conic section would be rotated.

That goes beyond the scope of this course; none of our equations will have Bxy terms which will make all the graphs square to the x and y axis.

Hyperbola

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

Notice the minus sign. This is what causes the graph to be a hyperbola instead of an ellipse.

SB

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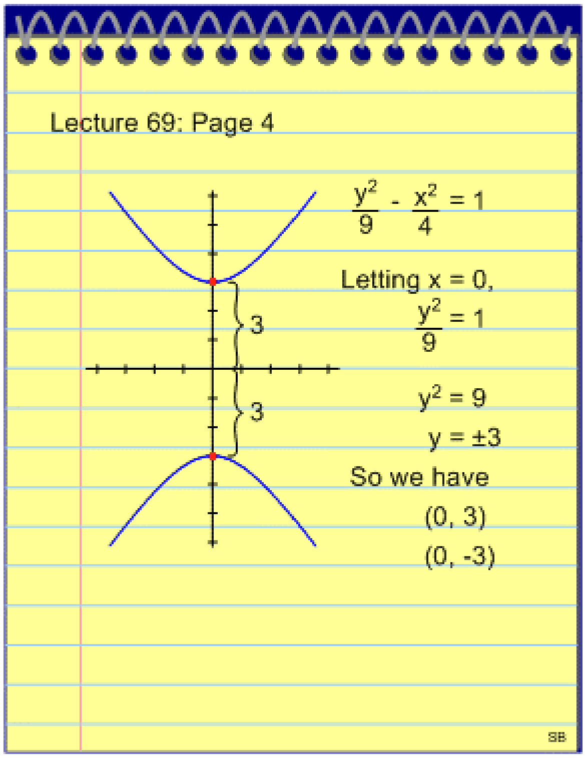


Lecture 69: Page 4

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

Letting $x = 0$,

$$\frac{y^2}{9} = 1$$

$$y^2 = 9$$

$$y = \pm 3$$

So we have

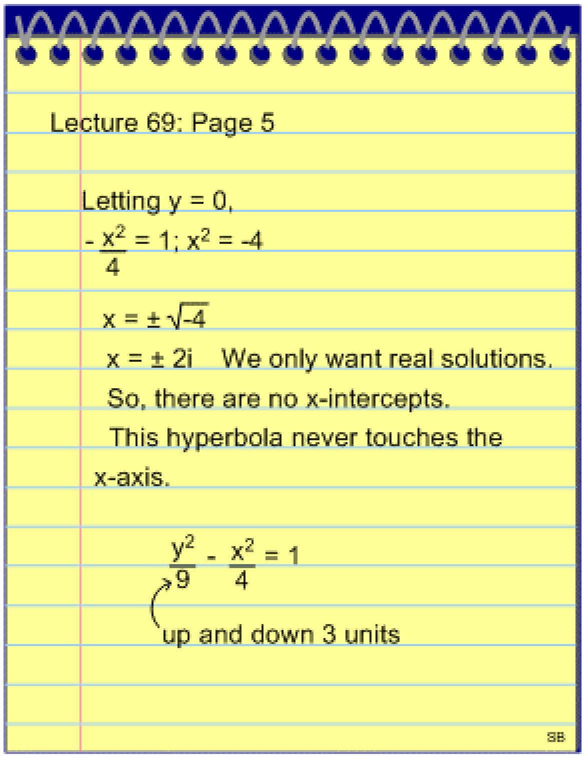
(0, 3)

(0, -3)

SB

Lecture 69 Notes, Continued

ALG2069-05



Lecture 69: Page 5

Letting y = 0,

$-\frac{x^2}{4} = 1;\ x^2 = -4$

$x = \pm\sqrt{-4}$

$x = \pm 2i$ We only want real solutions.

So, there are no x-intercepts.

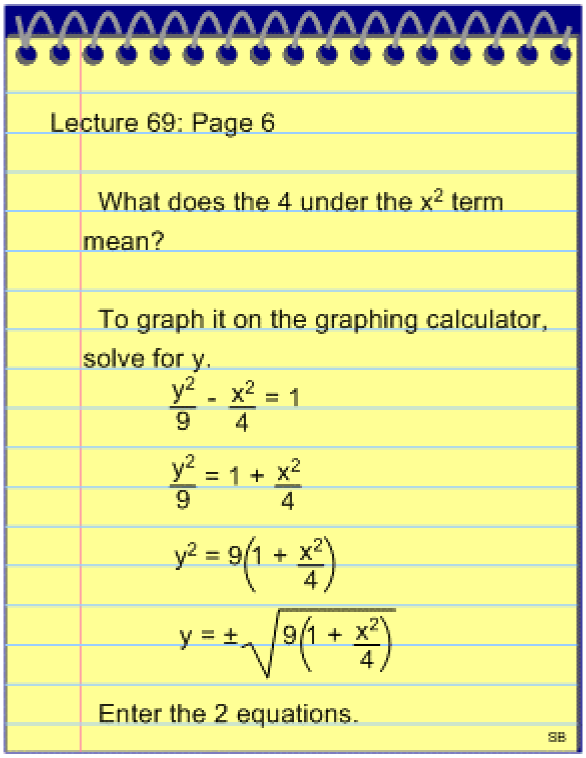
This hyperbola never touches the x-axis.

$\frac{y^2}{9} - \frac{x^2}{4} = 1$

up and down 3 units

SB

ALG2069-06



Lecture 69: Page 6

What does the 4 under the x^2 term mean?

To graph it on the graphing calculator, solve for y.

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

$$\frac{y^2}{9} = 1 + \frac{x^2}{4}$$

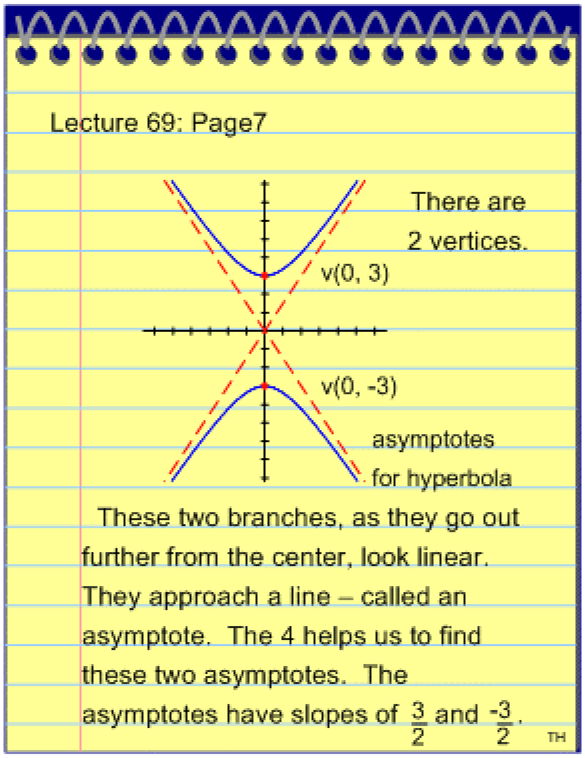
$$y^2 = 9\left(1 + \frac{x^2}{4}\right)$$

$$y = \pm\sqrt{9\left(1 + \frac{x^2}{4}\right)}$$

Enter the 2 equations.

SB

ALG2069-07



Lecture 69: Page7

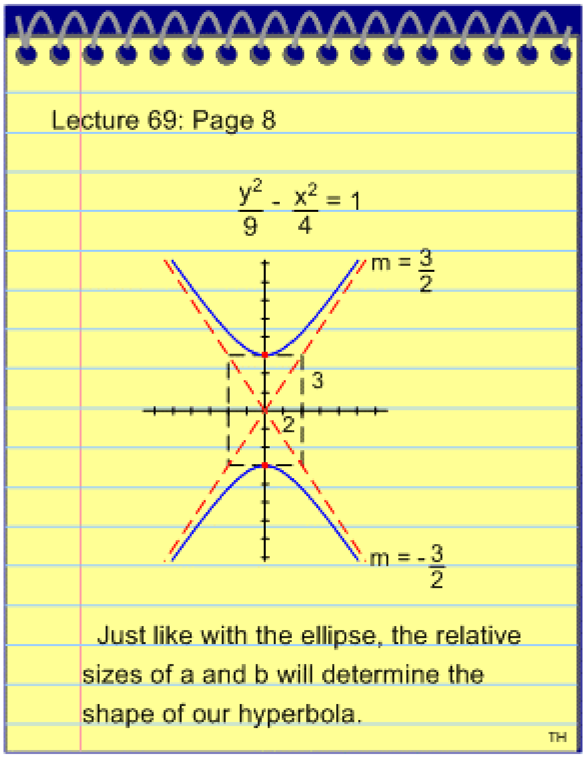
There are 2 vertices.

asymptotes for hyperbola

These two branches, as they go out further from the center, look linear. They approach a line – called an asymptote. The 4 helps us to find these two asymptotes. The asymptotes have slopes of $\frac{3}{2}$ and $\frac{-3}{2}$.

TH

ALG2069-08



Lecture 69: Page 8

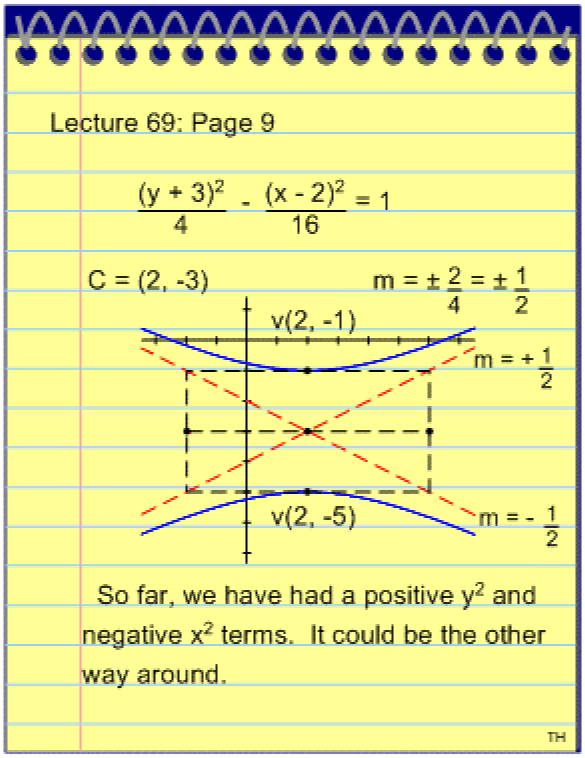
$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

Just like with the ellipse, the relative sizes of a and b will determine the shape of our hyperbola.

TH

Lecture 69 Notes, Continued

ALG2069-09



Lecture 69: Page 9

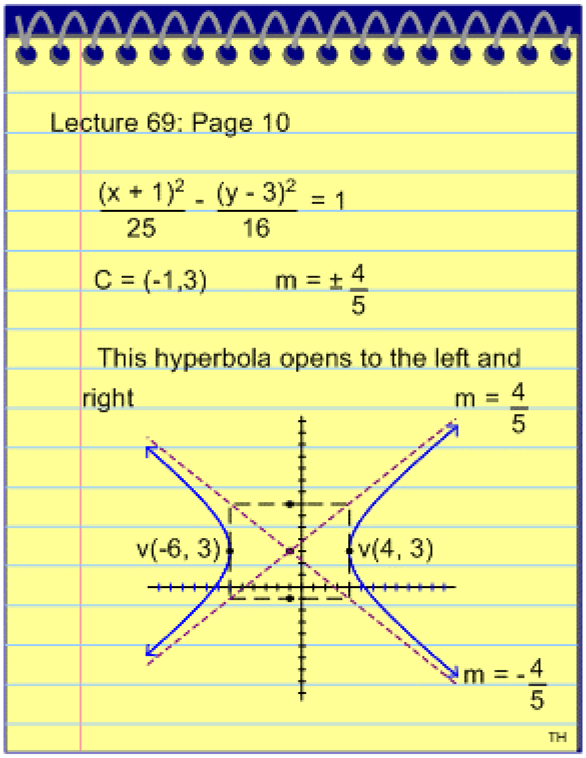
$$\frac{(y + 3)^2}{4} - \frac{(x - 2)^2}{16} = 1$$

C = (2, -3) $m = \pm\frac{2}{4} = \pm\frac{1}{2}$

So far, we have had a positive y^2 and negative x^2 terms. It could be the other way around.

TH

ALG2069-10



Lecture 69: Page 10

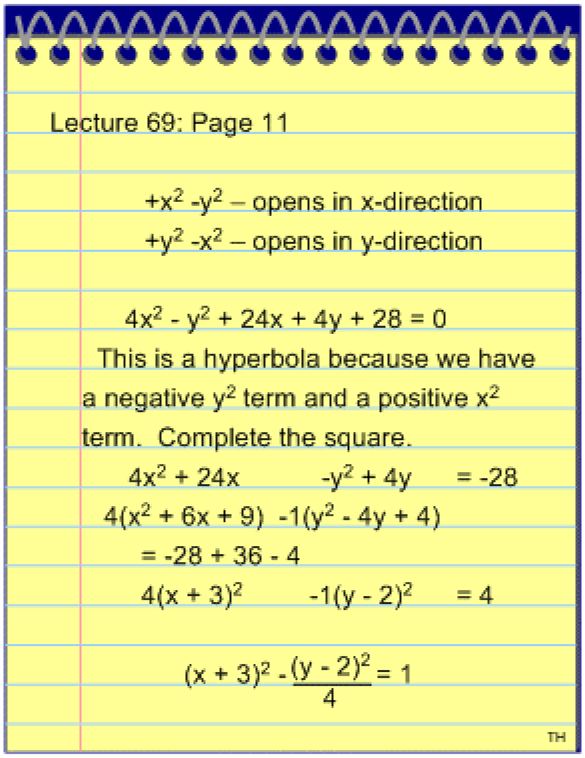
$$\frac{(x + 1)^2}{25} - \frac{(y - 3)^2}{16} = 1$$

C = (-1,3) $m = \pm\frac{4}{5}$

This hyperbola opens to the left and right

TH

ALG2069-11



Lecture 69: Page 11

$+x^2 -y^2$ – opens in x-direction

$+y^2 -x^2$ – opens in y-direction

$4x^2 - y^2 + 24x + 4y + 28 = 0$

This is a hyperbola because we have a negative y^2 term and a positive x^2 term. Complete the square.

$4x^2 + 24x \quad -y^2 + 4y \quad = -28$

$4(x^2 + 6x + 9) \ -1(y^2 - 4y + 4)$

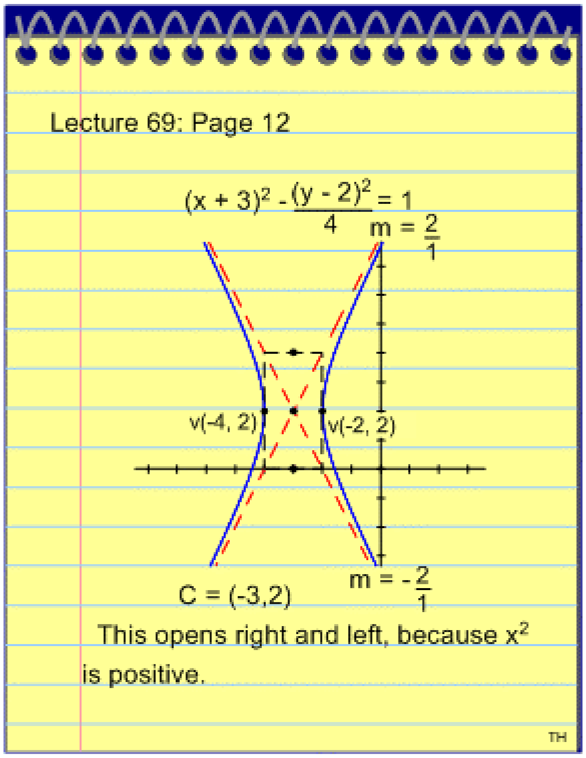
$= -28 + 36 - 4$

$4(x + 3)^2 \quad -1(y - 2)^2 \quad = 4$

$$(x + 3)^2 - \frac{(y - 2)^2}{4} = 1$$

TH

ALG2069-12



Lecture 69: Page 12

$$(x + 3)^2 - \frac{(y - 2)^2}{4} = 1$$

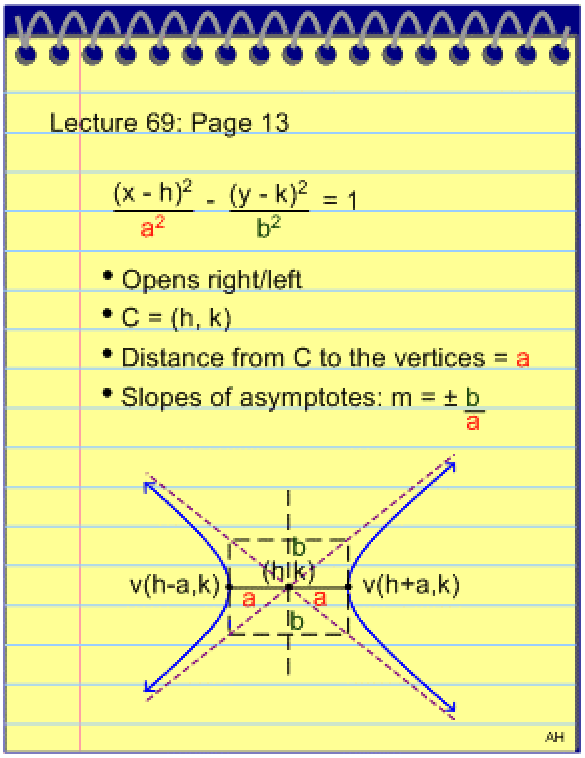
C = (-3,2)

This opens right and left, because x^2 is positive.

TH

Lecture 69 Notes, Continued

ALG2069-013

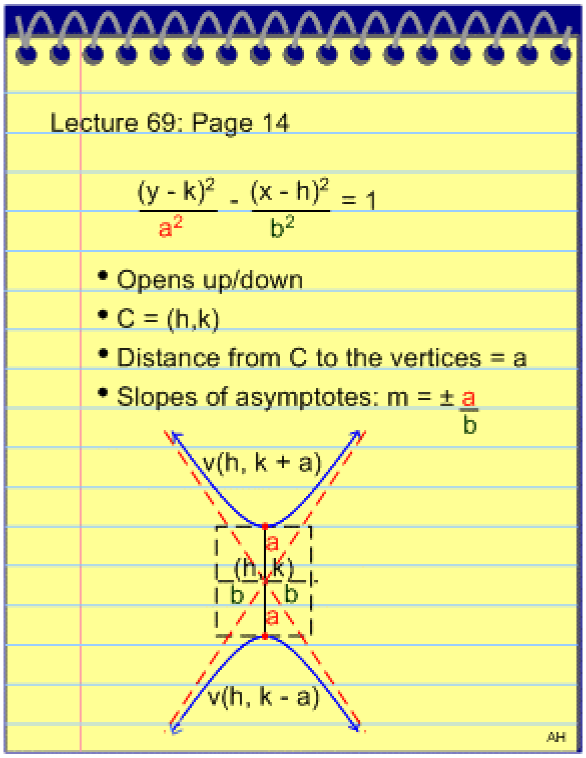


Lecture 69: Page 13

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

- Opens right/left
- C = (h, k)
- Distance from C to the vertices = a
- Slopes of asymptotes: $m = \pm \frac{b}{a}$

ALG2069-14



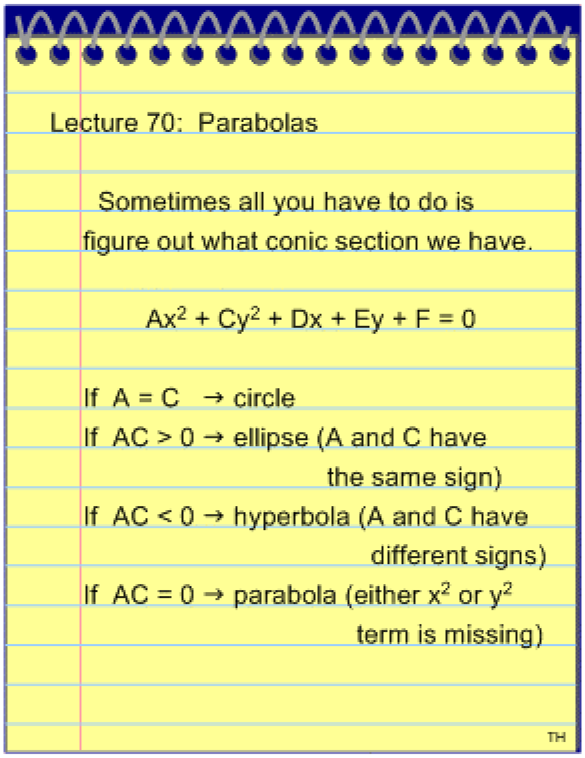
Lecture 69: Page 14

$$\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$$

- Opens up/down
- C = (h,k)
- Distance from C to the vertices = a
- Slopes of asymptotes: $m = \pm \frac{a}{b}$

Lecture 70 Notes

ALG2070-01



Lecture 70: Parabolas

Sometimes all you have to do is figure out what conic section we have.

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

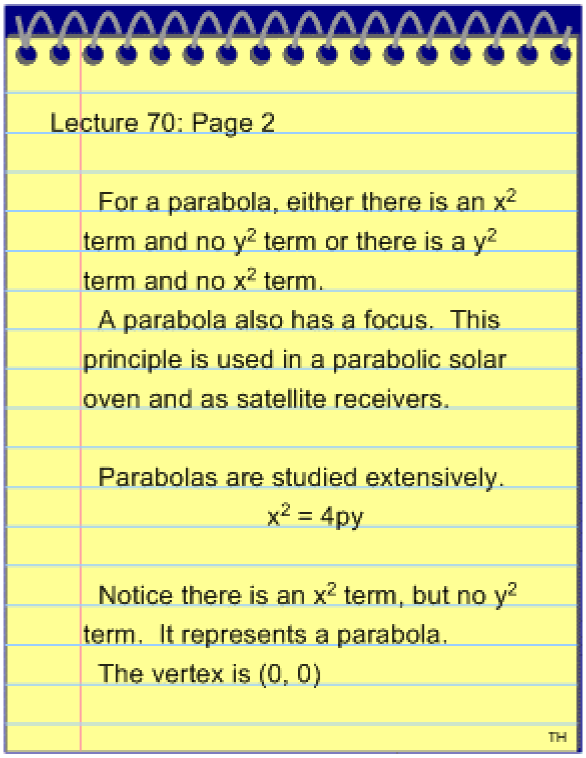
If $A = C$ → circle

If $AC > 0$ → ellipse (A and C have the same sign)

If $AC < 0$ → hyperbola (A and C have different signs)

If $AC = 0$ → parabola (either x^2 or y^2 term is missing)

ALG2070-02



Lecture 70: Page 2

For a parabola, either there is an x^2 term and no y^2 term or there is a y^2 term and no x^2 term.

A parabola also has a focus. This principle is used in a parabolic solar oven and as satellite receivers.

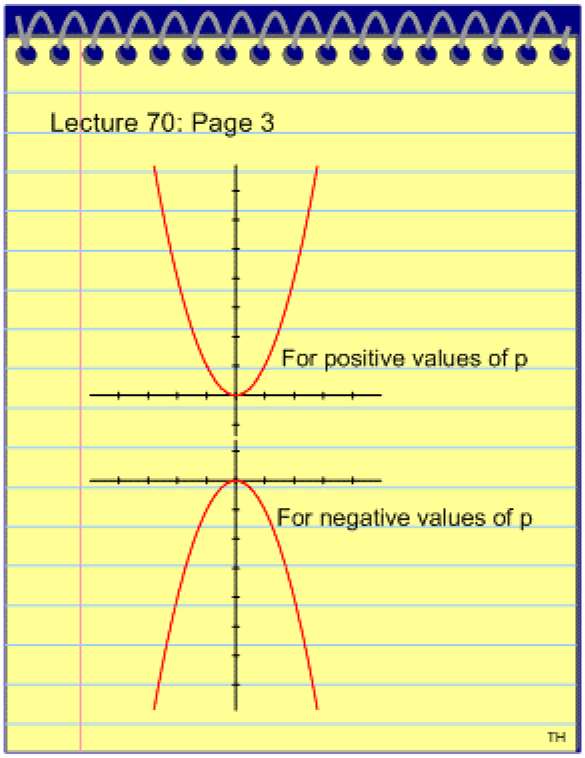
Parabolas are studied extensively.

$$x^2 = 4py$$

Notice there is an x^2 term, but no y^2 term. It represents a parabola.

The vertex is (0, 0)

ALG2070-03

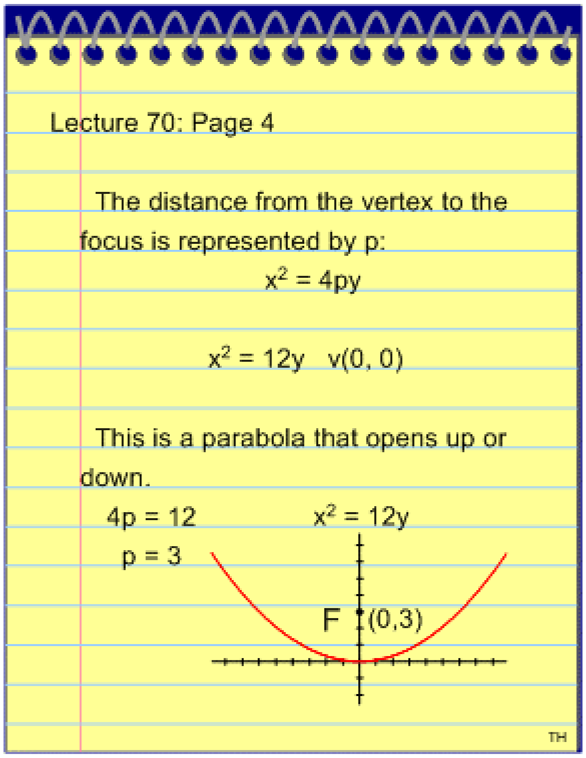


Lecture 70: Page 3

For positive values of p

For negative values of p

ALG2070-04



Lecture 70: Page 4

The distance from the vertex to the focus is represented by p:

$$x^2 = 4py$$

$$x^2 = 12y \quad v(0, 0)$$

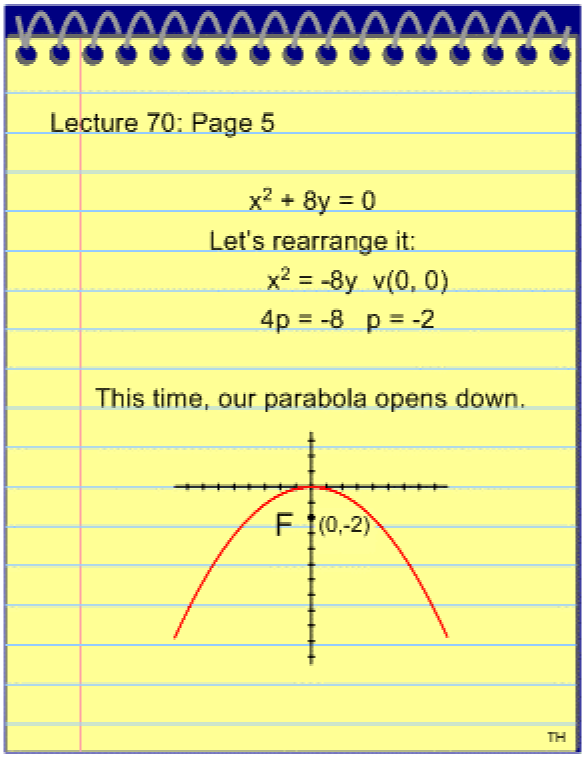
This is a parabola that opens up or down.

$4p = 12$

$p = 3$

Lecture 70 Notes, Continued

ALG2070-05



Lecture 70: Page 5

$x^2 + 8y = 0$

Let's rearrange it:

$x^2 = -8y$ v(0, 0)

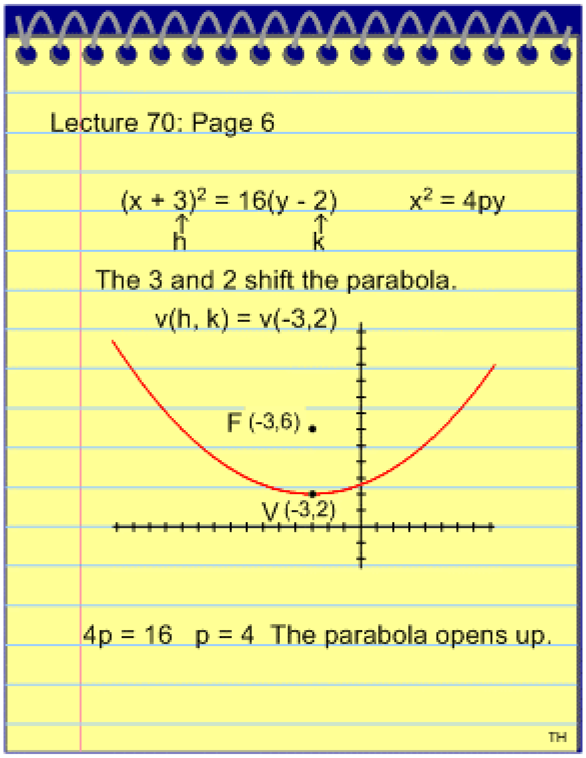
$4p = -8$ $p = -2$

This time, our parabola opens down.

F (0,-2)

TH

ALG2070-06



Lecture 70: Page 6

$(x + \underset{\overset{\uparrow}{h}}{3})^2 = 16(y - \underset{\overset{\uparrow}{k}}{2})$ $x^2 = 4py$

The 3 and 2 shift the parabola.

v(h, k) = v(-3,2)

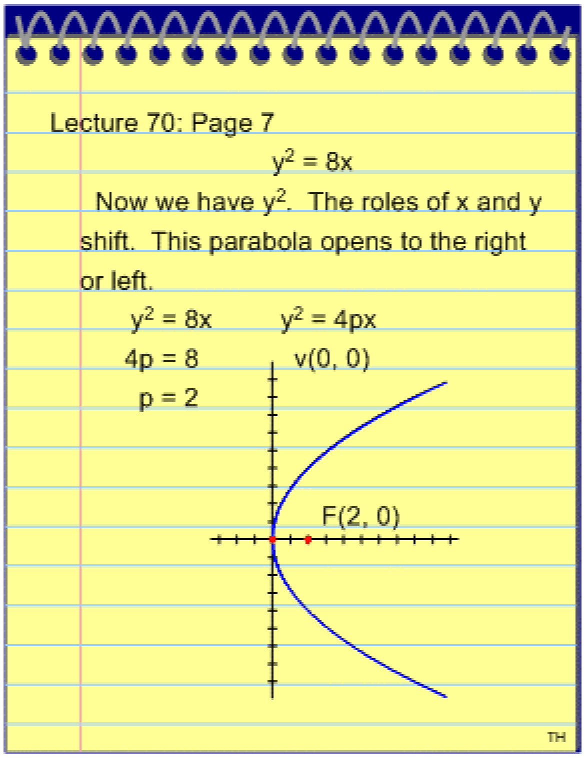
F (-3,6)

V (-3,2)

$4p = 16$ $p = 4$ The parabola opens up.

TH

ALG2070-07



Lecture 70: Page 7

$y^2 = 8x$

Now we have y^2. The roles of x and y shift. This parabola opens to the right or left.

$y^2 = 8x$ $y^2 = 4px$

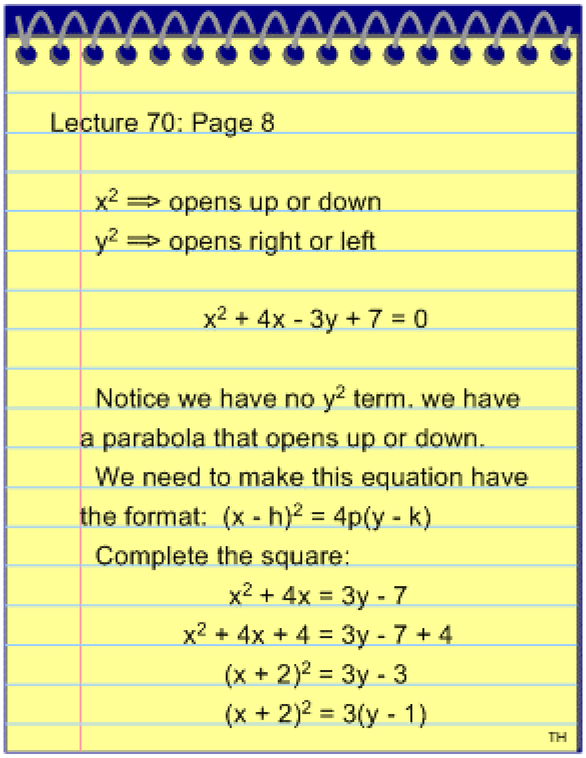
$4p = 8$ v(0, 0)

$p = 2$

F(2, 0)

TH

ALG2070-08



Lecture 70: Page 8

$x^2 \Longrightarrow$ opens up or down

$y^2 \Longrightarrow$ opens right or left

$x^2 + 4x - 3y + 7 = 0$

Notice we have no y^2 term, we have a parabola that opens up or down.

We need to make this equation have the format: $(x - h)^2 = 4p(y - k)$

Complete the square:

$x^2 + 4x = 3y - 7$

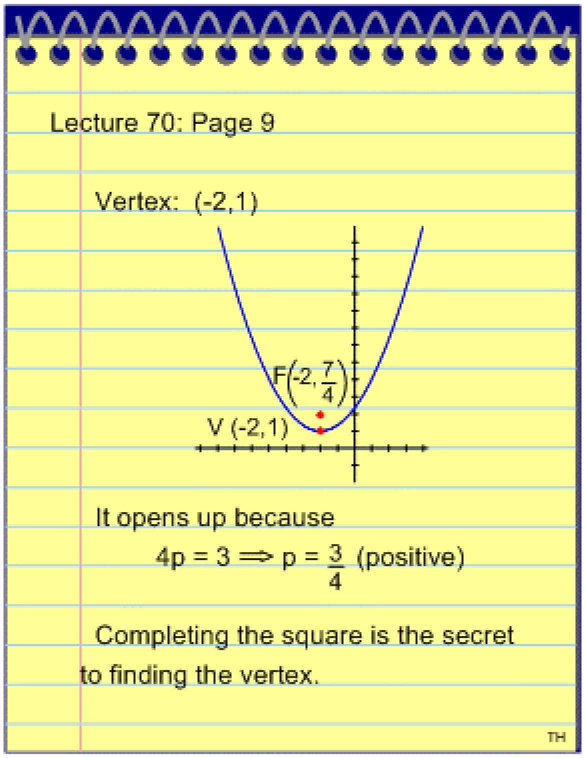
$x^2 + 4x + 4 = 3y - 7 + 4$

$(x + 2)^2 = 3y - 3$

$(x + 2)^2 = 3(y - 1)$

TH

ALG2070-09



Lecture 70: Page 9

Vertex: (-2,1)

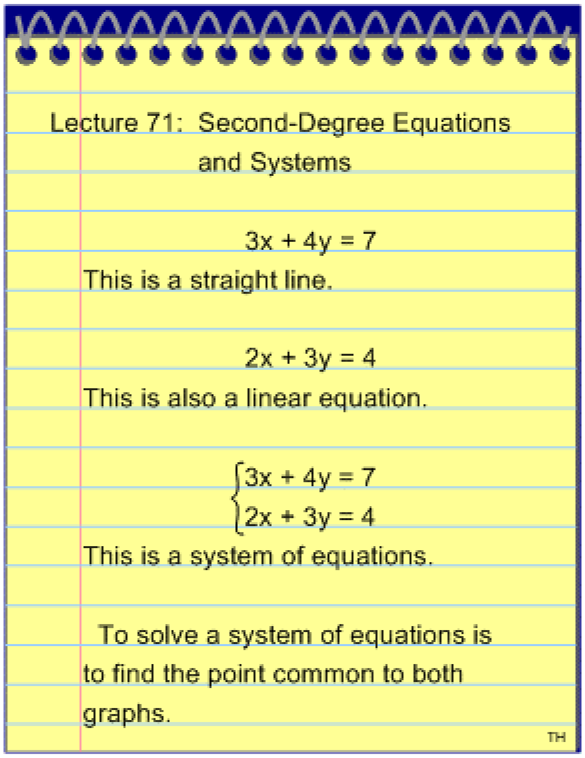
It opens up because

$$4p = 3 \Longrightarrow p = \frac{3}{4} \text{ (positive)}$$

Completing the square is the secret to finding the vertex.

Lecture 71 Notes

ALG2071-01



Lecture 71: Second-Degree Equations and Systems

$3x + 4y = 7$

This is a straight line.

$2x + 3y = 4$

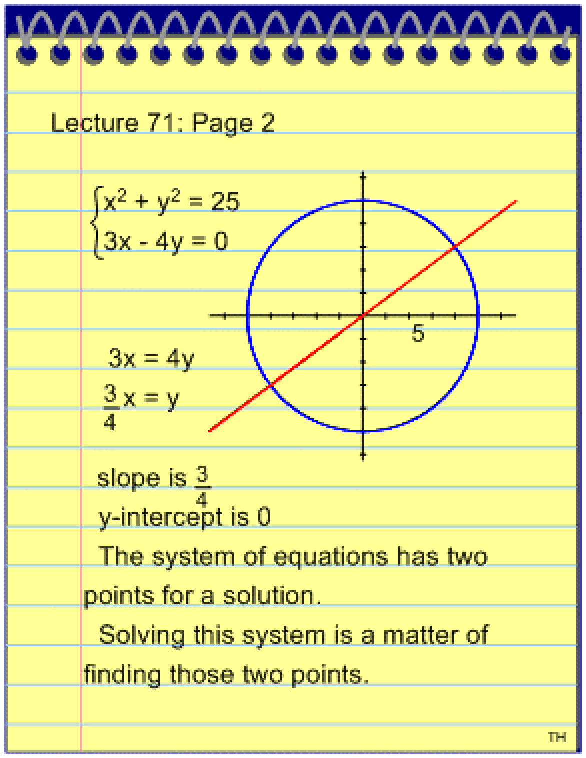
This is also a linear equation.

$$\begin{cases} 3x + 4y = 7 \\ 2x + 3y = 4 \end{cases}$$

This is a system of equations.

To solve a system of equations is to find the point common to both graphs.

ALG2071-02



Lecture 71: Page 2

$$\begin{cases} x^2 + y^2 = 25 \\ 3x - 4y = 0 \end{cases}$$

$3x = 4y$

$\frac{3}{4}x = y$

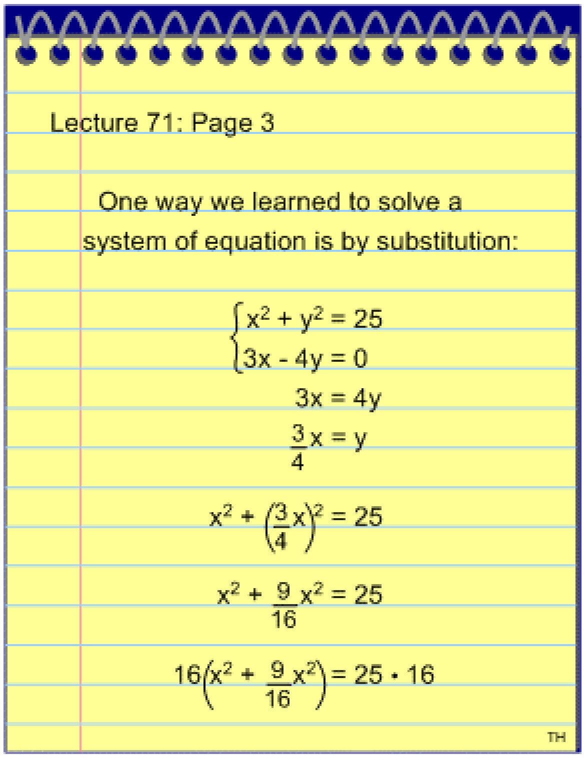
slope is $\frac{3}{4}$

y-intercept is 0

The system of equations has two points for a solution.

Solving this system is a matter of finding those two points.

ALG2071-03



Lecture 71: Page 3

One way we learned to solve a system of equation is by substitution:

$$\begin{cases} x^2 + y^2 = 25 \\ 3x - 4y = 0 \end{cases}$$

$$3x = 4y$$

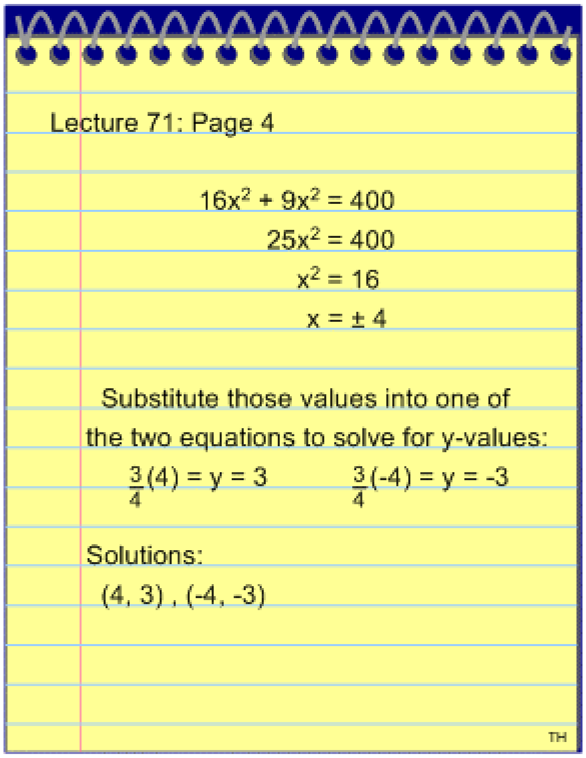
$$\frac{3}{4}x = y$$

$$x^2 + \left(\frac{3}{4}x\right)^2 = 25$$

$$x^2 + \frac{9}{16}x^2 = 25$$

$$16\left(x^2 + \frac{9}{16}x^2\right) = 25 \cdot 16$$

ALG2071-04



Lecture 71: Page 4

$$16x^2 + 9x^2 = 400$$

$$25x^2 = 400$$

$$x^2 = 16$$

$$x = \pm 4$$

Substitute those values into one of the two equations to solve for y-values:

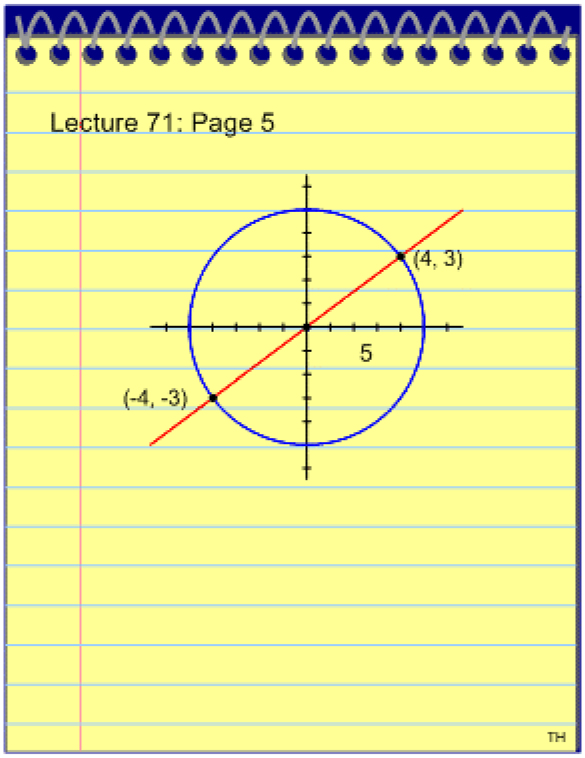
$\frac{3}{4}(4) = y = 3$ $\qquad$ $\frac{3}{4}(-4) = y = -3$

Solutions:

(4, 3) , (-4, -3)

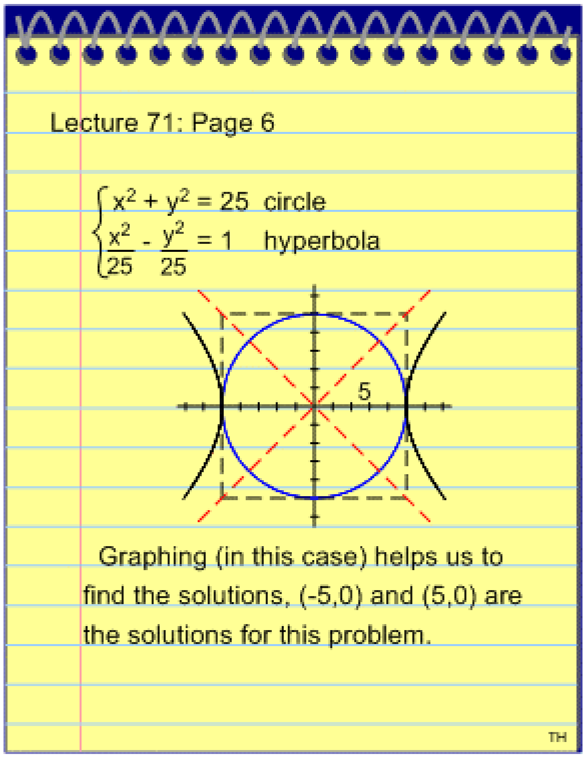
Lecture 71 Notes, Continued

ALG2071-05



Lecture 71: Page 5

ALG2071-06

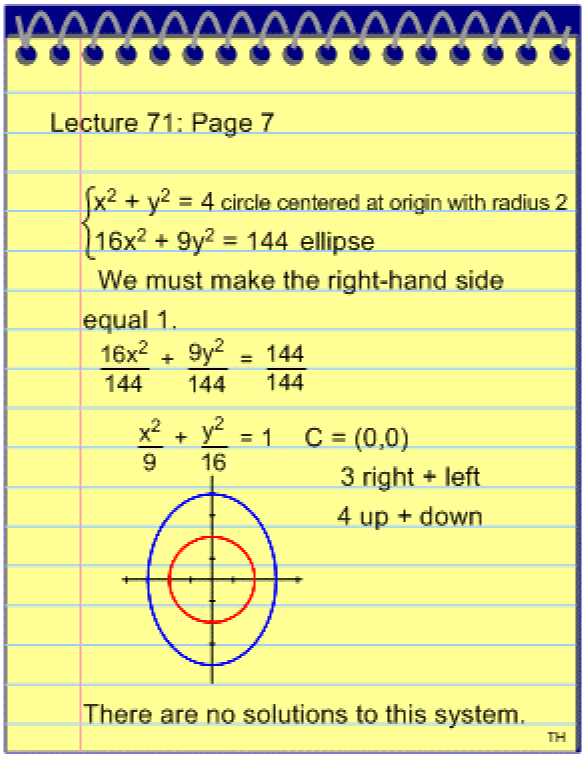


Lecture 71: Page 6

$$\begin{cases} x^2 + y^2 = 25 & \text{circle} \\ \dfrac{x^2}{25} - \dfrac{y^2}{25} = 1 & \text{hyperbola} \end{cases}$$

Graphing (in this case) helps us to find the solutions, (-5,0) and (5,0) are the solutions for this problem.

ALG2071-07



Lecture 71: Page 7

$$\begin{cases} x^2 + y^2 = 4 & \text{circle centered at origin with radius 2} \\ 16x^2 + 9y^2 = 144 & \text{ellipse} \end{cases}$$

We must make the right-hand side equal 1.

$$\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144}$$

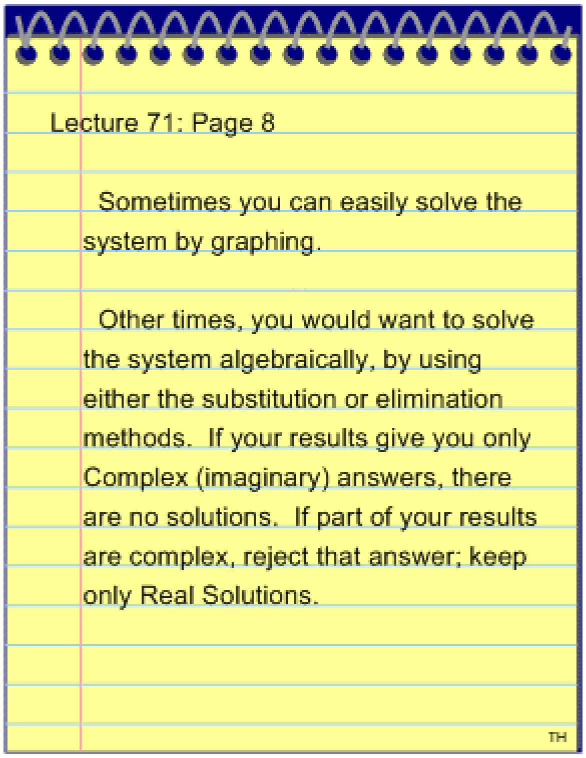
$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \quad C = (0,0)$$

3 right + left

4 up + down

There are no solutions to this system.

ALG2071-08



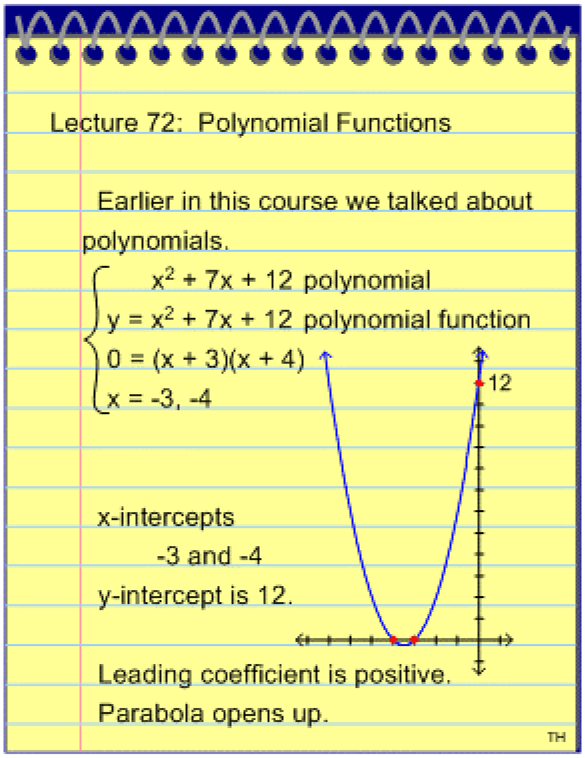
Lecture 71: Page 8

Sometimes you can easily solve the system by graphing.

Other times, you would want to solve the system algebraically, by using either the substitution or elimination methods. If your results give you only Complex (imaginary) answers, there are no solutions. If part of your results are complex, reject that answer; keep only Real Solutions.

Lecture 72 Notes

ALG2072-01



Lecture 72: Polynomial Functions

Earlier in this course we talked about polynomials.

$x^2 + 7x + 12$ polynomial

$y = x^2 + 7x + 12$ polynomial function

$0 = (x + 3)(x + 4)$

$x = -3, -4$

x-intercepts

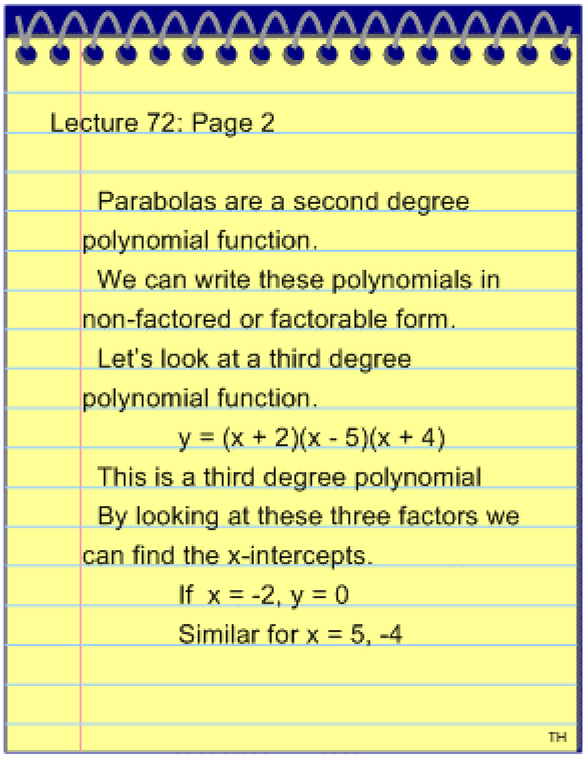
-3 and -4

y-intercept is 12.

Leading coefficient is positive.

Parabola opens up.

ALG2072-02



Lecture 72: Page 2

Parabolas are a second degree polynomial function.

We can write these polynomials in non-factored or factorable form.

Let's look at a third degree polynomial function.

$y = (x + 2)(x - 5)(x + 4)$

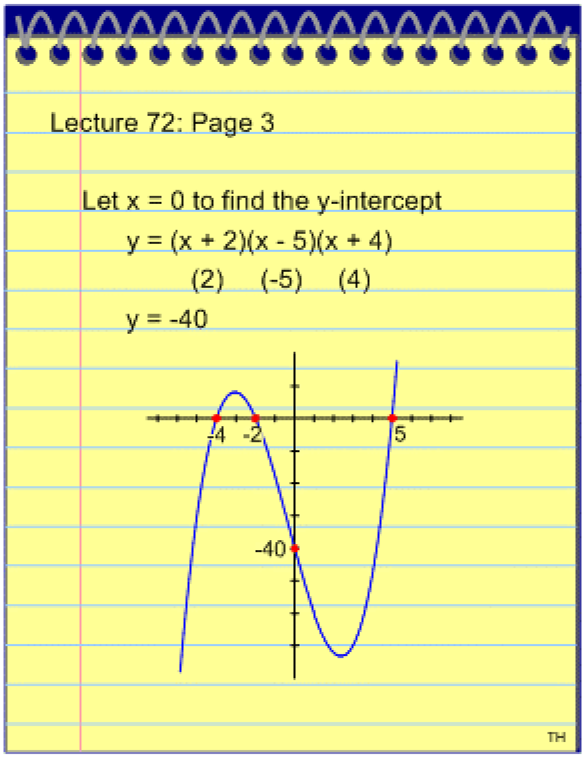
This is a third degree polynomial

By looking at these three factors we can find the x-intercepts.

If $x = -2$, $y = 0$

Similar for $x = 5, -4$

ALG2072-03



Lecture 72: Page 3

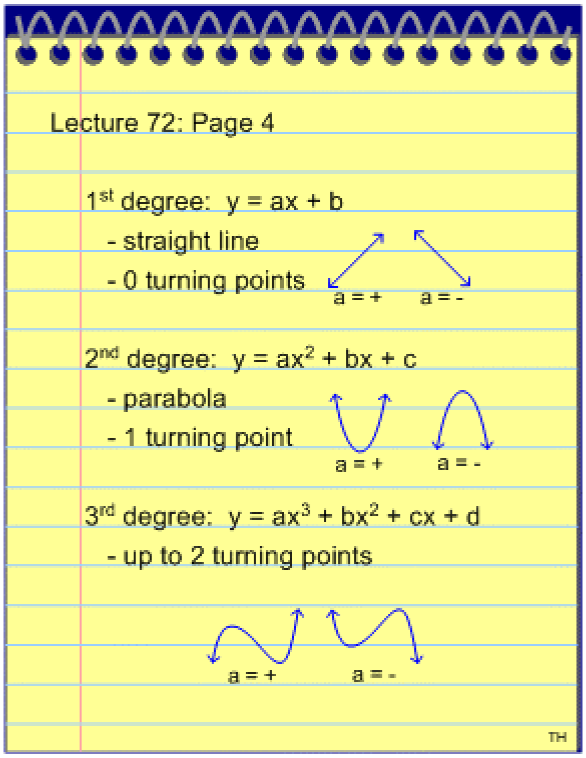
Let x = 0 to find the y-intercept

$y = (x + 2)(x - 5)(x + 4)$

(2) (-5) (4)

$y = -40$

ALG2072-04



Lecture 72: Page 4

1st degree: $y = ax + b$

- straight line
- 0 turning points

2nd degree: $y = ax^2 + bx + c$

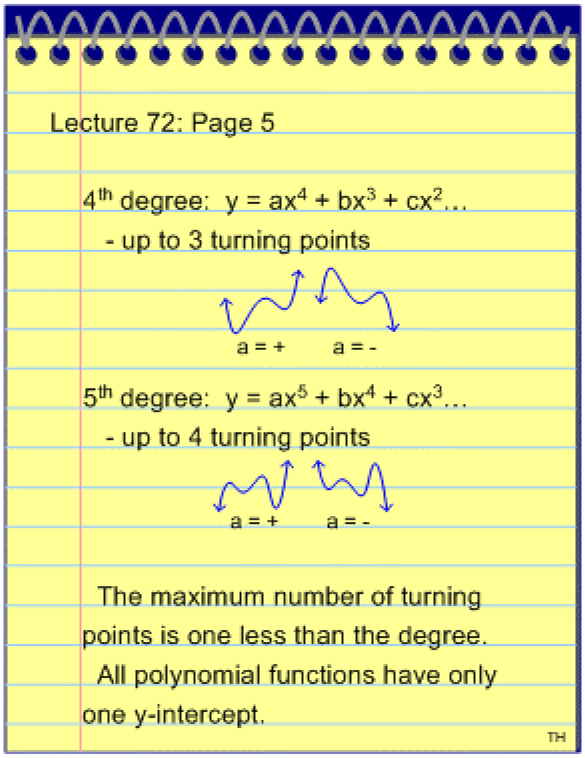
- parabola
- 1 turning point

3rd degree: $y = ax^3 + bx^2 + cx + d$

- up to 2 turning points

Lecture 72 Notes, Continued

ALG2072-05



Lecture 72: Page 5

4th degree: $y = ax^4 + bx^3 + cx^2 \ldots$

- up to 3 turning points

5th degree: $y = ax^5 + bx^4 + cx^3 \ldots$

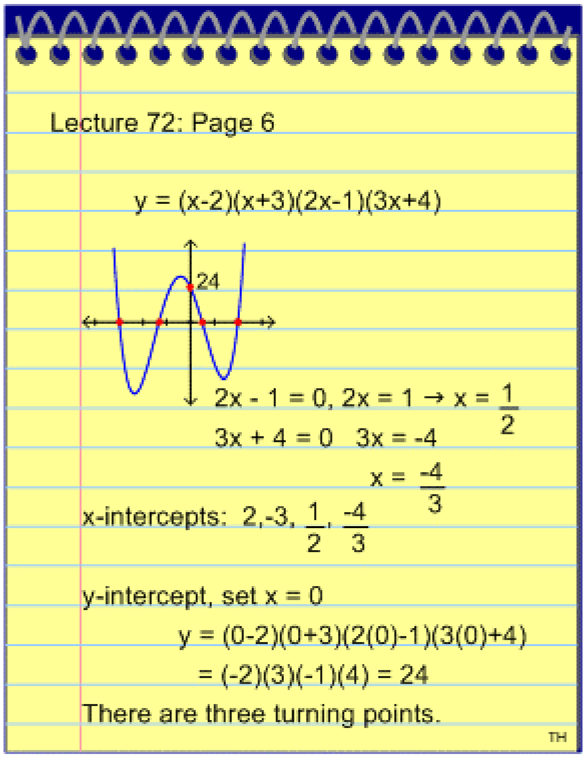
- up to 4 turning points

The maximum number of turning points is one less than the degree.

All polynomial functions have only one y-intercept.

TH

ALG2072-06



Lecture 72: Page 6

$y = (x-2)(x+3)(2x-1)(3x+4)$

$2x - 1 = 0,\ 2x = 1 \rightarrow x = \frac{1}{2}$

$3x + 4 = 0 \quad 3x = -4$

$x = \frac{-4}{3}$

x-intercepts: $2, -3, \frac{1}{2}, \frac{-4}{3}$

y-intercept, set x = 0

$y = (0-2)(0+3)(2(0)-1)(3(0)+4)$

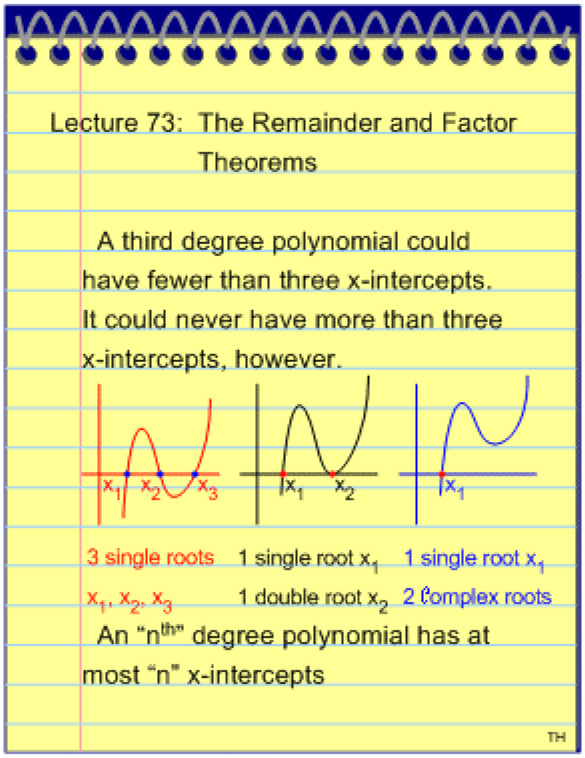
$= (-2)(3)(-1)(4) = 24$

There are three turning points.

TH

Lecture 73 Notes

ALG2073-01



Lecture 73: The Remainder and Factor Theorems

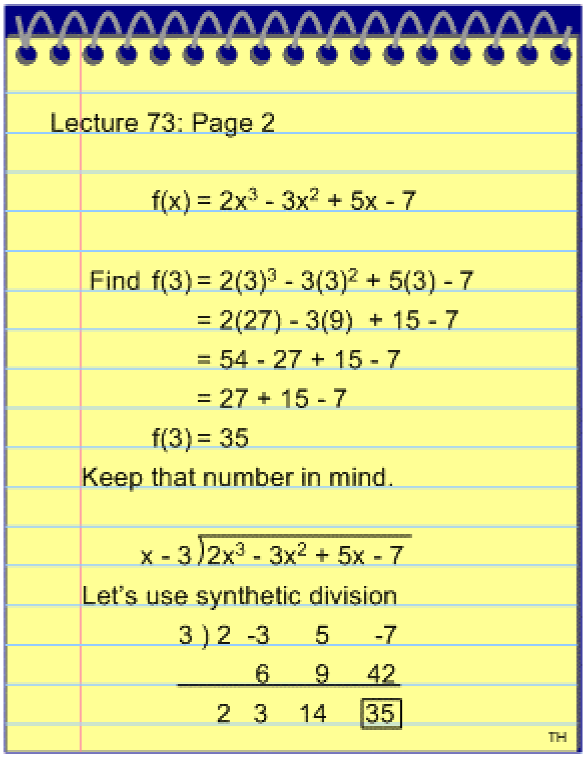
A third degree polynomial could have fewer than three x-intercepts. It could never have more than three x-intercepts, however.

3 single roots x_1, x_2, x_3	1 single root x_1 1 double root x_2	1 single root x_1 2 complex roots

An "nth" degree polynomial has at most "n" x-intercepts

TH

ALG2073-02



Lecture 73: Page 2

$f(x) = 2x^3 - 3x^2 + 5x - 7$

Find $f(3) = 2(3)^3 - 3(3)^2 + 5(3) - 7$

$= 2(27) - 3(9) + 15 - 7$

$= 54 - 27 + 15 - 7$

$= 27 + 15 - 7$

$f(3) = 35$

Keep that number in mind.

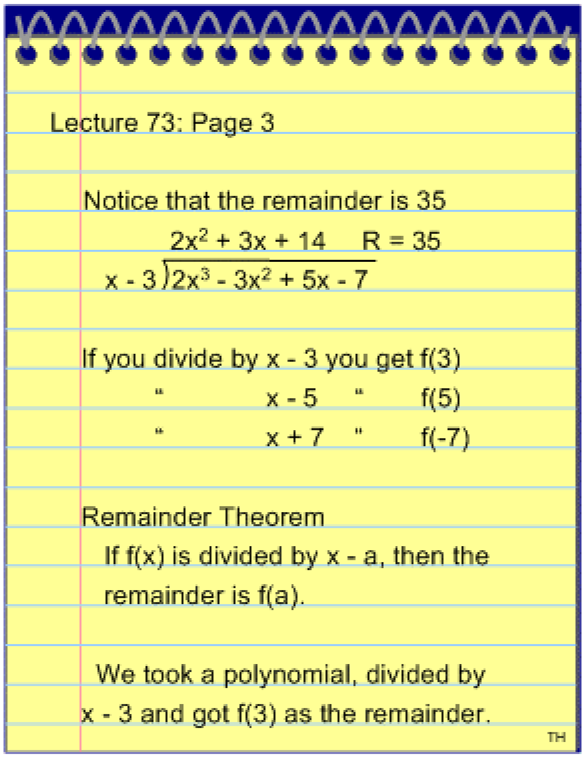
$x - 3\overline{)2x^3 - 3x^2 + 5x - 7}$

Let's use synthetic division

3)	2	-3	5	-7
		6	9	42
	2	3	14	$\boxed{35}$

TH

ALG2073-03



Lecture 73: Page 3

Notice that the remainder is 35

$$\begin{array}{l} \quad\ 2x^2 + 3x + 14 \quad R = 35 \\ x - 3\overline{)2x^3 - 3x^2 + 5x - 7} \end{array}$$

If you divide by x - 3 you get f(3)

" x - 5 " f(5)

" x + 7 " f(-7)

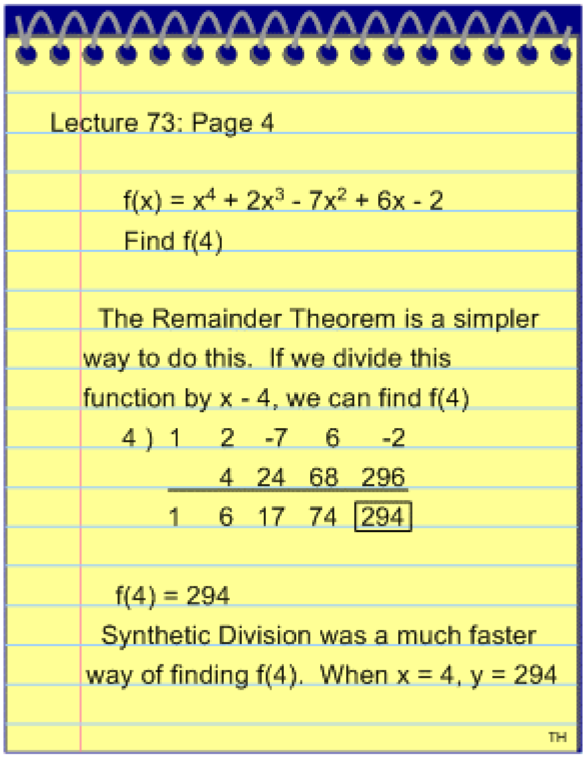
Remainder Theorem

If f(x) is divided by x - a, then the remainder is f(a).

We took a polynomial, divided by x - 3 and got f(3) as the remainder.

TH

ALG2073-04



Lecture 73: Page 4

$f(x) = x^4 + 2x^3 - 7x^2 + 6x - 2$

Find f(4)

The Remainder Theorem is a simpler way to do this. If we divide this function by x - 4, we can find f(4)

4)	1	2	-7	6	-2
		4	24	68	296
	1	6	17	74	$\boxed{294}$

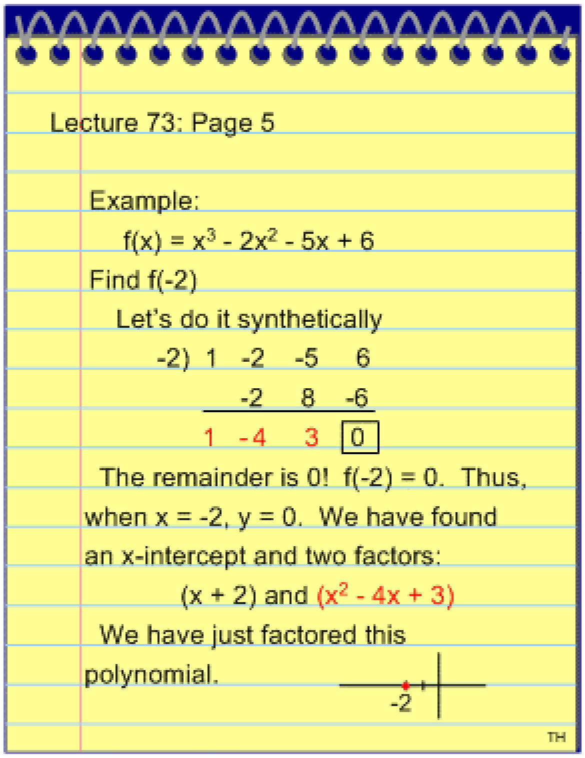
f(4) = 294

Synthetic Division was a much faster way of finding f(4). When x = 4, y = 294

TH

Lecture 73 Notes, Continued

ALG2073-05



Lecture 73: Page 5

Example:

$f(x) = x^3 - 2x^2 - 5x + 6$

Find f(-2)

Let's do it synthetically

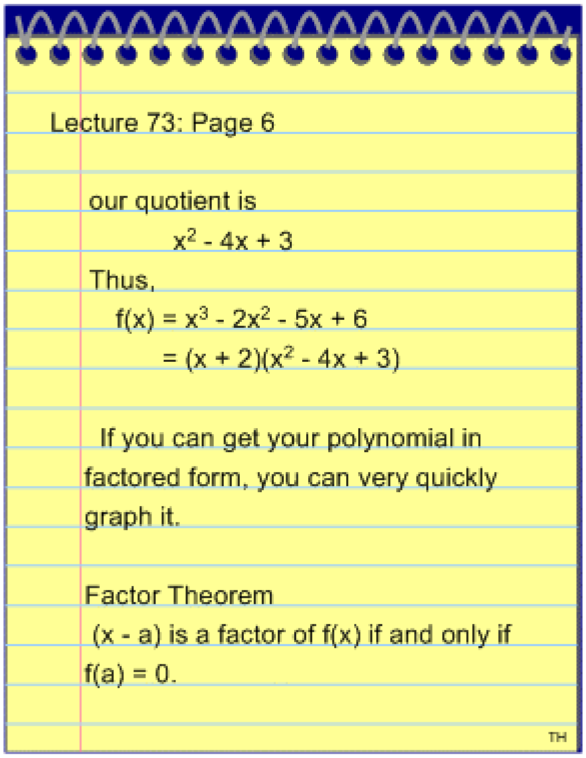
$$\begin{array}{r|rrrr} -2) & 1 & -2 & -5 & 6 \\ & & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array}$$

The remainder is 0! f(-2) = 0. Thus, when x = -2, y = 0. We have found an x-intercept and two factors:

$(x + 2)$ and $(x^2 - 4x + 3)$

We have just factored this polynomial.

ALG2073-06



Lecture 73: Page 6

our quotient is

$x^2 - 4x + 3$

Thus,

$f(x) = x^3 - 2x^2 - 5x + 6$

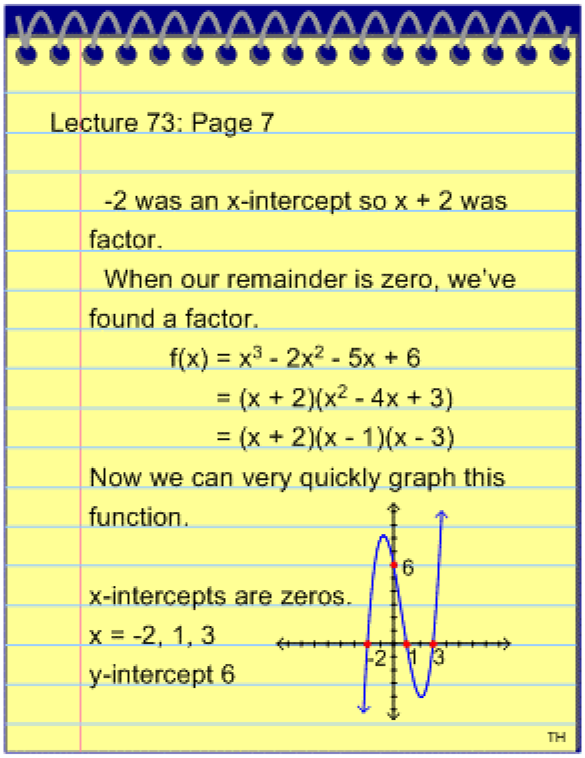
$= (x + 2)(x^2 - 4x + 3)$

If you can get your polynomial in factored form, you can very quickly graph it.

Factor Theorem

(x - a) is a factor of f(x) if and only if f(a) = 0.

ALG2073-07



Lecture 73: Page 7

-2 was an x-intercept so x + 2 was factor.

When our remainder is zero, we've found a factor.

$f(x) = x^3 - 2x^2 - 5x + 6$

$= (x + 2)(x^2 - 4x + 3)$

$= (x + 2)(x - 1)(x - 3)$

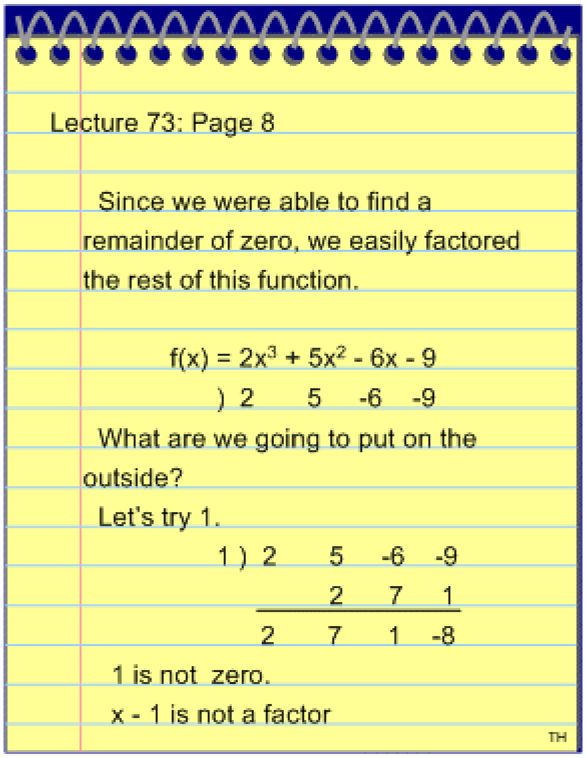
Now we can very quickly graph this function.

x-intercepts are zeros.

x = -2, 1, 3

y-intercept 6

ALG2073-08



Lecture 73: Page 8

Since we were able to find a remainder of zero, we easily factored the rest of this function.

$f(x) = 2x^3 + 5x^2 - 6x - 9$

$$\begin{array}{r|rrrr}) & 2 & 5 & -6 & -9 \end{array}$$

What are we going to put on the outside?

Let's try 1.

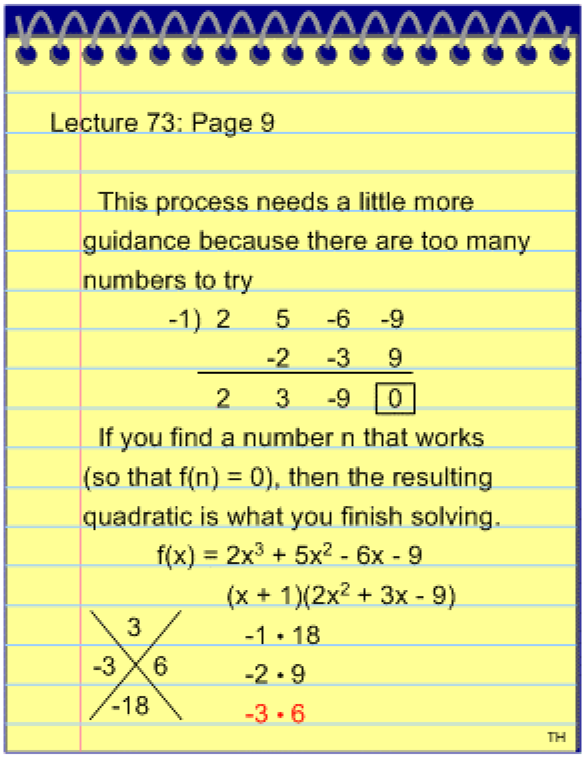
$$\begin{array}{r|rrrr} 1\ ) & 2 & 5 & -6 & -9 \\ & & 2 & 7 & 1 \\ \hline & 2 & 7 & 1 & -8 \end{array}$$

1 is not zero.

x - 1 is not a factor

Lecture 73 Notes, Continued

ALG2073-09



Lecture 73: Page 9

This process needs a little more guidance because there are too many numbers to try

$$\begin{array}{r|rrrr} -1) & 2 & 5 & -6 & -9 \\ & & -2 & -3 & 9 \\ \hline & 2 & 3 & -9 & \boxed{0} \end{array}$$

If you find a number n that works (so that f(n) = 0), then the resulting quadratic is what you finish solving.

$f(x) = 2x^3 + 5x^2 - 6x - 9$

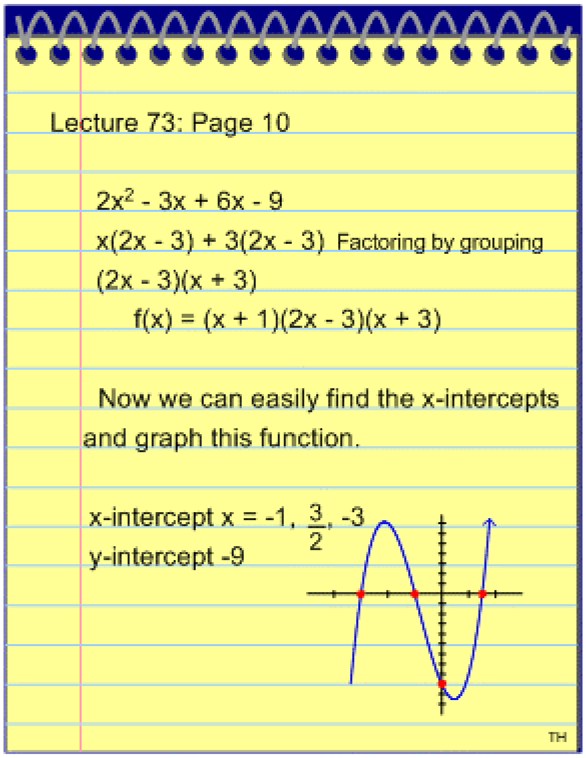
$(x + 1)(2x^2 + 3x - 9)$

$-1 \cdot 18$

$-2 \cdot 9$

$-3 \cdot 6$

ALG2073-10



Lecture 73: Page 10

$2x^2 - 3x + 6x - 9$

$x(2x - 3) + 3(2x - 3)$ Factoring by grouping

$(2x - 3)(x + 3)$

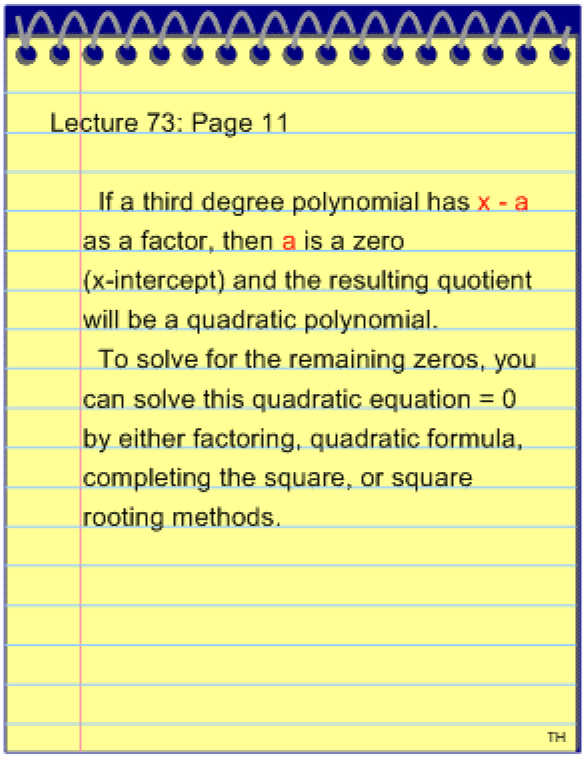
$f(x) = (x + 1)(2x - 3)(x + 3)$

Now we can easily find the x-intercepts and graph this function.

x-intercept $x = -1, \frac{3}{2}, -3$

y-intercept -9

ALG2073-11



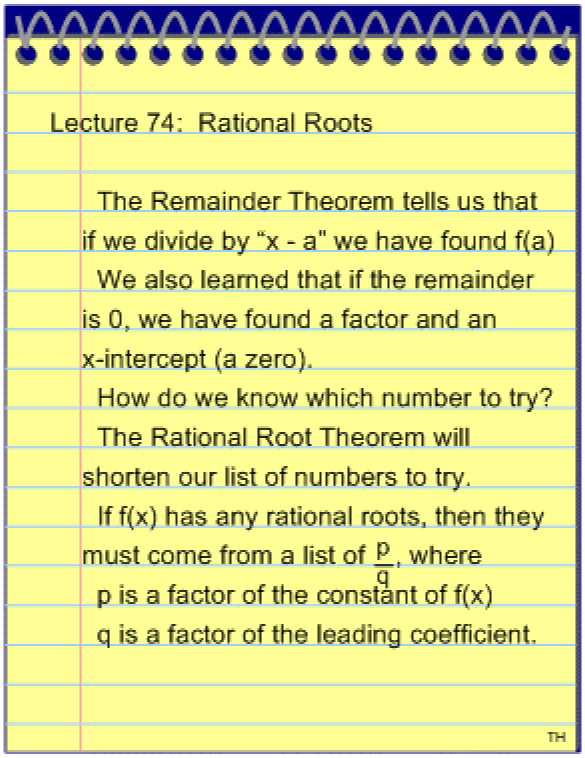
Lecture 73: Page 11

If a third degree polynomial has x - a as a factor, then a is a zero (x-intercept) and the resulting quotient will be a quadratic polynomial.

To solve for the remaining zeros, you can solve this quadratic equation = 0 by either factoring, quadratic formula, completing the square, or square rooting methods.

Lecture 74 Notes

ALG2074-01



Lecture 74: Rational Roots

The Remainder Theorem tells us that if we divide by “x - a" we have found f(a)

We also learned that if the remainder is 0, we have found a factor and an x-intercept (a zero).

How do we know which number to try?

The Rational Root Theorem will shorten our list of numbers to try.

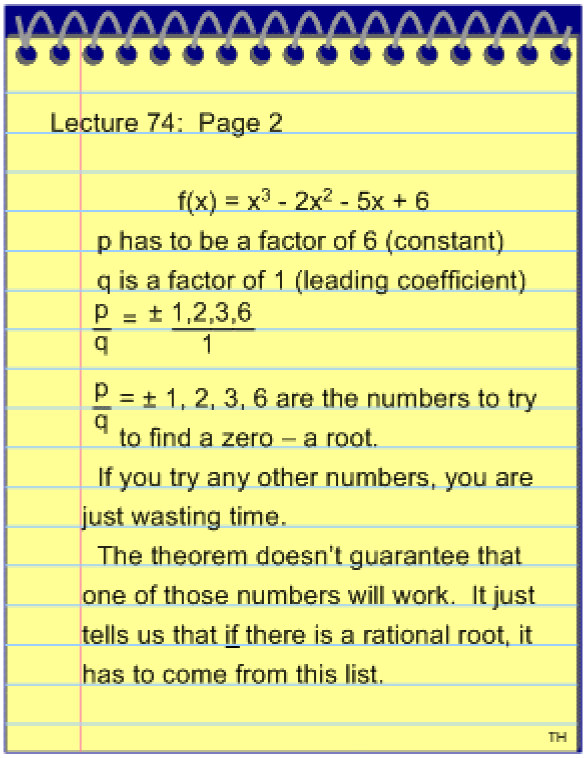
If f(x) has any rational roots, then they must come from a list of $\frac{p}{q}$, where

p is a factor of the constant of f(x)

q is a factor of the leading coefficient.

TH

ALG2074-02



Lecture 74: Page 2

$f(x) = x^3 - 2x^2 - 5x + 6$

p has to be a factor of 6 (constant)

q is a factor of 1 (leading coefficient)

$\frac{p}{q} = \pm\frac{1,2,3,6}{1}$

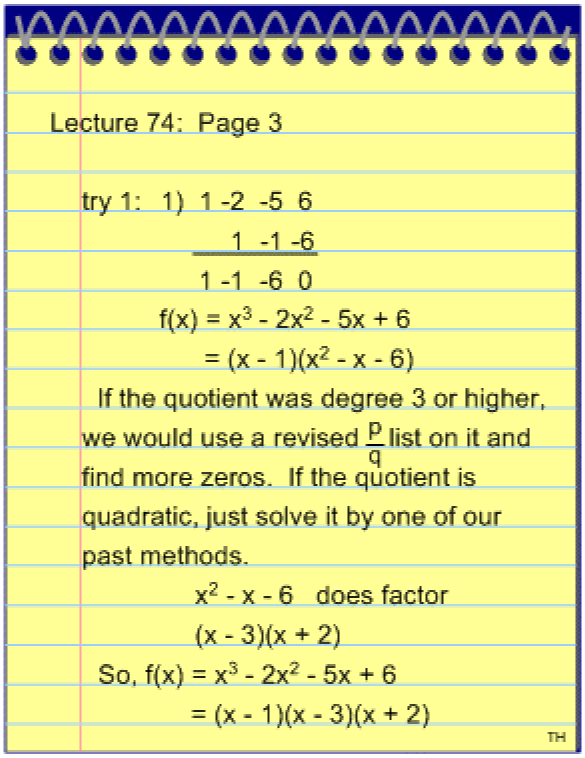
$\frac{p}{q}$ = ± 1, 2, 3, 6 are the numbers to try to find a zero – a root.

If you try any other numbers, you are just wasting time.

The theorem doesn't guarantee that one of those numbers will work. It just tells us that if there is a rational root, it has to come from this list.

TH

ALG2074-03



Lecture 74: Page 3

try 1: 1) 1 -2 -5 6
1 -1 -6
1 -1 -6 0

$f(x) = x^3 - 2x^2 - 5x + 6$
$= (x - 1)(x^2 - x - 6)$

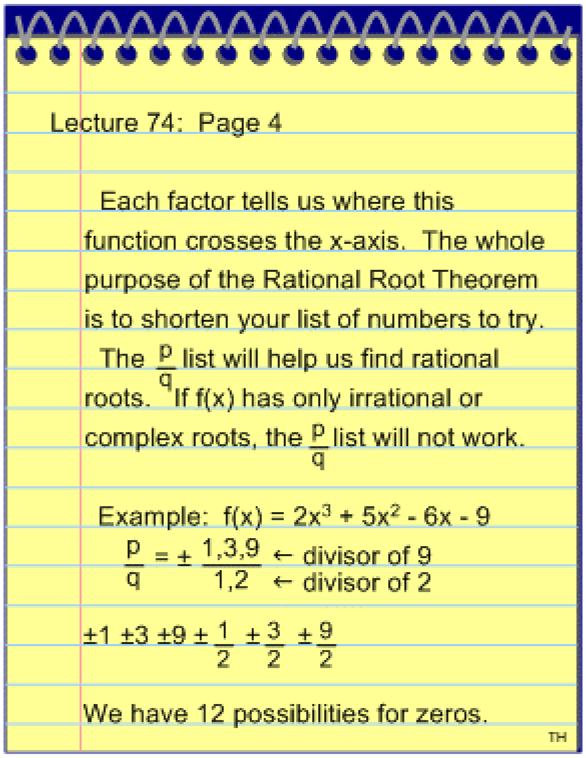
If the quotient was degree 3 or higher, we would use a revised $\frac{p}{q}$ list on it and find more zeros. If the quotient is quadratic, just solve it by one of our past methods.

$x^2 - x - 6$ does factor
$(x - 3)(x + 2)$

So, $f(x) = x^3 - 2x^2 - 5x + 6$
$= (x - 1)(x - 3)(x + 2)$

TH

ALG2074-04



Lecture 74: Page 4

Each factor tells us where this function crosses the x-axis. The whole purpose of the Rational Root Theorem is to shorten your list of numbers to try.

The $\frac{p}{q}$ list will help us find rational roots. If f(x) has only irrational or complex roots, the $\frac{p}{q}$ list will not work.

Example: $f(x) = 2x^3 + 5x^2 - 6x - 9$

$\frac{p}{q} = \pm\frac{1,3,9}{1,2}$ ← divisor of 9
← divisor of 2

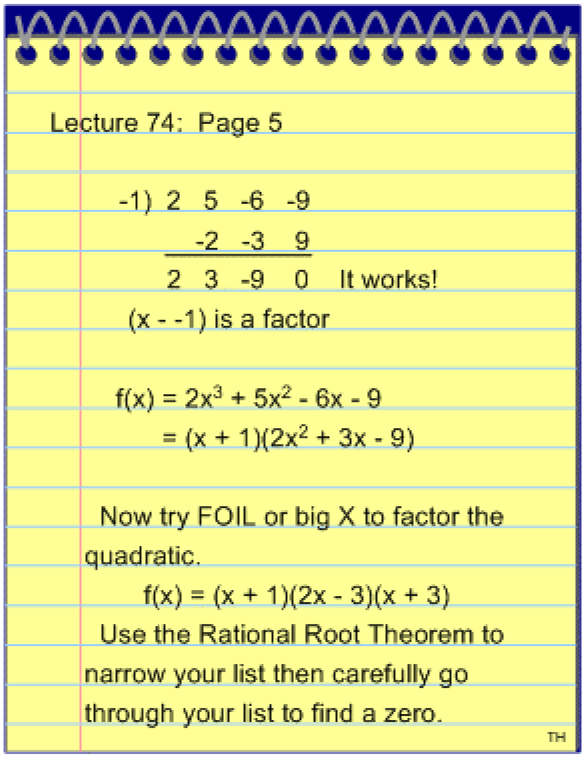
$\pm1 \ \pm3 \ \pm9 \ \pm\frac{1}{2} \ \pm\frac{3}{2} \ \pm\frac{9}{2}$

We have 12 possibilities for zeros.

TH

Lecture 74 Notes, Continued

ALG2074-05



Lecture 74: Page 5

-1) 2 5 -6 -9
-2 -3 9
2 3 -9 0 It works!

(x - -1) is a factor

$f(x) = 2x^3 + 5x^2 - 6x - 9$
$= (x + 1)(2x^2 + 3x - 9)$

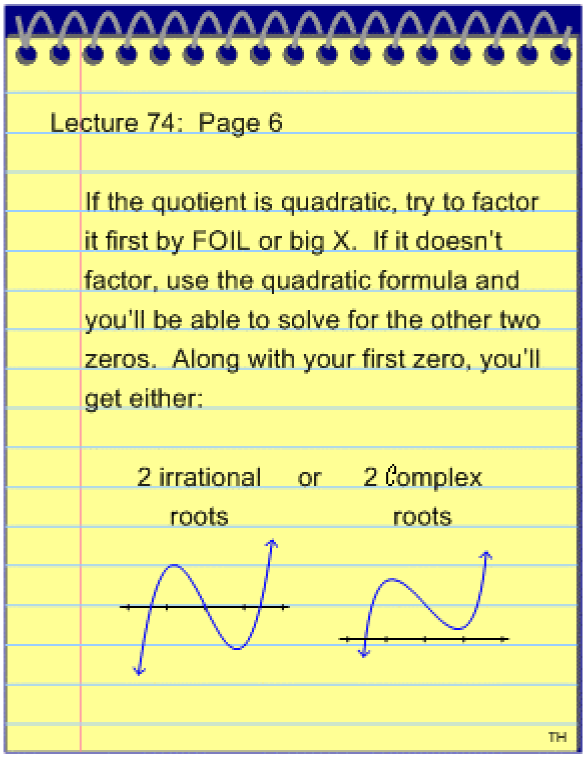
Now try FOIL or big X to factor the quadratic.

$f(x) = (x + 1)(2x - 3)(x + 3)$

Use the Rational Root Theorem to narrow your list then carefully go through your list to find a zero.

TH

ALG2074-06



Lecture 74: Page 6

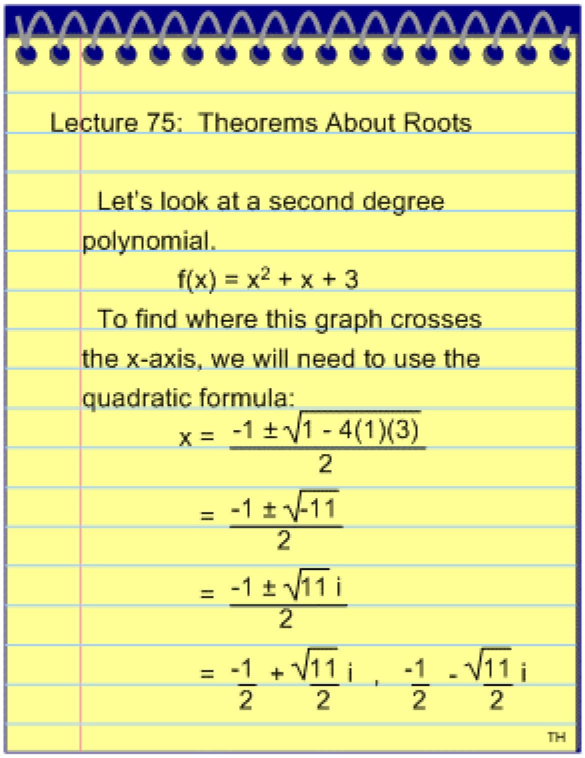
If the quotient is quadratic, try to factor it first by FOIL or big X. If it doesn't factor, use the quadratic formula and you'll be able to solve for the other two zeros. Along with your first zero, you'll get either:

2 irrational roots or 2 complex roots

TH

Lecture 75 Notes

ALG2075-01



Lecture 75: Theorems About Roots

Let's look at a second degree polynomial.

$f(x) = x^2 + x + 3$

To find where this graph crosses the x-axis, we will need to use the quadratic formula:

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(3)}}{2}$$

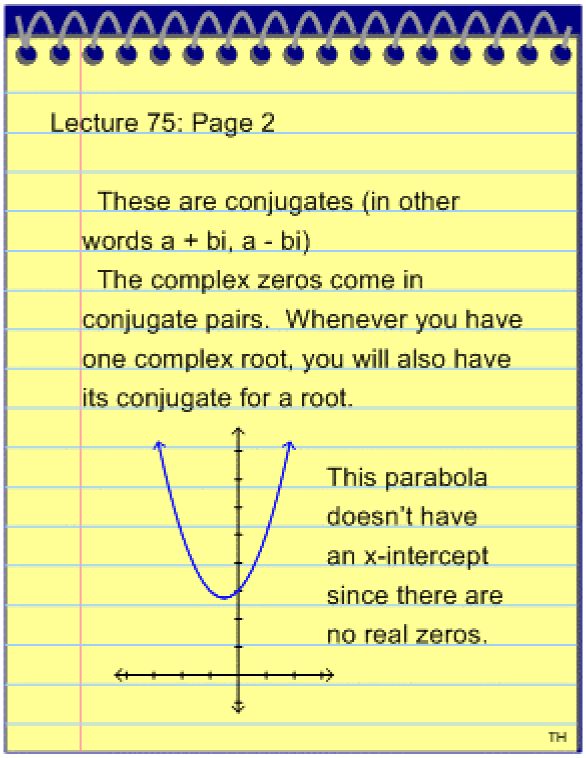
$$= \frac{-1 \pm \sqrt{-11}}{2}$$

$$= \frac{-1 \pm \sqrt{11}\, i}{2}$$

$$= \frac{-1}{2} + \frac{\sqrt{11}\, i}{2} \;,\; \frac{-1}{2} - \frac{\sqrt{11}\, i}{2}$$

TH

ALG2075-02



Lecture 75: Page 2

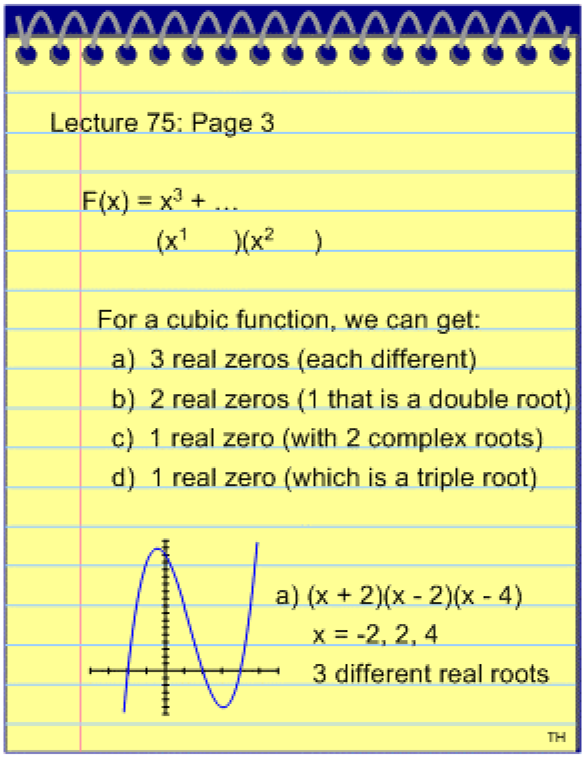
These are conjugates (in other words a + bi, a - bi)

The complex zeros come in conjugate pairs. Whenever you have one complex root, you will also have its conjugate for a root.

This parabola doesn't have an x-intercept since there are no real zeros.

TH

ALG2075-03



Lecture 75: Page 3

$F(x) = x^3 + \ldots$

$(x^1 \quad)(x^2 \quad)$

For a cubic function, we can get:

a) 3 real zeros (each different)

b) 2 real zeros (1 that is a double root)

c) 1 real zero (with 2 complex roots)

d) 1 real zero (which is a triple root)

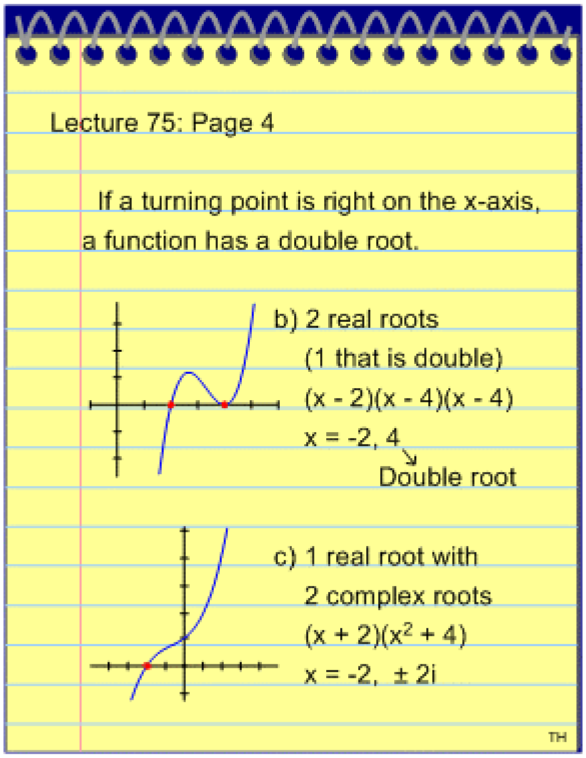
a) $(x + 2)(x - 2)(x - 4)$

$x = -2, 2, 4$

3 different real roots

TH

ALG2075-04



Lecture 75: Page 4

If a turning point is right on the x-axis, a function has a double root.

b) 2 real roots

(1 that is double)

$(x - 2)(x - 4)(x - 4)$

$x = -2, \underset{\searrow}{4}$

Double root

c) 1 real root with

2 complex roots

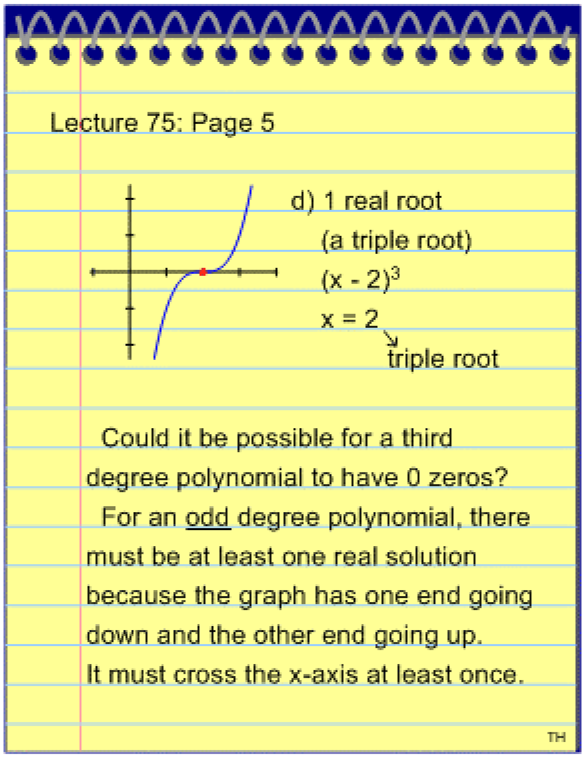
$(x + 2)(x^2 + 4)$

$x = -2, \pm 2i$

TH

Lecture 75 Notes, Continued

ALG2075-05



Lecture 75: Page 5

d) 1 real root

(a triple root)

$(x - 2)^3$

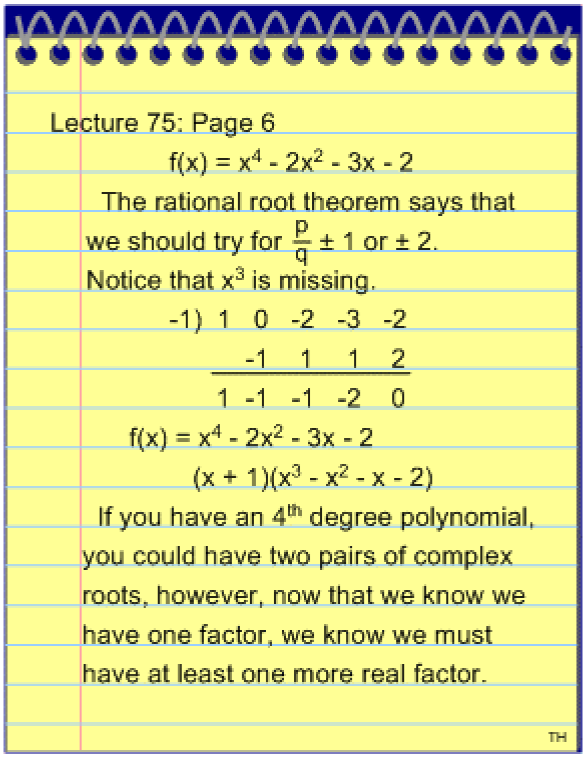
$x = 2$ ↘

triple root

Could it be possible for a third degree polynomial to have 0 zeros?

For an odd degree polynomial, there must be at least one real solution because the graph has one end going down and the other end going up. It must cross the x-axis at least once.

ALG2075-06



Lecture 75: Page 6

$f(x) = x^4 - 2x^2 - 3x - 2$

The rational root theorem says that we should try for $\frac{p}{q}$ ± 1 or ± 2.

Notice that x^3 is missing.

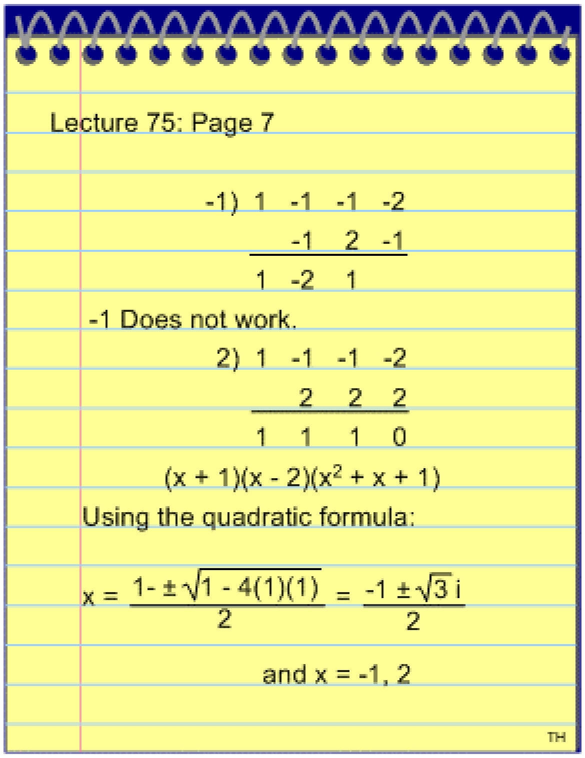
$$\begin{array}{rrrrrr} -1) & 1 & 0 & -2 & -3 & -2 \\ & & -1 & 1 & 1 & 2 \\ \hline & 1 & -1 & -1 & -2 & 0 \end{array}$$

$f(x) = x^4 - 2x^2 - 3x - 2$

$(x + 1)(x^3 - x^2 - x - 2)$

If you have an 4th degree polynomial, you could have two pairs of complex roots, however, now that we know we have one factor, we know we must have at least one more real factor.

ALG2075-07



Lecture 75: Page 7

$$\begin{array}{rrrrr} -1) & 1 & -1 & -1 & -2 \\ & & -1 & 2 & -1 \\ \hline & 1 & -2 & 1 & \end{array}$$

-1 Does not work.

$$\begin{array}{rrrrr} 2) & 1 & -1 & -1 & -2 \\ & & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

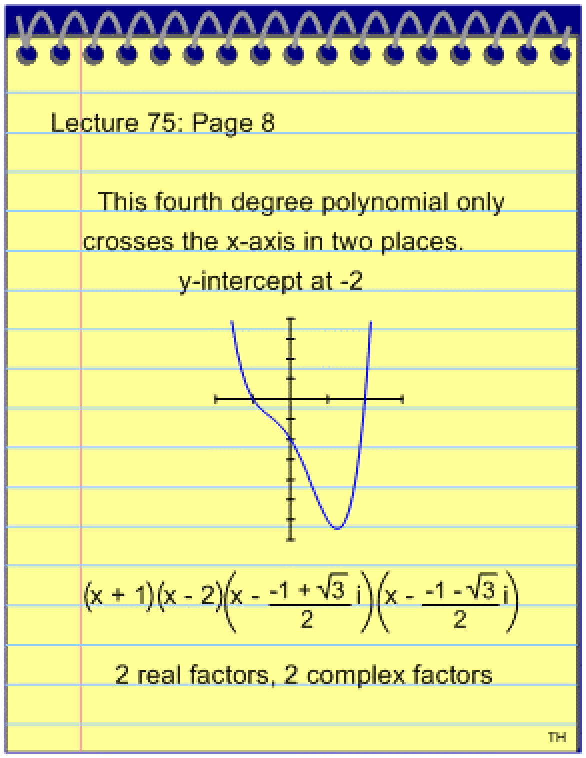
$(x + 1)(x - 2)(x^2 + x + 1)$

Using the quadratic formula:

$$x = \frac{1- \pm\sqrt{1 - 4(1)(1)}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2}$$

and $x = -1, 2$

ALG2075-08



Lecture 75: Page 8

This fourth degree polynomial only crosses the x-axis in two places.

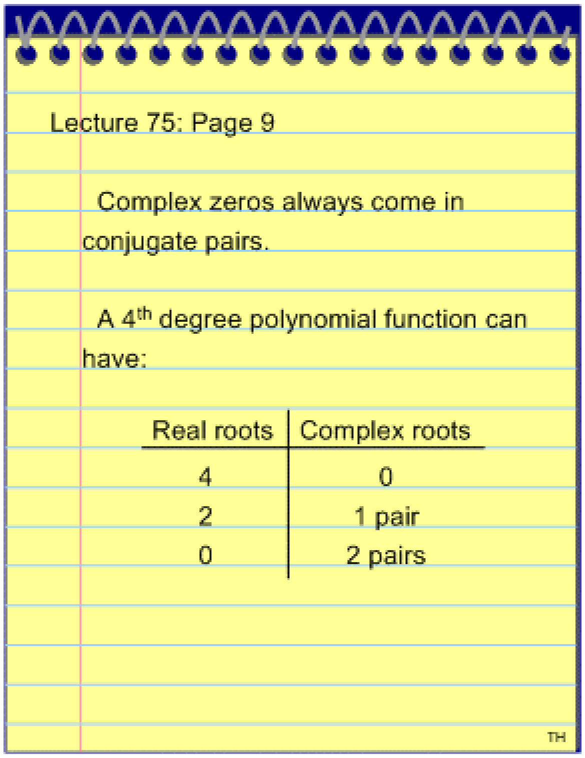
y-intercept at -2

$$(x + 1)(x - 2)\left(x - \frac{-1 + \sqrt{3}}{2}i\right)\left(x - \frac{-1 - \sqrt{3}}{2}i\right)$$

2 real factors, 2 complex factors

Lecture 75 Notes, Continued

ALG2075-09



Lecture 75: Page 9

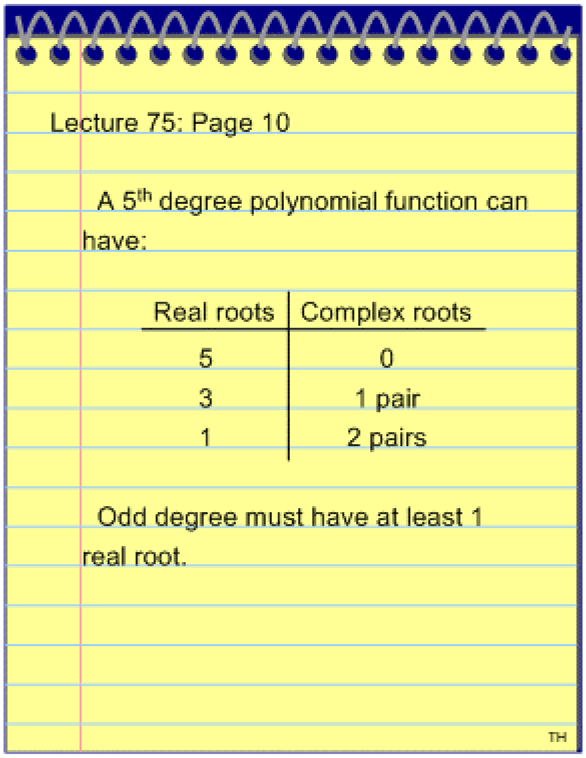
Complex zeros always come in conjugate pairs.

A 4th degree polynomial function can have:

Real roots	Complex roots
4	0
2	1 pair
0	2 pairs

TH

ALG2075-10



Lecture 75: Page 10

A 5th degree polynomial function can have:

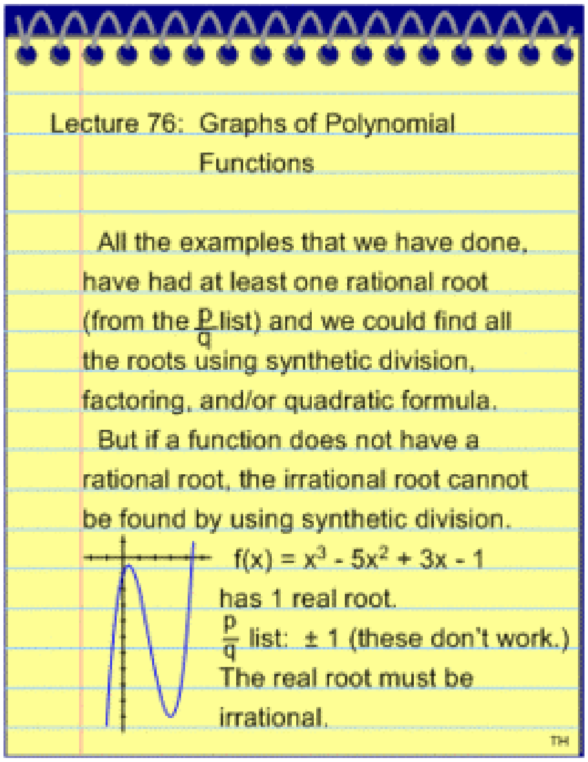
Real roots	Complex roots
5	0
3	1 pair
1	2 pairs

Odd degree must have at least 1 real root.

TH

Lecture 76 Notes

ALG2076-01



Lecture 76: Graphs of Polynomial Functions

All the examples that we have done, have had at least one rational root (from the $\frac{p}{q}$ list) and we could find all the roots using synthetic division, factoring, and/or quadratic formula.

But if a function does not have a rational root, the irrational root cannot be found by using synthetic division.

$f(x) = x^3 - 5x^2 + 3x - 1$

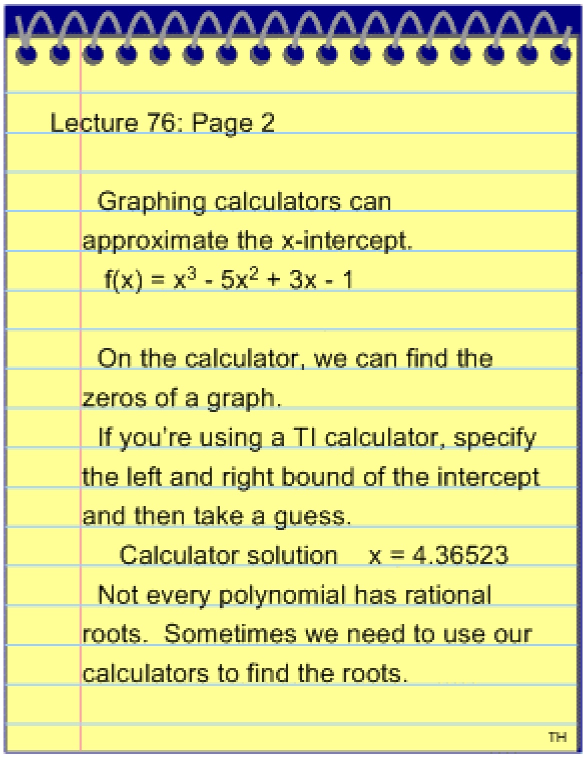
has 1 real root.

$\frac{p}{q}$ list: ± 1 (these don't work.)

The real root must be irrational.

TH

ALG2076-02



Lecture 76: Page 2

Graphing calculators can approximate the x-intercept.

$f(x) = x^3 - 5x^2 + 3x - 1$

On the calculator, we can find the zeros of a graph.

If you're using a TI calculator, specify the left and right bound of the intercept and then take a guess.

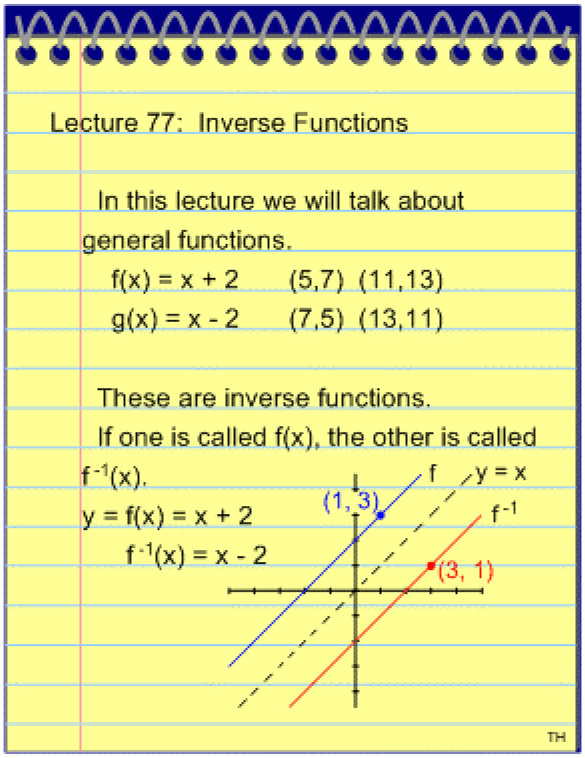
Calculator solution $x = 4.36523$

Not every polynomial has rational roots. Sometimes we need to use our calculators to find the roots.

TH

Lecture 77 Notes

ALG2077-01



Lecture 77: Inverse Functions

In this lecture we will talk about general functions.

$f(x) = x + 2$ (5,7) (11,13)

$g(x) = x - 2$ (7,5) (13,11)

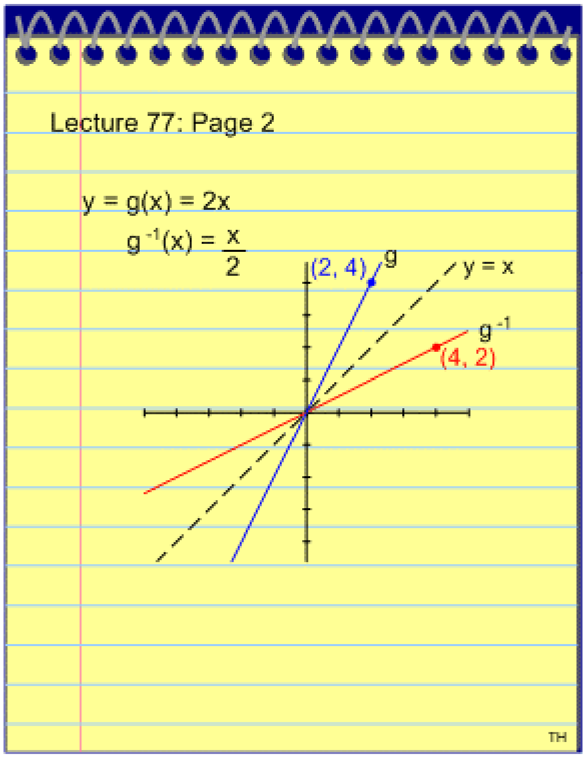
These are inverse functions.

If one is called $f(x)$, the other is called $f^{-1}(x)$.

$y = f(x) = x + 2$

$f^{-1}(x) = x - 2$

ALG2077-02

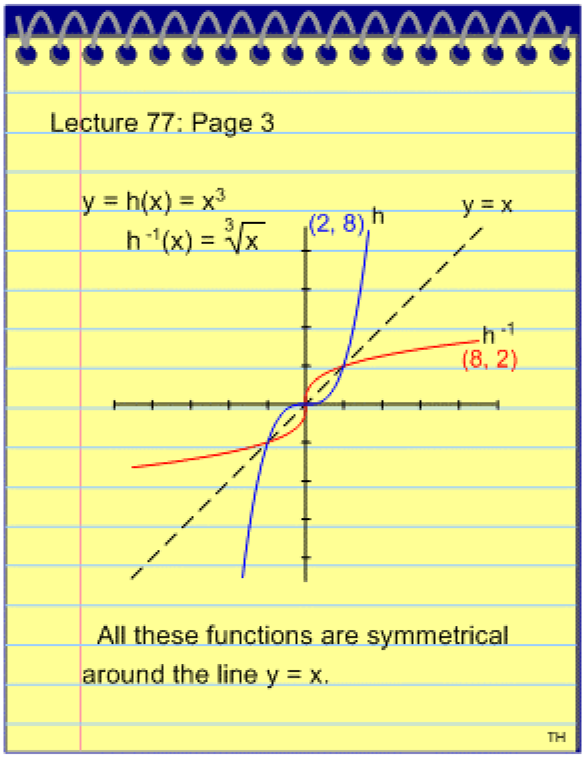


Lecture 77: Page 2

$y = g(x) = 2x$

$g^{-1}(x) = \frac{x}{2}$

ALG2077-03



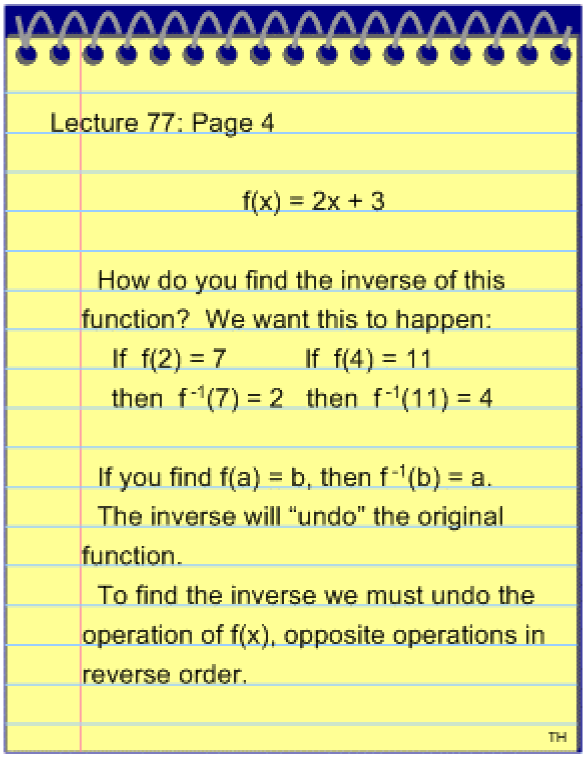
Lecture 77: Page 3

$y = h(x) = x^3$

$h^{-1}(x) = \sqrt[3]{x}$

All these functions are symmetrical around the line $y = x$.

ALG2077-04



Lecture 77: Page 4

$f(x) = 2x + 3$

How do you find the inverse of this function? We want this to happen:

If $f(2) = 7$ If $f(4) = 11$

then $f^{-1}(7) = 2$ then $f^{-1}(11) = 4$

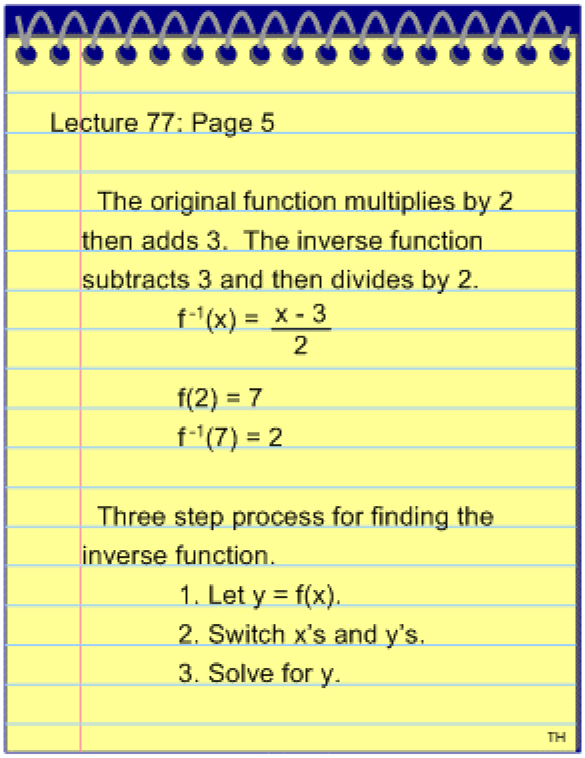
If you find $f(a) = b$, then $f^{-1}(b) = a$.

The inverse will "undo" the original function.

To find the inverse we must undo the operation of $f(x)$, opposite operations in reverse order.

Lecture 77 Notes, Continued

ALG2077-05



Lecture 77: Page 5

The original function multiplies by 2 then adds 3. The inverse function subtracts 3 and then divides by 2.

$$f^{-1}(x) = \frac{x - 3}{2}$$

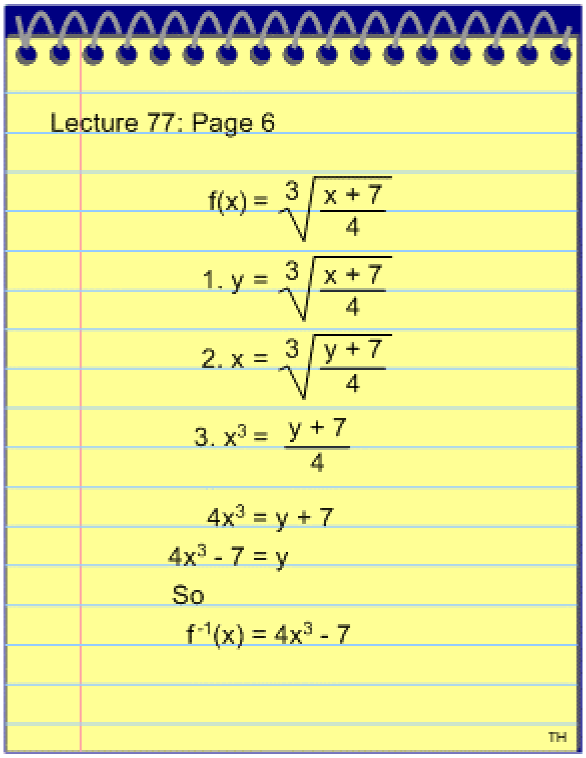
$f(2) = 7$

$f^{-1}(7) = 2$

Three step process for finding the inverse function.

1. Let $y = f(x)$.
2. Switch x's and y's.
3. Solve for y.

ALG2077-06



Lecture 77: Page 6

$$f(x) = \sqrt[3]{\frac{x + 7}{4}}$$

$$1.\ y = \sqrt[3]{\frac{x + 7}{4}}$$

$$2.\ x = \sqrt[3]{\frac{y + 7}{4}}$$

$$3.\ x^3 = \frac{y + 7}{4}$$

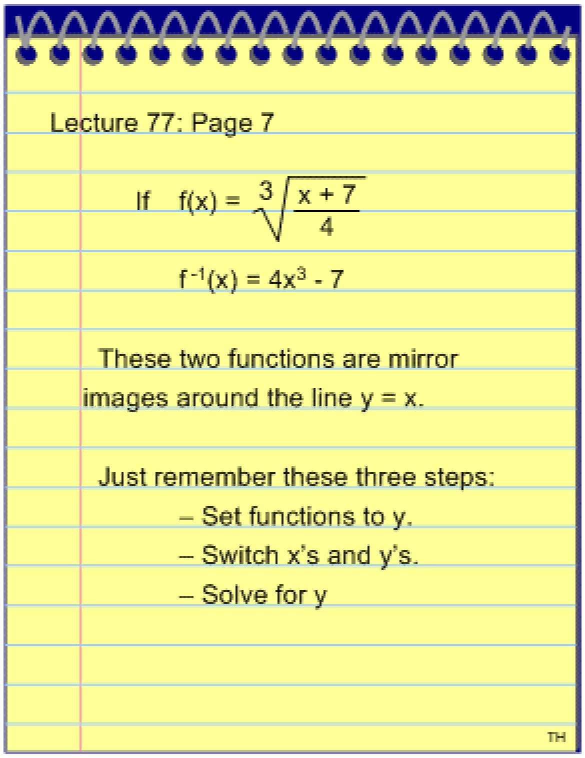
$4x^3 = y + 7$

$4x^3 - 7 = y$

So

$f^{-1}(x) = 4x^3 - 7$

ALG2077-07



Lecture 77: Page 7

If $f(x) = \sqrt[3]{\frac{x + 7}{4}}$

$f^{-1}(x) = 4x^3 - 7$

These two functions are mirror images around the line $y = x$.

Just remember these three steps:

- Set functions to y.
- Switch x's and y's.
- Solve for y

Lecture 78 Notes

ALG2078-01

Lecture 78: Exponential and Logarithmic Functions

If y is two bigger than x, isn't y two less than x?

$$\begin{cases} y = x + 2 \\ x = y - 2 \end{cases}$$

If y is twice as big as x, x is half as big as y.

$$\begin{cases} y = 2x \\ x = \dfrac{y}{2} \end{cases} \qquad \begin{cases} y = x^3 \\ x = \sqrt[3]{y} \end{cases}$$

Each of these pairs of equations are equivalent.

ALG2078-02

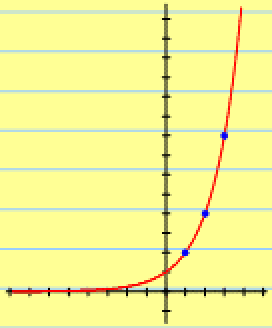
Lecture 78: Page 2

$y = 2^x$ This equation has a variable as an exponent. We've never seen these functions before. This is an exponential function.

This is the exponential function base 2.

$y = 2^x$

x	y
3	8
2	4
1	2
0	1
-1	1/2
-2	1/4
-3	1/8



ALG2078-03

Lecture 78: Page 3

The y-intercept on exponential functions is always 1 since anything with an exponent of 0 equals one.

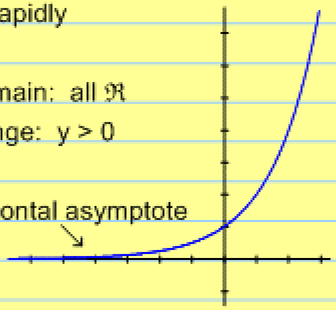
On the left side, this function gets very close to zero, but never touches it.

On the right side, this function grows very rapidly

Domain: all $\Re$

Range: $y > 0$

horizontal asymptote ↘



ALG2078-04

Lecture 78: Page 4

Horizontal asymptote – a line that a function approaches but never touches.

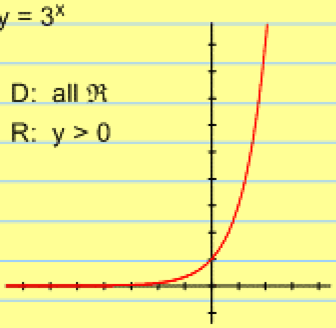
The horizontal asymptote of an exponential function is the x-axis.

$y = 3^x$

x	y
3	27
2	9
1	3
0	1
-1	1/3
-2	1/9
-3	1/27

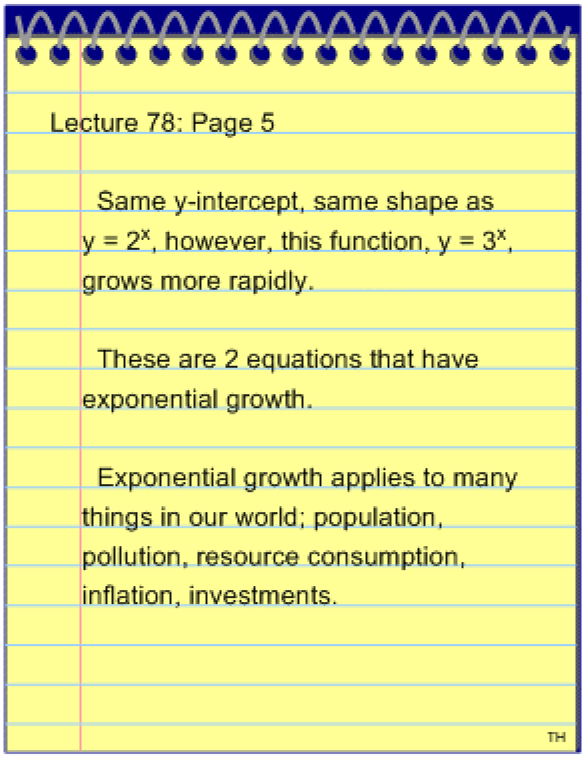
D: all $\Re$

R: $y > 0$



Lecture 78 Notes, Continued

ALG2078-05



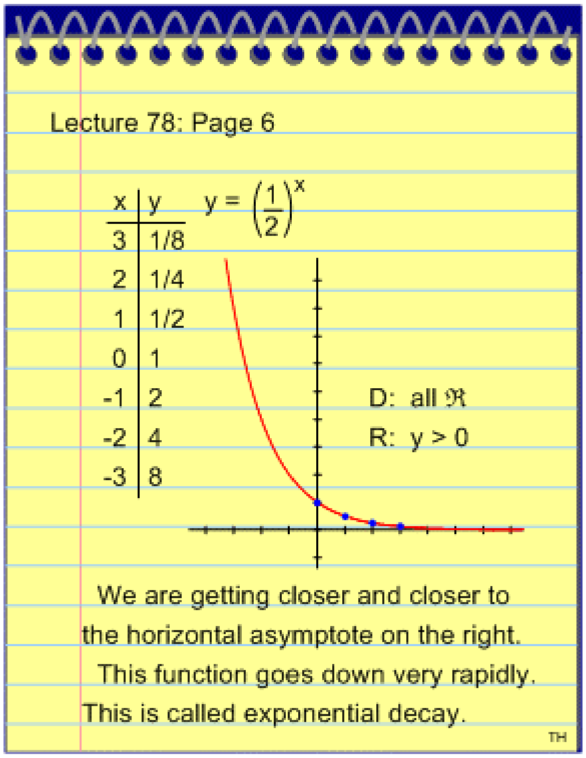
Lecture 78: Page 5

Same y-intercept, same shape as $y = 2^x$, however, this function, $y = 3^x$, grows more rapidly.

These are 2 equations that have exponential growth.

Exponential growth applies to many things in our world; population, pollution, resource consumption, inflation, investments.

ALG2078-06



Lecture 78: Page 6

$y = \left(\frac{1}{2}\right)^x$

x	y
3	1/8
2	1/4
1	1/2
0	1
-1	2
-2	4
-3	8

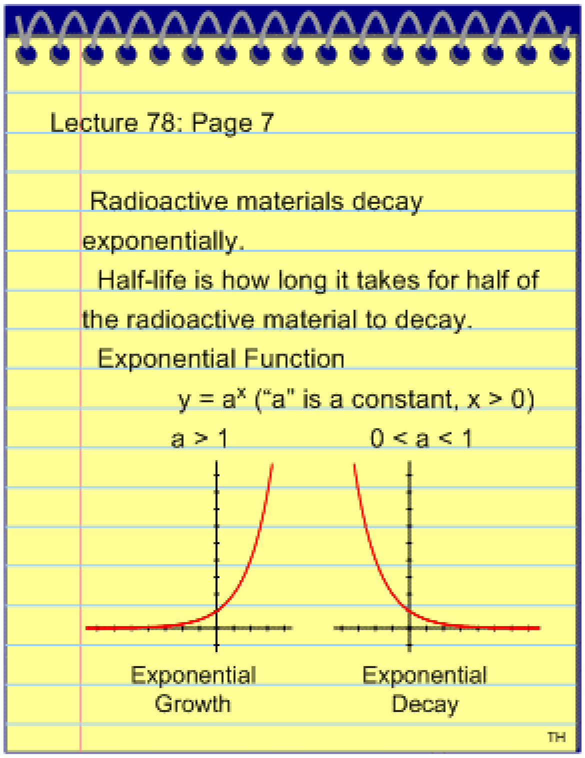
D: all $\Re$

R: $y > 0$

We are getting closer and closer to the horizontal asymptote on the right.

This function goes down very rapidly. This is called exponential decay.

ALG2078-07



Lecture 78: Page 7

Radioactive materials decay exponentially.

Half-life is how long it takes for half of the radioactive material to decay.

Exponential Function

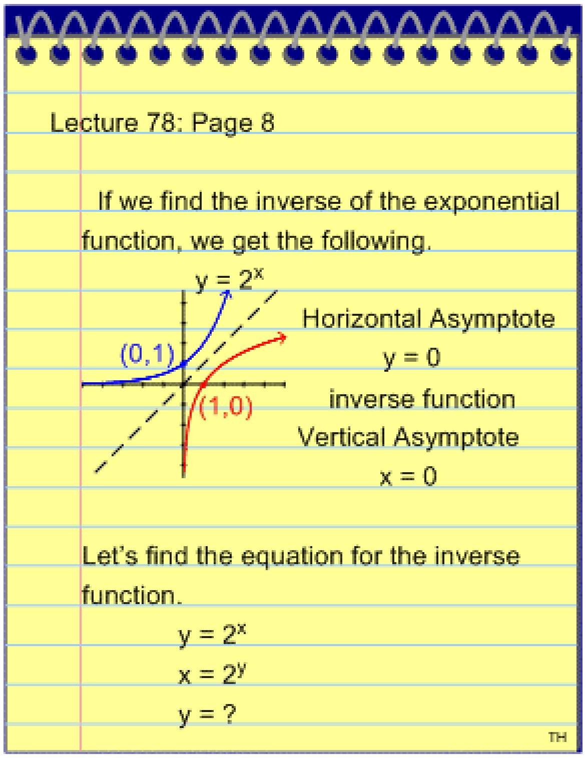
$y = a^x$ ("a" is a constant, $x > 0$)

$a > 1$ $0 < a < 1$

Exponential Growth

Exponential Decay

ALG2078-08



Lecture 78: Page 8

If we find the inverse of the exponential function, we get the following.

Horizontal Asymptote

$y = 0$

inverse function

Vertical Asymptote

$x = 0$

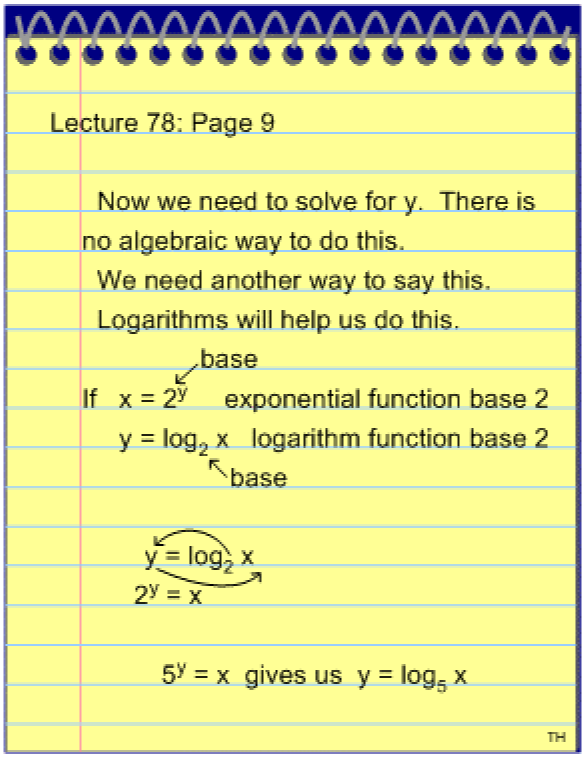
Let's find the equation for the inverse function.

$y = 2^x$

$x = 2^y$

$y = ?$

ALG2078-09



Lecture 78: Page 9

Now we need to solve for y. There is no algebraic way to do this.

We need another way to say this.

Logarithms will help us do this.

base

If $x = 2^y$ exponential function base 2

$y = \log_2 x$ logarithm function base 2

base

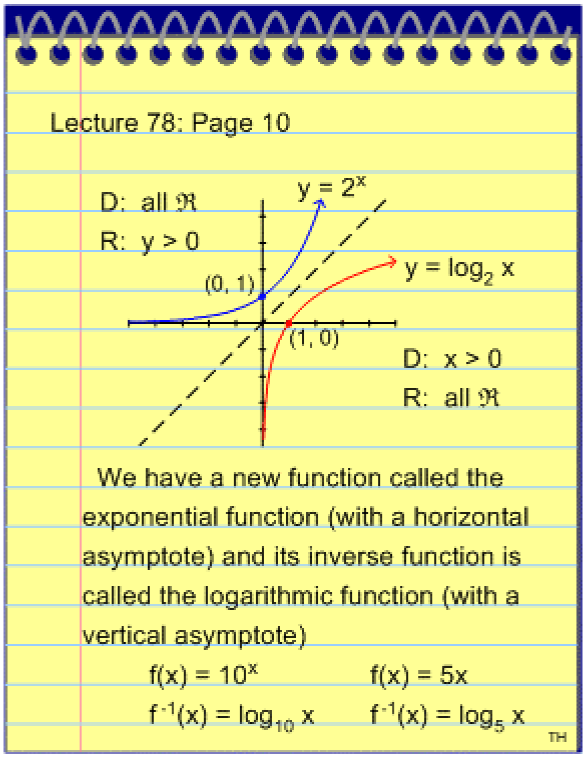
$y = \log_2 x$

$2^y = x$

$5^y = x$ gives us $y = \log_5 x$

TH

ALG2078-10



Lecture 78: Page 10

D: all $\mathfrak{R}$

R: $y > 0$

D: $x > 0$

R: all $\mathfrak{R}$

We have a new function called the exponential function (with a horizontal asymptote) and its inverse function is called the logarithmic function (with a vertical asymptote)

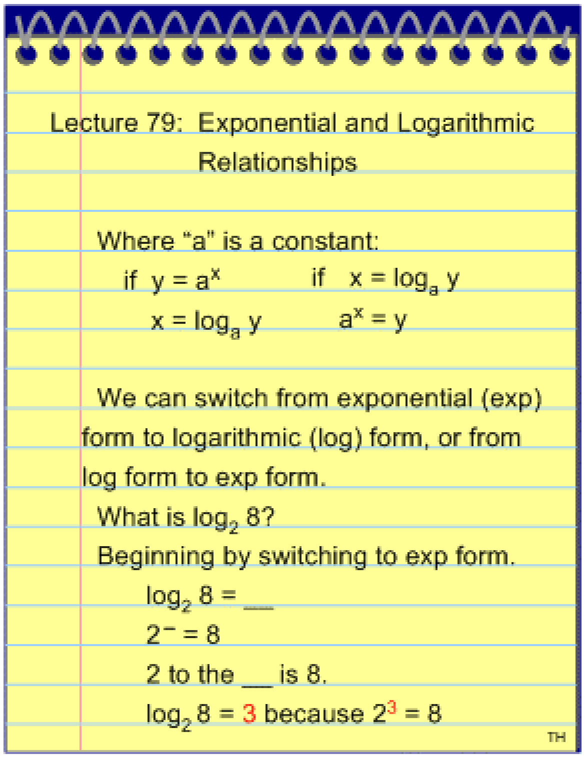
$f(x) = 10^x$ $f(x) = 5x$

$f^{-1}(x) = \log_{10} x$ $f^{-1}(x) = \log_5 x$

TH

Lecture 79 Notes

ALG2079-01



Lecture 79: Exponential and Logarithmic Relationships

Where “a” is a constant:

if $y = a^x$ if $x = \log_a y$

$x = \log_a y$ $a^x = y$

We can switch from exponential (exp) form to logarithmic (log) form, or from log form to exp form.

What is $\log_2 8$?

Beginning by switching to exp form.

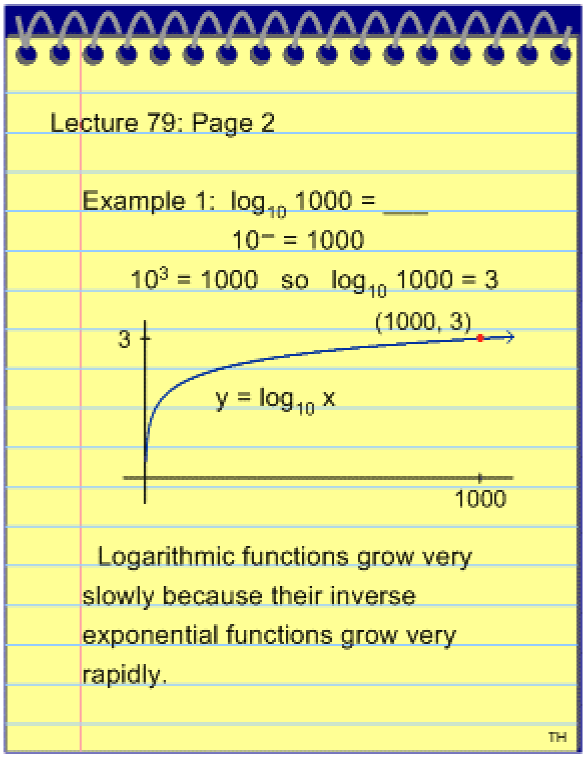
$\log_2 8 = \_\_$

$2^{\_} = 8$

2 to the __ is 8.

$\log_2 8 = 3$ because $2^3 = 8$

ALG2079-02



Lecture 79: Page 2

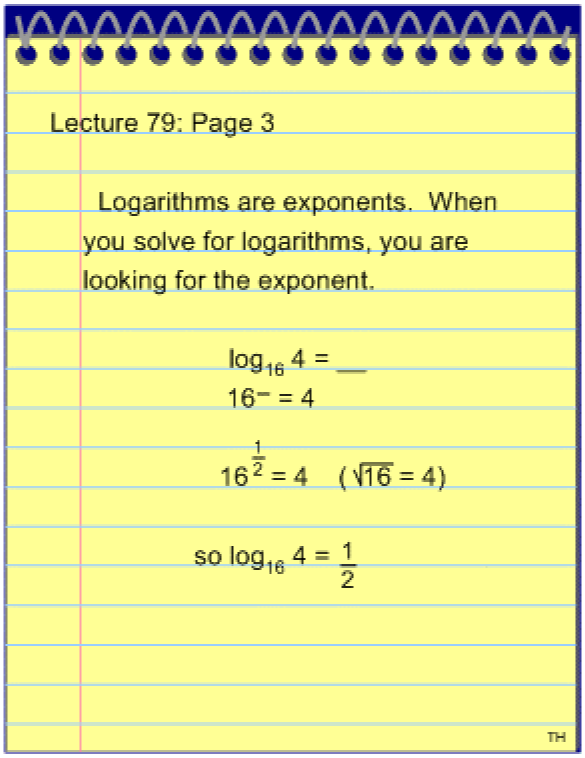
Example 1: $\log_{10} 1000 = \_\_\_$

$10^{\_} = 1000$

$10^3 = 1000$ so $\log_{10} 1000 = 3$

Logarithmic functions grow very slowly because their inverse exponential functions grow very rapidly.

ALG2079-03



Lecture 79: Page 3

Logarithms are exponents. When you solve for logarithms, you are looking for the exponent.

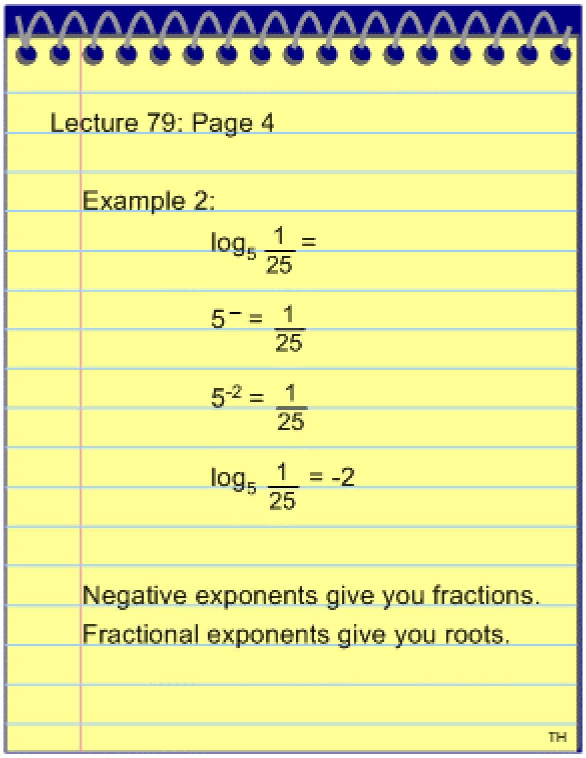
$\log_{16} 4 = \_\_$

$16^{\_} = 4$

$16^{\frac{1}{2}} = 4$ ($\sqrt{16} = 4$)

so $\log_{16} 4 = \frac{1}{2}$

ALG2079-04



Lecture 79: Page 4

Example 2:

$\log_5 \frac{1}{25} =$

$5^{\_} = \frac{1}{25}$

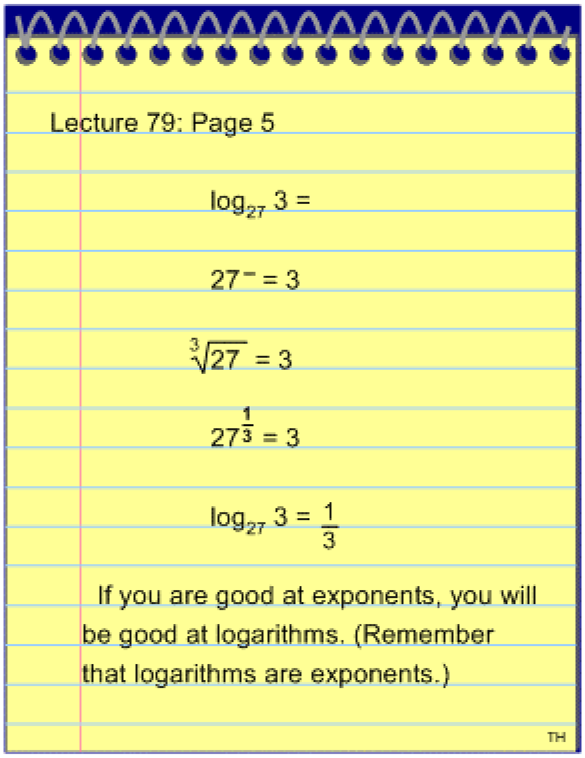
$5^{-2} = \frac{1}{25}$

$\log_5 \frac{1}{25} = -2$

Negative exponents give you fractions. Fractional exponents give you roots.

Lecture 79 Notes, Continued

ALG2079-05



Lecture 79: Page 5

$\log_{27} 3 =$

$27^{-} = 3$

$\sqrt[3]{27} = 3$

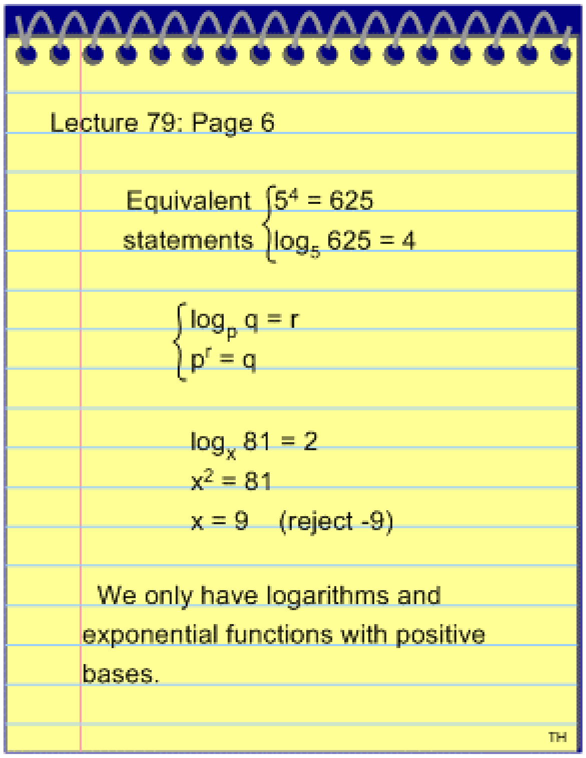
$27^{\frac{1}{3}} = 3$

$\log_{27} 3 = \frac{1}{3}$

If you are good at exponents, you will be good at logarithms. (Remember that logarithms are exponents.)

TH

ALG2079-06



Lecture 79: Page 6

$$\text{Equivalent statements} \begin{cases} 5^4 = 625 \\ \log_5 625 = 4 \end{cases}$$

$$\begin{cases} \log_p q = r \\ p^r = q \end{cases}$$

$\log_x 81 = 2$

$x^2 = 81$

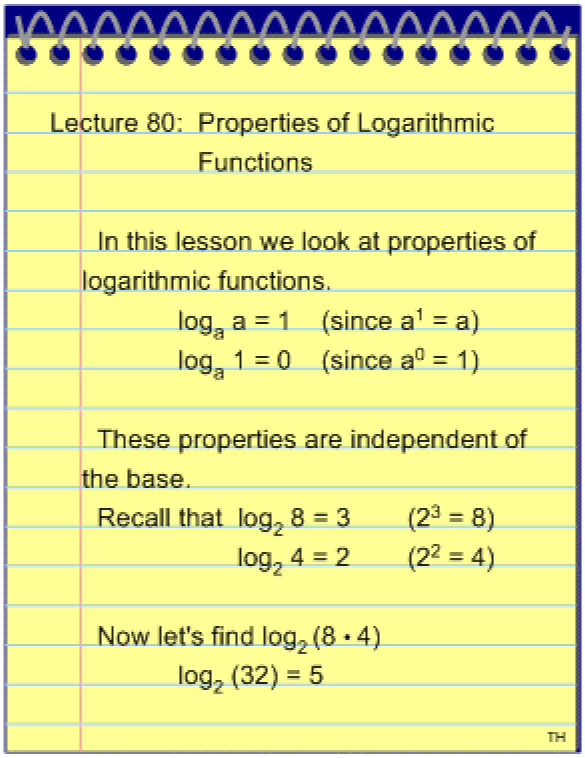
$x = 9$ (reject -9)

We only have logarithms and exponential functions with positive bases.

TH

Lecture 80 Notes

ALG2080-01



Lecture 80: Properties of Logarithmic Functions

In this lesson we look at properties of logarithmic functions.

$\log_a a = 1$ (since $a^1 = a$)

$\log_a 1 = 0$ (since $a^0 = 1$)

These properties are independent of the base.

Recall that $\log_2 8 = 3$ $(2^3 = 8)$

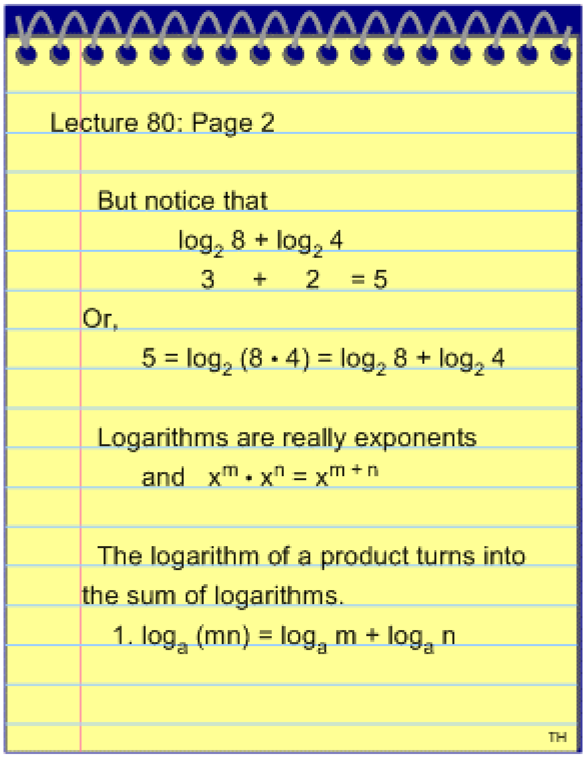
$\log_2 4 = 2$ $(2^2 = 4)$

Now let's find $\log_2 (8 \cdot 4)$

$\log_2 (32) = 5$

TH

ALG2080-02



Lecture 80: Page 2

But notice that

$\log_2 8 + \log_2 4$

$3 \quad + \quad 2 \quad = 5$

Or,

$5 = \log_2 (8 \cdot 4) = \log_2 8 + \log_2 4$

Logarithms are really exponents

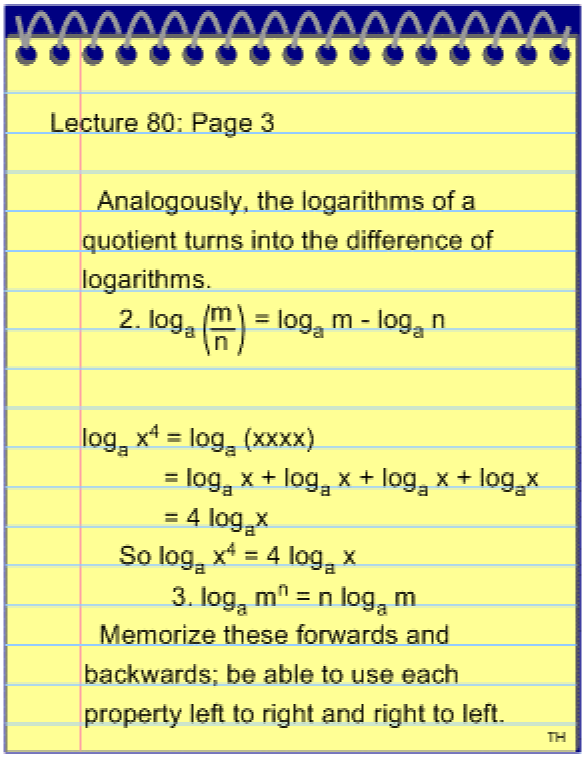
and $x^m \cdot x^n = x^{m+n}$

The logarithm of a product turns into the sum of logarithms.

1. $\log_a (mn) = \log_a m + \log_a n$

TH

ALG2080-03



Lecture 80: Page 3

Analogously, the logarithms of a quotient turns into the difference of logarithms.

2. $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$

$\log_a x^4 = \log_a (xxxx)$

$= \log_a x + \log_a x + \log_a x + \log_a x$

$= 4 \log_a x$

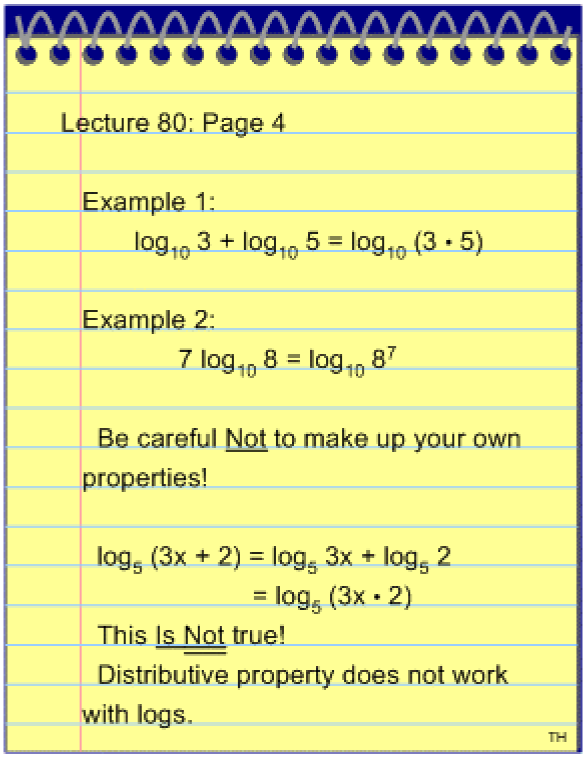
So $\log_a x^4 = 4 \log_a x$

3. $\log_a m^n = n \log_a m$

Memorize these forwards and backwards; be able to use each property left to right and right to left.

TH

ALG2080-04



Lecture 80: Page 4

Example 1:

$\log_{10} 3 + \log_{10} 5 = \log_{10} (3 \cdot 5)$

Example 2:

$7 \log_{10} 8 = \log_{10} 8^7$

Be careful Not to make up your own properties!

$\log_5 (3x + 2) = \log_5 3x + \log_5 2$

$= \log_5 (3x \cdot 2)$

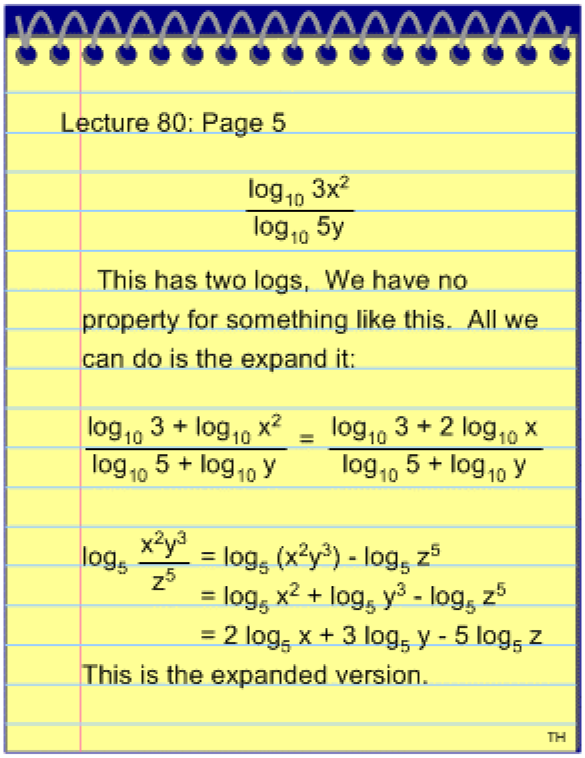
This Is Not true!

Distributive property does not work with logs.

TH

Lecture 80 Notes, Continued

ALG2080-05



Lecture 80: Page 5

$$\frac{\log_{10} 3x^2}{\log_{10} 5y}$$

This has two logs, We have no property for something like this. All we can do is the expand it:

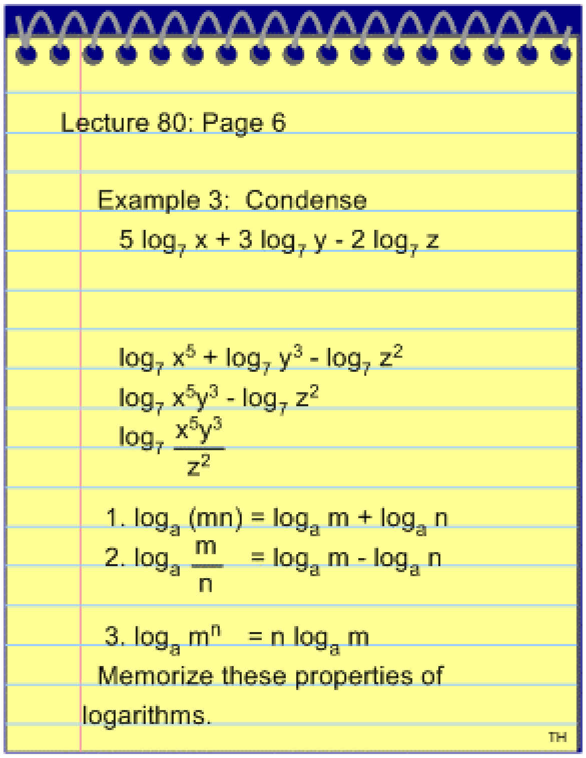
$$\frac{\log_{10} 3 + \log_{10} x^2}{\log_{10} 5 + \log_{10} y} = \frac{\log_{10} 3 + 2 \log_{10} x}{\log_{10} 5 + \log_{10} y}$$

$$\begin{aligned}\log_5 \frac{x^2y^3}{z^5} &= \log_5 (x^2y^3) - \log_5 z^5 \\ &= \log_5 x^2 + \log_5 y^3 - \log_5 z^5 \\ &= 2 \log_5 x + 3 \log_5 y - 5 \log_5 z\end{aligned}$$

This is the expanded version.

TH

ALG2080-06



Lecture 80: Page 6

Example 3: Condense

$5 \log_7 x + 3 \log_7 y - 2 \log_7 z$

$\log_7 x^5 + \log_7 y^3 - \log_7 z^2$

$\log_7 x^5y^3 - \log_7 z^2$

$\log_7 \frac{x^5y^3}{z^2}$

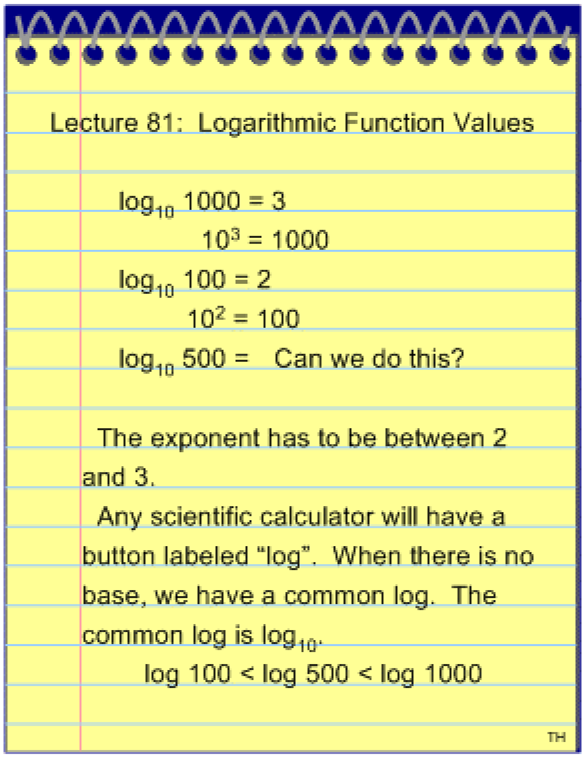
1. $\log_a (mn) = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n \log_a m$

Memorize these properties of logarithms.

TH

Lecture 81 Notes

ALG2081-01



Lecture 81: Logarithmic Function Values

$\log_{10} 1000 = 3$

$10^3 = 1000$

$\log_{10} 100 = 2$

$10^2 = 100$

$\log_{10} 500 =$ Can we do this?

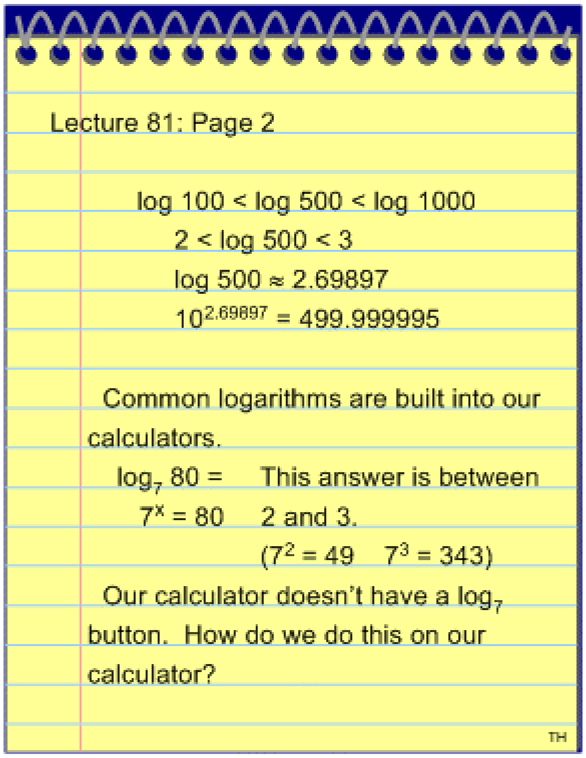
The exponent has to be between 2 and 3.

Any scientific calculator will have a button labeled "log". When there is no base, we have a common log. The common log is $\log_{10}$.

$\log 100 < \log 500 < \log 1000$

TH

ALG2081-02



Lecture 81: Page 2

$\log 100 < \log 500 < \log 1000$

$2 < \log 500 < 3$

$\log 500 \approx 2.69897$

$10^{2.69897} = 499.999995$

Common logarithms are built into our calculators.

$\log_7 80 =$ This answer is between

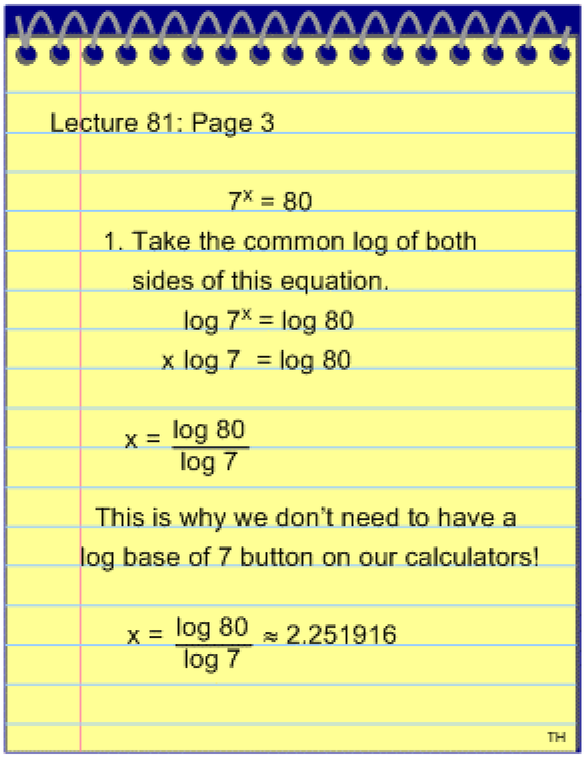
$7^x = 80$ 2 and 3.

$(7^2 = 49 \quad 7^3 = 343)$

Our calculator doesn't have a $\log_7$ button. How do we do this on our calculator?

TH

ALG2081-03



Lecture 81: Page 3

$7^x = 80$

1. Take the common log of both sides of this equation.

$\log 7^x = \log 80$

$x \log 7 = \log 80$

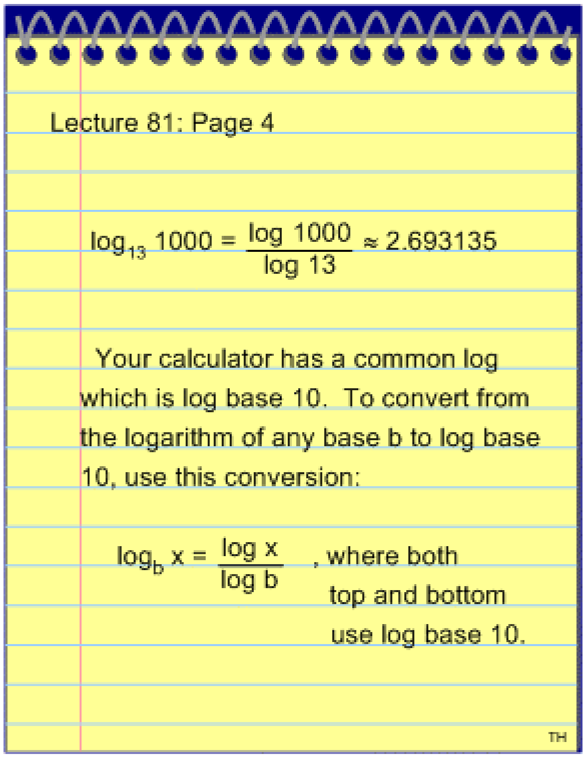
$$x = \frac{\log 80}{\log 7}$$

This is why we don't need to have a log base of 7 button on our calculators!

$$x = \frac{\log 80}{\log 7} \approx 2.251916$$

TH

ALG2081-04



Lecture 81: Page 4

$$\log_{13} 1000 = \frac{\log 1000}{\log 13} \approx 2.693135$$

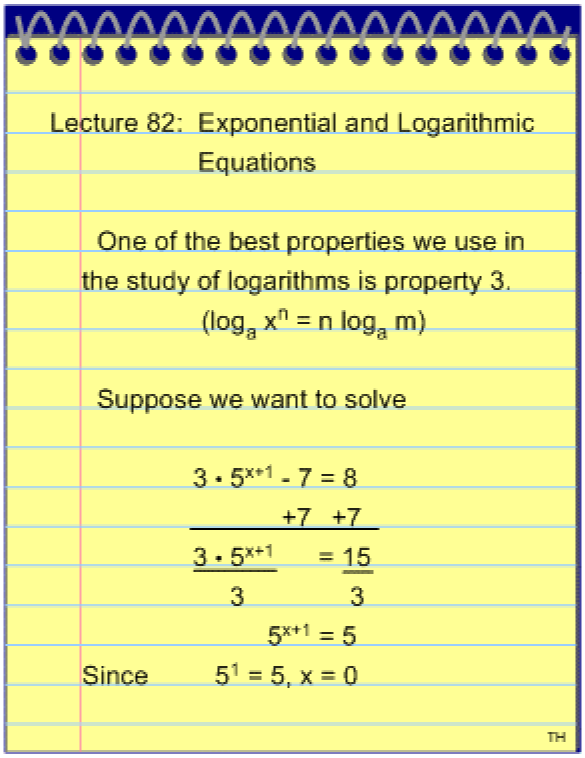
Your calculator has a common log which is log base 10. To convert from the logarithm of any base b to log base 10, use this conversion:

$$\log_b x = \frac{\log x}{\log b}$$, where both top and bottom use log base 10.

TH

Lecture 82 Notes

ALG2082-01



Lecture 82: Exponential and Logarithmic Equations

One of the best properties we use in the study of logarithms is property 3.

$(\log_a x^n = n \log_a m)$

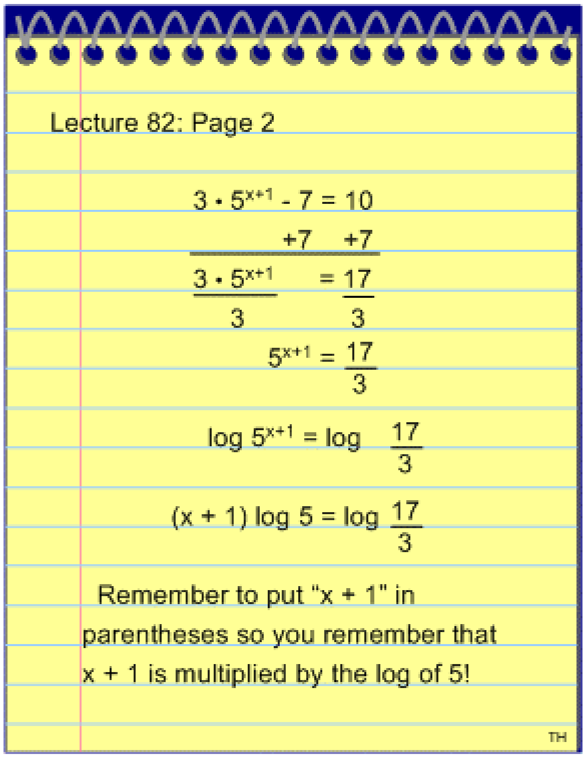
Suppose we want to solve

$$3 \cdot 5^{x+1} - 7 = 8$$
$$+7 \quad +7$$
$$\frac{3 \cdot 5^{x+1}}{3} = \frac{15}{3}$$
$$5^{x+1} = 5$$

Since $5^1 = 5$, $x = 0$

TH

ALG2082-02



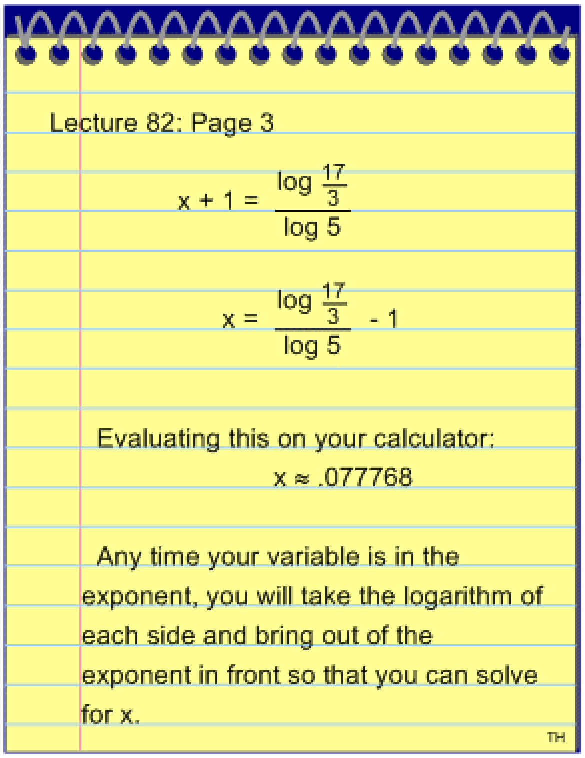
Lecture 82: Page 2

$$3 \cdot 5^{x+1} - 7 = 10$$
$$+7 \quad +7$$
$$\frac{3 \cdot 5^{x+1}}{3} = \frac{17}{3}$$
$$5^{x+1} = \frac{17}{3}$$
$$\log 5^{x+1} = \log \frac{17}{3}$$
$$(x + 1) \log 5 = \log \frac{17}{3}$$

Remember to put "x + 1" in parentheses so you remember that x + 1 is multiplied by the log of 5!

TH

ALG2082-03



Lecture 82: Page 3

$$x + 1 = \frac{\log \frac{17}{3}}{\log 5}$$

$$x = \frac{\log \frac{17}{3}}{\log 5} - 1$$

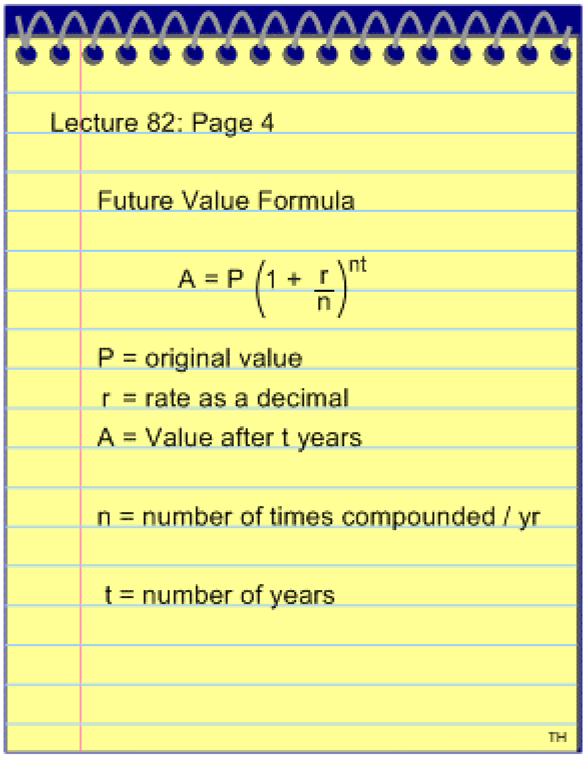
Evaluating this on your calculator:

$x \approx .077768$

Any time your variable is in the exponent, you will take the logarithm of each side and bring out the exponent in front so that you can solve for x.

TH

ALG2082-04



Lecture 82: Page 4

Future Value Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

P = original value

r = rate as a decimal

A = Value after t years

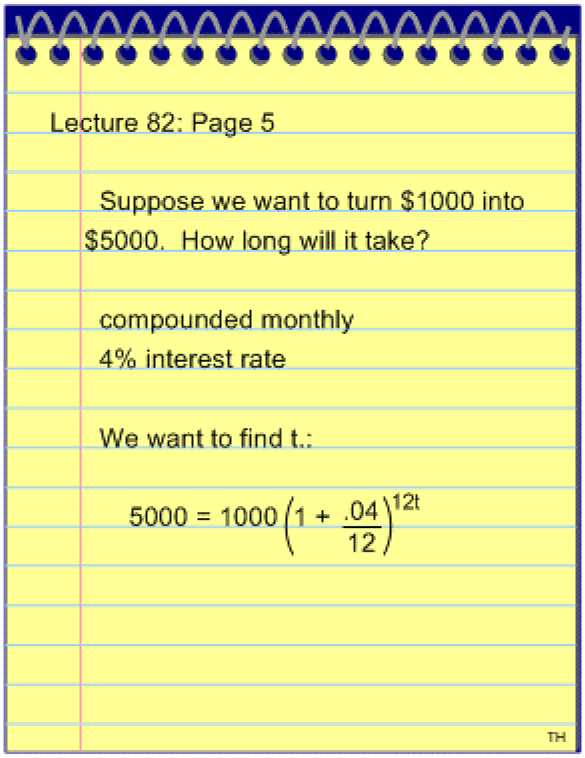
n = number of times compounded / yr

t = number of years

TH

Lecture 82 Notes

ALG2082-05



Lecture 82: Page 5

Suppose we want to turn \$1000 into \$5000. How long will it take?

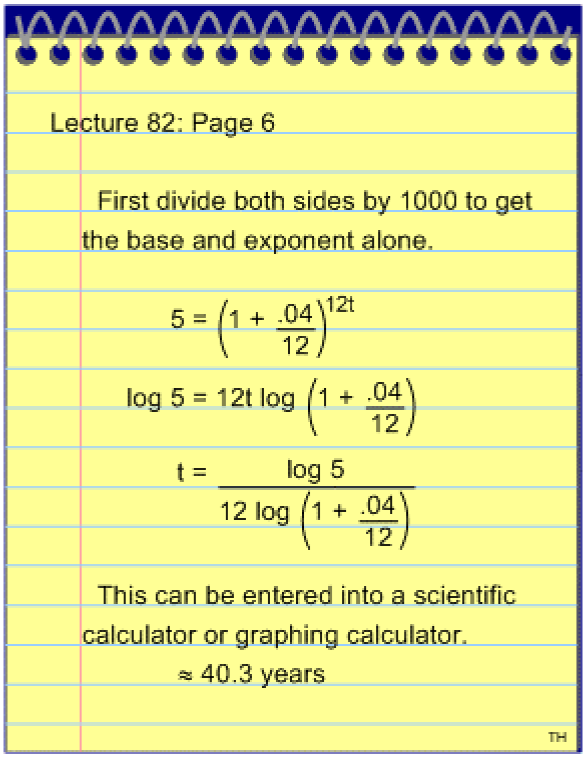
compounded monthly
4% interest rate

We want to find t.:

$$5000 = 1000\left(1 + \frac{.04}{12}\right)^{12t}$$

TH

ALG2082-06



Lecture 82: Page 6

First divide both sides by 1000 to get the base and exponent alone.

$$5 = \left(1 + \frac{.04}{12}\right)^{12t}$$

$$\log 5 = 12t \log\left(1 + \frac{.04}{12}\right)$$

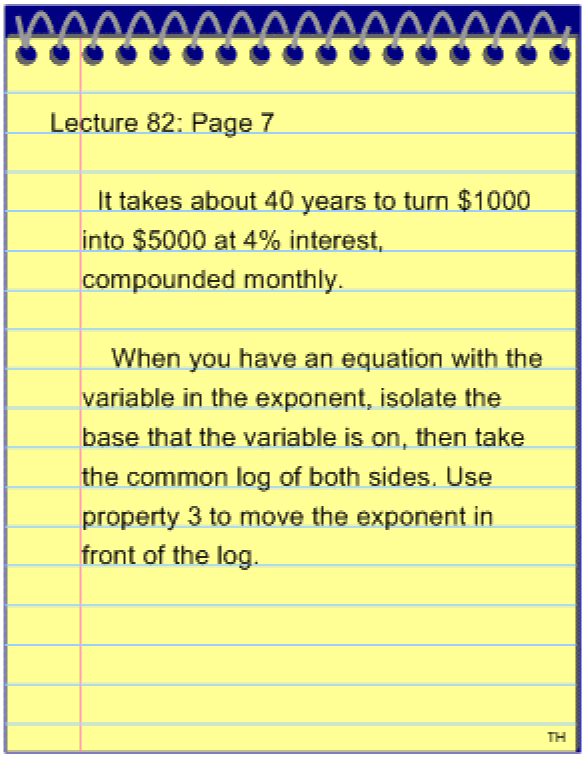
$$t = \frac{\log 5}{12 \log\left(1 + \frac{.04}{12}\right)}$$

This can be entered into a scientific calculator or graphing calculator.

$\approx$ 40.3 years

TH

ALG2082-07



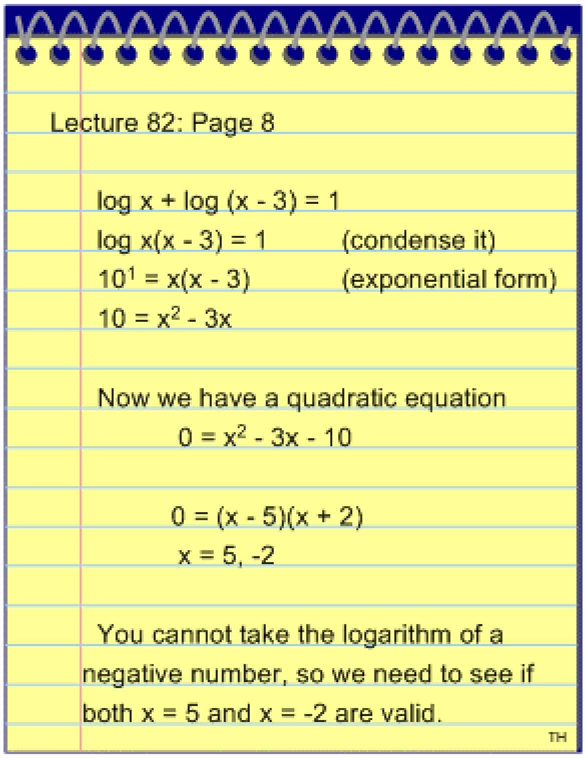
Lecture 82: Page 7

It takes about 40 years to turn \$1000 into \$5000 at 4% interest, compounded monthly.

When you have an equation with the variable in the exponent, isolate the base that the variable is on, then take the common log of both sides. Use property 3 to move the exponent in front of the log.

TH

ALG2082-08



Lecture 82: Page 8

$\log x + \log(x - 3) = 1$
$\log x(x - 3) = 1$ (condense it)
$10^1 = x(x - 3)$ (exponential form)
$10 = x^2 - 3x$

Now we have a quadratic equation

$$0 = x^2 - 3x - 10$$

$$0 = (x - 5)(x + 2)$$

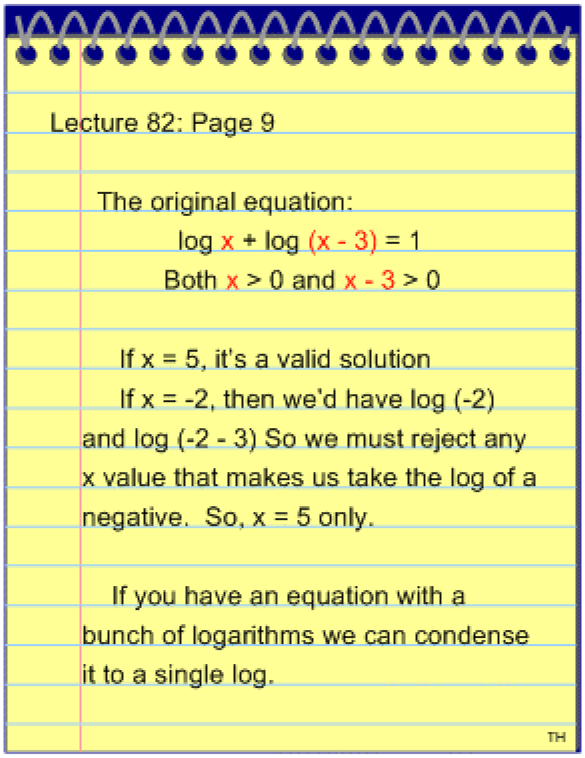
$$x = 5, -2$$

You cannot take the logarithm of a negative number, so we need to see if both x = 5 and x = -2 are valid.

TH

Lecture 82 Notes

ALG2082-09



Lecture 82: Page 9

The original equation:

log x + log (x - 3) = 1

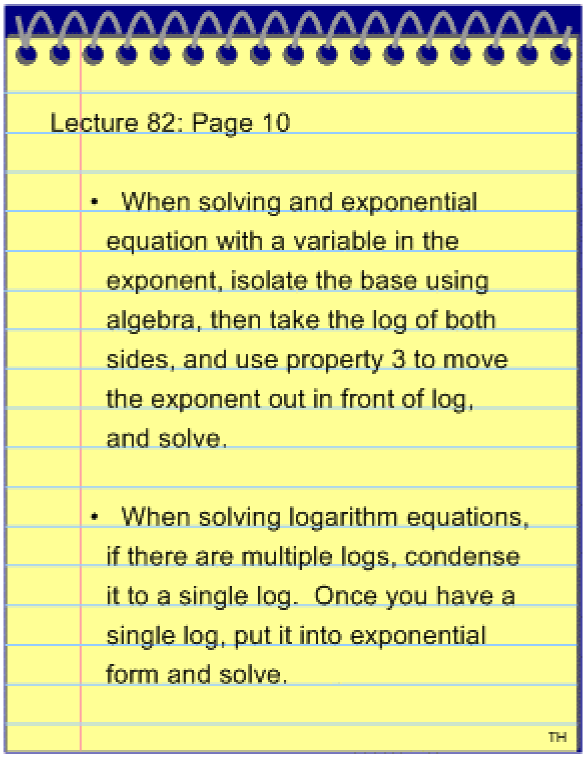
Both x > 0 and x - 3 > 0

If x = 5, it's a valid solution

If x = -2, then we'd have log (-2) and log (-2 - 3) So we must reject any x value that makes us take the log of a negative. So, x = 5 only.

If you have an equation with a bunch of logarithms we can condense it to a single log.

ALG2082-10



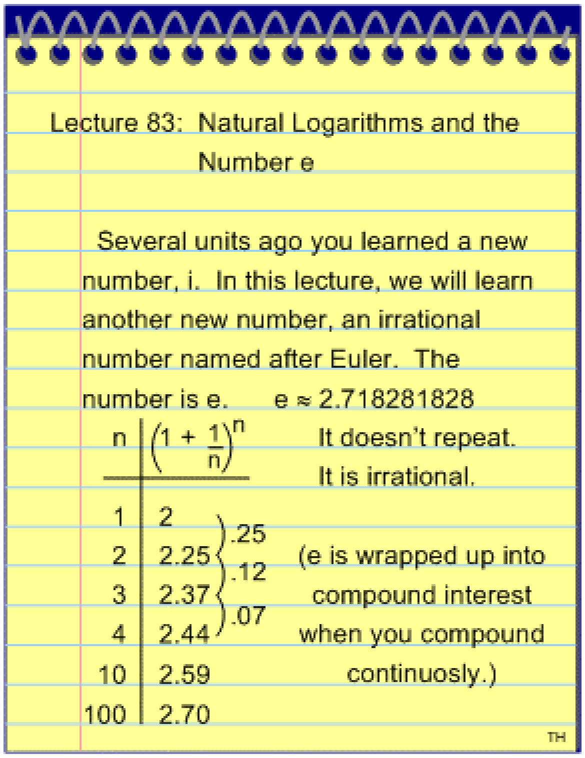
Lecture 82: Page 10

- When solving and exponential equation with a variable in the exponent, isolate the base using algebra, then take the log of both sides, and use property 3 to move the exponent out in front of log, and solve.

- When solving logarithm equations, if there are multiple logs, condense it to a single log. Once you have a single log, put it into exponential form and solve.

Lecture 83 Notes

ALG2083-01



Lecture 83: Natural Logarithms and the Number e

Several units ago you learned a new number, i. In this lecture, we will learn another new number, an irrational number named after Euler. The number is e. $e \approx 2.718281828$

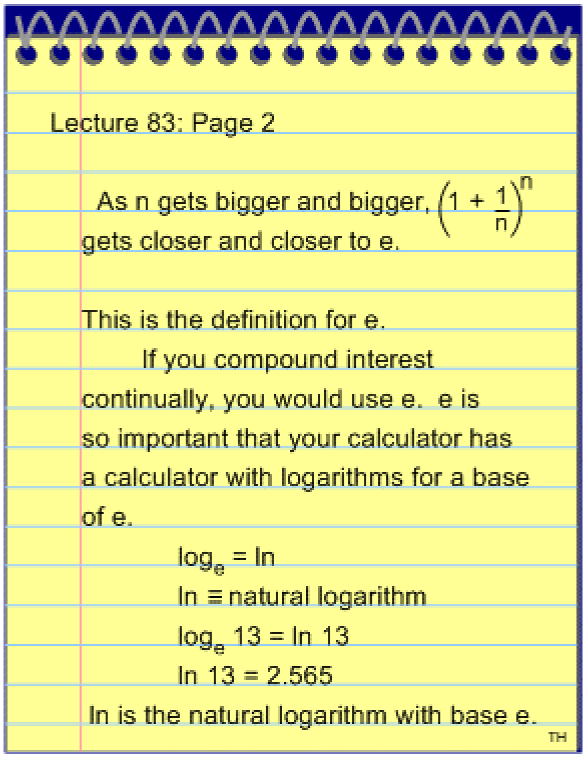
n	$\left(1 + \frac{1}{n}\right)^n$	
1	2	.25
2	2.25	.12
3	2.37	.07
4	2.44	
10	2.59	
100	2.70	

It doesn't repeat.
It is irrational.

(e is wrapped up into compound interest when you compound continuosly.)

TH

ALG2083-02



Lecture 83: Page 2

As n gets bigger and bigger, $\left(1 + \frac{1}{n}\right)^n$ gets closer and closer to e.

This is the definition for e.

If you compound interest continually, you would use e. e is so important that your calculator has a calculator with logarithms for a base of e.

$\log_e = \ln$

$\ln \equiv$ natural logarithm

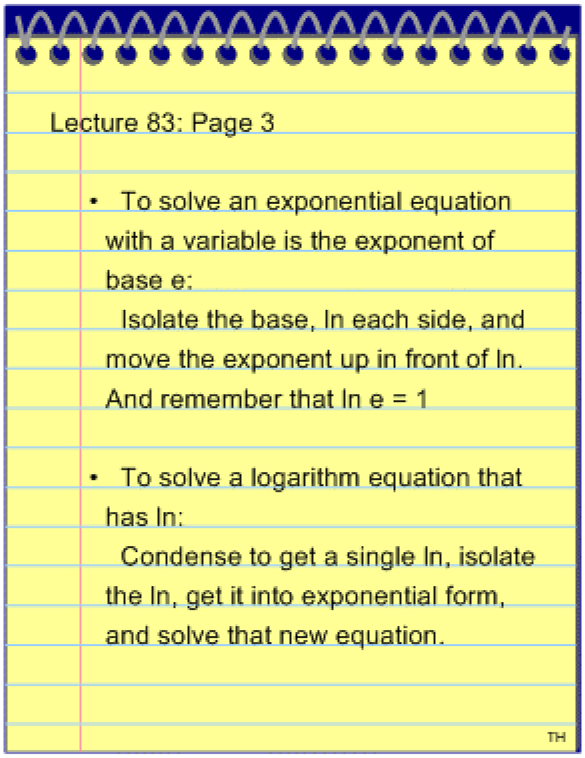
$\log_e 13 = \ln 13$

$\ln 13 = 2.565$

ln is the natural logarithm with base e.

TH

ALG2083-03



Lecture 83: Page 3

- To solve an exponential equation with a variable is the exponent of base e:

  Isolate the base, ln each side, and move the exponent up in front of ln. And remember that ln e = 1

- To solve a logarithm equation that has ln:

  Condense to get a single ln, isolate the ln, get it into exponential form, and solve that new equation.

TH