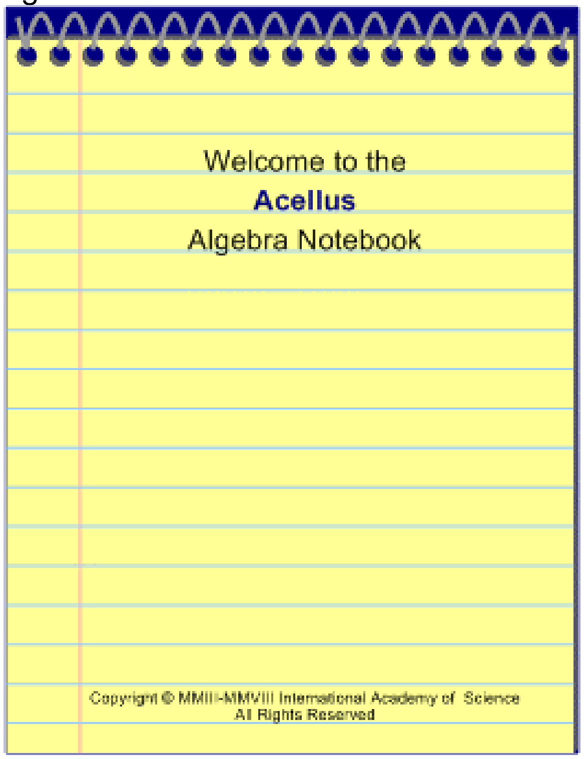
ALGEBRA I NOTES

Introduction Notes

Alg000-00

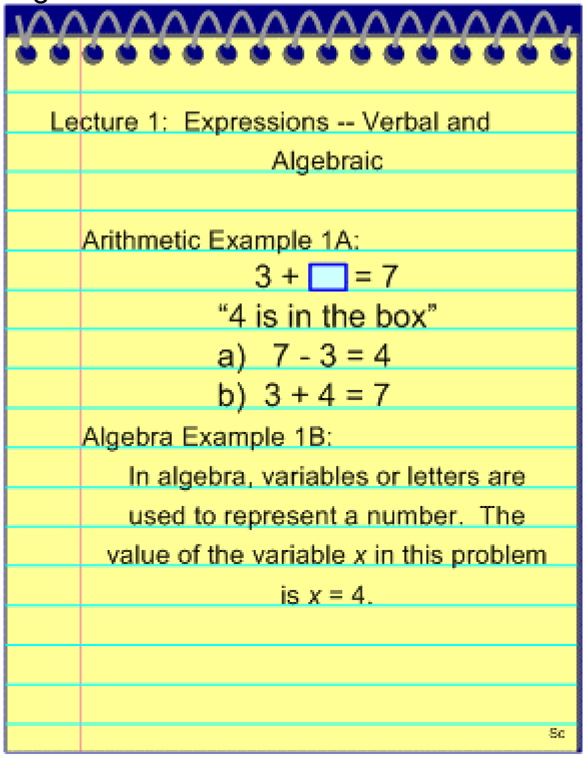


Welcome to the
Acellus
Algebra Notebook

Copyright © MMIII-MMVIII International Academy of Science
All Rights Reserved

Lecture 1 Notes

Alg001-01



Lecture 1: Expressions -- Verbal and Algebraic

Arithmetic Example 1A:

$$3 + \boxed{} = 7$$

"4 is in the box"

a) $7 - 3 = 4$

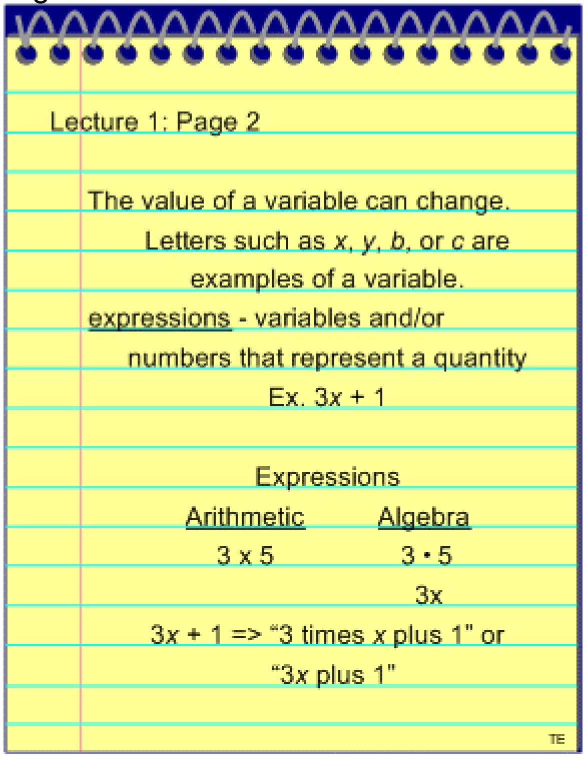
b) $3 + 4 = 7$

Algebra Example 1B:

In algebra, variables or letters are used to represent a number. The value of the variable x in this problem is $x = 4$.

Sc

Alg001-02



Lecture 1: Page 2

The value of a variable can change.
Letters such as x, y, b, or c are examples of a variable.

expressions - variables and/or numbers that represent a quantity

Ex. $3x + 1$

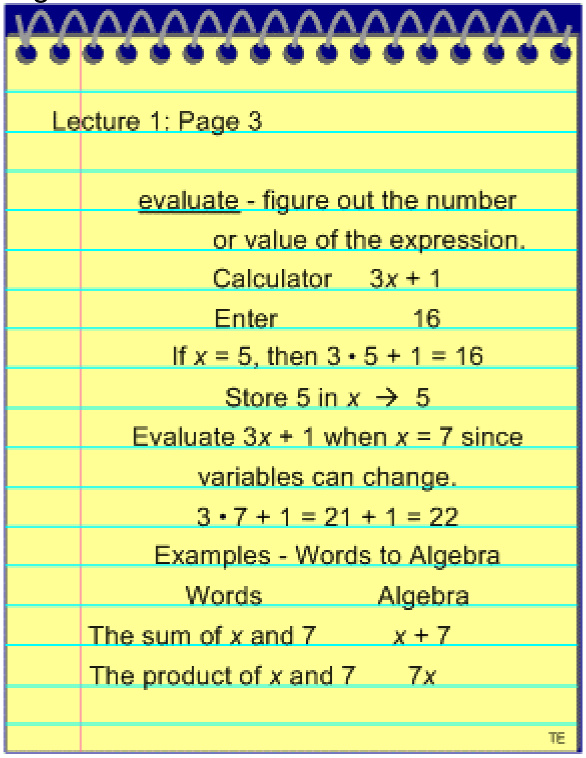
Expressions

Arithmetic	Algebra
3 x 5	$3 \cdot 5$
	3x

$3x + 1$ => "3 times x plus 1" or "3x plus 1"

TE

Alg001-03



Lecture 1: Page 3

evaluate - figure out the number or value of the expression.

Calculator $3x + 1$

Enter 16

If $x = 5$, then $3 \cdot 5 + 1 = 16$

Store 5 in x → 5

Evaluate $3x + 1$ when $x = 7$ since variables can change.

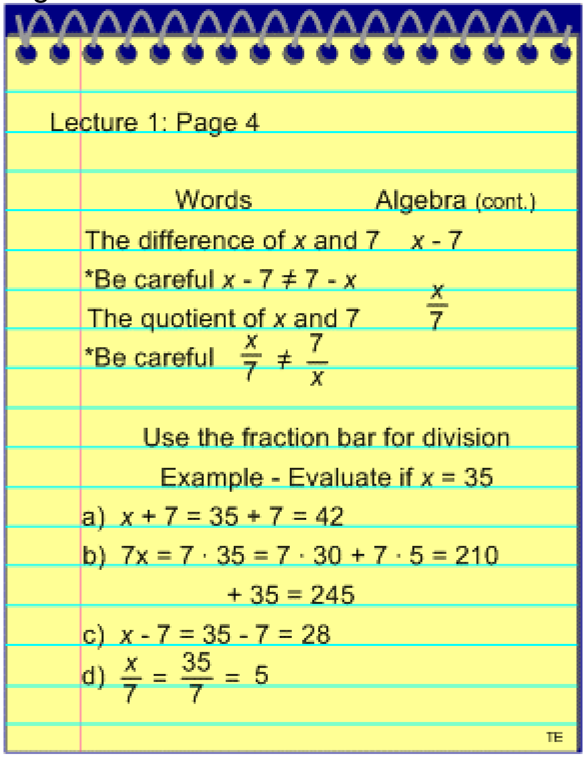
$3 \cdot 7 + 1 = 21 + 1 = 22$

Examples - Words to Algebra

Words	Algebra
The sum of x and 7	$x + 7$
The product of x and 7	$7x$

TE

Alg001-04



Lecture 1: Page 4

Words	Algebra (cont.)
The difference of x and 7	$x - 7$
*Be careful $x - 7 \neq 7 - x$	
The quotient of x and 7	$\frac{x}{7}$
*Be careful $\frac{x}{7} \neq \frac{7}{x}$	

Use the fraction bar for division

Example - Evaluate if $x = 35$

a) $x + 7 = 35 + 7 = 42$

b) $7x = 7 \cdot 35 = 7 \cdot 30 + 7 \cdot 5 = 210$
$+ 35 = 245$

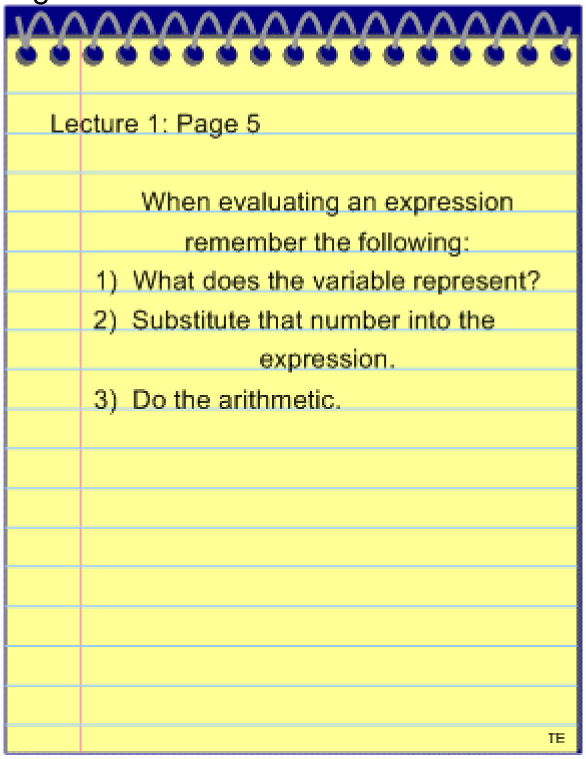
c) $x - 7 = 35 - 7 = 28$

d) $\frac{x}{7} = \frac{35}{7} = 5$

TE

Lecture 1 Notes, Continued

Alg001-05



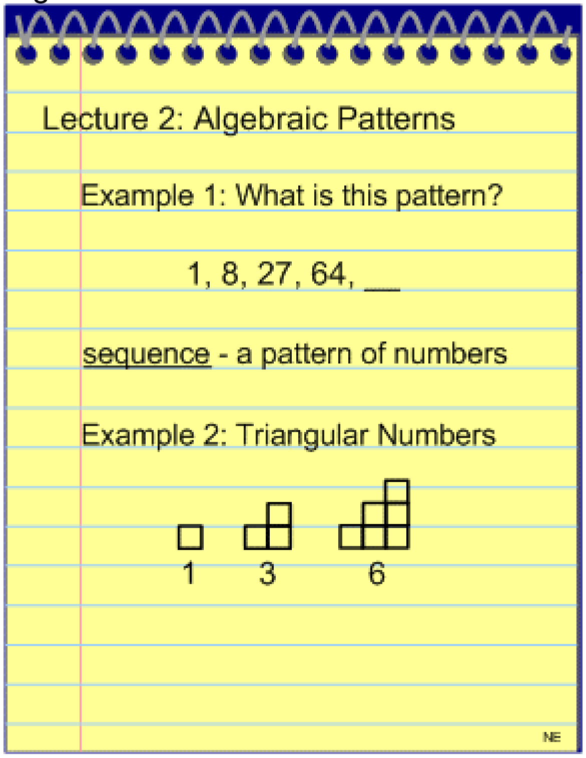
Lecture 1: Page 5

When evaluating an expression remember the following:

1) What does the variable represent?
2) Substitute that number into the expression.
3) Do the arithmetic.

Lecture 2 Notes

Alg002-01



Lecture 2: Algebraic Patterns

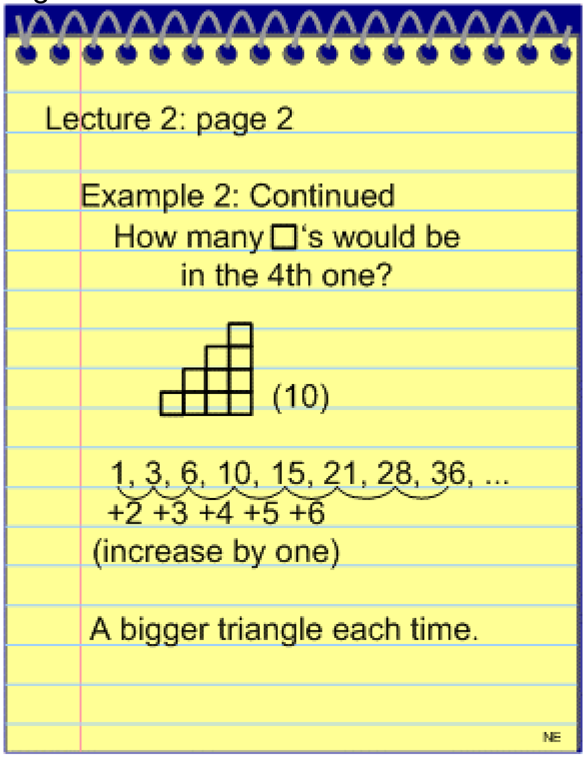
Example 1: What is this pattern?

1, 8, 27, 64, ___

<u>sequence</u> - a pattern of numbers

Example 2: Triangular Numbers

Alg002-02



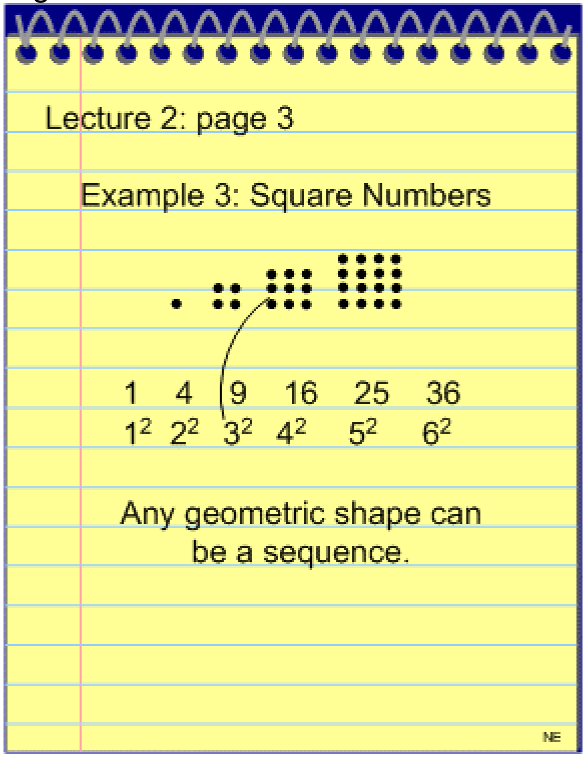
Lecture 2: page 2

Example 2: Continued
How many □'s would be
in the 4th one?

1, 3, 6, 10, 15, 21, 28, 36, ...
+2 +3 +4 +5 +6
(increase by one)

A bigger triangle each time.

Alg002-03



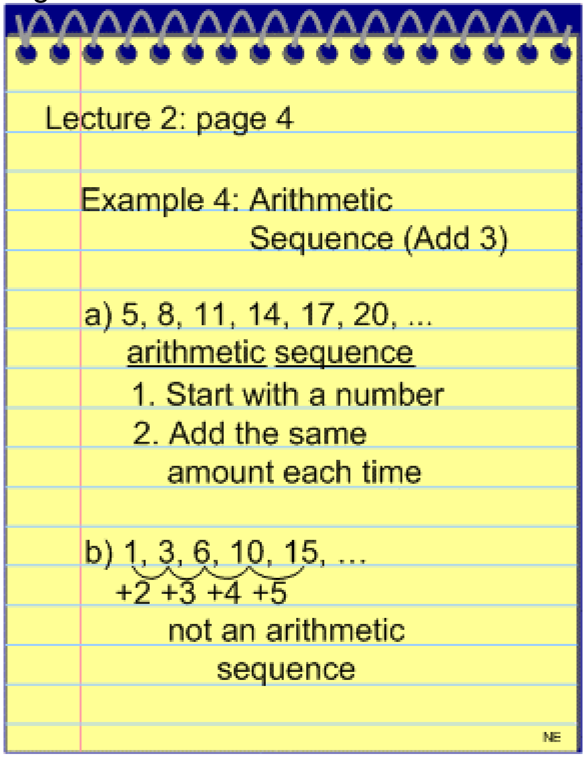
Lecture 2: page 3

Example 3: Square Numbers

1 4 9 16 25 36
1^2 2^2 3^2 4^2 5^2 6^2

Any geometric shape can
be a sequence.

Alg002-04



Lecture 2: page 4

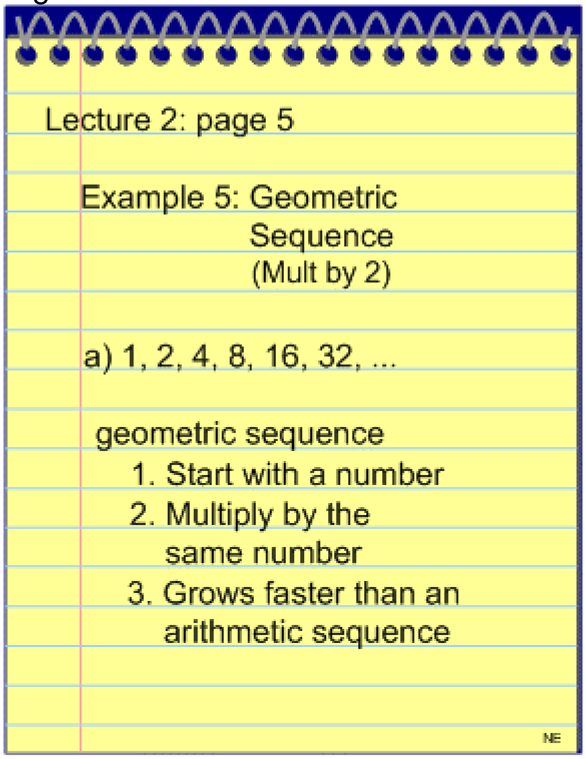
Example 4: Arithmetic
Sequence (Add 3)

a) 5, 8, 11, 14, 17, 20, ...
<u>arithmetic sequence</u>
1. Start with a number
2. Add the same
amount each time

b) 1, 3, 6, 10, 15, ...
+2 +3 +4 +5
not an arithmetic
sequence

Lecture 2 Notes, Continued

Alg002-05



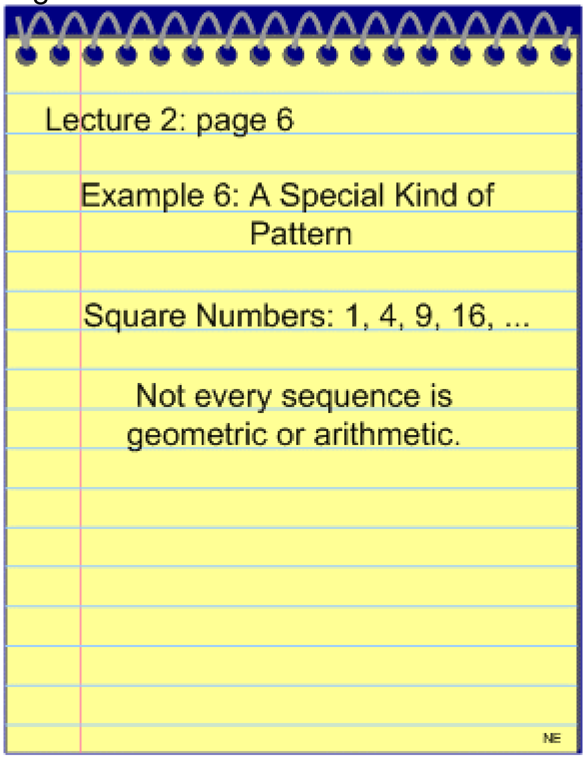
Lecture 2: page 5

Example 5: Geometric Sequence
(Mult by 2)

a) 1, 2, 4, 8, 16, 32, ...

geometric sequence
1. Start with a number
2. Multiply by the same number
3. Grows faster than an arithmetic sequence

Alg002-06



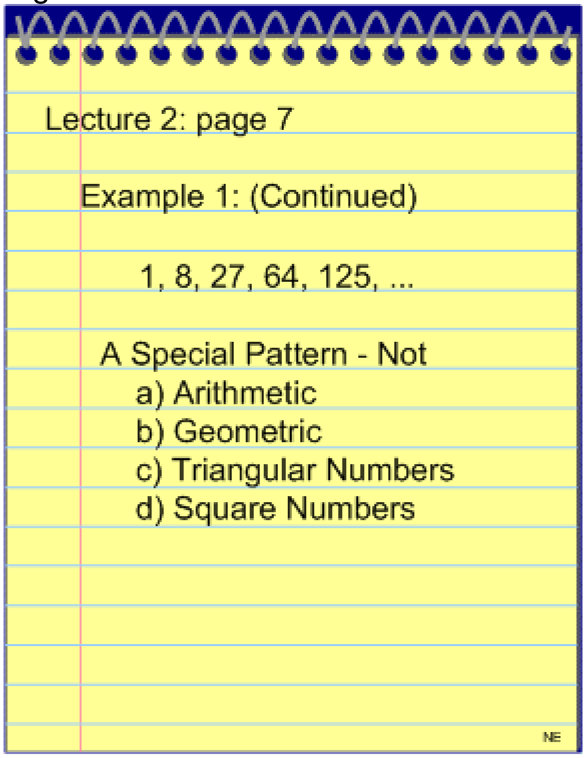
Lecture 2: page 6

Example 6: A Special Kind of Pattern

Square Numbers: 1, 4, 9, 16, ...

Not every sequence is geometric or arithmetic.

Alg002-07



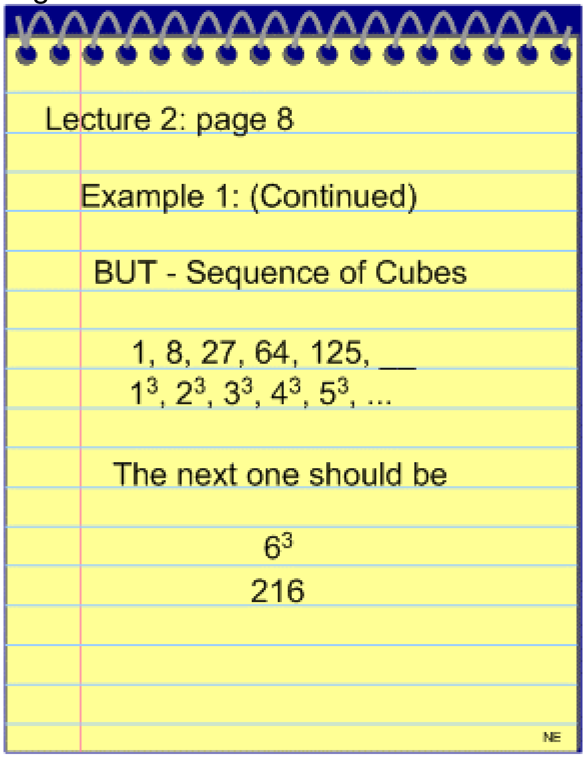
Lecture 2: page 7

Example 1: (Continued)

1, 8, 27, 64, 125, ...

A Special Pattern - Not
a) Arithmetic
b) Geometric
c) Triangular Numbers
d) Square Numbers

Alg002-08



Lecture 2: page 8

Example 1: (Continued)

BUT - Sequence of Cubes

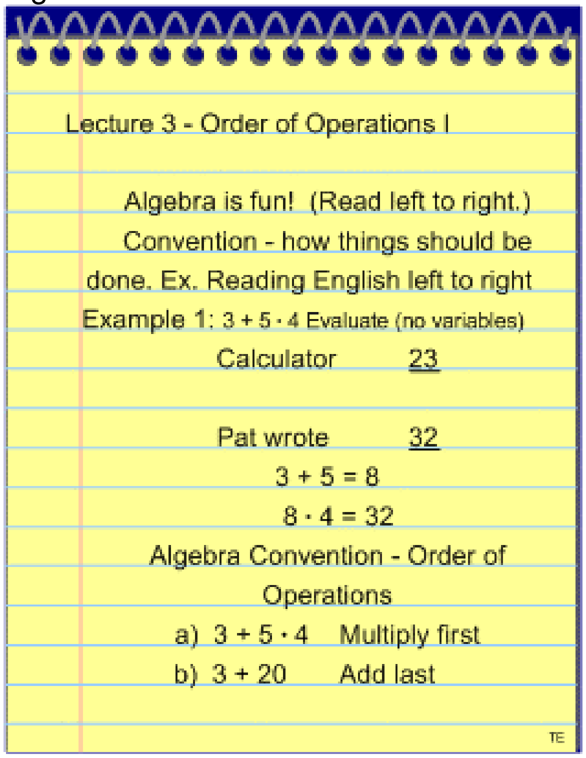
1, 8, 27, 64, 125, ___
$1^3, 2^3, 3^3, 4^3, 5^3, \ldots$

The next one should be

6^3
216

Lecture 3 Notes

Alg003-01



Lecture 3 - Order of Operations I

Algebra is fun! (Read left to right.)

Convention - how things should be done. Ex. Reading English left to right

Example 1: 3 + 5 · 4 Evaluate (no variables)

Calculator <u>23</u>

Pat wrote <u>32</u>

3 + 5 = 8

8 · 4 = 32

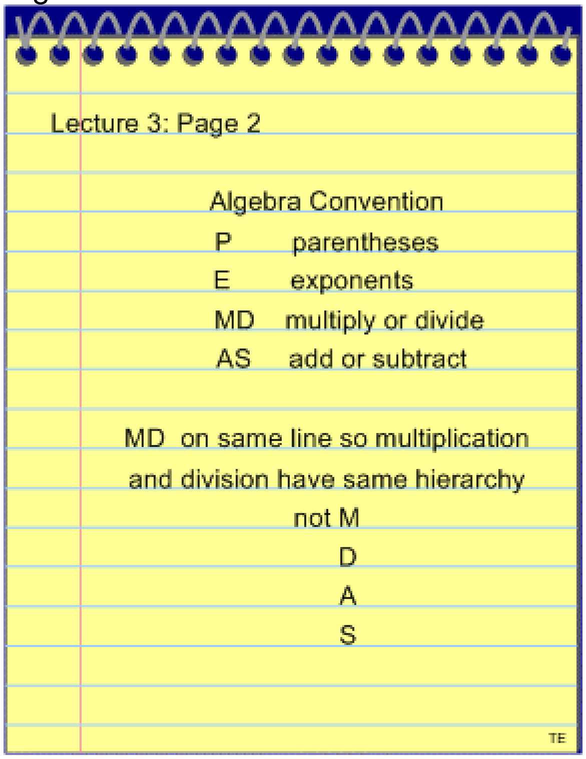
Algebra Convention - Order of Operations

a) 3 + 5 · 4 Multiply first

b) 3 + 20 Add last

TE

Alg003-02



Lecture 3: Page 2

Algebra Convention

P parentheses

E exponents

MD multiply or divide

AS add or subtract

MD on same line so multiplication and division have same hierarchy

not M

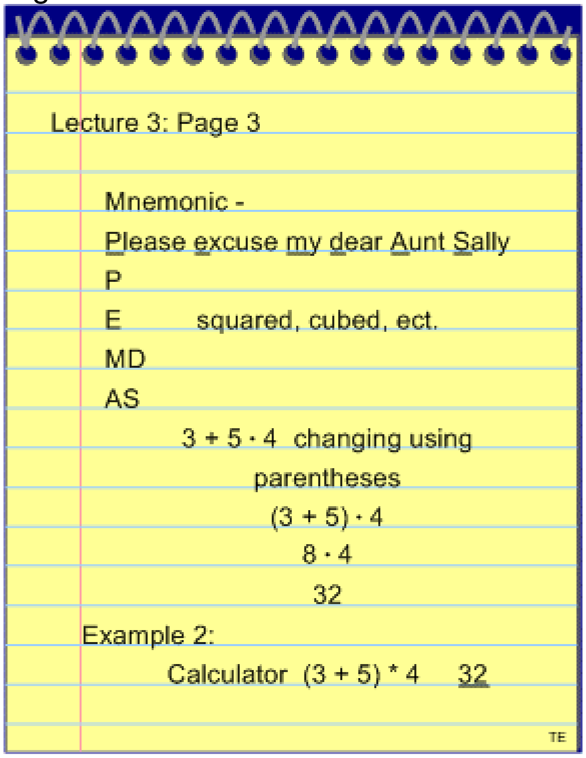
D

A

S

TE

Alg003-03



Lecture 3: Page 3

Mnemonic -

<u>P</u>lease <u>e</u>xcuse <u>m</u>y <u>d</u>ear <u>A</u>unt <u>S</u>ally

P

E squared, cubed, ect.

MD

AS

3 + 5 · 4 changing using parentheses

(3 + 5) · 4

8 · 4

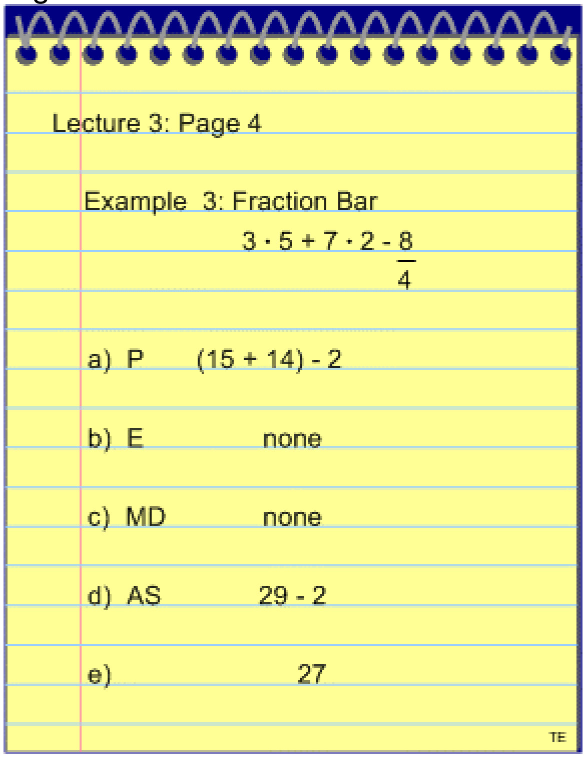
32

Example 2:

Calculator (3 + 5) * 4 <u>32</u>

TE

Alg003-04



Lecture 3: Page 4

Example 3: Fraction Bar

$3 \cdot 5 + 7 \cdot 2 - \frac{8}{4}$

a) P (15 + 14) - 2

b) E none

c) MD none

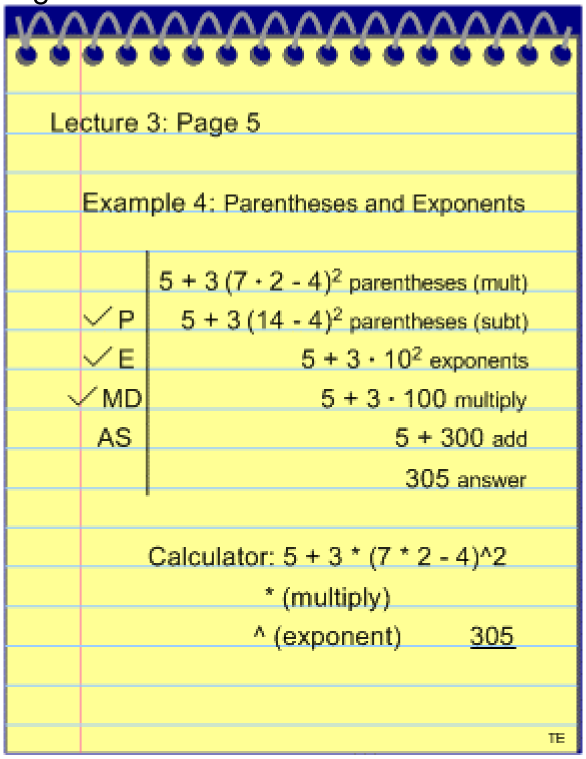
d) AS 29 - 2

e) 27

TE

Lecture 3 Notes, Continued

Alg003-05



Lecture 3: Page 5

Example 4: Parentheses and Exponents

$5 + 3\,(7 \cdot 2 - 4)^2$ parentheses (mult)

✓ P $5 + 3\,(14 - 4)^2$ parentheses (subt)

✓ E $5 + 3 \cdot 10^2$ exponents

✓ MD $5 + 3 \cdot 100$ multiply

AS $5 + 300$ add

305 answer

Calculator: 5 + 3 * (7 * 2 - 4)^2

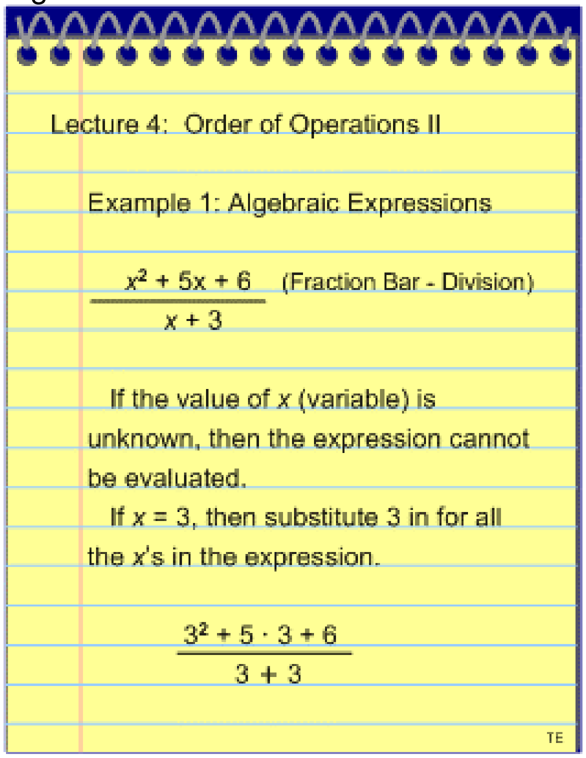
* (multiply)

^ (exponent) 305

TE

Lecture 4 Notes

Alg004-01



Lecture 4: Order of Operations II

Example 1: Algebraic Expressions

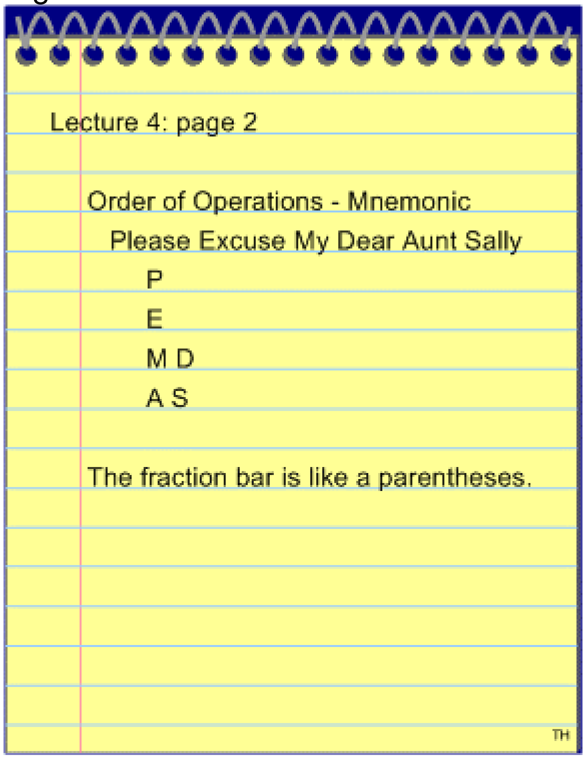
$$\frac{x^2 + 5x + 6}{x + 3}$$ (Fraction Bar - Division)

If the value of *x* (variable) is unknown, then the expression cannot be evaluated.

If $x = 3$, then substitute 3 in for all the *x*'s in the expression.

$$\frac{3^2 + 5 \cdot 3 + 6}{3 + 3}$$

Alg004-02



Lecture 4: page 2

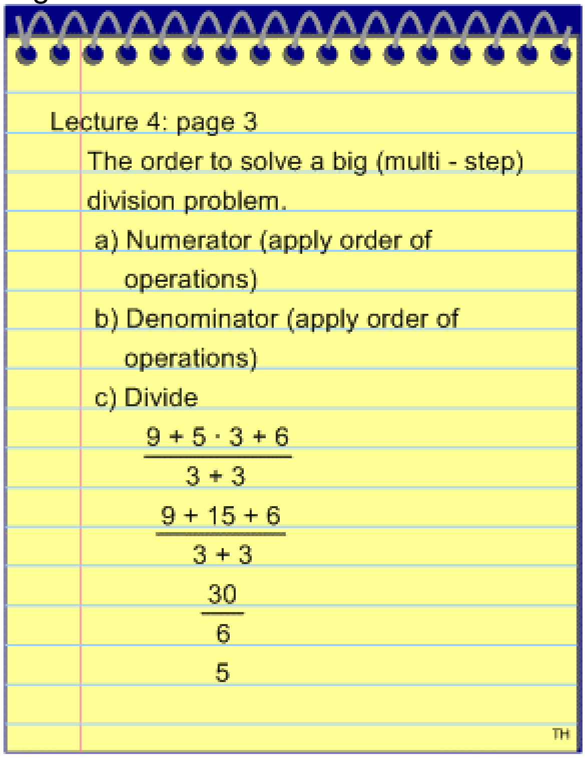
Order of Operations - Mnemonic

Please Excuse My Dear Aunt Sally

P
E
M D
A S

The fraction bar is like a parentheses.

Alg004-03



Lecture 4: page 3

The order to solve a big (multi - step) division problem.

a) Numerator (apply order of operations)

b) Denominator (apply order of operations)

c) Divide

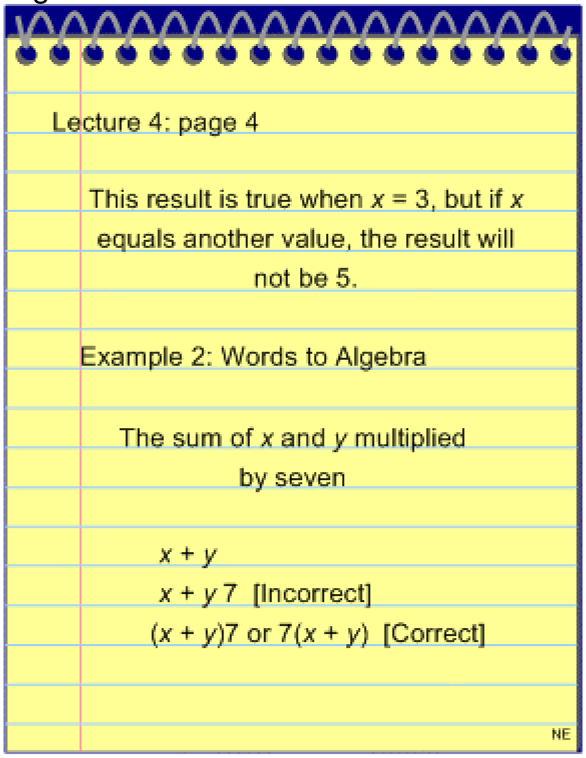
$$\frac{9 + 5 \cdot 3 + 6}{3 + 3}$$

$$\frac{9 + 15 + 6}{3 + 3}$$

$$\frac{30}{6}$$

$$5$$

Alg004-04



Lecture 4: page 4

This result is true when $x = 3$, but if *x* equals another value, the result will not be 5.

Example 2: Words to Algebra

The sum of *x* and *y* multiplied by seven

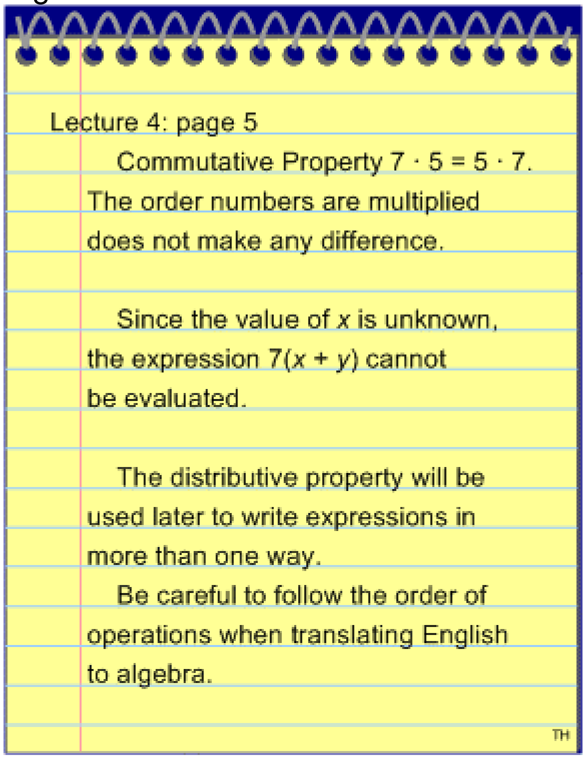
$x + y$

$x + y\,7$ [Incorrect]

$(x + y)7$ or $7(x + y)$ [Correct]

Lecture 4 Notes, Continued

Alg004-05



Lecture 4: page 5

Commutative Property $7 \cdot 5 = 5 \cdot 7$. The order numbers are multiplied does not make any difference.

Since the value of x is unknown, the expression $7(x + y)$ cannot be evaluated.

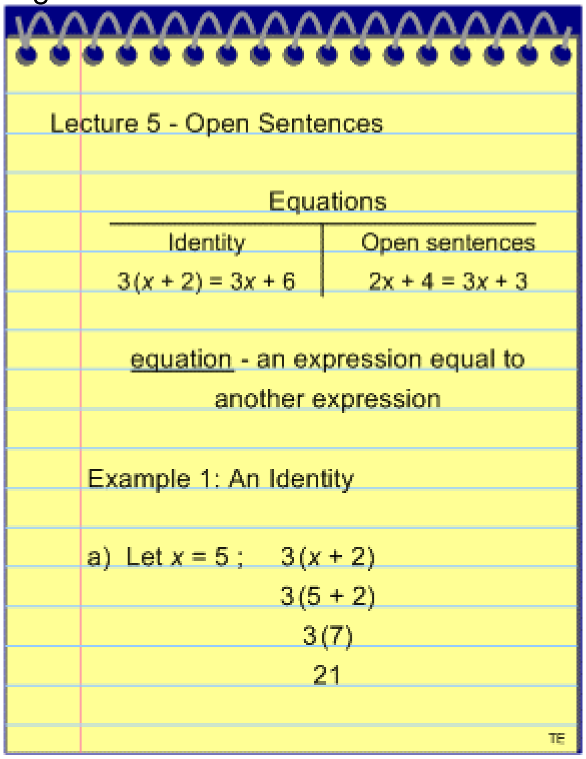
The distributive property will be used later to write expressions in more than one way.

Be careful to follow the order of operations when translating English to algebra.

TH

Lecture 5 Notes

Alg005-01



Lecture 5 - Open Sentences

Equations	
Identity	Open sentences
$3(x+2) = 3x + 6$	$2x + 4 = 3x + 3$

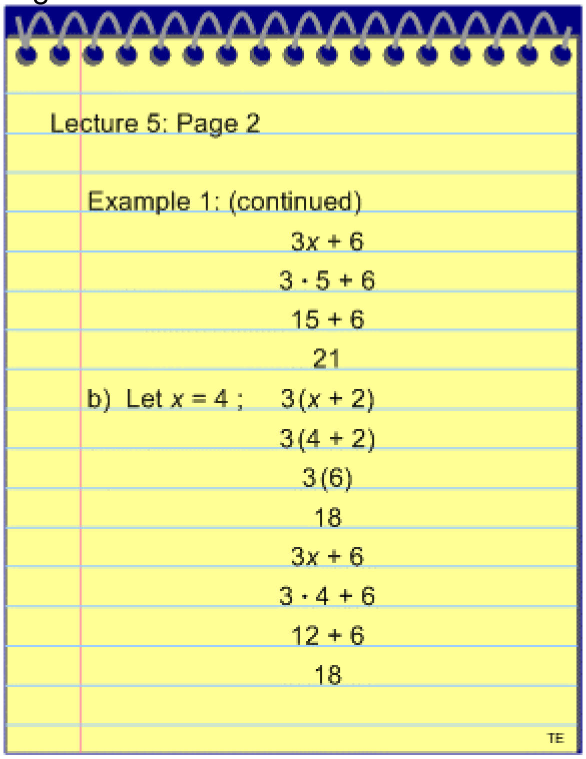
equation - an expression equal to another expression

Example 1: An Identity

a) Let $x = 5$; $3(x+2)$
$3(5+2)$
$3(7)$
21

TE

Alg005-02



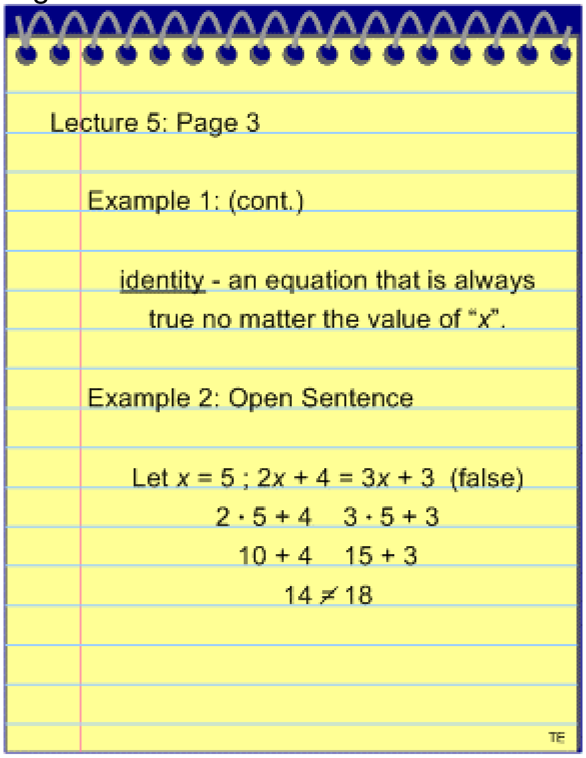
Lecture 5: Page 2

Example 1: (continued)
$3x + 6$
$3 \cdot 5 + 6$
$15 + 6$
21

b) Let $x = 4$; $3(x+2)$
$3(4+2)$
$3(6)$
18
$3x + 6$
$3 \cdot 4 + 6$
$12 + 6$
18

TE

Alg005-03



Lecture 5: Page 3

Example 1: (cont.)

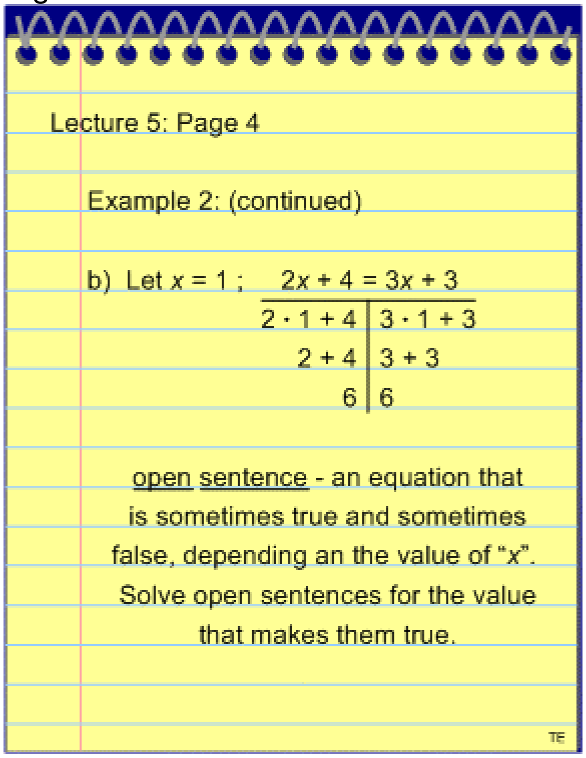
identity - an equation that is always true no matter the value of "x".

Example 2: Open Sentence

Let $x = 5$; $2x + 4 = 3x + 3$ (false)
$2 \cdot 5 + 4 \quad 3 \cdot 5 + 3$
$10 + 4 \quad 15 + 3$
$14 \neq 18$

TE

Alg005-04



Lecture 5: Page 4

Example 2: (continued)

b) Let $x = 1$; $2x + 4 = 3x + 3$

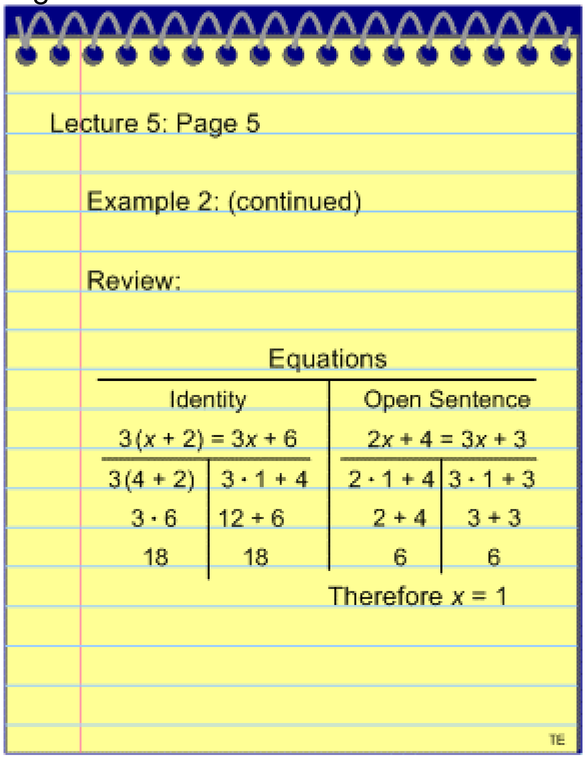
$2 \cdot 1 + 4$	$3 \cdot 1 + 3$
$2 + 4$	$3 + 3$
6	6

open sentence - an equation that is sometimes true and sometimes false, depending an the value of "x". Solve open sentences for the value that makes them true.

TE

Lecture 5 Notes, Continued

Alg005-05



Lecture 5: Page 5

Example 2: (continued)

Review:

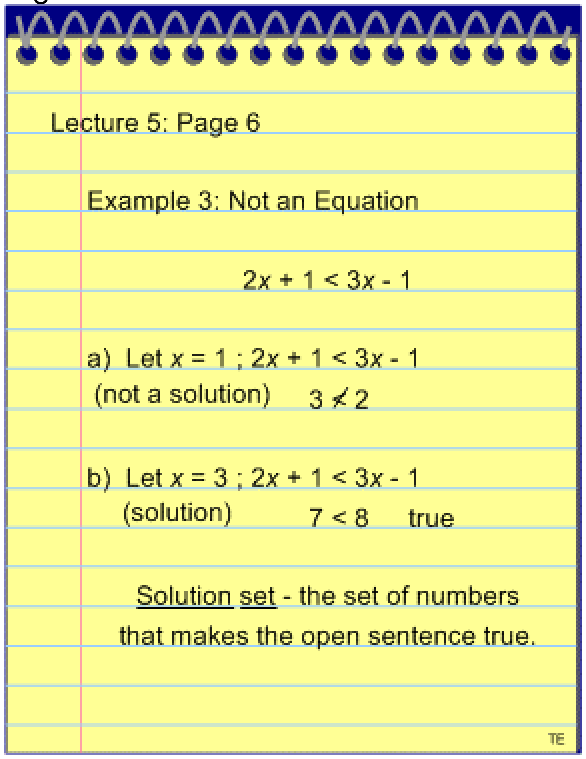
Equations

Identity		Open Sentence	
$3(x + 2) = 3x + 6$		$2x + 4 = 3x + 3$	
$3(4 + 2)$	$3 \cdot 1 + 4$	$2 \cdot 1 + 4$	$3 \cdot 1 + 3$
$3 \cdot 6$	$12 + 6$	$2 + 4$	$3 + 3$
18	18	6	6

Therefore $x = 1$

TE

Alg005-06



Lecture 5: Page 6

Example 3: Not an Equation

$$2x + 1 < 3x - 1$$

a) Let $x = 1$; $2x + 1 < 3x - 1$

(not a solution) $3 \nless 2$

b) Let $x = 3$; $2x + 1 < 3x - 1$

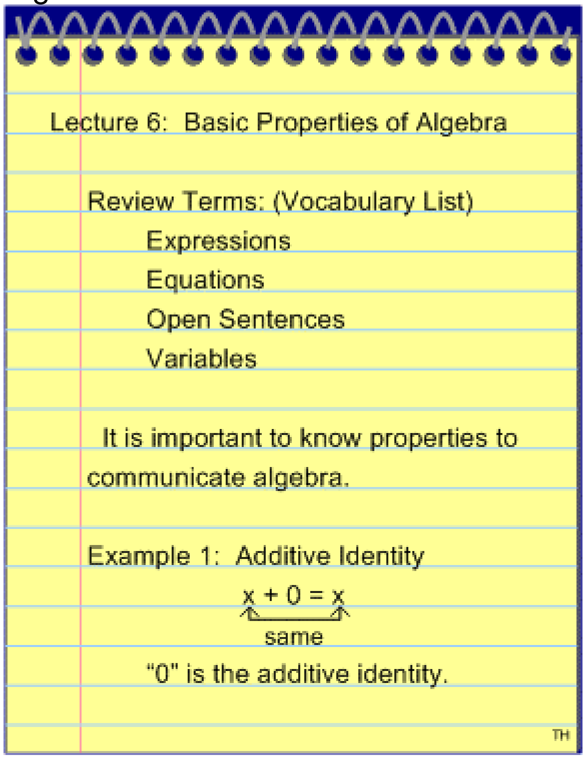
(solution) $7 < 8$ true

<u>Solution set</u> - the set of numbers that makes the open sentence true.

TE

Lecture 6 Notes

Alg006-01



Lecture 6: Basic Properties of Algebra

Review Terms: (Vocabulary List)

- Expressions
- Equations
- Open Sentences
- Variables

It is important to know properties to communicate algebra.

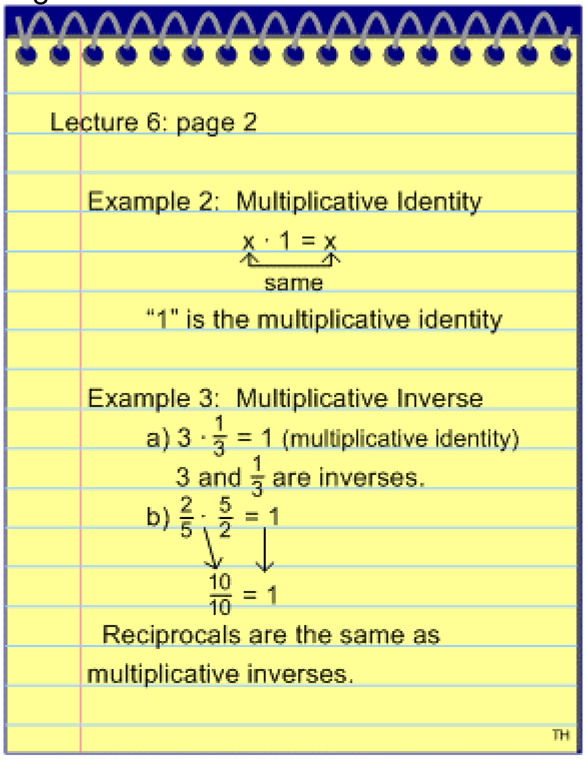
Example 1: Additive Identity

$$\underbrace{x + 0 = x}_{\text{same}}$$

"0" is the additive identity.

TH

Alg006-02



Lecture 6: page 2

Example 2: Multiplicative Identity

$$\underbrace{x \cdot 1 = x}_{\text{same}}$$

"1" is the multiplicative identity

Example 3: Multiplicative Inverse

a) $3 \cdot \frac{1}{3} = 1$ (multiplicative identity)

3 and $\frac{1}{3}$ are inverses.

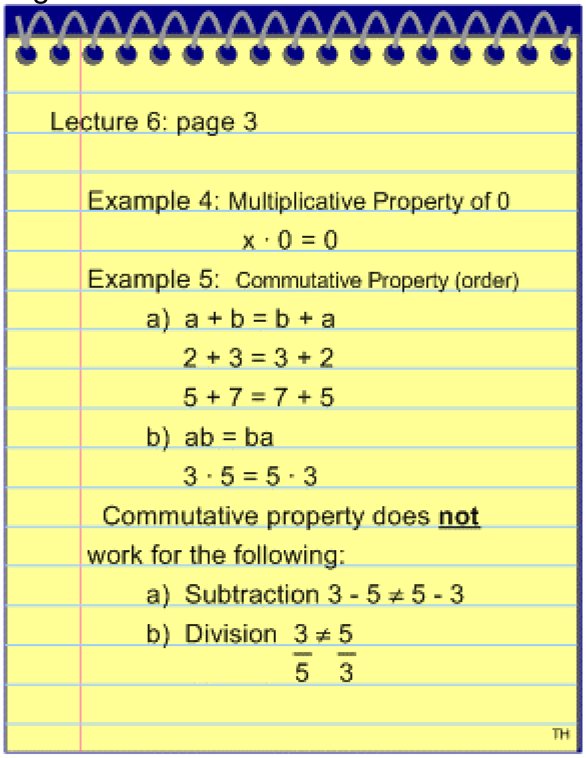
b) $\frac{2}{5} \cdot \frac{5}{2} = 1$

$$\frac{10}{10} = 1$$

Reciprocals are the same as multiplicative inverses.

TH

Alg006-03



Lecture 6: page 3

Example 4: Multiplicative Property of 0

$$x \cdot 0 = 0$$

Example 5: Commutative Property (order)

a) $a + b = b + a$

$2 + 3 = 3 + 2$

$5 + 7 = 7 + 5$

b) $ab = ba$

$3 \cdot 5 = 5 \cdot 3$

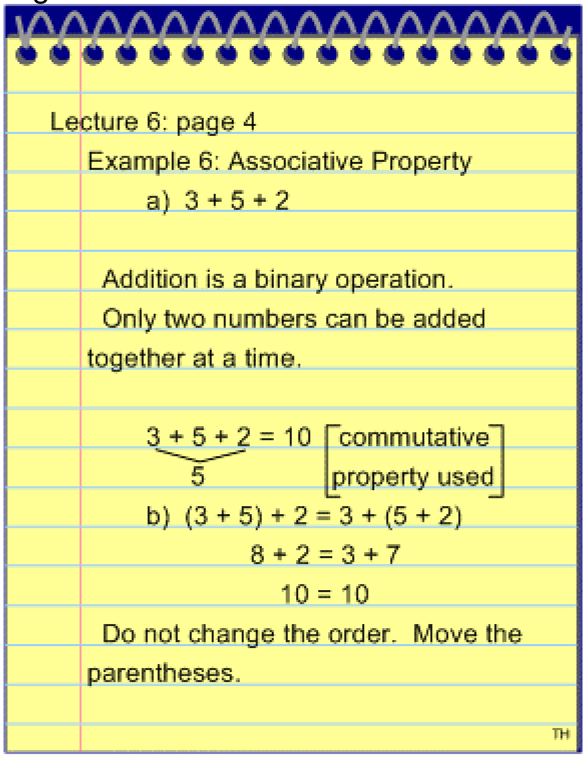
Commutative property does **not** work for the following:

a) Subtraction $3 - 5 \neq 5 - 3$

b) Division $\frac{3}{5} \neq \frac{5}{3}$

TH

Alg006-04



Lecture 6: page 4

Example 6: Associative Property

a) $3 + 5 + 2$

Addition is a binary operation. Only two numbers can be added together at a time.

$$\underbrace{3 + 5}_{5} + 2 = 10 \quad \left[\begin{array}{l}\text{commutative}\\ \text{property used}\end{array}\right]$$

b) $(3 + 5) + 2 = 3 + (5 + 2)$

$8 + 2 = 3 + 7$

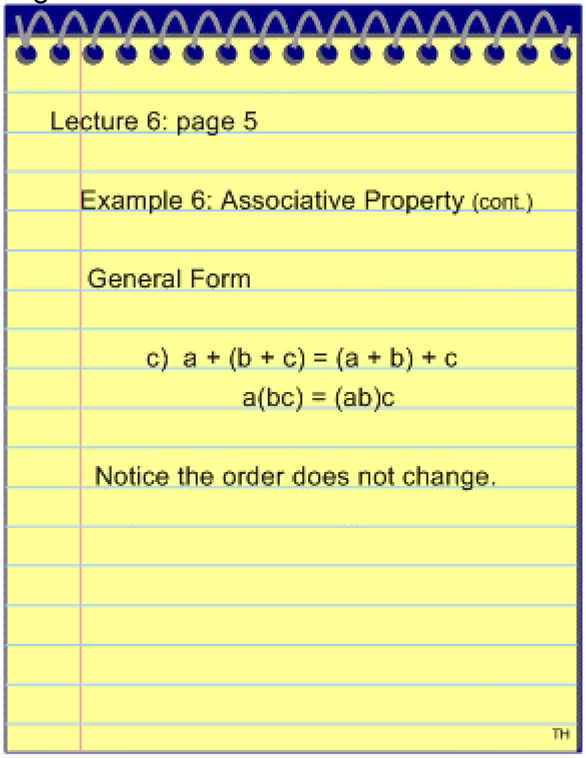
$10 = 10$

Do not change the order. Move the parentheses.

TH

Lecture 6 Notes, Continued

Alg006-05



Lecture 6: page 5

Example 6: Associative Property (cont.)

General Form

c) $a + (b + c) = (a + b) + c$

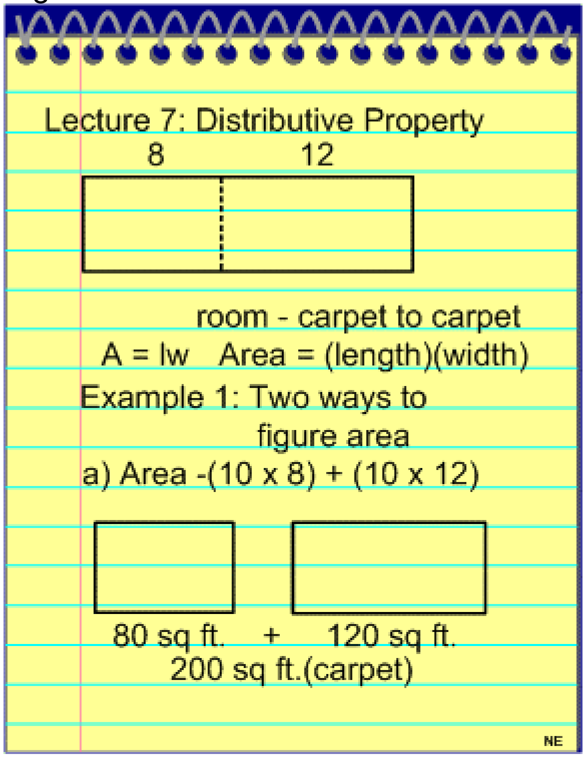
$a(bc) = (ab)c$

Notice the order does not change.

TH

Lecture 7 Notes

Alg007-01



Lecture 7: Distributive Property

8 12

room - carpet to carpet

A = lw Area = (length)(width)

Example 1: Two ways to figure area

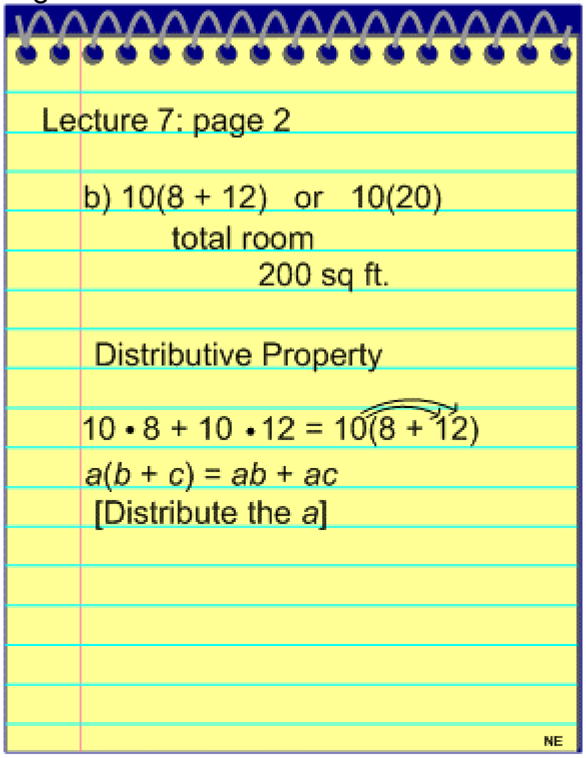
a) Area -(10 x 8) + (10 x 12)

80 sq ft. + 120 sq ft.

200 sq ft.(carpet)

NE

Alg007-02



Lecture 7: page 2

b) 10(8 + 12) or 10(20)

total room

200 sq ft.

Distributive Property

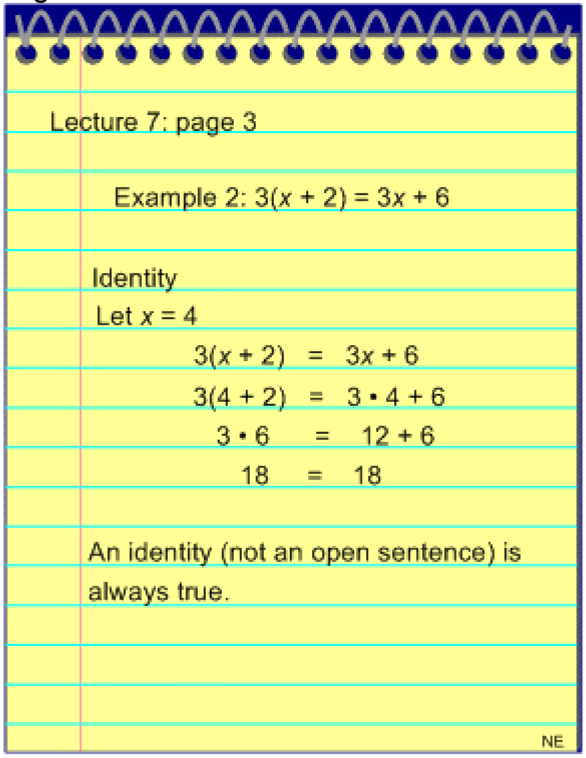
$10 \cdot 8 + 10 \cdot 12 = 10(8 + 12)$

$a(b + c) = ab + ac$

[Distribute the *a*]

NE

Alg007-03



Lecture 7: page 3

Example 2: $3(x + 2) = 3x + 6$

Identity

Let $x = 4$

$$3(x + 2) = 3x + 6$$

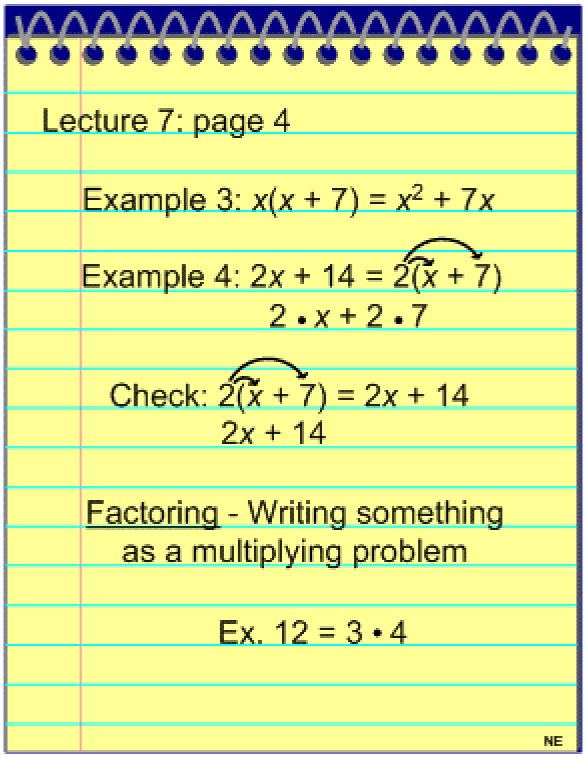
$$3(4 + 2) = 3 \cdot 4 + 6$$

$$3 \cdot 6 = 12 + 6$$

$$18 = 18$$

An identity (not an open sentence) is always true.

NE



Lecture 7: page 4

Example 3: $x(x + 7) = x^2 + 7x$

Example 4: $2x + 14 = 2(x + 7)$

$2 \cdot x + 2 \cdot 7$

Check: $2(x + 7) = 2x + 14$

$2x + 14$

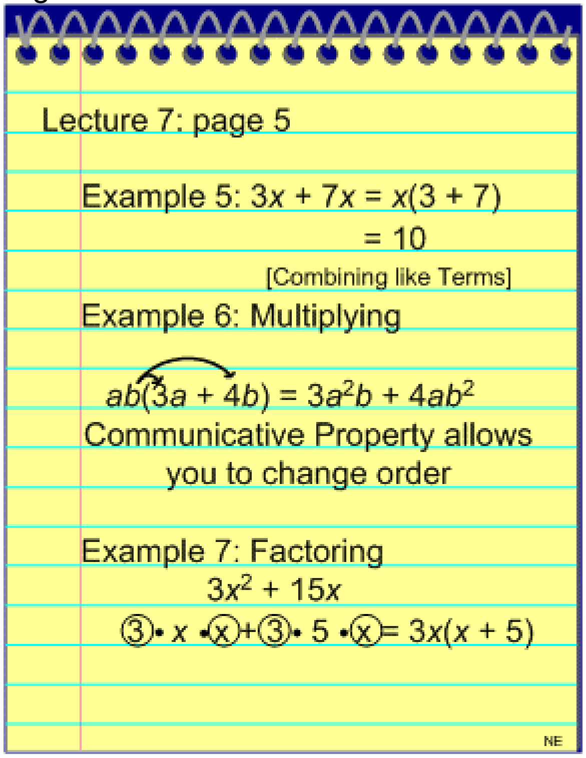
<u>Factoring</u> - Writing something as a multiplying problem

Ex. $12 = 3 \cdot 4$

NE

Lecture 7 Notes, Continued

Alg007-05



Lecture 7: page 5

Example 5: $3x + 7x = x(3 + 7)$

$= 10$

[Combining like Terms]

Example 6: Multiplying

$ab(3a + 4b) = 3a^2b + 4ab^2$

Communicative Property allows you to change order

Example 7: Factoring

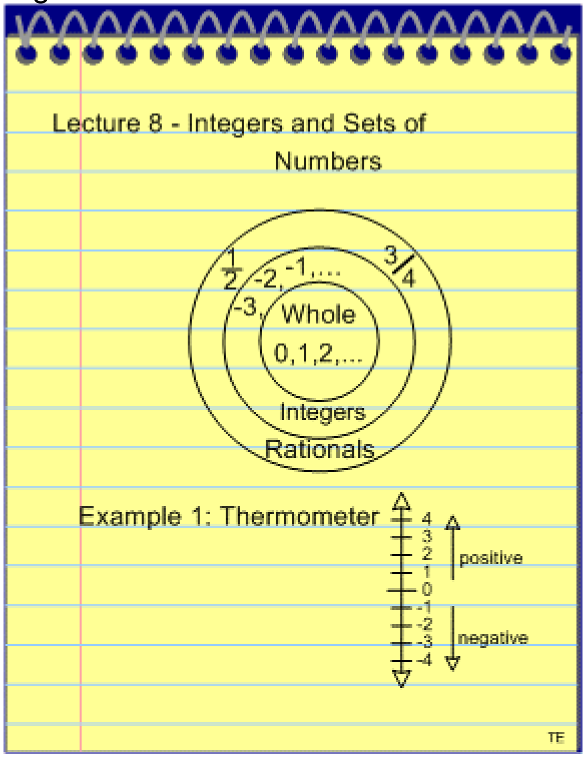
$3x^2 + 15x$

$\textcircled{3} \cdot x \cdot \textcircled{x} + \textcircled{3} \cdot 5 \cdot \textcircled{x} = 3x(x + 5)$

NE

Lecture 8 Notes

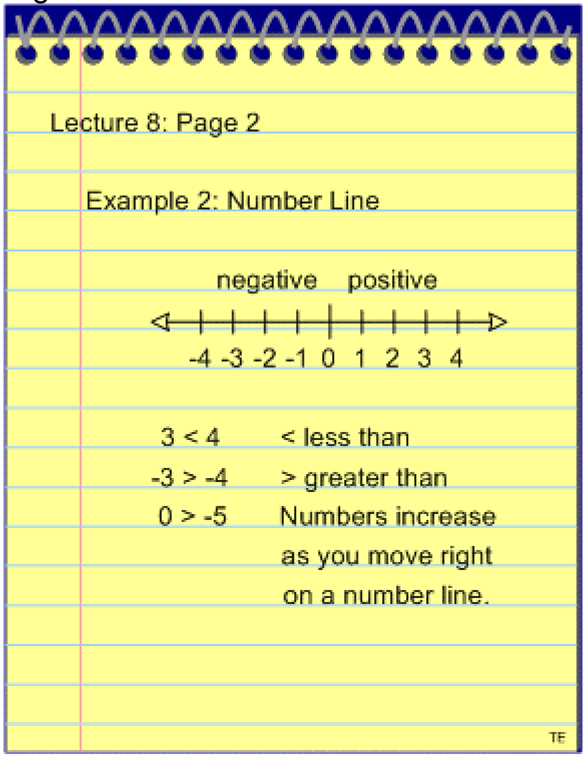
Alg008-01



Lecture 8 - Integers and Sets of Numbers

Example 1: Thermometer

Alg008-02



Lecture 8: Page 2

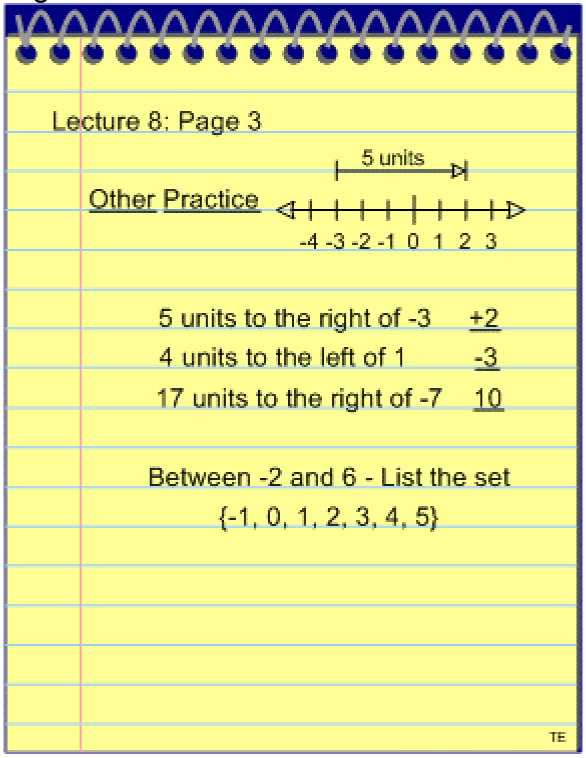
Example 2: Number Line

$3 < 4$ < less than

$-3 > -4$ > greater than

$0 > -5$ Numbers increase as you move right on a number line.

Alg008-03



Lecture 8: Page 3

Other Practice

5 units to the right of -3 +2

4 units to the left of 1 -3

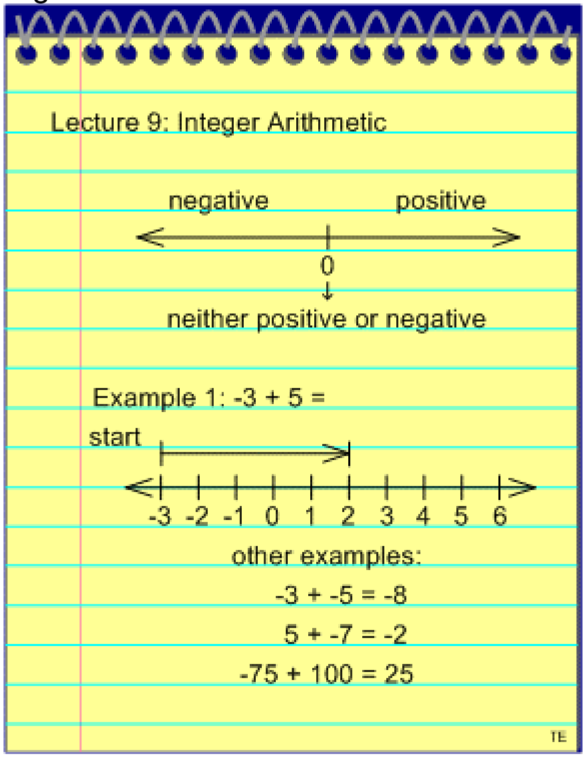
17 units to the right of -7 10

Between -2 and 6 - List the set

{-1, 0, 1, 2, 3, 4, 5}

Lecture 9 Notes

Alg009-01



Lecture 9: Integer Arithmetic

negative positive

0
↓
neither positive or negative

Example 1: -3 + 5 =

start

-3 -2 -1 0 1 2 3 4 5 6

other examples:

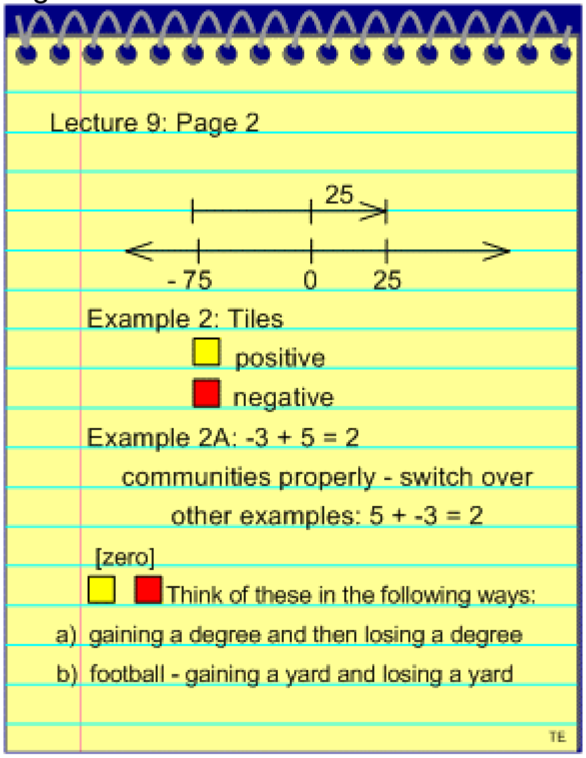
-3 + -5 = -8

5 + -7 = -2

-75 + 100 = 25

TE

Alg009-02



Lecture 9: Page 2

25

- 75 0 25

Example 2: Tiles

🟨 positive

🟥 negative

Example 2A: -3 + 5 = 2

communities properly - switch over

other examples: 5 + -3 = 2

[zero]

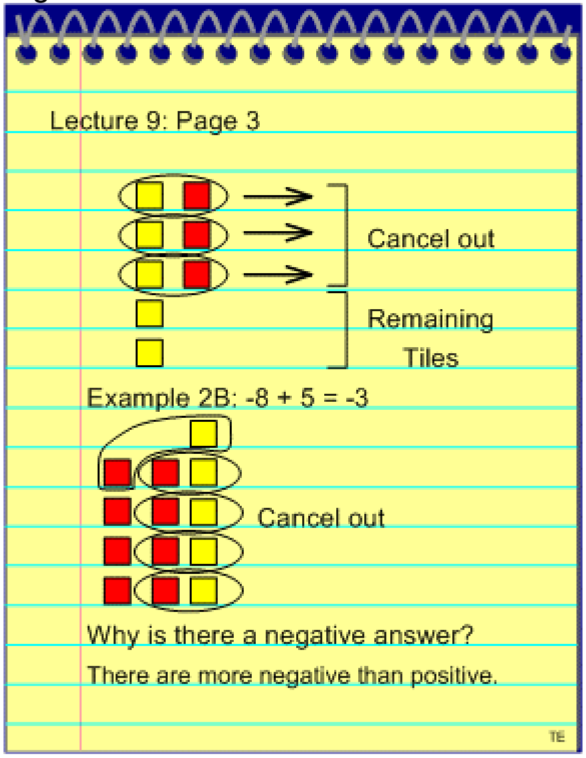
🟨 🟥 Think of these in the following ways:

a) gaining a degree and then losing a degree

b) football - gaining a yard and losing a yard

TE

Alg009-03



Lecture 9: Page 3

🟨 🟥 →
🟨 🟥 →
🟨 🟥 →
Cancel out

🟨
🟨
Remaining Tiles

Example 2B: -8 + 5 = -3

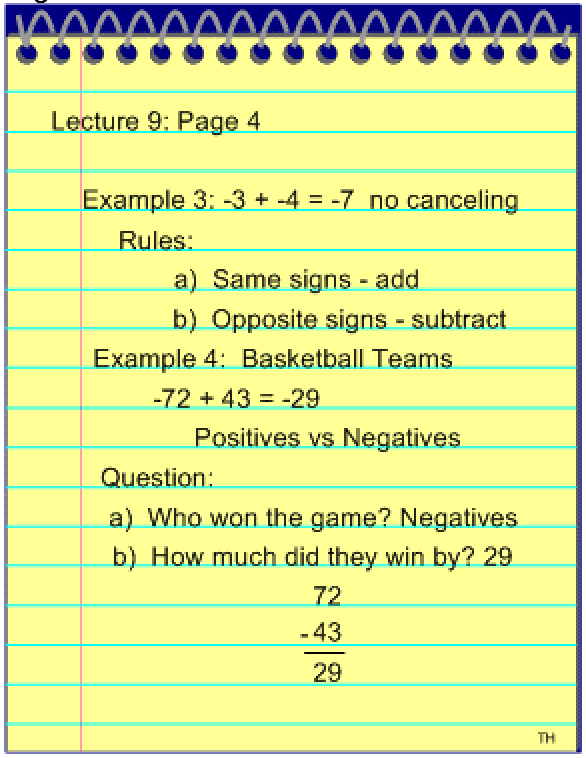
🟨
🟥 🟥 🟨
🟥 🟥 🟨
🟥 🟥 🟨
🟥 🟥 🟨
Cancel out

Why is there a negative answer?

There are more negative than positive.

TE

Alg009-04



Lecture 9: Page 4

Example 3: -3 + -4 = -7 no canceling

Rules:

a) Same signs - add

b) Opposite signs - subtract

Example 4: Basketball Teams

-72 + 43 = -29

Positives vs Negatives

Question:

a) Who won the game? Negatives

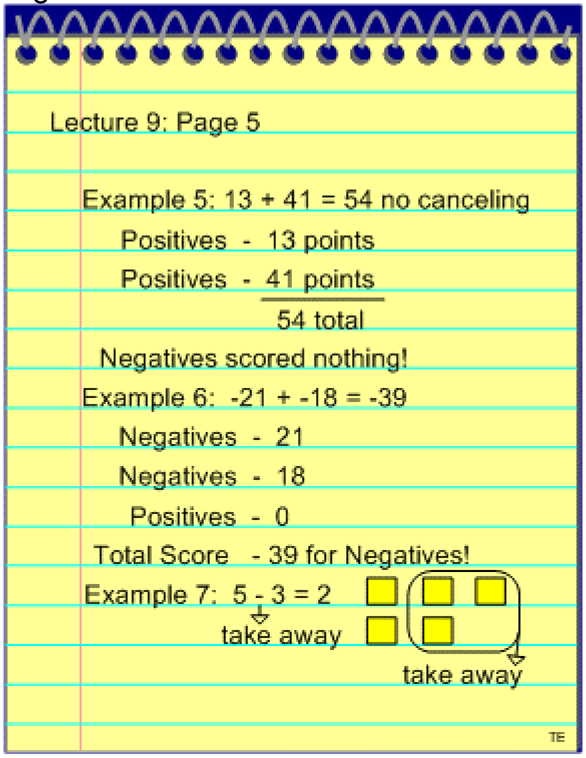
b) How much did they win by? 29

$$\begin{array}{r} 72 \\ -43 \\ \hline 29 \end{array}$$

TH

Lecture 9 Notes, Continued

Alg009-05



Lecture 9: Page 5

Example 5: 13 + 41 = 54 no canceling

Positives - 13 points

Positives - 41 points

54 total

Negatives scored nothing!

Example 6: -21 + -18 = -39

Negatives - 21

Negatives - 18

Positives - 0

Total Score - 39 for Negatives!

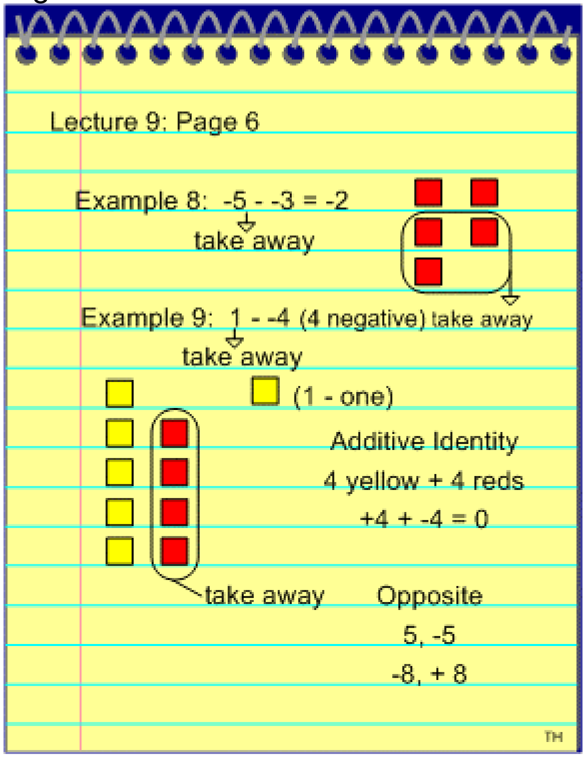
Example 7: 5 - 3 = 2

take away

take away

TE

Alg009-06



Lecture 9: Page 6

Example 8: -5 - -3 = -2

take away

Example 9: 1 - -4 (4 negative) take away

take away

(1 - one)

Additive Identity

4 yellow + 4 reds

+4 + -4 = 0

take away

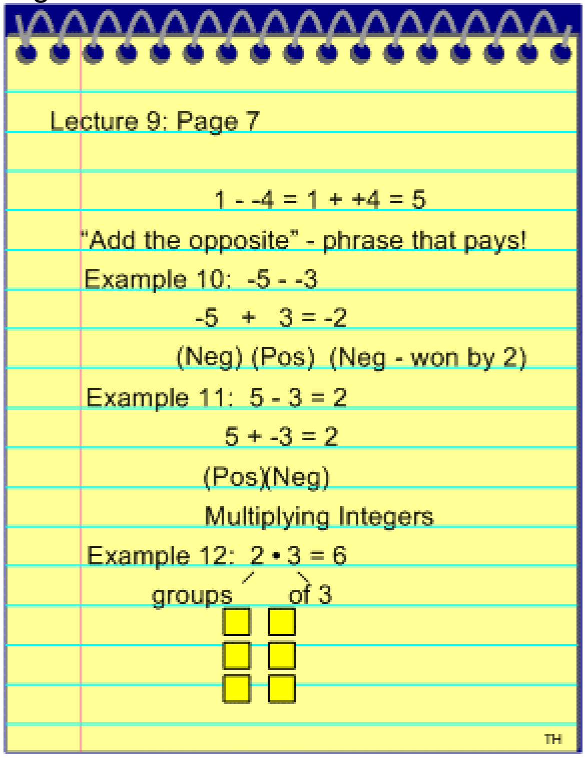
Opposite

5, -5

-8, + 8

TH

Alg009-07



Lecture 9: Page 7

1 - -4 = 1 + +4 = 5

"Add the opposite" - phrase that pays!

Example 10: -5 - -3

-5 + 3 = -2

(Neg) (Pos) (Neg - won by 2)

Example 11: 5 - 3 = 2

5 + -3 = 2

(Pos)(Neg)

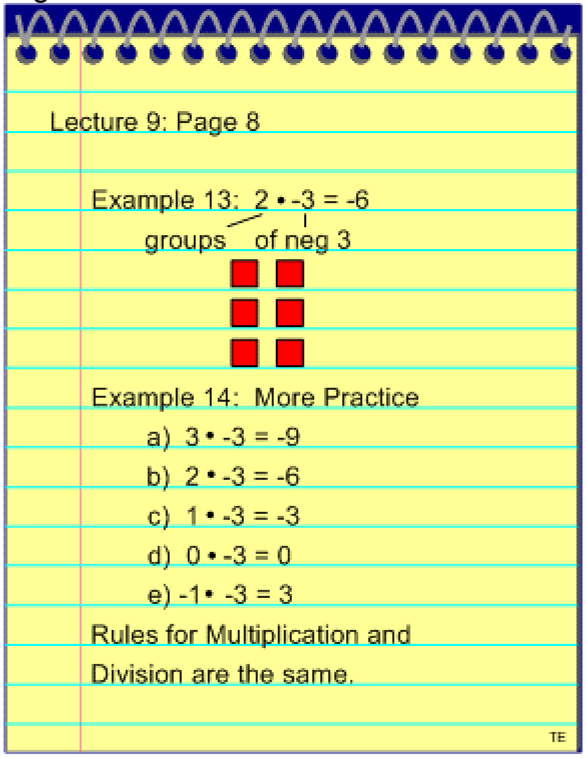
Multiplying Integers

Example 12: 2 • 3 = 6

groups of 3

TH

Alg009-08



Lecture 9: Page 8

Example 13: 2 • -3 = -6

groups of neg 3

Example 14: More Practice

a) 3 • -3 = -9

b) 2 • -3 = -6

c) 1 • -3 = -3

d) 0 • -3 = 0

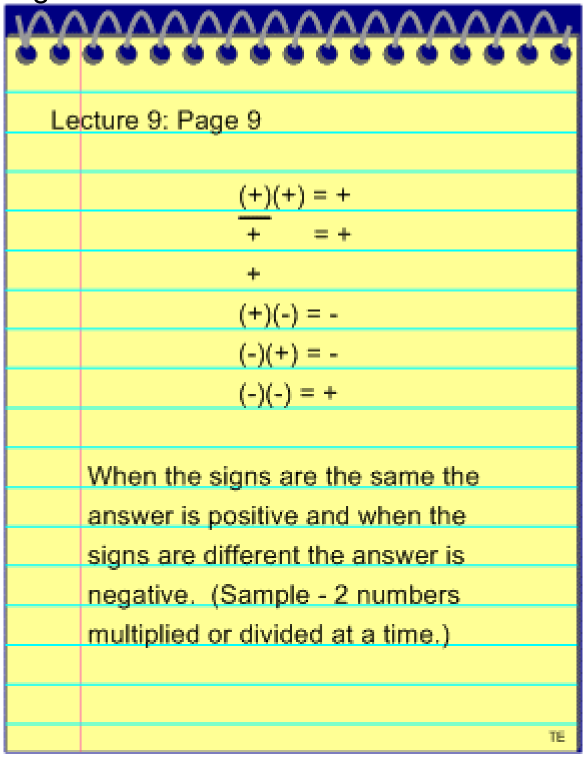
e) -1 • -3 = 3

Rules for Multiplication and

Division are the same.

TE

Alg009-09



Lecture 9: Page 9

$(+)(+) = +$

$\frac{+}{+} = +$

$(+)(-) = -$

$(-)(+) = -$

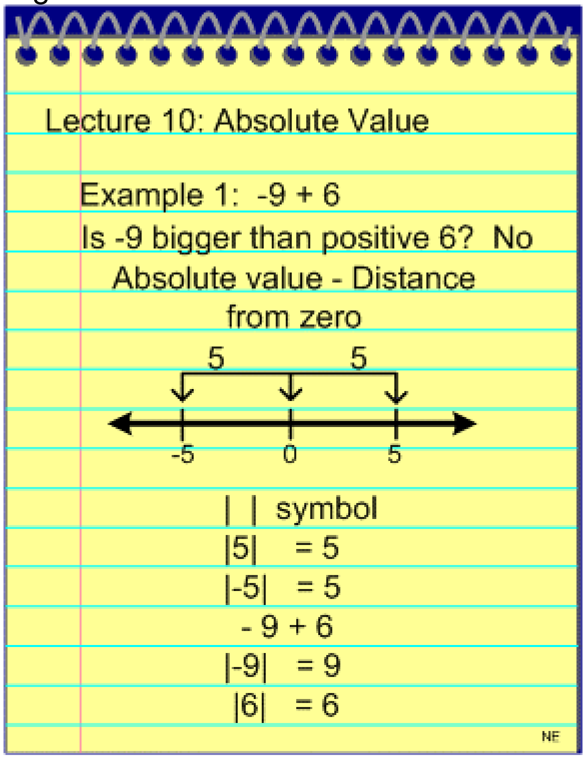
$(-)(-) = +$

When the signs are the same the answer is positive and when the signs are different the answer is negative. (Sample - 2 numbers multiplied or divided at a time.)

TE

Lecture 10 Notes

Alg010-01



Lecture 10: Absolute Value

Example 1: $-9 + 6$

Is -9 bigger than positive 6? No

Absolute value - Distance from zero

| | symbol

$|5| = 5$

$|-5| = 5$

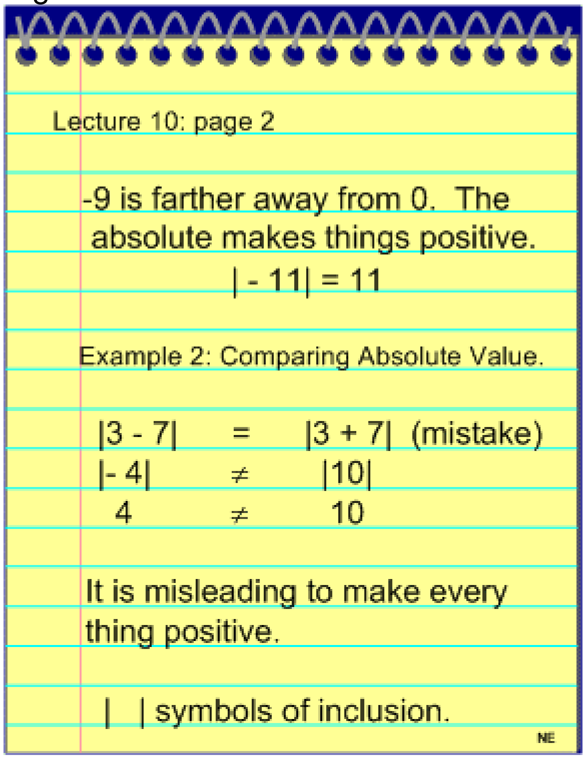
$-9 + 6$

$|-9| = 9$

$|6| = 6$

NE

Alg010-02



Lecture 10: page 2

-9 is farther away from 0. The absolute makes things positive.

$|-11| = 11$

Example 2: Comparing Absolute Value.

$|3 - 7| = |3 + 7|$ (mistake)

$|-4| \neq |10|$

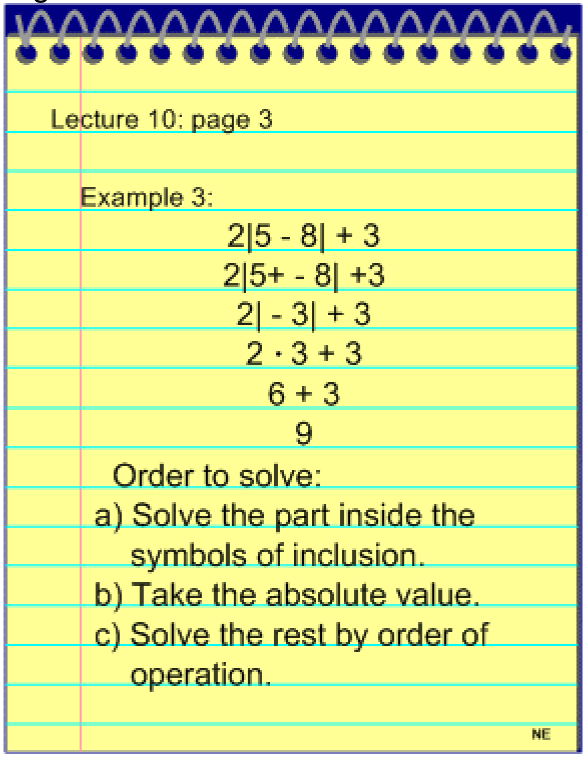
$4 \neq 10$

It is misleading to make every thing positive.

| | symbols of inclusion.

NE

Alg010-03



Lecture 10: page 3

Example 3:

$2|5 - 8| + 3$

$2|5+ - 8| +3$

$2|-3| + 3$

$2 \cdot 3 + 3$

$6 + 3$

9

Order to solve:

a) Solve the part inside the symbols of inclusion.

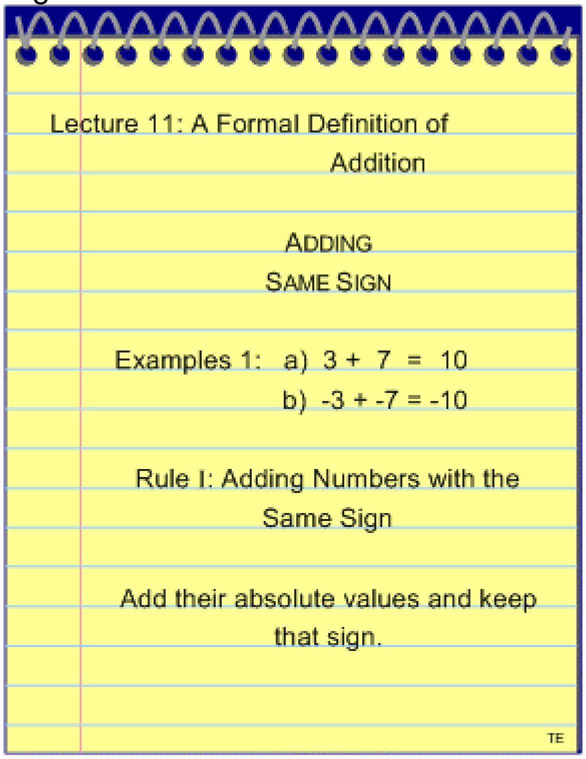
b) Take the absolute value.

c) Solve the rest by order of operation.

NE

Lecture 11 Notes

Alg011-01



Lecture 11: A Formal Definition of Addition

ADDING

SAME SIGN

Examples 1: a) 3 + 7 = 10

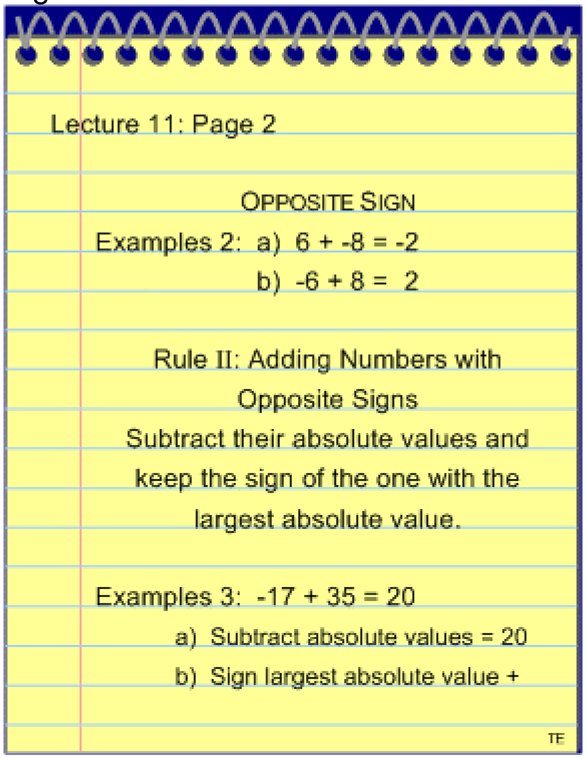
b) -3 + -7 = -10

Rule I: Adding Numbers with the Same Sign

Add their absolute values and keep that sign.

TE

Alg011-02



Lecture 11: Page 2

OPPOSITE SIGN

Examples 2: a) 6 + -8 = -2

b) -6 + 8 = 2

Rule II: Adding Numbers with Opposite Signs

Subtract their absolute values and keep the sign of the one with the largest absolute value.

Examples 3: -17 + 35 = 20

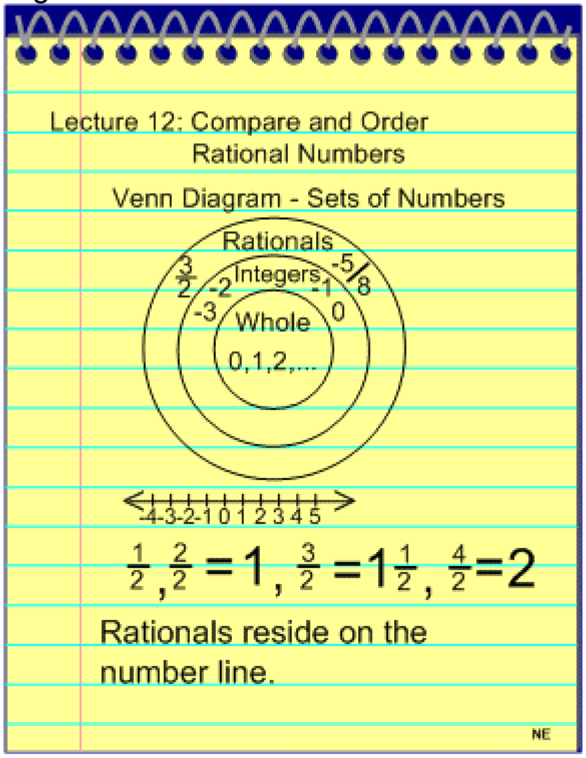
a) Subtract absolute values = 20

b) Sign largest absolute value +

TE

Lecture 12 Notes

Alg012-01



Lecture 12: Compare and Order Rational Numbers

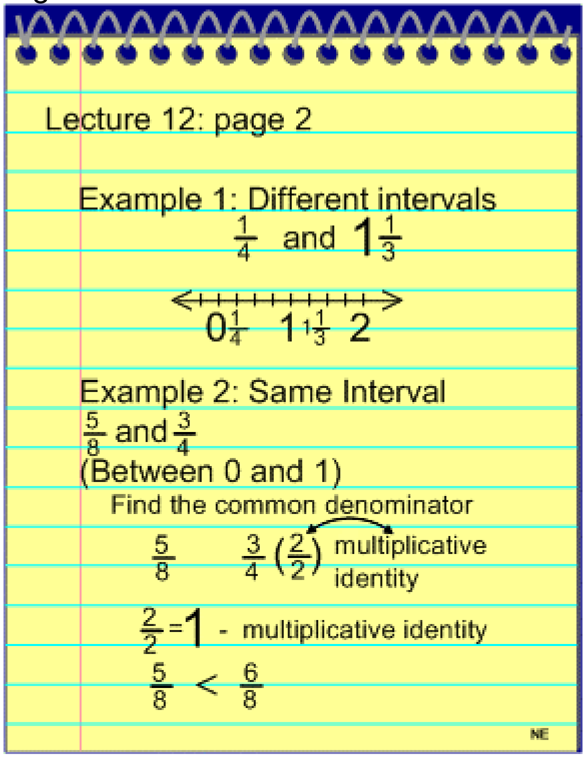
Venn Diagram - Sets of Numbers

$\frac{1}{2}, \frac{2}{2} = 1, \frac{3}{2} = 1\frac{1}{2}, \frac{4}{2} = 2$

Rationals reside on the number line.

NE

Alg012-02



Lecture 12: page 2

Example 1: Different intervals

$\frac{1}{4}$ and $1\frac{1}{3}$

Example 2: Same Interval

$\frac{5}{8}$ and $\frac{3}{4}$

(Between 0 and 1)

Find the common denominator

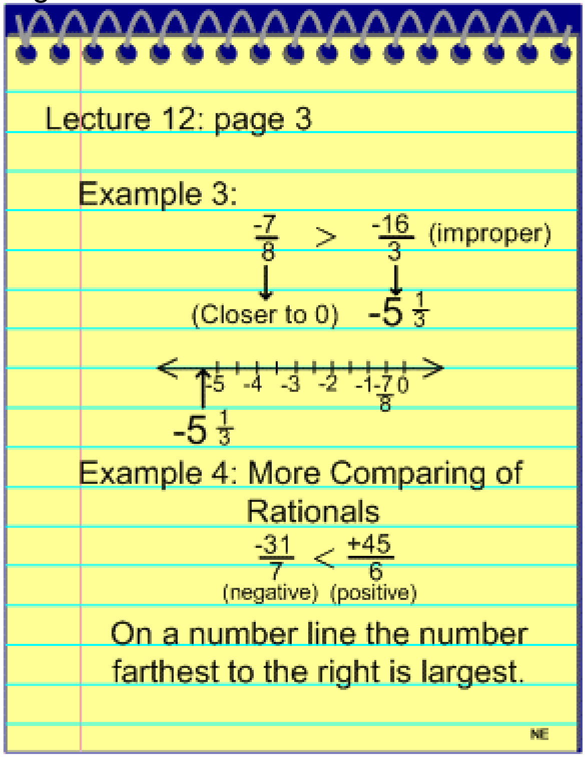
$\frac{5}{8} \qquad \frac{3}{4}\left(\frac{2}{2}\right)$ multiplicative identity

$\frac{2}{2} = 1$ - multiplicative identity

$\frac{5}{8} < \frac{6}{8}$

NE

Alg012-03



Lecture 12: page 3

Example 3:

$-\frac{7}{8} > -\frac{16}{3}$ (improper)

$\downarrow \qquad \downarrow$

(Closer to 0) $-5\frac{1}{3}$

Example 4: More Comparing of Rationals

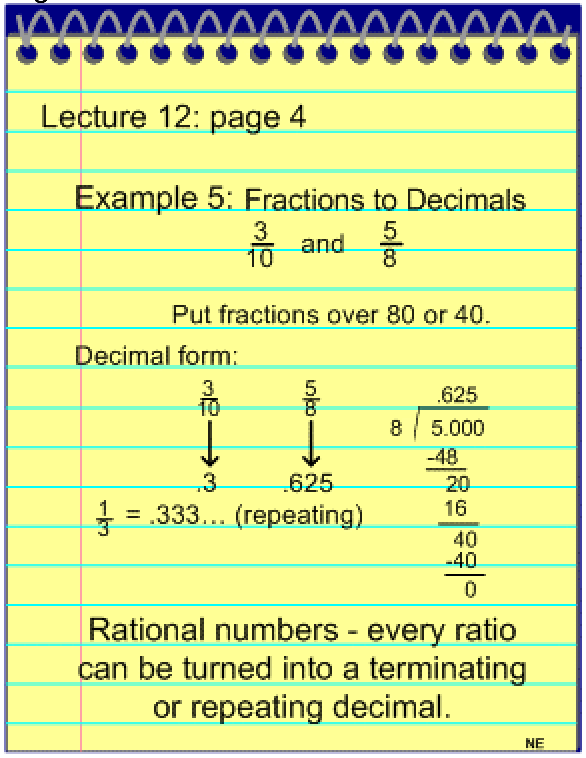
$\frac{-31}{7} < \frac{+45}{6}$

(negative) (positive)

On a number line the number farthest to the right is largest.

NE

Alg012-04



Lecture 12: page 4

Example 5: Fractions to Decimals

$\frac{3}{10}$ and $\frac{5}{8}$

Put fractions over 80 or 40.

Decimal form:

$\frac{3}{10} \downarrow .3 \qquad \frac{5}{8} \downarrow .625$

$$\begin{array}{r} .625 \\ 8\,\overline{)\,5.000} \\ -48 \\ \hline 20 \\ 16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

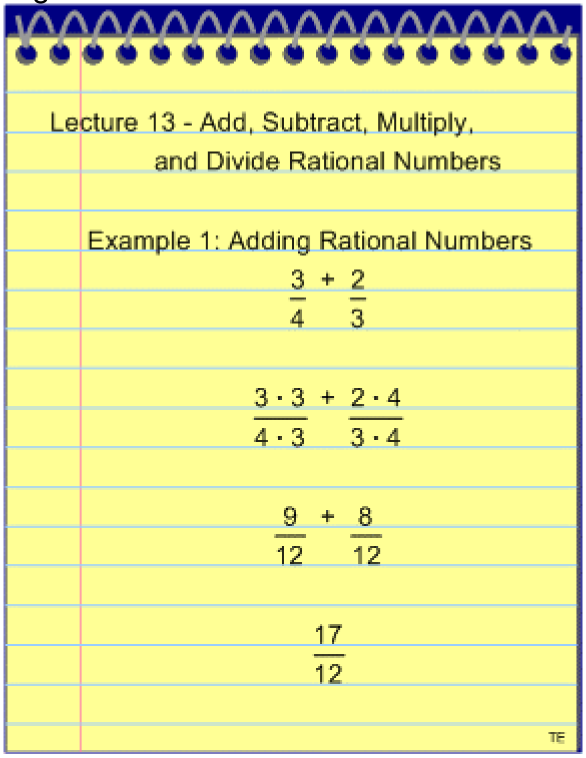
$\frac{1}{3} = .333\ldots$ (repeating)

Rational numbers - every ratio can be turned into a terminating or repeating decimal.

NE

Lecture 13 Notes

Alg013-01



Lecture 13 - Add, Subtract, Multiply, and Divide Rational Numbers

Example 1: Adding Rational Numbers

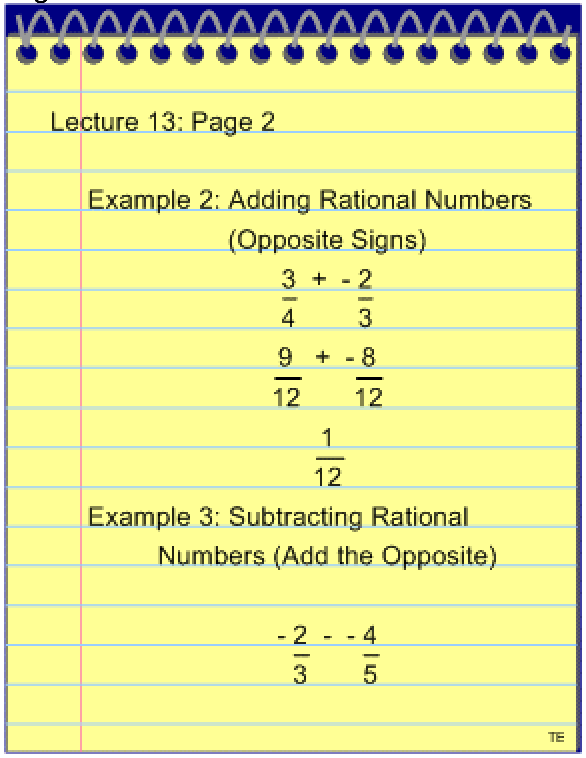
$$\frac{3}{4} + \frac{2}{3}$$

$$\frac{3 \cdot 3}{4 \cdot 3} + \frac{2 \cdot 4}{3 \cdot 4}$$

$$\frac{9}{12} + \frac{8}{12}$$

$$\frac{17}{12}$$

Alg013-02



Lecture 13: Page 2

Example 2: Adding Rational Numbers (Opposite Signs)

$$\frac{3}{4} + -\frac{2}{3}$$

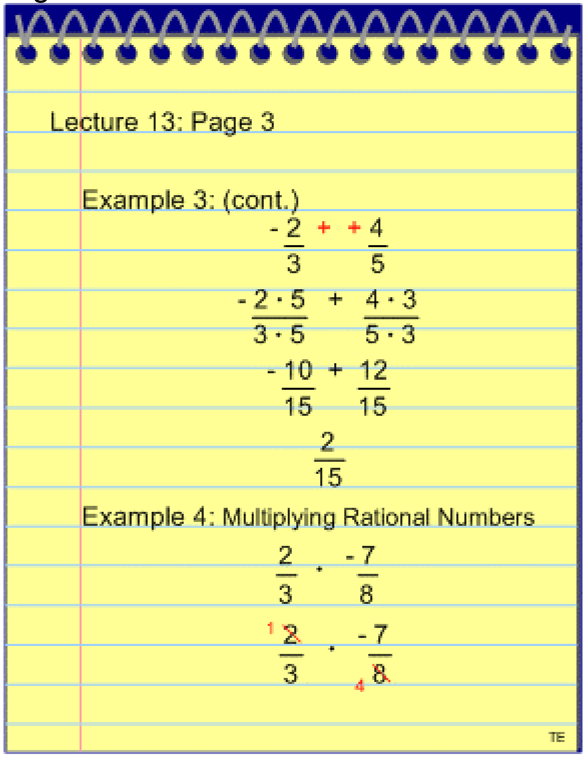
$$\frac{9}{12} + -\frac{8}{12}$$

$$\frac{1}{12}$$

Example 3: Subtracting Rational Numbers (Add the Opposite)

$$-\frac{2}{3} - -\frac{4}{5}$$

Alg013-03



Lecture 13: Page 3

Example 3: (cont.)

$$-\frac{2}{3} + +\frac{4}{5}$$

$$-\frac{2 \cdot 5}{3 \cdot 5} + \frac{4 \cdot 3}{5 \cdot 3}$$

$$-\frac{10}{15} + \frac{12}{15}$$

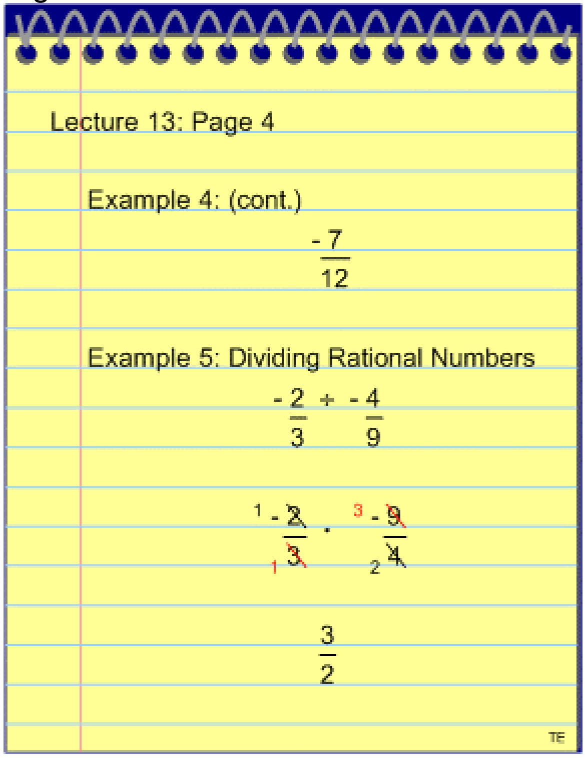
$$\frac{2}{15}$$

Example 4: Multiplying Rational Numbers

$$\frac{2}{3} \cdot -\frac{7}{8}$$

$$\frac{{}^{1}\cancel{2}}{3} \cdot -\frac{7}{{}_{4}\cancel{8}}$$

Alg013-04



Lecture 13: Page 4

Example 4: (cont.)

$$-\frac{7}{12}$$

Example 5: Dividing Rational Numbers

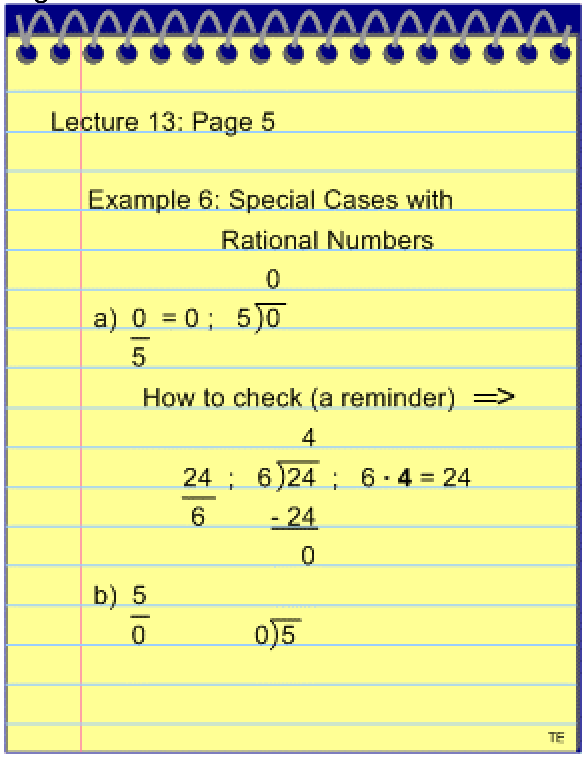
$$-\frac{2}{3} \div -\frac{4}{9}$$

$$-\frac{{}^{1}\cancel{2}}{{}_{1}\cancel{3}} \cdot -\frac{{}^{3}\cancel{9}}{{}_{2}\cancel{4}}$$

$$\frac{3}{2}$$

Lecture 13 Notes, Continued

Alg013-05



Lecture 13: Page 5

Example 6: Special Cases with Rational Numbers

a) $\frac{0}{5} = 0$; $\begin{array}{r} 0 \\ 5\overline{)0} \end{array}$

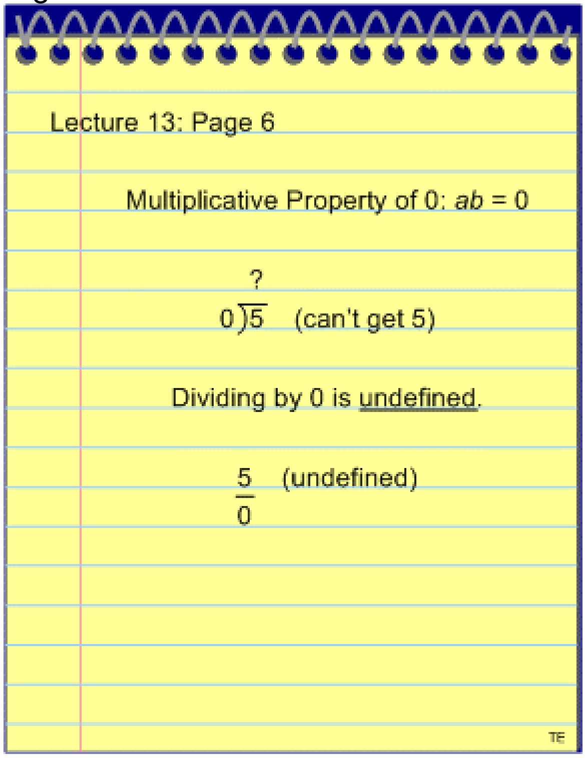
How to check (a reminder) =>

$\frac{24}{6}$; $\begin{array}{r} 4 \\ 6\overline{)24} \\ \underline{-24} \\ 0 \end{array}$; $6 \cdot \mathbf{4} = 24$

b) $\frac{5}{0}$ $\qquad 0\overline{)5}$

TE

Alg013-06



Lecture 13: Page 6

Multiplicative Property of 0: $ab = 0$

$\begin{array}{r} ? \\ 0\overline{)5} \end{array}$ (can't get 5)

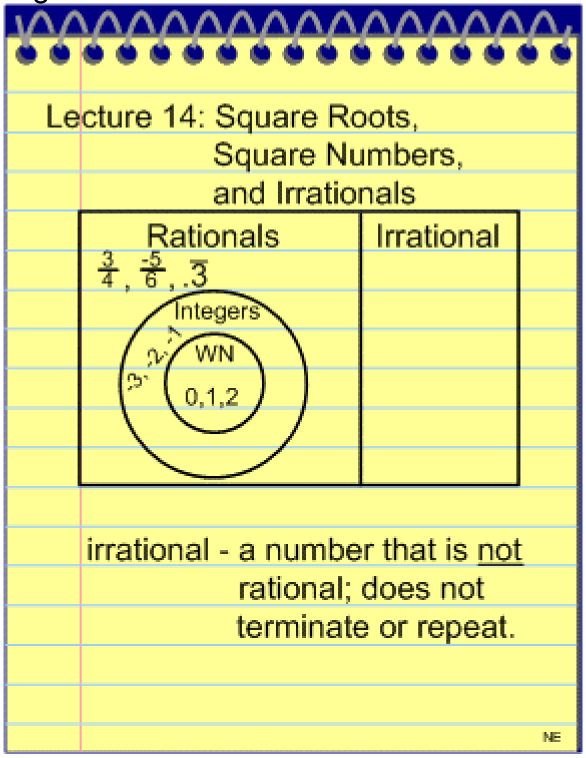
Dividing by 0 is undefined.

$\frac{5}{0}$ (undefined)

TE

Lecture 14 Notes

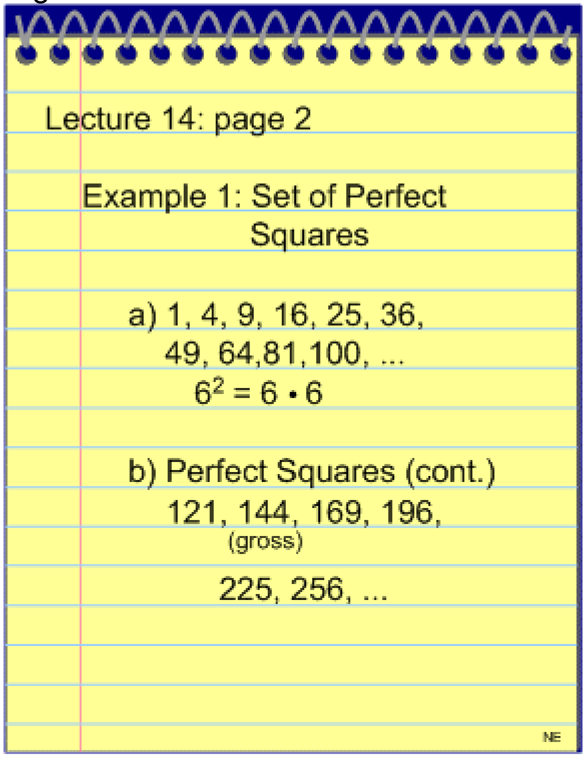
Alg014-01



Lecture 14: Square Roots,
Square Numbers,
and Irrationals

irrational - a number that is not rational; does not terminate or repeat.

Alg014-02



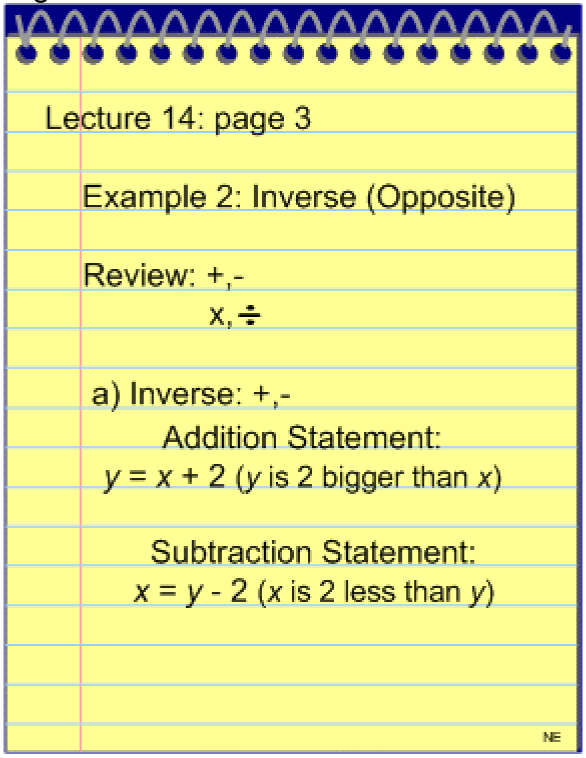
Lecture 14: page 2

Example 1: Set of Perfect Squares

a) 1, 4, 9, 16, 25, 36, 49, 64,81,100, ...
$6^2 = 6 \cdot 6$

b) Perfect Squares (cont.)
121, 144, 169, 196,
(gross)
225, 256, ...

Alg14-03



Lecture 14: page 3

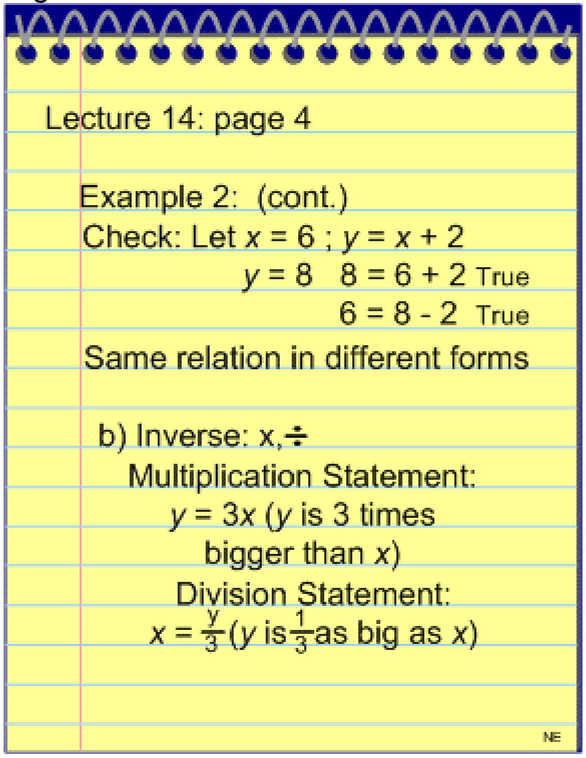
Example 2: Inverse (Opposite)

Review: +,-
x, ÷

a) Inverse: +,-
Addition Statement:
$y = x + 2$ (y is 2 bigger than x)

Subtraction Statement:
$x = y - 2$ (x is 2 less than y)

Alg14-04

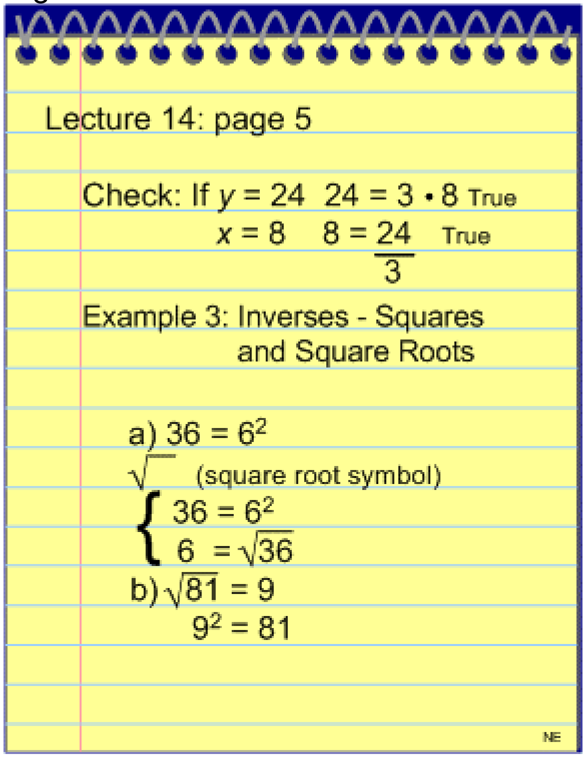


Lecture 14: page 4

Example 2: (cont.)
Check: Let $x = 6$; $y = x + 2$
$y = 8$ $8 = 6 + 2$ True
$6 = 8 - 2$ True
Same relation in different forms

b) Inverse: x,÷
Multiplication Statement:
$y = 3x$ (y is 3 times bigger than x)
Division Statement:
$x = \frac{y}{3}$ (y is $\frac{1}{3}$ as big as x)

Alg014-05



Lecture 14: page 5

Check: If $y = 24$ $24 = 3 \cdot 8$ True
$x = 8$ $8 = \frac{24}{3}$ True

Example 3: Inverses - Squares and Square Roots

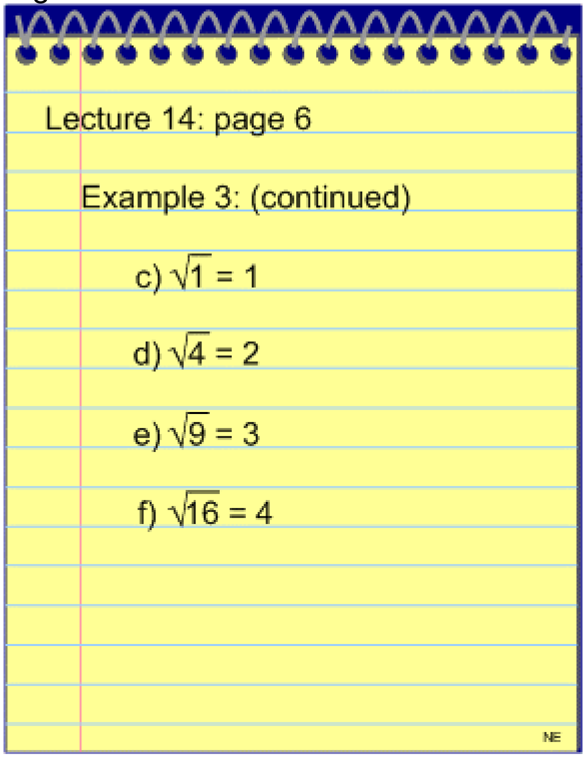
a) $36 = 6^2$

$\sqrt{\ }$ (square root symbol)

$$\left\{ \begin{array}{l} 36 = 6^2 \\ 6 = \sqrt{36} \end{array} \right.$$

b) $\sqrt{81} = 9$
$9^2 = 81$

Alg014-06



Lecture 14: page 6

Example 3: (continued)

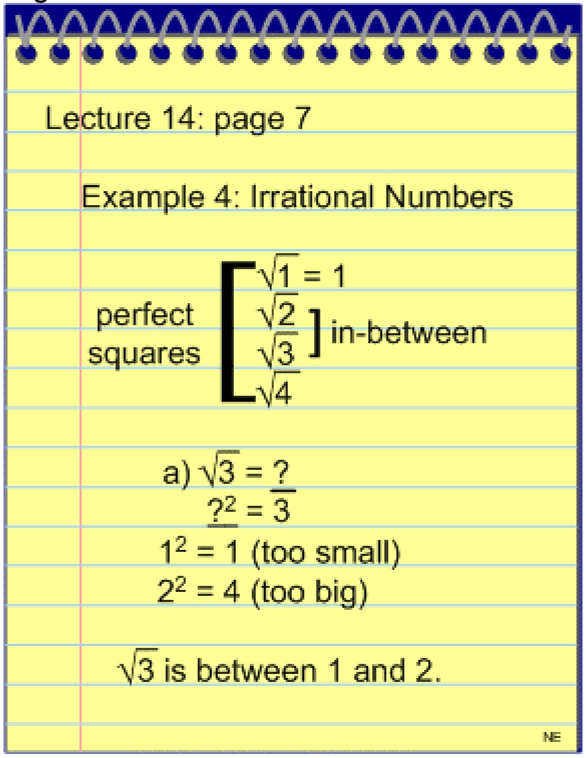
c) $\sqrt{1} = 1$

d) $\sqrt{4} = 2$

e) $\sqrt{9} = 3$

f) $\sqrt{16} = 4$

Alg014-07



Lecture 14: page 7

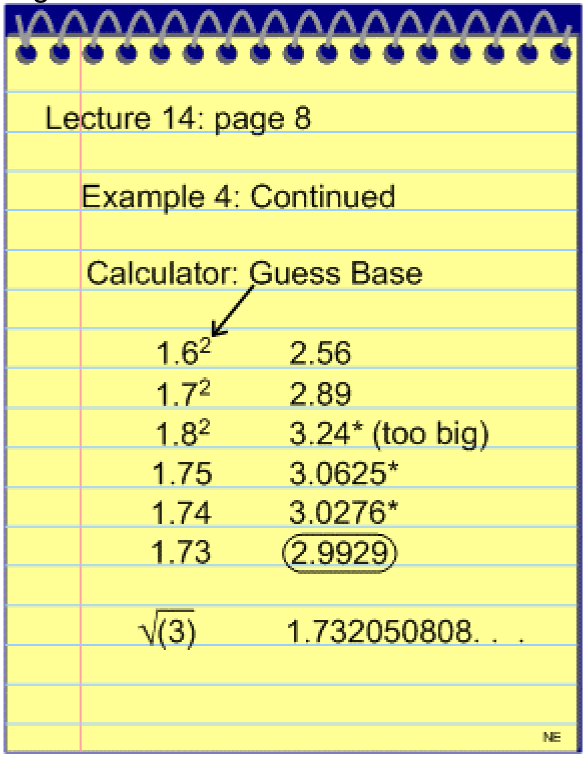
Example 4: Irrational Numbers

$$\text{perfect squares} \left[\begin{array}{l} \sqrt{1} = 1 \\ \left. \begin{array}{l} \sqrt{2} \\ \sqrt{3} \end{array} \right] \text{in-between} \\ \sqrt{4} \end{array} \right.$$

a) $\sqrt{3} = \underline{?}$
$\underline{?}^2 = 3$
$1^2 = 1$ (too small)
$2^2 = 4$ (too big)

$\sqrt{3}$ is between 1 and 2.

Alg014-08



Lecture 14: page 8

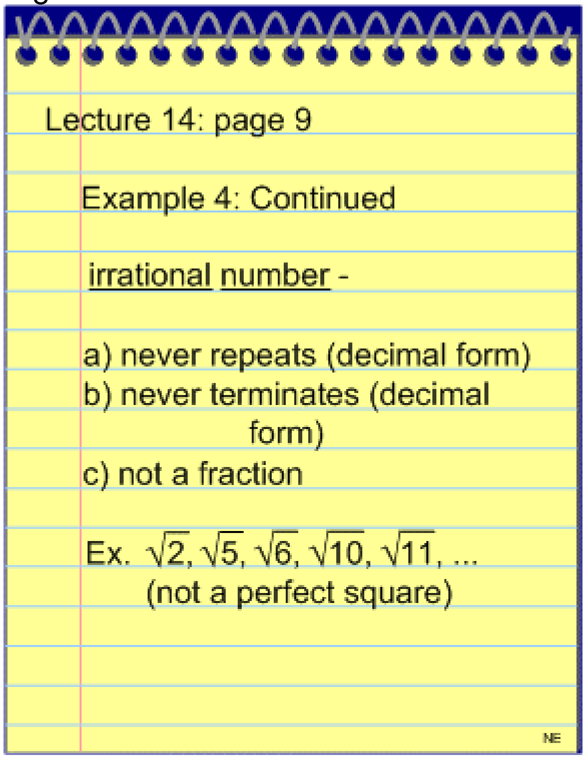
Example 4: Continued

Calculator: Guess Base

1.6^2	2.56
1.7^2	2.89
1.8^2	3.24* (too big)
1.75	3.0625*
1.74	3.0276*
1.73	$\boxed{2.9929}$
$\sqrt{(3)}$	1.732050808. . .

Lecture 14 Notes, Continued

Alg014-09



Lecture 14: page 9

Example 4: Continued

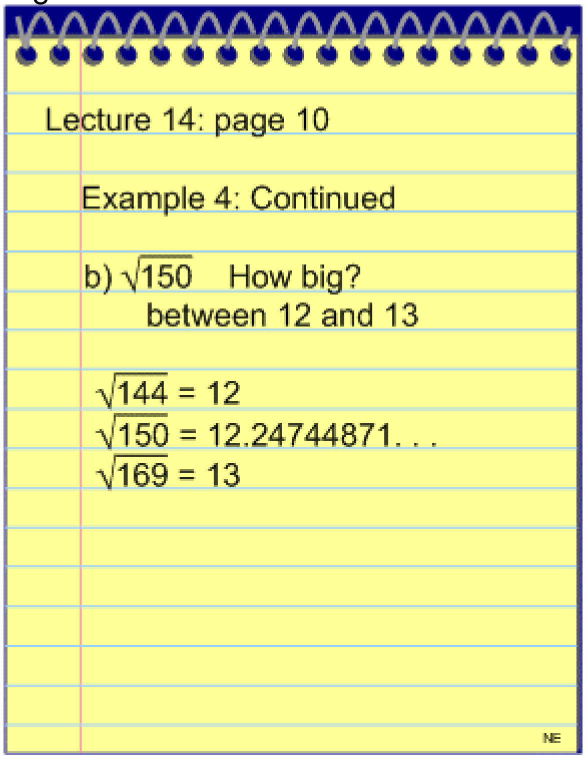
<u>irrational</u> <u>number</u> -

a) never repeats (decimal form)
b) never terminates (decimal form)
c) not a fraction

Ex. $\sqrt{2}, \sqrt{5}, \sqrt{6}, \sqrt{10}, \sqrt{11}, \ldots$
(not a perfect square)

NE

Alg014-10



Lecture 14: page 10

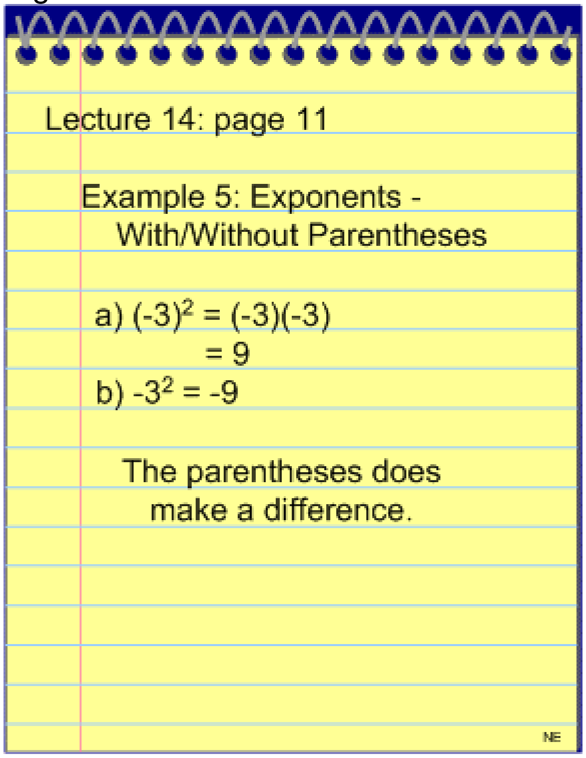
Example 4: Continued

b) $\sqrt{150}$ How big?
between 12 and 13

$\sqrt{144} = 12$
$\sqrt{150} = 12.24744871\ldots$
$\sqrt{169} = 13$

NE

Alg14-11



Lecture 14: page 11

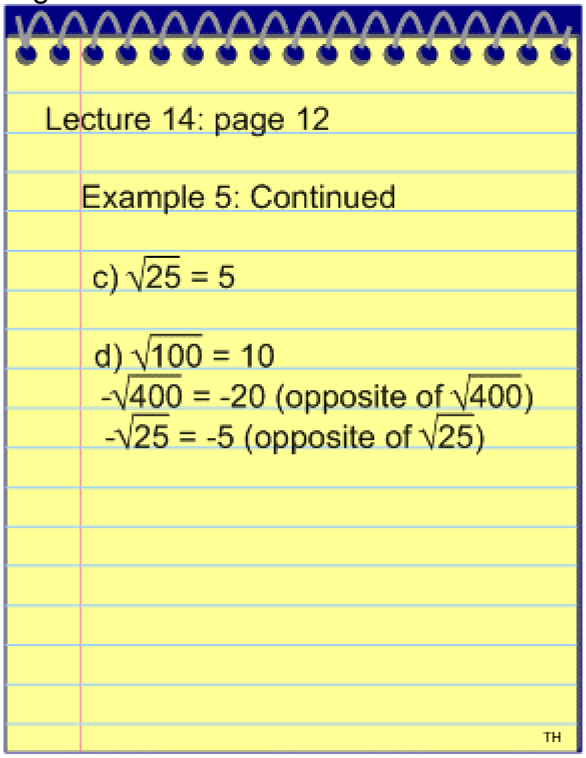
Example 5: Exponents - With/Without Parentheses

a) $(-3)^2 = (-3)(-3)$
$= 9$
b) $-3^2 = -9$

The parentheses does make a difference.

NE

Alg14-12



Lecture 14: page 12

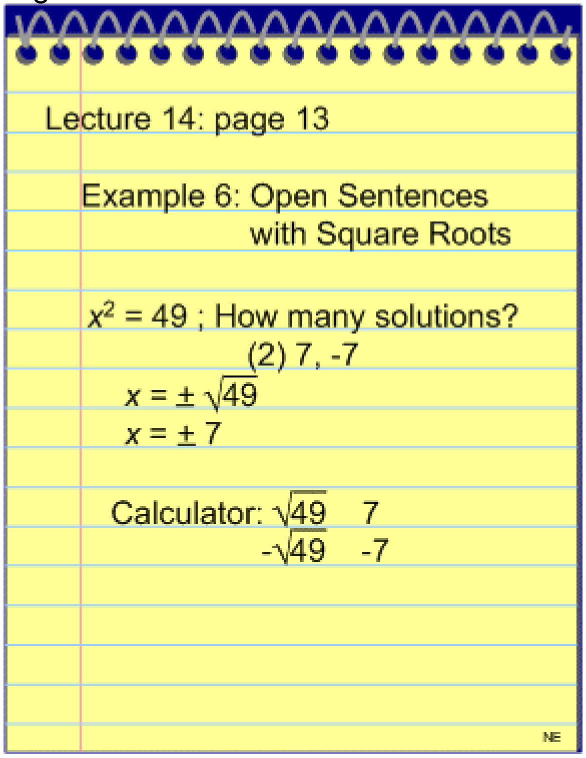
Example 5: Continued

c) $\sqrt{25} = 5$

d) $\sqrt{100} = 10$
$-\sqrt{400} = -20$ (opposite of $\sqrt{400}$)
$-\sqrt{25} = -5$ (opposite of $\sqrt{25}$)

TH

Alg014-13



Lecture 14: page 13

Example 6: Open Sentences with Square Roots

$x^2 = 49$; How many solutions?

(2) 7, -7

$x = \pm \sqrt{49}$

$x = \pm 7$

Calculator: $\sqrt{49}$ 7

$-\sqrt{49}$ -7

NE

Lecture 15 Notes

Alg015-01

Lecture 15: One - Step Equations Using Addition and Subtraction

Example 1: Open Sentence

$5x + 7 = 3x + 1$

a) $x = 2$

$5 \cdot 2 + 7 \; ? \; 3 \cdot 2 + 1$

$10 + 7 \; ? \; 6 + 1$

$17 \neq 7$

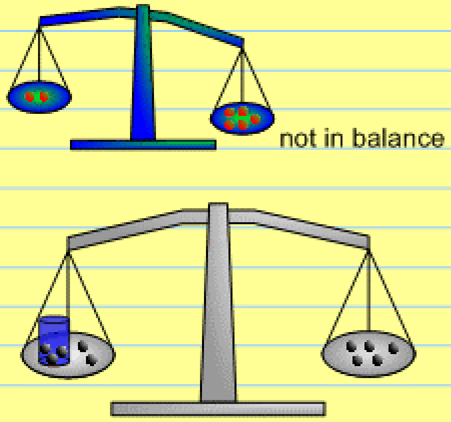
2 is NOT a solution.

NE

Alg015-02

Lecture 15: page 2

Example 2: Pan Balance



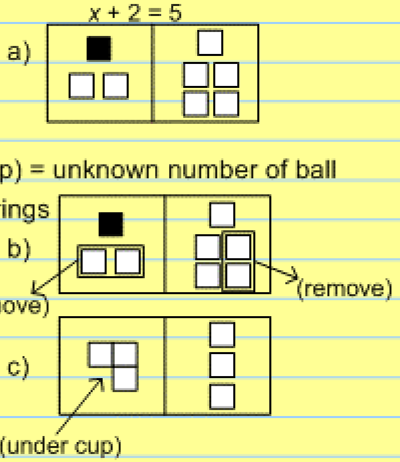
TH

Alg015-03

Lecture 15: page 3

OVER HEAD MODEL:

$x + 2 = 5$



TH

Alg015-04

Lecture 15: page 4

$$\begin{array}{r} x + 2 = 5 \\ -2 \;\; -2 \\ \hline x = 3 \end{array} \quad \text{(remove opposites)}$$

Check:

$$\begin{array}{r} x + 2 = 5 \\ (3) + 2 = 5 \\ \hline 5 = 5 \checkmark \end{array}$$

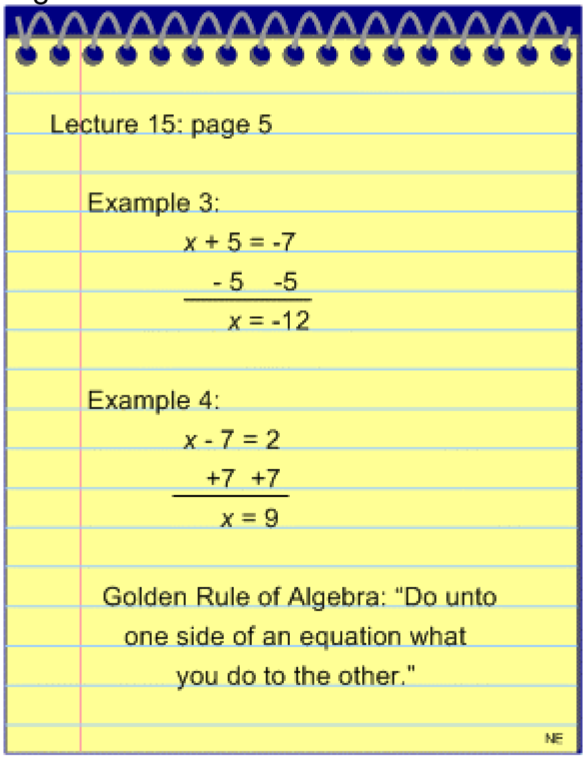
{3} is the solution.

Review:	$2 + -2 = 0$	Additive Inverse
	$x + 0 = x$	Additive Identity

TH

Lecture 15 Notes, Continued

Alg015-05



Lecture 15: page 5

Example 3:

$$\begin{array}{r} x + 5 = -7 \\ -5 \quad -5 \\ \hline x = -12 \end{array}$$

Example 4:

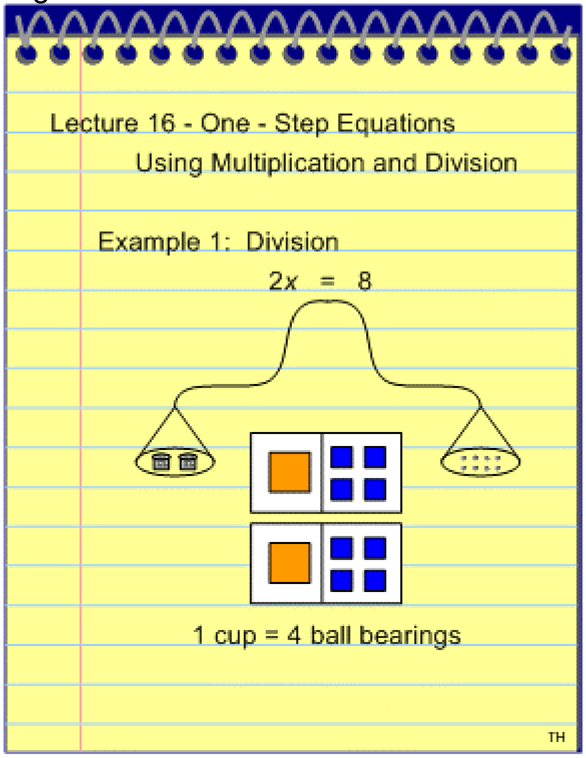
$$\begin{array}{r} x - 7 = 2 \\ +7 \quad +7 \\ \hline x = 9 \end{array}$$

Golden Rule of Algebra: "Do unto one side of an equation what you do to the other."

NE

Lecture 16 Notes

Alg016-01



Lecture 16 - One - Step Equations

Using Multiplication and Division

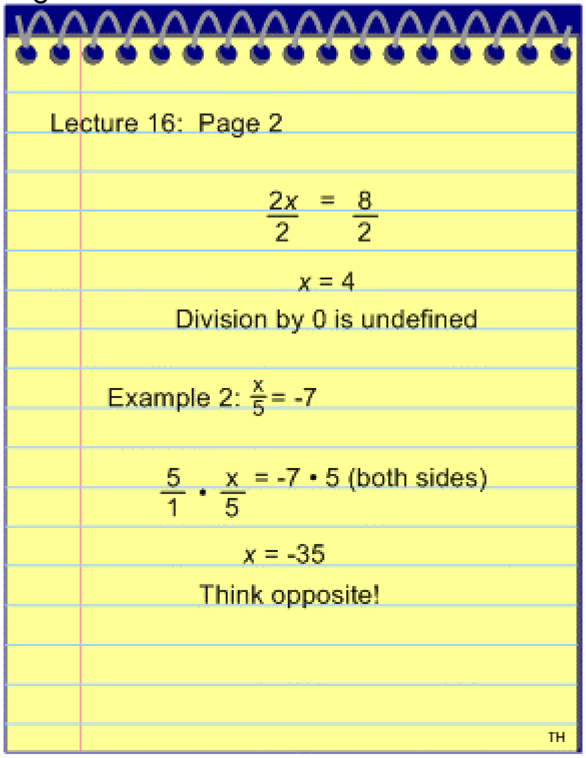
Example 1: Division

$2x = 8$

1 cup = 4 ball bearings

TH

Alg016-02



Lecture 16: Page 2

$$\frac{2x}{2} = \frac{8}{2}$$

$x = 4$

Division by 0 is undefined

Example 2: $\frac{x}{5} = -7$

$\frac{5}{1} \cdot \frac{x}{5} = -7 \cdot 5$ (both sides)

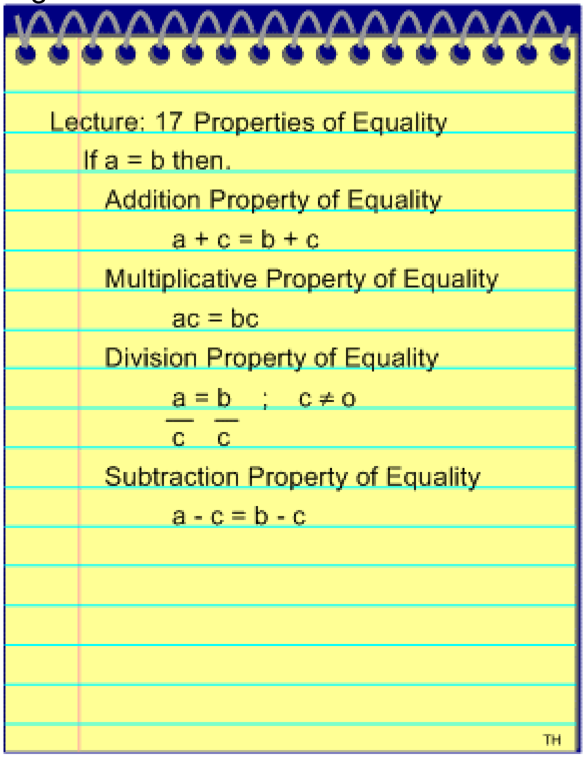
$x = -35$

Think opposite!

TH

Lecture 17 Notes

Alg017-01



Lecture: 17 Properties of Equality

If $a = b$ then.

Addition Property of Equality

$a + c = b + c$

Multiplicative Property of Equality

$ac = bc$

Division Property of Equality

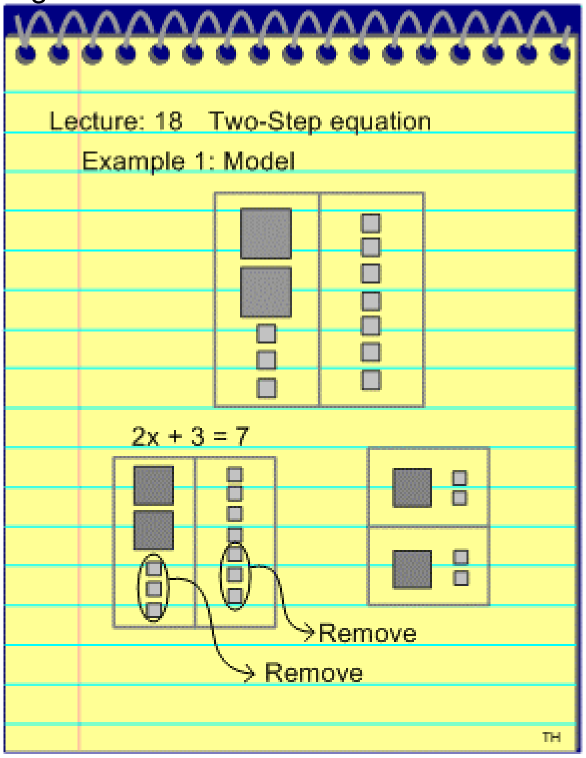
$\frac{a}{c} = \frac{b}{c}$; $c \neq o$

Subtraction Property of Equality

$a - c = b - c$

Lecture 18 Notes

Alg018-01



Lecture: 18 Two-Step equation

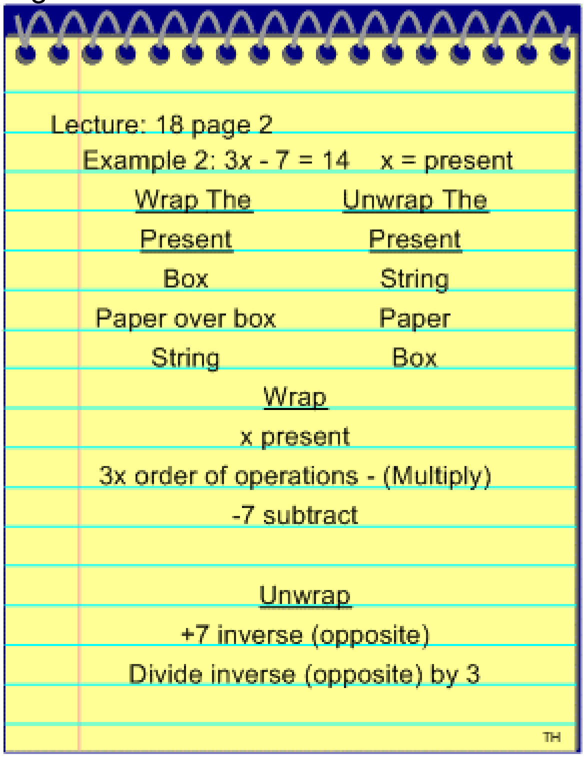
Example 1: Model

$2x + 3 = 7$

Remove

Remove

Alg018-02



Lecture: 18 page 2

Example 2: $3x - 7 = 14$ x = present

Wrap The Present	Unwrap The Present
Box	String
Paper over box	Paper
String	Box

Wrap

x present

3x order of operations - (Multiply)

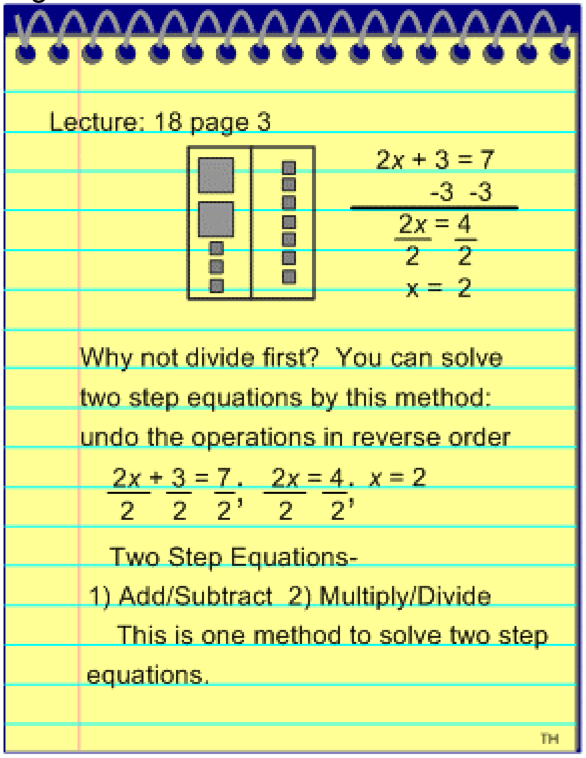
-7 subtract

Unwrap

+7 inverse (opposite)

Divide inverse (opposite) by 3

Alg018-03



Lecture: 18 page 3

$$\begin{array}{r} 2x + 3 = 7 \\ -3 \ \ -3 \\ \hline \frac{2x}{2} = \frac{4}{2} \\ x = 2 \end{array}$$

Why not divide first? You can solve two step equations by this method: undo the operations in reverse order

$\frac{2x}{2} + \frac{3}{2} = \frac{7}{2}, \ \frac{2x}{2} = \frac{4}{2}, \ x = 2$

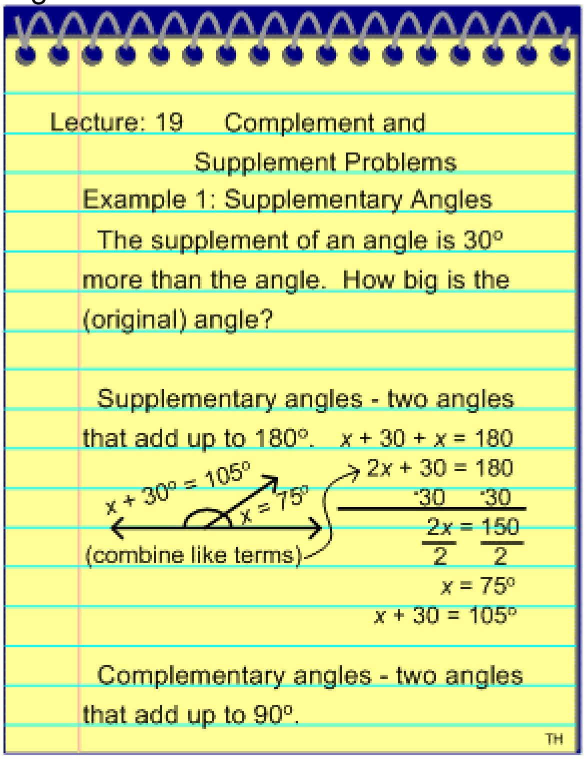
Two Step Equations-

1) Add/Subtract 2) Multiply/Divide

This is one method to solve two step equations.

Lecture 19 Notes

Alg019-01



Lecture: 19 Complement and Supplement Problems

Example 1: Supplementary Angles

The supplement of an angle is 30° more than the angle. How big is the (original) angle?

Supplementary angles - two angles that add up to 180°.

$x + 30^\circ = 105^\circ$

$x = 75^\circ$

(combine like terms)

$x + 30 + x = 180$

$2x + 30 = 180$

$-30 \quad -30$

$2x = 150$

$\frac{2x}{2} = \frac{150}{2}$

$x = 75^\circ$

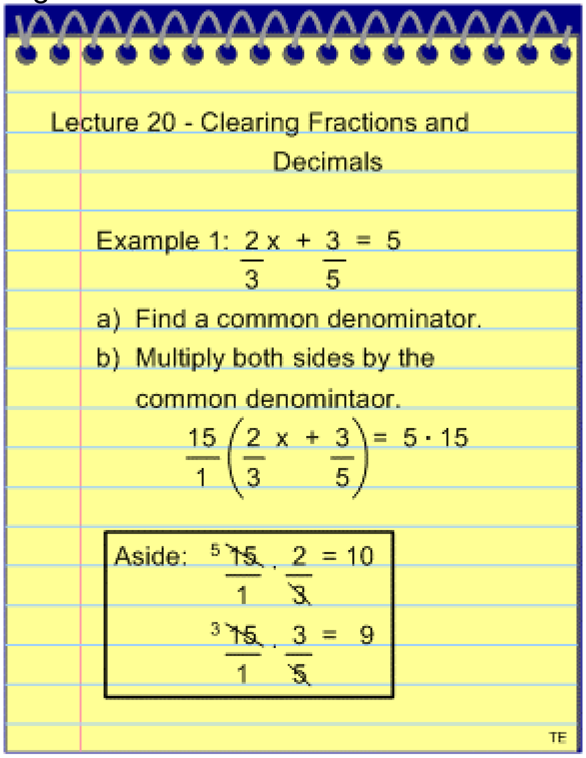
$x + 30 = 105^\circ$

Complementary angles - two angles that add up to 90°.

TH

Lecture 20 Notes

Alg020-01



Lecture 20 - Clearing Fractions and Decimals

Example 1: $\frac{2}{3}x + \frac{3}{5} = 5$

a) Find a common denominator.

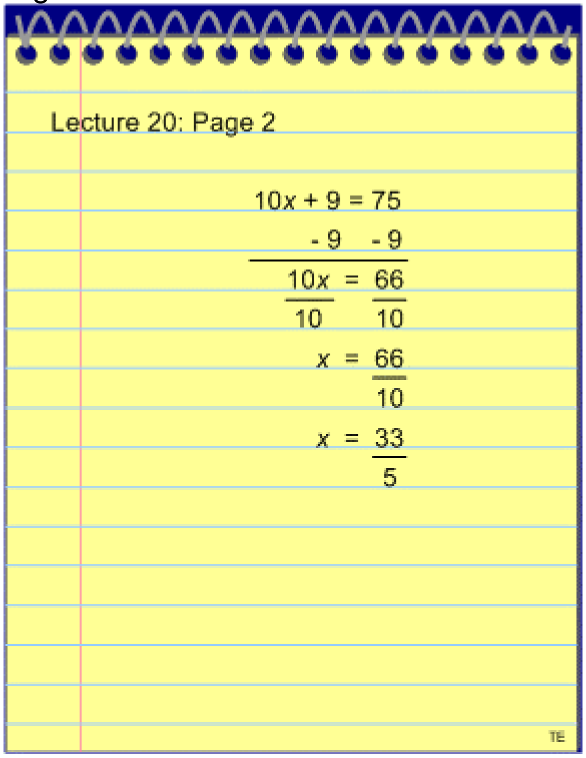
b) Multiply both sides by the common denomintaor.

$$\frac{15}{1}\left(\frac{2}{3}x + \frac{3}{5}\right) = 5 \cdot 15$$

Aside: $\frac{{}^{5}\cancel{15}}{1} \cdot \frac{2}{\cancel{3}} = 10$

$\frac{{}^{3}\cancel{15}}{1} \cdot \frac{3}{\cancel{5}} = 9$

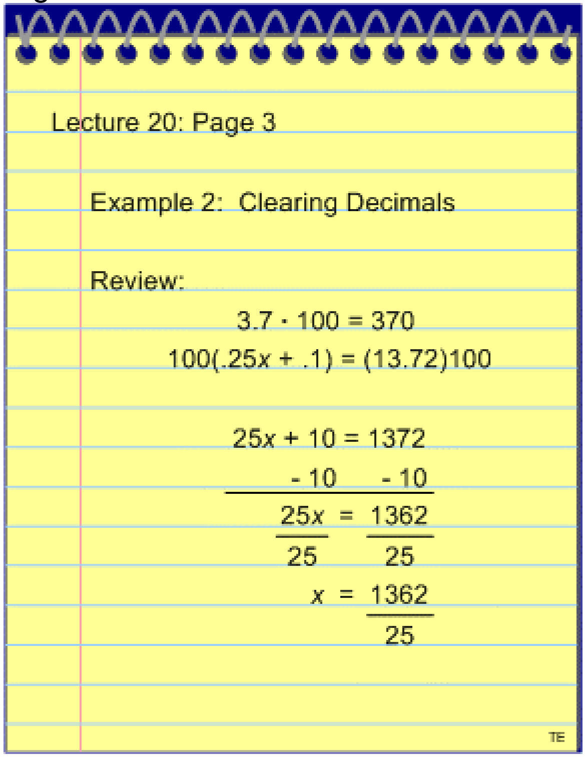
Alg020-02



Lecture 20: Page 2

$$10x + 9 = 75$$
$$-9 \quad -9$$
$$\frac{10x}{10} = \frac{66}{10}$$
$$x = \frac{66}{10}$$
$$x = \frac{33}{5}$$

Alg020-03



Lecture 20: Page 3

Example 2: Clearing Decimals

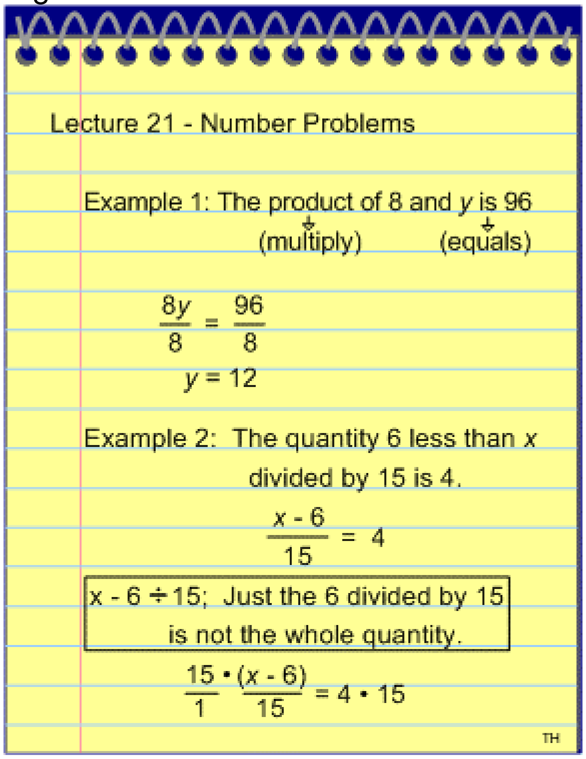
Review:

$$3.7 \cdot 100 = 370$$
$$100(.25x + .1) = (13.72)100$$

$$25x + 10 = 1372$$
$$-10 \quad -10$$
$$\frac{25x}{25} = \frac{1362}{25}$$
$$x = \frac{1362}{25}$$

Lecture 21 Notes

Alg021-01



Lecture 21 - Number Problems

Example 1: The product of 8 and *y* is 96

(multiply) (equals)

$$\frac{8y}{8} = \frac{96}{8}$$

$$y = 12$$

Example 2: The quantity 6 less than *x* divided by 15 is 4.

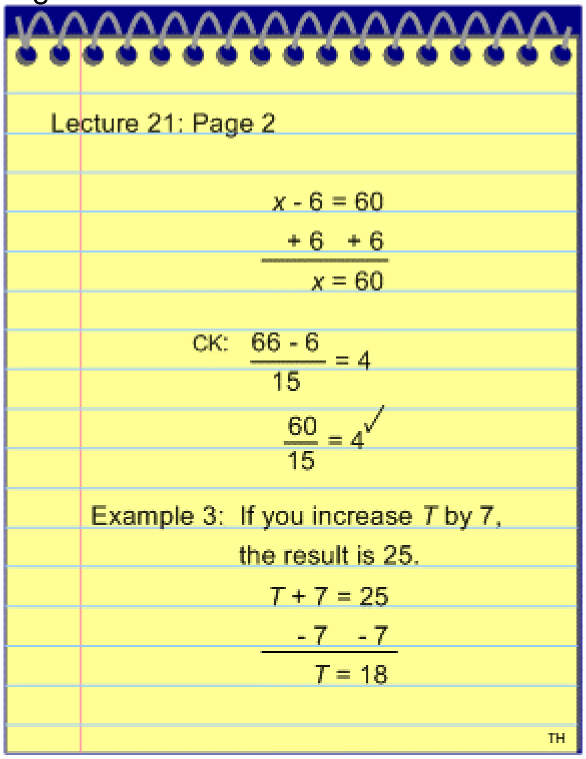
$$\frac{x - 6}{15} = 4$$

x - 6 ÷ 15; Just the 6 divided by 15 is not the whole quantity.

$$\frac{15}{1} \cdot \frac{(x - 6)}{15} = 4 \cdot 15$$

TH

Alg021-02



Lecture 21: Page 2

$$\begin{array}{r} x - 6 = 60 \\ +6 \quad +6 \\ \hline x = 60 \end{array}$$

CK: $\frac{66 - 6}{15} = 4$

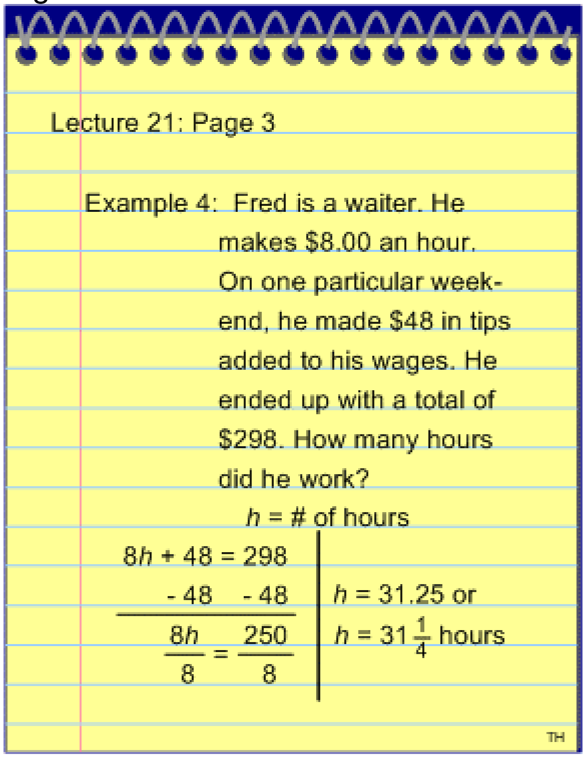
$\frac{60}{15} = 4$ ✓

Example 3: If you increase *T* by 7, the result is 25.

$$\begin{array}{r} T + 7 = 25 \\ -7 \quad -7 \\ \hline T = 18 \end{array}$$

TH

Alg021-03



Lecture 21: Page 3

Example 4: Fred is a waiter. He makes $8.00 an hour. On one particular weekend, he made $48 in tips added to his wages. He ended up with a total of $298. How many hours did he work?

h = # of hours

$$\begin{array}{r} 8h + 48 = 298 \\ -48 \quad -48 \\ \hline \frac{8h}{8} = \frac{250}{8} \end{array}$$

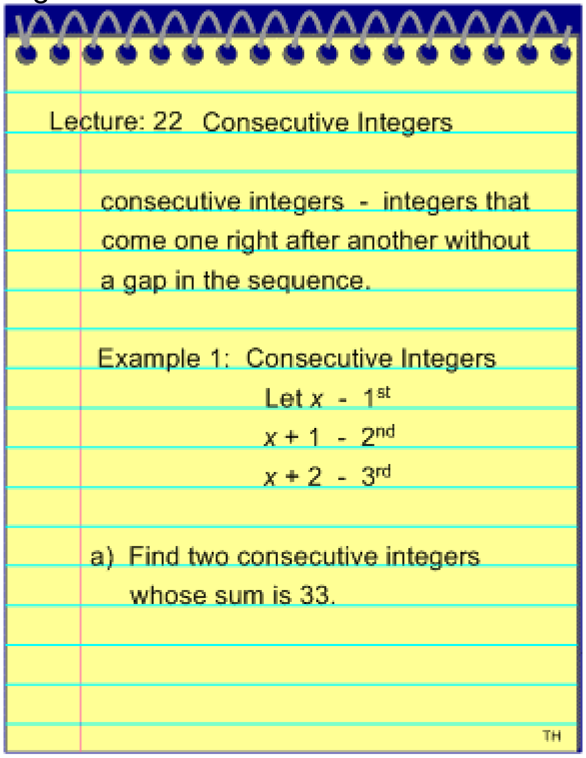
$h = 31.25$ or

$h = 31\frac{1}{4}$ hours

TH

Lecture 22 Notes

Alg022-01



Lecture: 22 Consecutive Integers

consecutive integers - integers that come one right after another without a gap in the sequence.

Example 1: Consecutive Integers

Let x - 1st

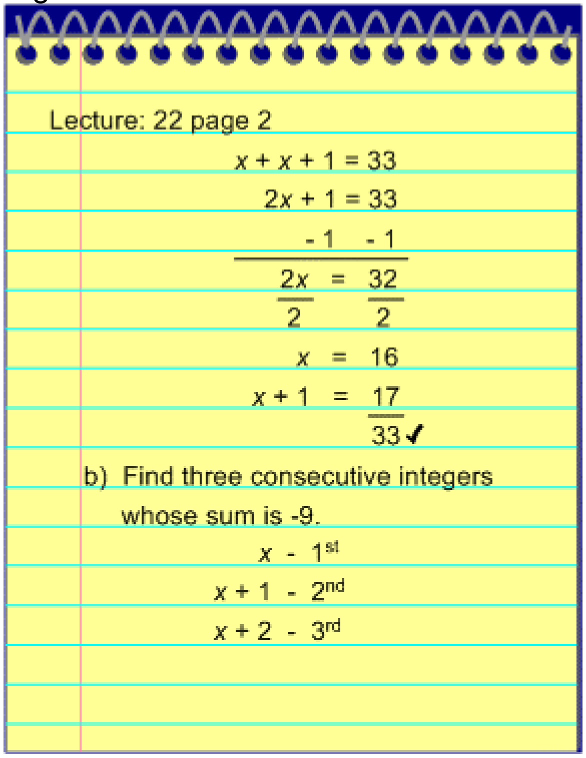
$x + 1$ - 2nd

$x + 2$ - 3rd

a) Find two consecutive integers whose sum is 33.

TH

Alg022-02



Lecture: 22 page 2

$$x + x + 1 = 33$$
$$2x + 1 = 33$$
$$-1 \quad -1$$
$$\frac{2x}{2} = \frac{32}{2}$$
$$x = 16$$
$$x + 1 = \underline{17}$$
$$33 ✓$$

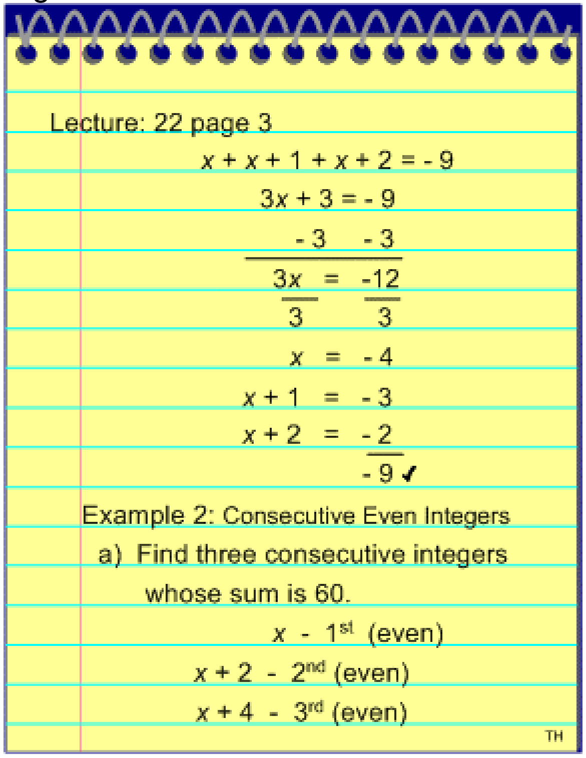
b) Find three consecutive integers whose sum is -9.

x - 1st

$x + 1$ - 2nd

$x + 2$ - 3rd

Alg022-03



Lecture: 22 page 3

$$x + x + 1 + x + 2 = -9$$
$$3x + 3 = -9$$
$$-3 \quad -3$$
$$\frac{3x}{3} = \frac{-12}{3}$$
$$x = -4$$
$$x + 1 = -3$$
$$x + 2 = \underline{-2}$$
$$-9 ✓$$

Example 2: Consecutive Even Integers

a) Find three consecutive integers whose sum is 60.

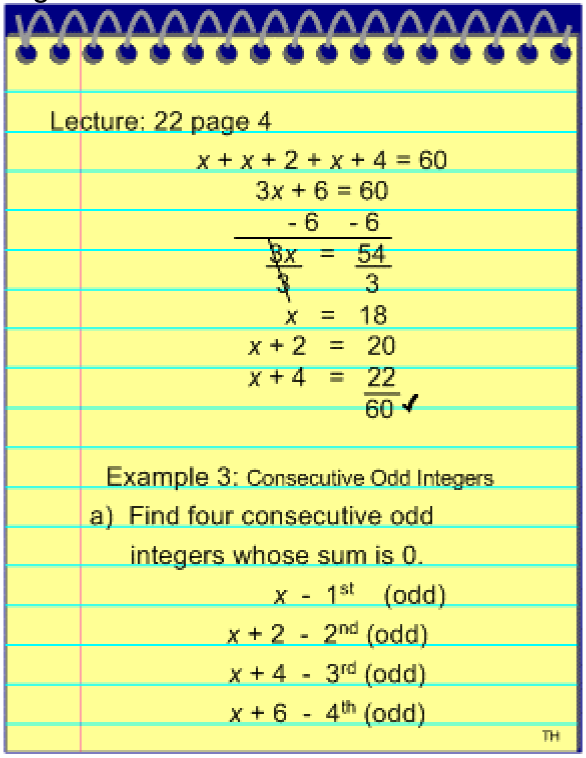
x - 1st (even)

$x + 2$ - 2nd (even)

$x + 4$ - 3rd (even)

TH

Alg022-04



Lecture: 22 page 4

$$x + x + 2 + x + 4 = 60$$
$$3x + 6 = 60$$
$$-6 \quad -6$$
$$\frac{\cancel{3}x}{\cancel{3}} = \frac{54}{3}$$
$$x = 18$$
$$x + 2 = 20$$
$$x + 4 = \underline{22}$$
$$60 ✓$$

Example 3: Consecutive Odd Integers

a) Find four consecutive odd integers whose sum is 0.

x - 1st (odd)

$x + 2$ - 2nd (odd)

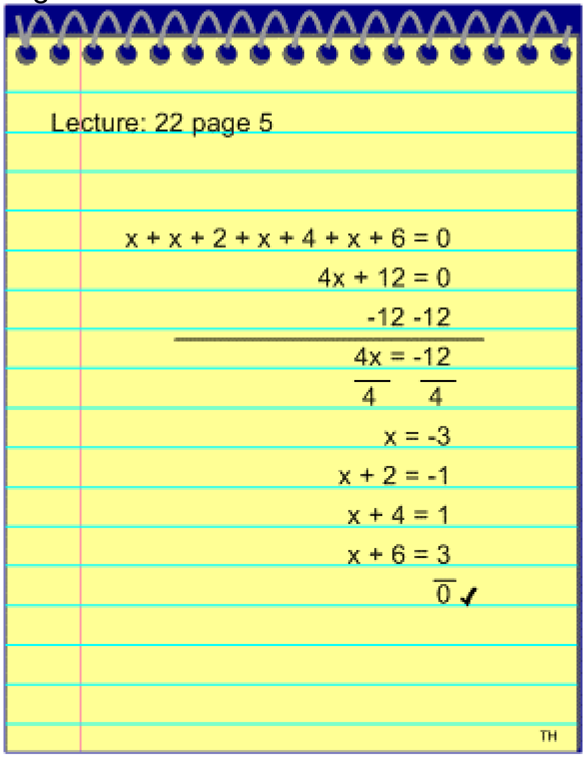
$x + 4$ - 3rd (odd)

$x + 6$ - 4th (odd)

TH

Lecture 22 Notes, Continued

Alg022-05



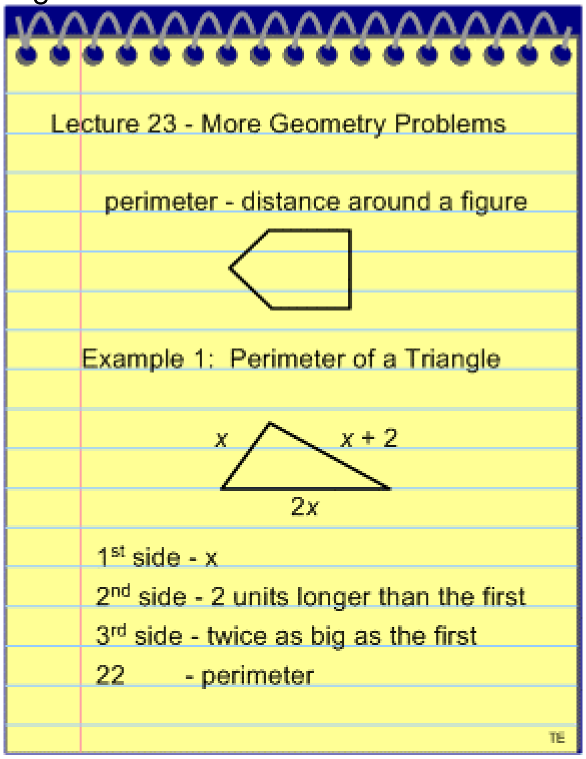
Lecture: 22 page 5

$$
\begin{array}{r}
x + x + 2 + x + 4 + x + 6 = 0 \\
4x + 12 = 0 \\
-12 \;\; -12 \\
\hline
\frac{4x}{4} = \frac{-12}{4} \\
x = -3 \\
x + 2 = -1 \\
x + 4 = 1 \\
x + 6 = 3 \\
\overline{0} \checkmark
\end{array}
$$

TH

Lecture 23 Notes

Alg023-01



Lecture 23 - More Geometry Problems

perimeter - distance around a figure

Example 1: Perimeter of a Triangle

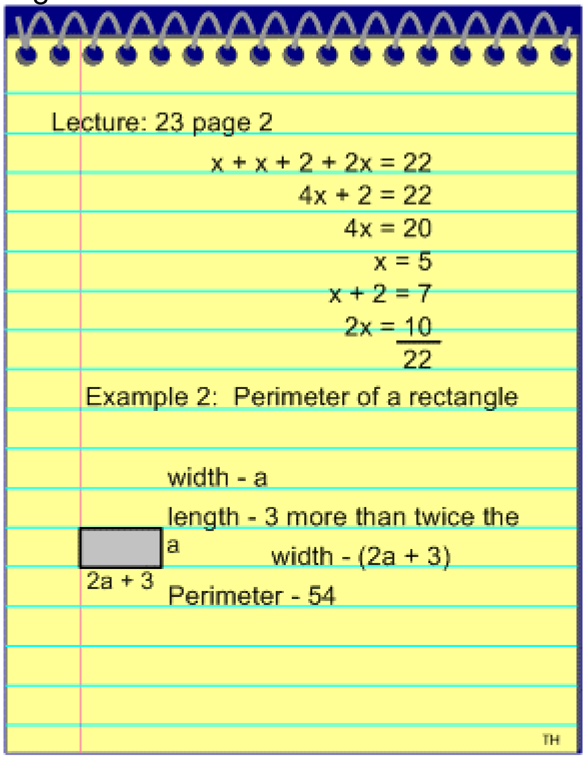
1st side - x

2nd side - 2 units longer than the first

3rd side - twice as big as the first

22 - perimeter

Alg023-02



Lecture: 23 page 2

$$x + x + 2 + 2x = 22$$
$$4x + 2 = 22$$
$$4x = 20$$
$$x = 5$$
$$x + 2 = 7$$
$$2x = \underline{10}$$
$$22$$

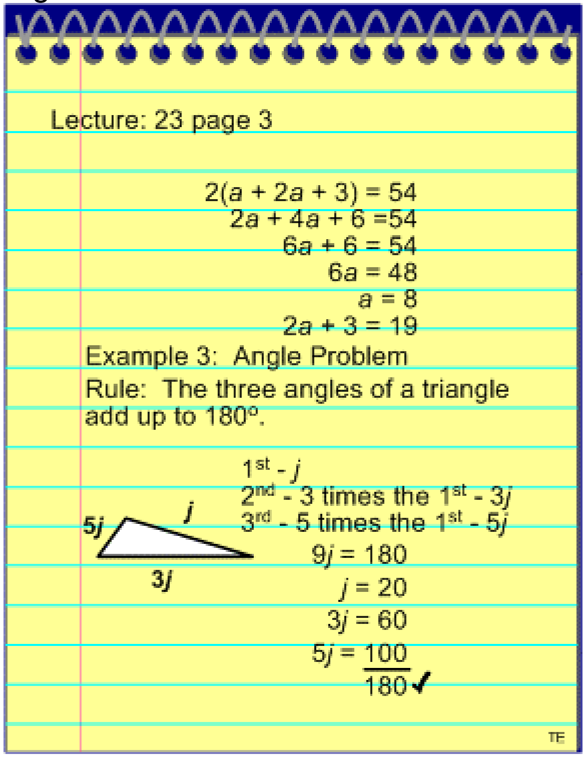
Example 2: Perimeter of a rectangle

width - a

length - 3 more than twice the width - (2a + 3)

Perimeter - 54

Alg023-03



Lecture: 23 page 3

$$2(a + 2a + 3) = 54$$
$$2a + 4a + 6 = 54$$
$$6a + 6 = 54$$
$$6a = 48$$
$$a = 8$$
$$2a + 3 = 19$$

Example 3: Angle Problem

Rule: The three angles of a triangle add up to 180°.

1st - j

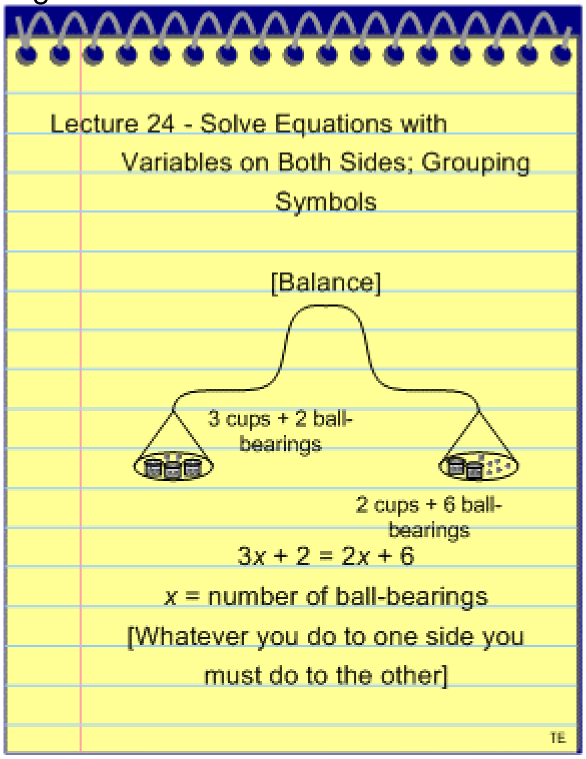
2nd - 3 times the 1st - $3j$

3rd - 5 times the 1st - $5j$

$$9j = 180$$
$$j = 20$$
$$3j = 60$$
$$5j = \underline{100}$$
$$180 \checkmark$$

Lecture 24 Notes

Alg024-01



Lecture 24 - Solve Equations with Variables on Both Sides; Grouping Symbols

[Balance]

3 cups + 2 ball-bearings

2 cups + 6 ball-bearings

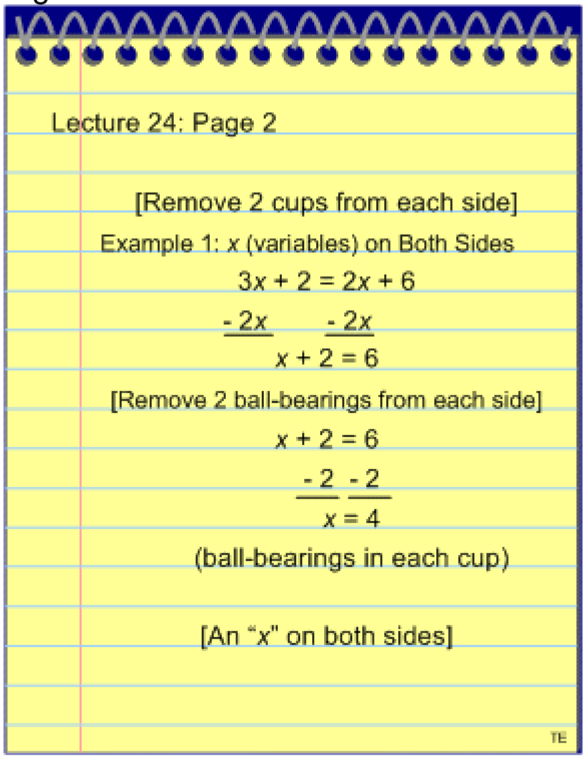
$3x + 2 = 2x + 6$

x = number of ball-bearings

[Whatever you do to one side you must do to the other]

TE

Alg024-02



Lecture 24: Page 2

[Remove 2 cups from each side]

Example 1: x (variables) on Both Sides

$$3x + 2 = 2x + 6$$
$$\underline{-2x} \qquad \underline{-2x}$$
$$x + 2 = 6$$

[Remove 2 ball-bearings from each side]

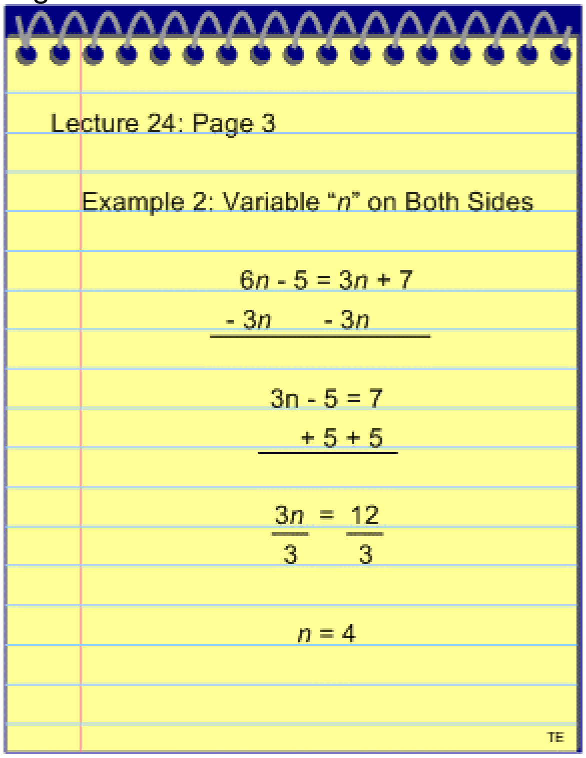
$$x + 2 = 6$$
$$\underline{-2} \quad \underline{-2}$$
$$x = 4$$

(ball-bearings in each cup)

[An “x” on both sides]

TE

Alg024-03



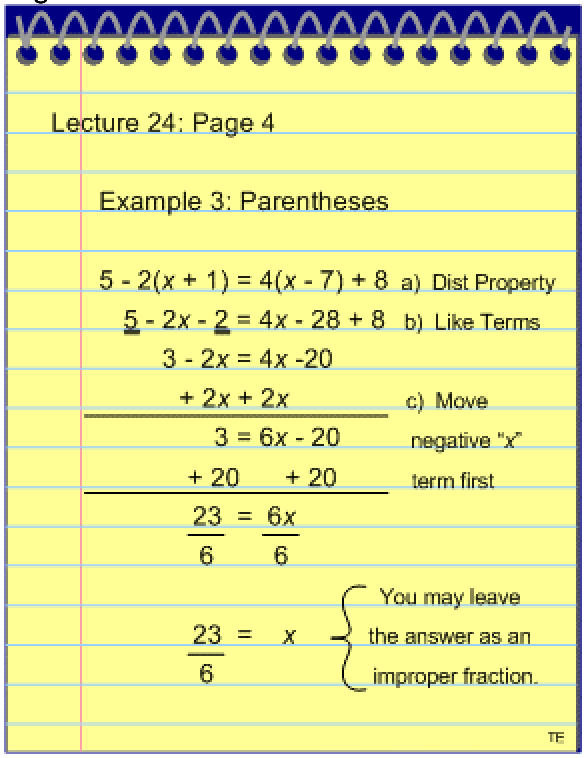
Lecture 24: Page 3

Example 2: Variable “n” on Both Sides

$$6n - 5 = 3n + 7$$
$$\underline{-3n \qquad -3n}$$
$$3n - 5 = 7$$
$$\underline{+5 + 5}$$
$$\frac{3n}{3} = \frac{12}{3}$$
$$n = 4$$

TE

Alg024-04



Lecture 24: Page 4

Example 3: Parentheses

$5 - 2(x + 1) = 4(x - 7) + 8$ a) Dist Property

$\underline{5} - 2x - \underline{2} = 4x - 28 + 8$ b) Like Terms

$3 - 2x = 4x - 20$

$\underline{+2x + 2x}$ c) Move negative “x” term first

$3 = 6x - 20$

$\underline{+20 \qquad +20}$

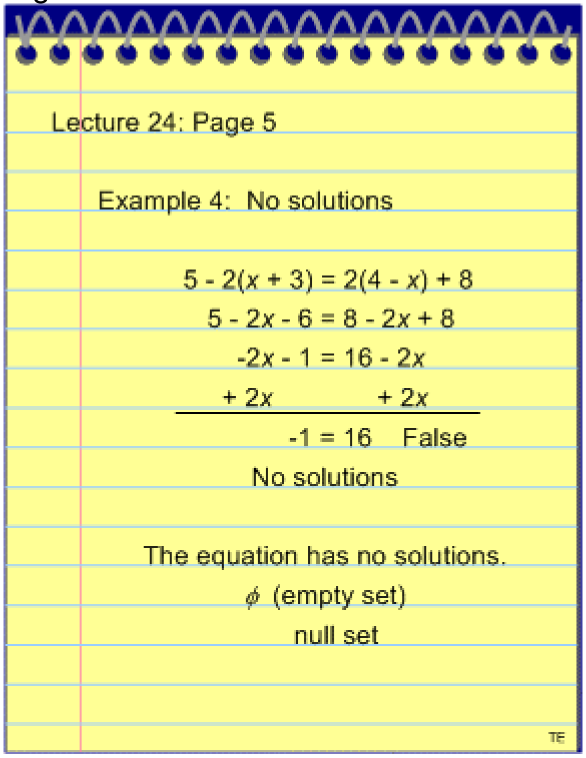
$\frac{23}{6} = \frac{6x}{6}$

$\frac{23}{6} = x$ $\Big\}$ You may leave the answer as an improper fraction.

TE

Lecture 24 Notes, Continued

Alg024-05



Lecture 24: Page 5

Example 4: No solutions

$$5 - 2(x + 3) = 2(4 - x) + 8$$
$$5 - 2x - 6 = 8 - 2x + 8$$
$$-2x - 1 = 16 - 2x$$
$$\underline{+\,2x \qquad\qquad +\,2x}$$
$$-1 = 16 \quad \text{False}$$

No solutions

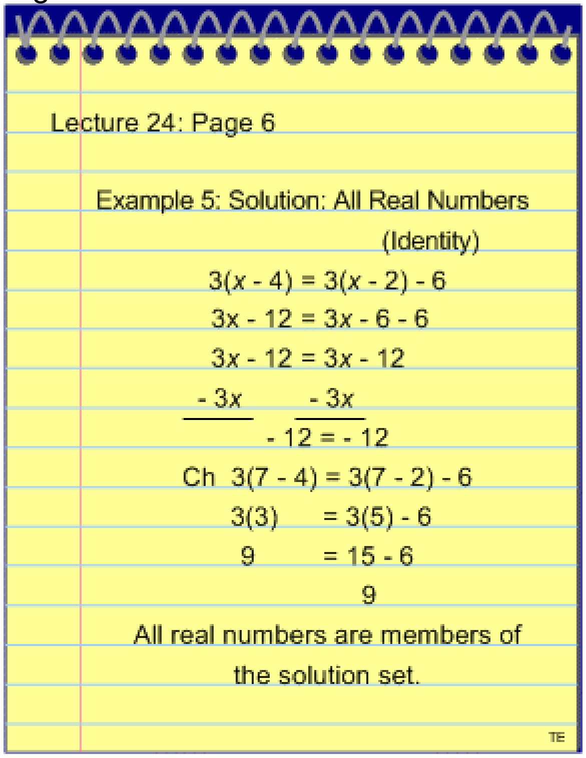
The equation has no solutions.

ϕ (empty set)

null set

TE

Alg024-06



Lecture 24: Page 6

Example 5: Solution: All Real Numbers

(Identity)

$$3(x - 4) = 3(x - 2) - 6$$
$$3x - 12 = 3x - 6 - 6$$
$$3x - 12 = 3x - 12$$
$$\underline{-\,3x} \qquad \underline{-\,3x}$$
$$-\,12 = -\,12$$

Ch $3(7 - 4) = 3(7 - 2) - 6$

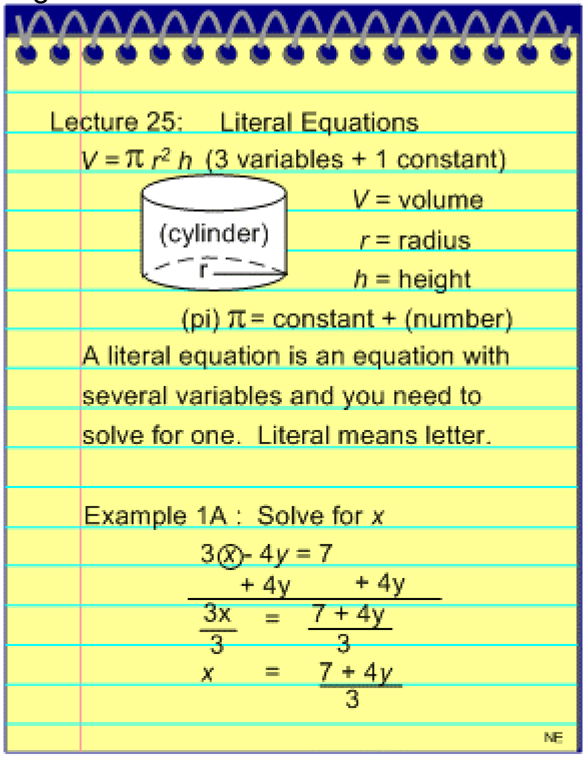
$$3(3) \quad = 3(5) - 6$$
$$9 \quad = 15 - 6$$
$$9$$

All real numbers are members of the solution set.

TE

Lecture 25 Notes

Alg025-01



Lecture 25: Literal Equations

$V = \pi r^2 h$ (3 variables + 1 constant)

(cylinder)

r

V = volume

r = radius

h = height

(pi) π = constant + (number)

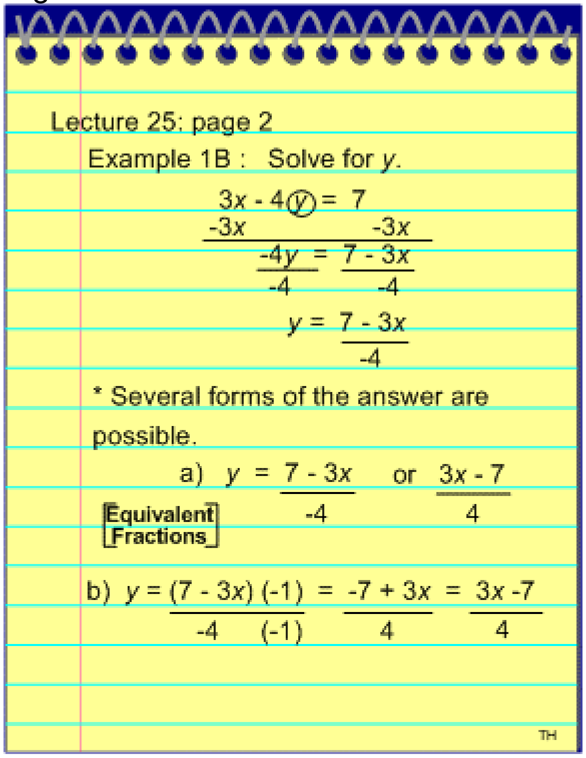
A literal equation is an equation with several variables and you need to solve for one. Literal means letter.

Example 1A : Solve for x

$$\begin{aligned} 3\textcircled{x} - 4y &= 7 \\ +4y \quad & \quad +4y \\ \frac{3x}{3} &= \frac{7+4y}{3} \\ x &= \frac{7+4y}{3} \end{aligned}$$

NE

Alg025-02



Lecture 25: page 2

Example 1B : Solve for y.

$$\begin{aligned} 3x - 4\textcircled{y} &= 7 \\ -3x \quad & \quad -3x \\ \frac{-4y}{-4} &= \frac{7-3x}{-4} \\ y &= \frac{7-3x}{-4} \end{aligned}$$

* Several forms of the answer are possible.

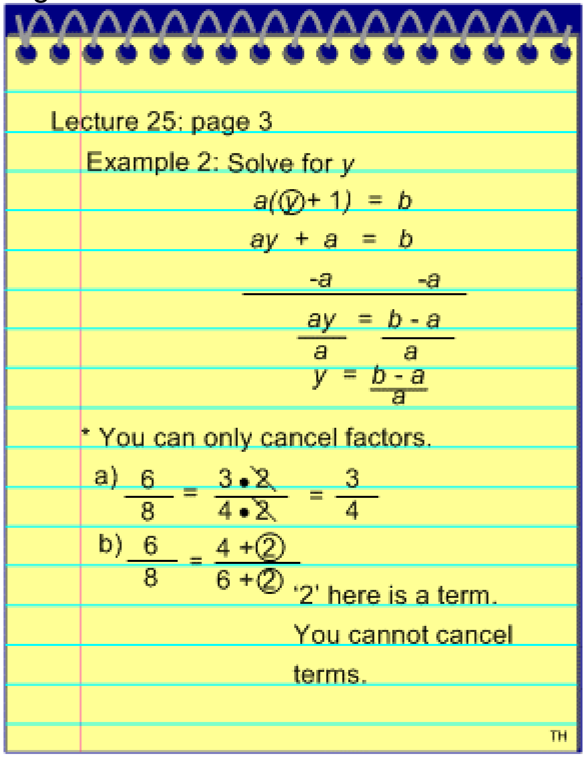
a) $y = \frac{7-3x}{-4}$ or $\frac{3x-7}{4}$

Equivalent Fractions

b) $y = \frac{(7-3x)\,(-1)}{-4 \quad (-1)} = \frac{-7+3x}{4} = \frac{3x-7}{4}$

TH

Alg025-03



Lecture 25: page 3

Example 2: Solve for y

$$\begin{aligned} a(\textcircled{y} + 1) &= b \\ ay + a &= b \\ -a \quad & \quad -a \\ \frac{ay}{a} &= \frac{b-a}{a} \\ y &= \frac{b-a}{a} \end{aligned}$$

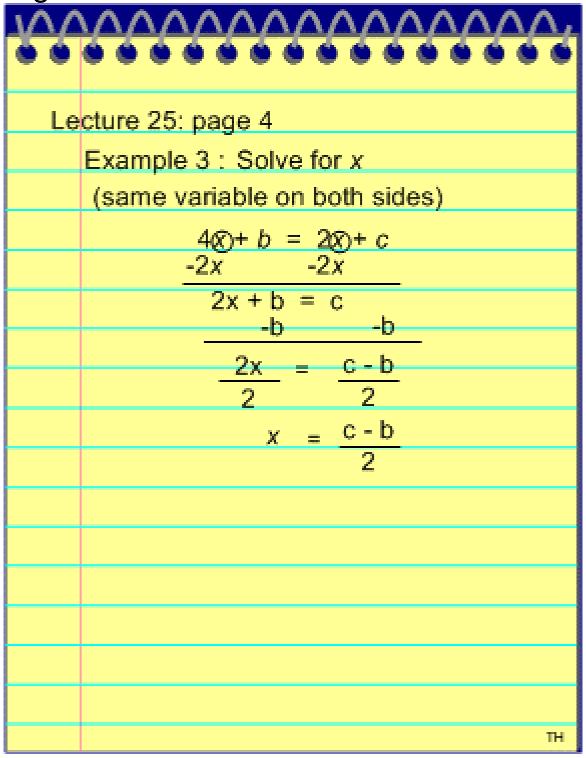
* You can only cancel factors.

a) $\frac{6}{8} = \frac{3 \bullet \cancel{2}}{4 \bullet \cancel{2}} = \frac{3}{4}$

b) $\frac{6}{8} = \frac{4 + \textcircled{2}}{6 + \textcircled{2}}$ '2' here is a term. You cannot cancel terms.

TH

Alg025-04



Lecture 25: page 4

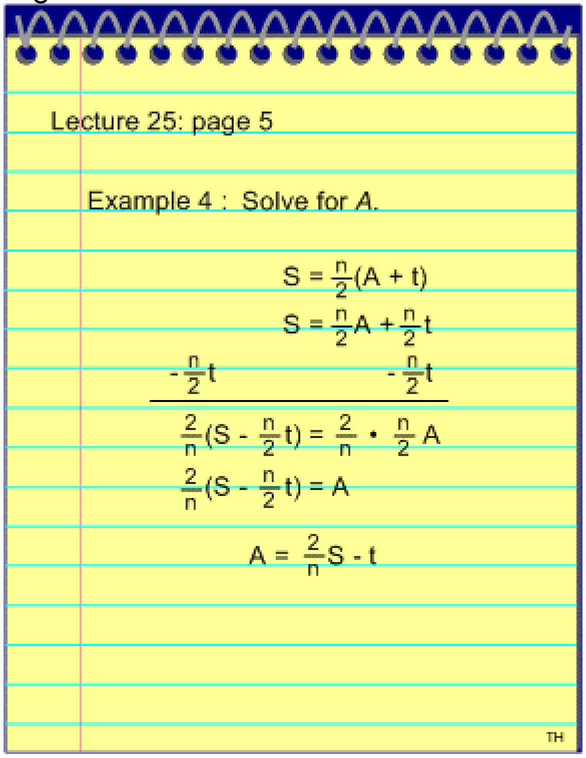
Example 3 : Solve for x

(same variable on both sides)

$$\begin{aligned} 4\textcircled{x} + b &= 2\textcircled{x} + c \\ -2x \quad & \quad -2x \\ 2x + b &= c \\ -b \quad & \quad -b \\ \frac{2x}{2} &= \frac{c-b}{2} \\ x &= \frac{c-b}{2} \end{aligned}$$

TH

Alg025-05



Lecture 25: page 5

Example 4 : Solve for *A*.

$$S = \frac{n}{2}(A + t)$$

$$S = \frac{n}{2}A + \frac{n}{2}t$$

$$\underline{-\frac{n}{2}t \qquad\qquad\qquad -\frac{n}{2}t}$$

$$\frac{2}{n}(S - \frac{n}{2}t) = \frac{2}{n} \cdot \frac{n}{2}A$$

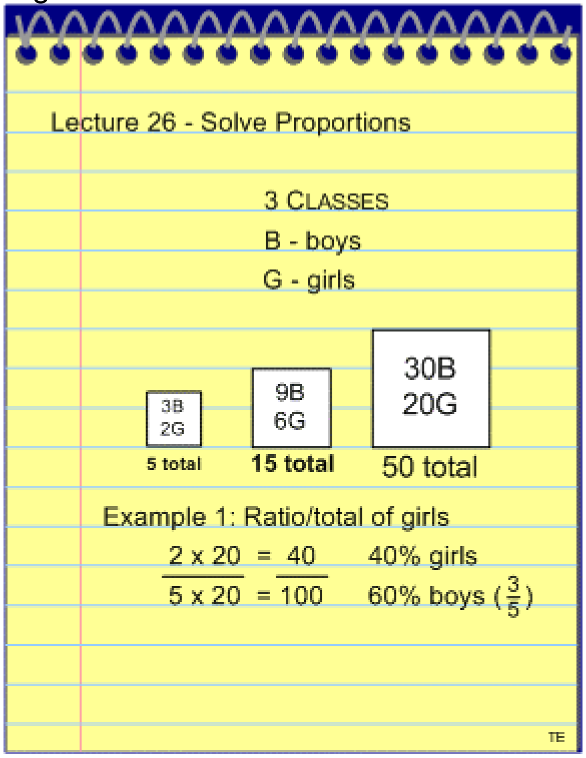
$$\frac{2}{n}(S - \frac{n}{2}t) = A$$

$$A = \frac{2}{n}S - t$$

TH

Lecture 26 Notes

Alg026-01



Lecture 26 - Solve Proportions

3 CLASSES
B - boys
G - girls

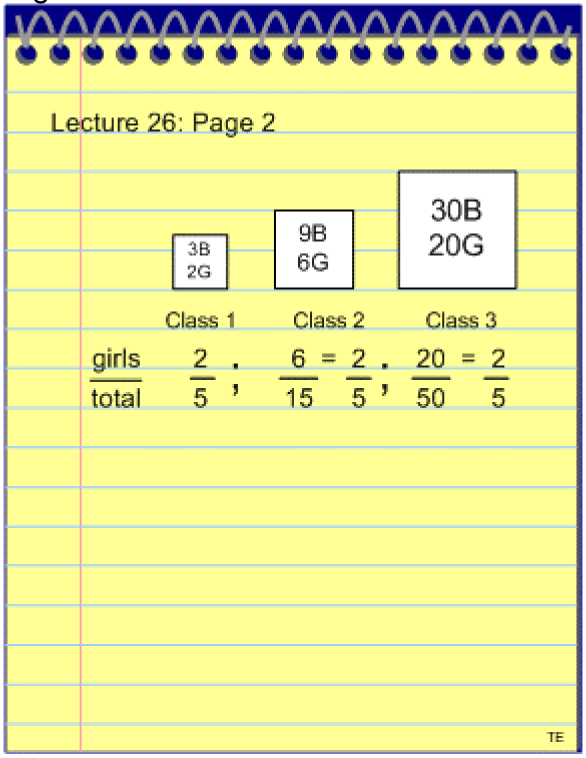
3B 2G	9B 6G	30B 20G
5 total	**15 total**	50 total

Example 1: Ratio/total of girls

$$\frac{2 \times 20}{5 \times 20} = \frac{40}{100}$$

40% girls
60% boys ($\frac{3}{5}$)

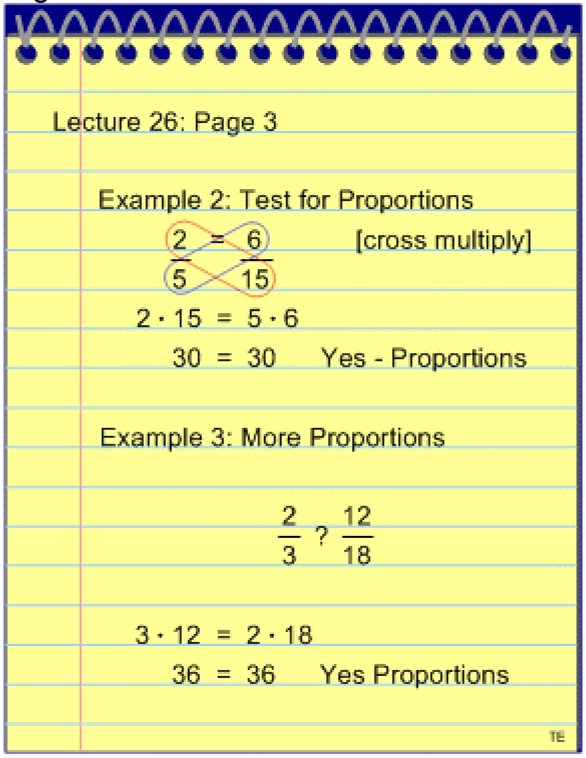
Alg026-02



Lecture 26: Page 2

	3B 2G	9B 6G	30B 20G
	Class 1	Class 2	Class 3
$\frac{\text{girls}}{\text{total}}$	$\frac{2}{5}$,	$\frac{6}{15} = \frac{2}{5}$,	$\frac{20}{50} = \frac{2}{5}$

Alg026-03



Lecture 26: Page 3

Example 2: Test for Proportions

$$\frac{2}{5} = \frac{6}{15}$$

[cross multiply]

$2 \cdot 15 = 5 \cdot 6$

$30 = 30$ Yes - Proportions

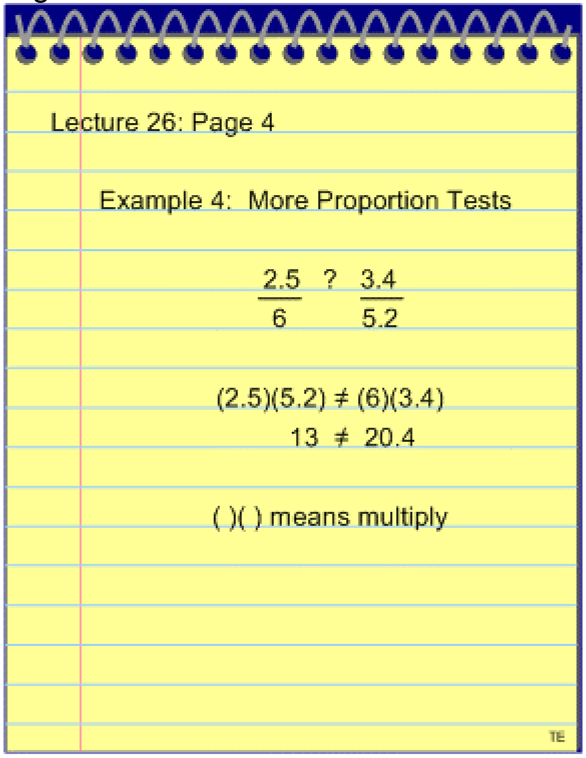
Example 3: More Proportions

$$\frac{2}{3} \; ? \; \frac{12}{18}$$

$3 \cdot 12 = 2 \cdot 18$

$36 = 36$ Yes Proportions

Alg026-04



Lecture 26: Page 4

Example 4: More Proportion Tests

$$\frac{2.5}{6} \; ? \; \frac{3.4}{5.2}$$

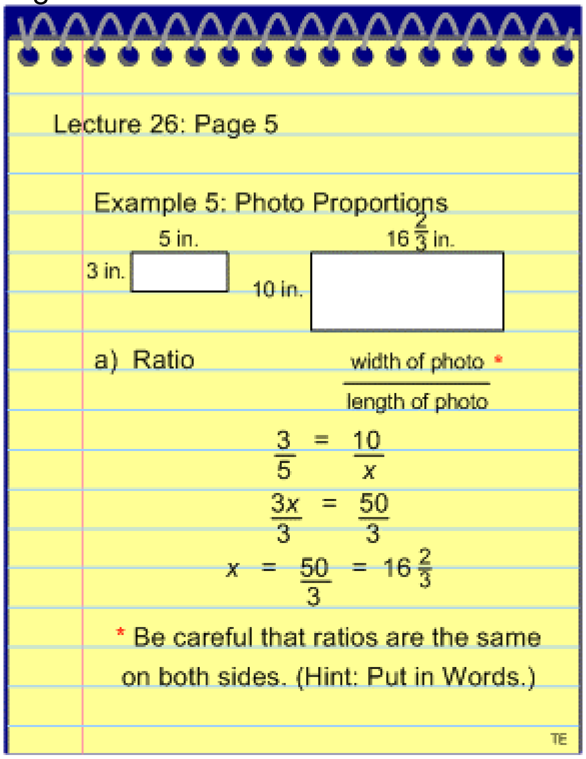
$(2.5)(5.2) \neq (6)(3.4)$

$13 \neq 20.4$

( )( ) means multiply

Lecture 26 Notes, Continued

Alg026-05



Lecture 26: Page 5

Example 5: Photo Proportions

5 in.

3 in.

$16\frac{2}{3}$ in.

10 in.

a) Ratio $\dfrac{\text{width of photo}}{\text{length of photo}}$ *

$$\frac{3}{5} = \frac{10}{x}$$

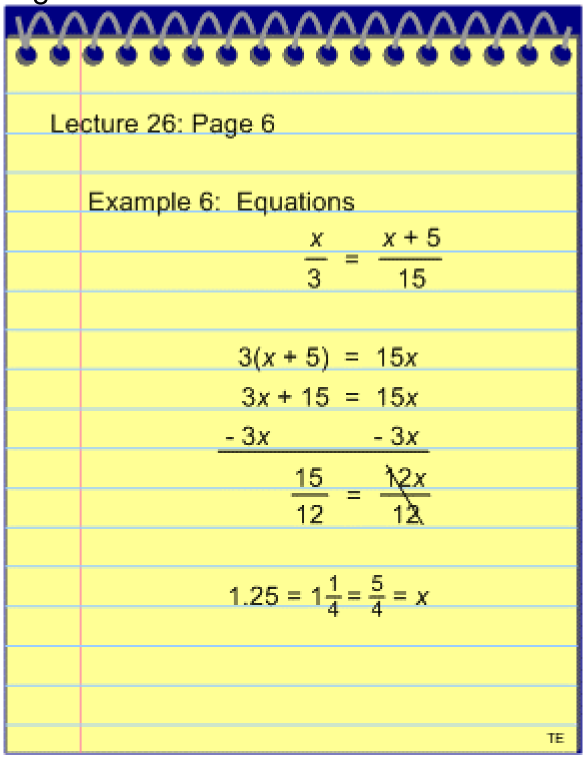
$$\frac{3x}{3} = \frac{50}{3}$$

$$x = \frac{50}{3} = 16\frac{2}{3}$$

* Be careful that ratios are the same on both sides. (Hint: Put in Words.)

TE

Alg026-06



Lecture 26: Page 6

Example 6: Equations

$$\frac{x}{3} = \frac{x+5}{15}$$

$$3(x + 5) = 15x$$

$$3x + 15 = 15x$$

$$-3x \qquad -3x$$

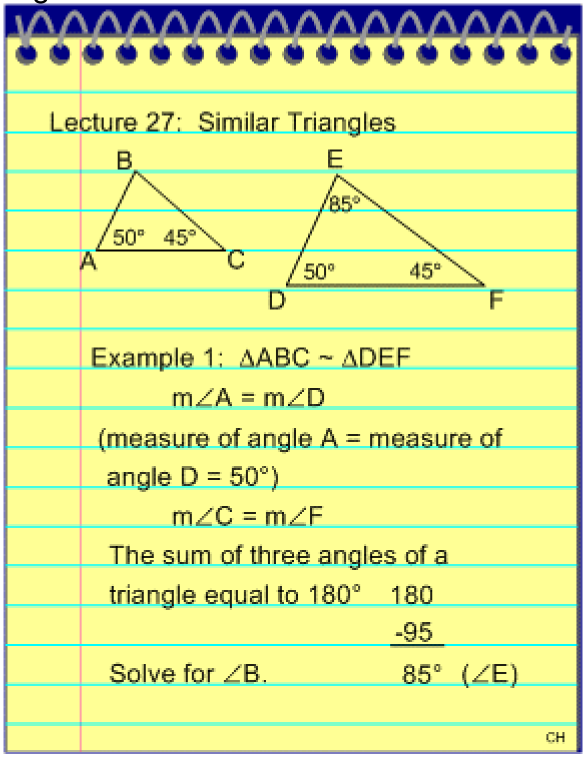
$$\frac{15}{12} = \frac{\cancel{12}x}{\cancel{12}}$$

$$1.25 = 1\frac{1}{4} = \frac{5}{4} = x$$

TE

Lecture 27 Notes

Alg027-01



Lecture 27: Similar Triangles

Example 1: ΔABC ~ ΔDEF

m∠A = m∠D

(measure of angle A = measure of angle D = 50°)

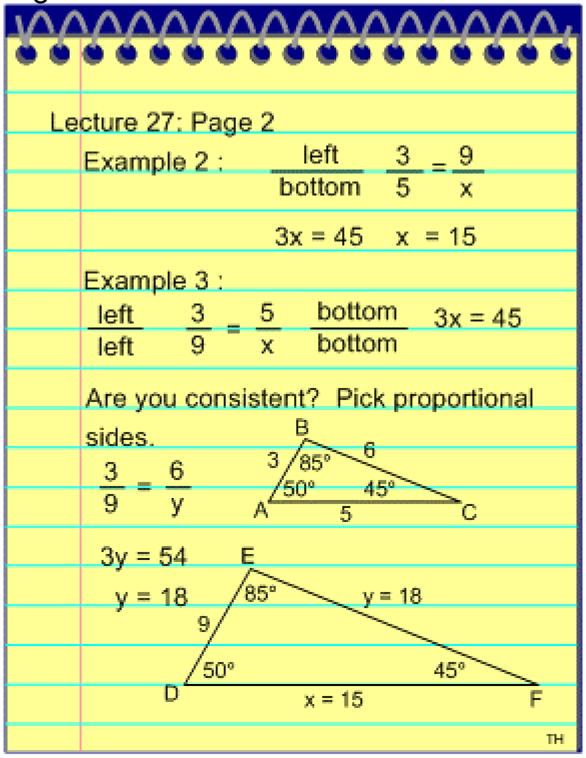
m∠C = m∠F

The sum of three angles of a triangle equal to 180°

$$\begin{array}{r} 180 \\ \underline{-95} \\ 85° \end{array}$$

Solve for ∠B. 85° (∠E)

Alg027-02



Lecture 27: Page 2

Example 2 : $\frac{\text{left}}{\text{bottom}}$ $\frac{3}{5} = \frac{9}{x}$

3x = 45 x = 15

Example 3 :

$\frac{\text{left}}{\text{left}}$ $\frac{3}{9} = \frac{5}{x}$ $\frac{\text{bottom}}{\text{bottom}}$ 3x = 45

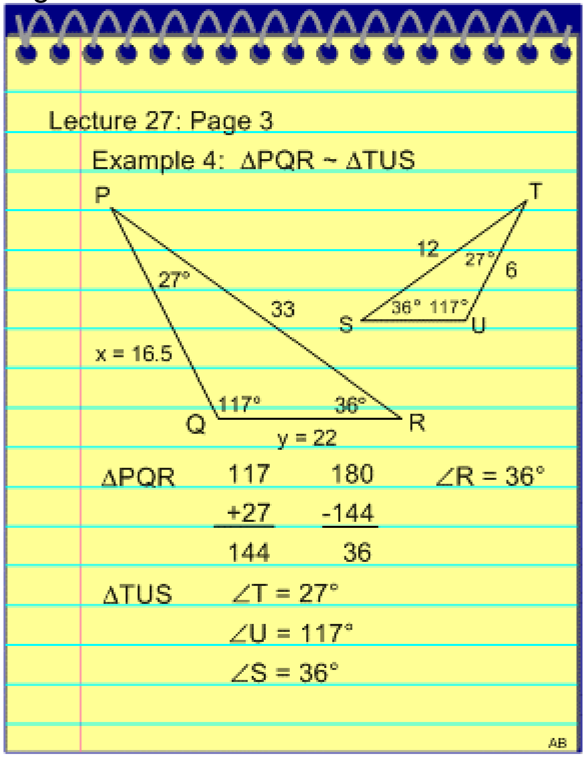
Are you consistent? Pick proportional sides.

$\frac{3}{9} = \frac{6}{y}$

3y = 54

y = 18

Alg027-03



Lecture 27: Page 3

Example 4: ΔPQR ~ ΔTUS

ΔPQR

$$\begin{array}{r} 117 \\ \underline{+27} \\ 144 \end{array} \qquad \begin{array}{r} 180 \\ \underline{-144} \\ 36 \end{array}$$

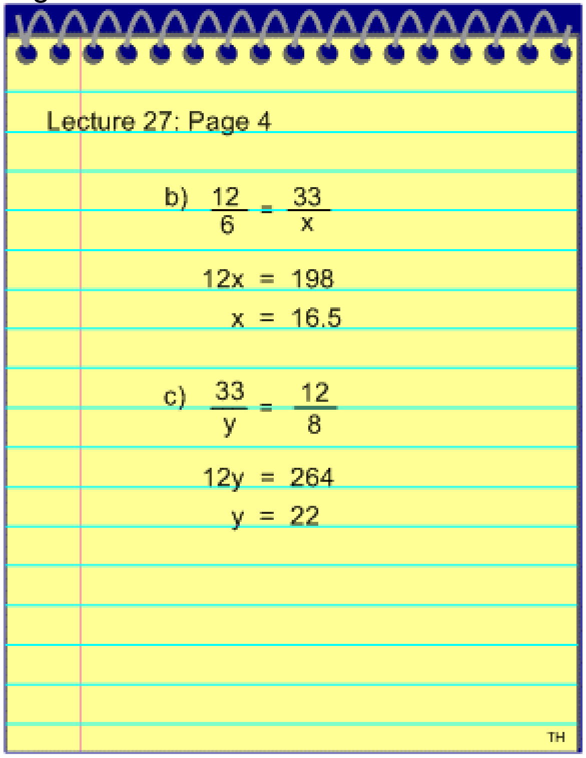
∠R = 36°

ΔTUS ∠T = 27°

∠U = 117°

∠S = 36°

Alg027-04



Lecture 27: Page 4

b) $\frac{12}{6} = \frac{33}{x}$

12x = 198

x = 16.5

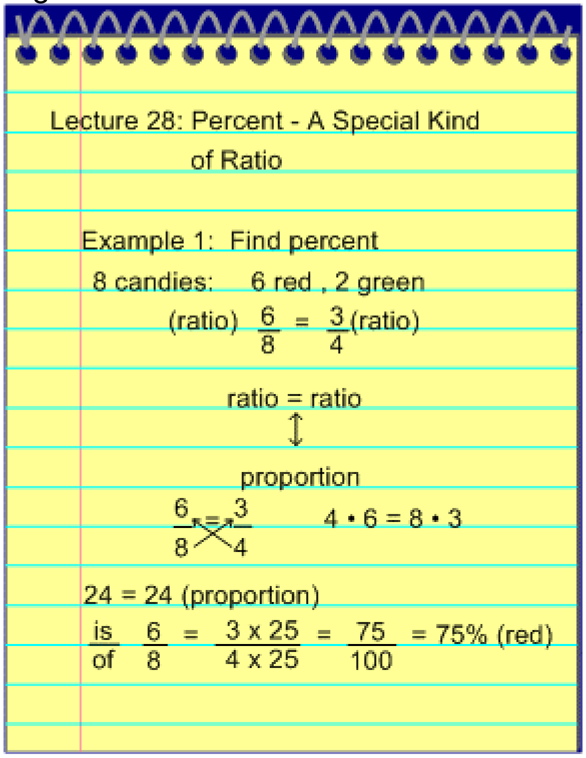
c) $\frac{33}{y} = \frac{12}{8}$

12y = 264

y = 22

Lecture 28 Notes

Alg028-01



Lecture 28: Percent - A Special Kind of Ratio

Example 1: Find percent

8 candies: 6 red , 2 green

(ratio) $\frac{6}{8} = \frac{3}{4}$ (ratio)

ratio = ratio

↕

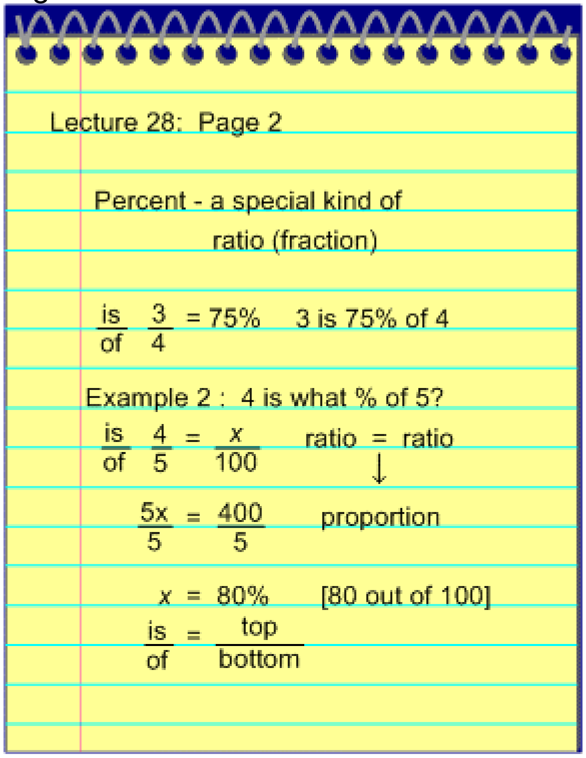
proportion

$\frac{6}{8} = \frac{3}{4}$ 4 • 6 = 8 • 3

24 = 24 (proportion)

$\frac{\text{is}}{\text{of}} \quad \frac{6}{8} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\%$ (red)

Alg028-02



Lecture 28: Page 2

Percent - a special kind of ratio (fraction)

$\frac{\text{is}}{\text{of}} \quad \frac{3}{4} = 75\%$ 3 is 75% of 4

Example 2 : 4 is what % of 5?

$\frac{\text{is}}{\text{of}} \quad \frac{4}{5} = \frac{x}{100}$ ratio = ratio

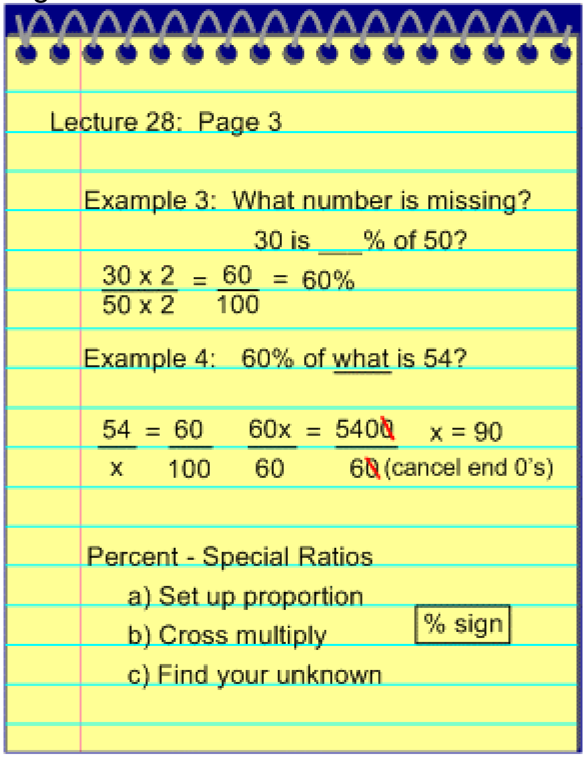
↓

$\frac{5x}{5} = \frac{400}{5}$ proportion

$x = 80\%$ [80 out of 100]

$\frac{\text{is}}{\text{of}} = \frac{\text{top}}{\text{bottom}}$

Alg028-03



Lecture 28: Page 3

Example 3: What number is missing?

30 is ___% of 50?

$\frac{30 \times 2}{50 \times 2} = \frac{60}{100} = 60\%$

Example 4: 60% of <u>what</u> is 54?

$\frac{54}{x} = \frac{60}{100} \quad \frac{60x}{60} = \frac{540\cancel{0}}{6\cancel{0}}$ (cancel end 0's) $x = 90$

Percent - Special Ratios

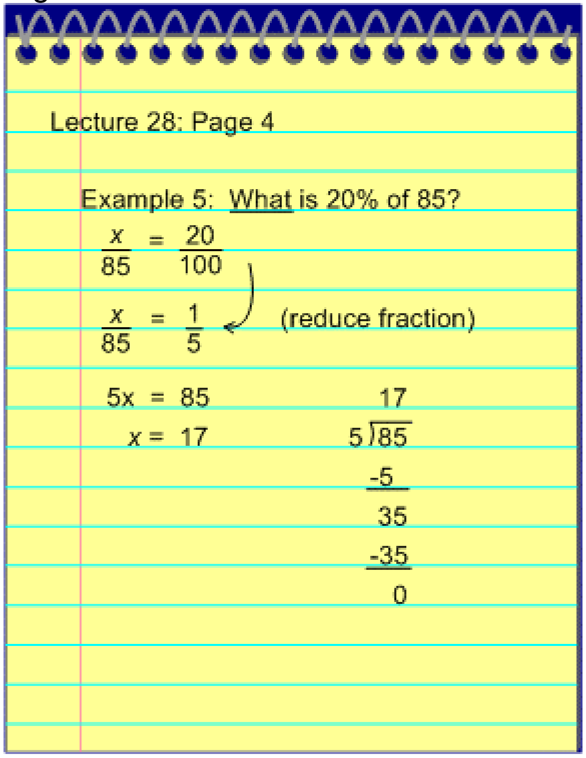
a) Set up proportion

b) Cross multiply

c) Find your unknown

$\boxed{\%\text{ sign}}$

Alg028-04



Lecture 28: Page 4

Example 5: What is 20% of 85?

$\frac{x}{85} = \frac{20}{100}$

$\frac{x}{85} = \frac{1}{5}$ (reduce fraction)

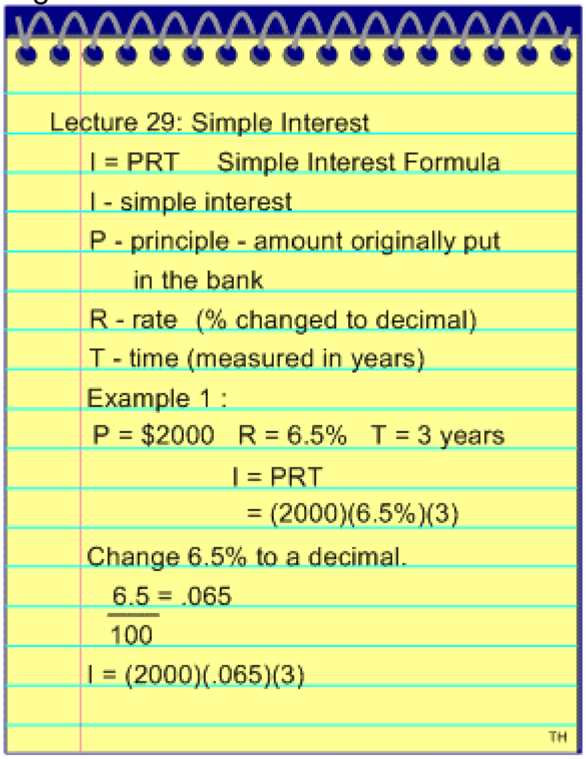
$5x = 85$

$x = 17$

$$\begin{array}{r} 17 \\ 5\overline{)85} \\ \underline{-5} \\ 35 \\ \underline{-35} \\ 0 \end{array}$$

Lecture 29 Notes

Alg029-01



Lecture 29: Simple Interest

I = PRT Simple Interest Formula

I - simple interest

P - principle - amount originally put in the bank

R - rate (% changed to decimal)

T - time (measured in years)

Example 1 :

P = \$2000 R = 6.5% T = 3 years

$I = PRT$

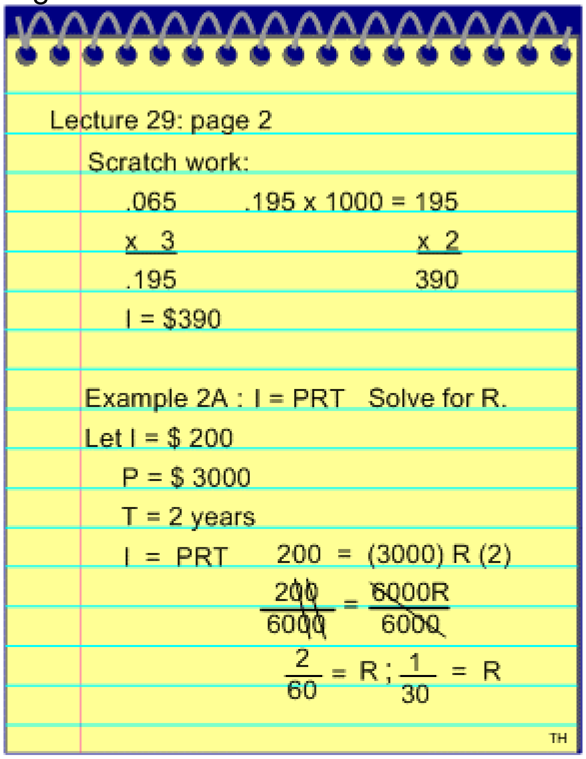
$= (2000)(6.5\%)(3)$

Change 6.5% to a decimal.

$\frac{6.5}{100} = .065$

$I = (2000)(.065)(3)$

Alg029-02



Lecture 29: page 2

Scratch work:

$$\begin{array}{r} .065 \\ \underline{x\ \ 3} \\ .195 \end{array} \qquad .195 \text{ x } 1000 = \begin{array}{r} 195 \\ \underline{x\ 2} \\ 390 \end{array}$$

I = \$390

Example 2A : I = PRT Solve for R.

Let I = \$ 200

P = \$ 3000

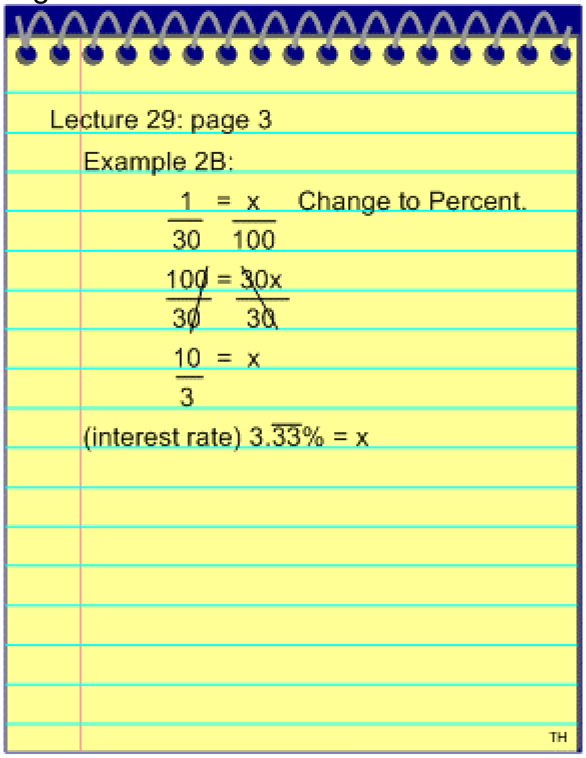
T = 2 years

$I = PRT \qquad 200 = (3000)\,R\,(2)$

$$\frac{20\cancel{0}}{600\cancel{0}} = \frac{\cancel{6}000R}{600\cancel{0}}$$

$$\frac{2}{60} = R\,;\ \frac{1}{30} = R$$

Alg029-03



Lecture 29: page 3

Example 2B:

$\frac{1}{30} = \frac{x}{100}$ Change to Percent.

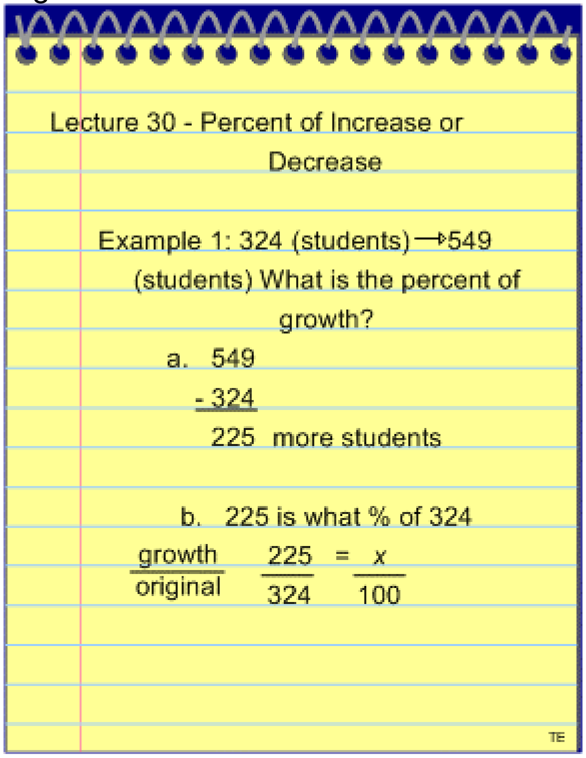
$$\frac{10\cancel{0}}{3\cancel{0}} = \frac{\cancel{3}0x}{3\cancel{0}}$$

$$\frac{10}{3} = x$$

(interest rate) $3.\overline{33}\% = x$

Lecture 30 Notes

Alg030-01



Lecture 30 - Percent of Increase or Decrease

Example 1: 324 (students) → 549 (students) What is the percent of growth?

a. 549

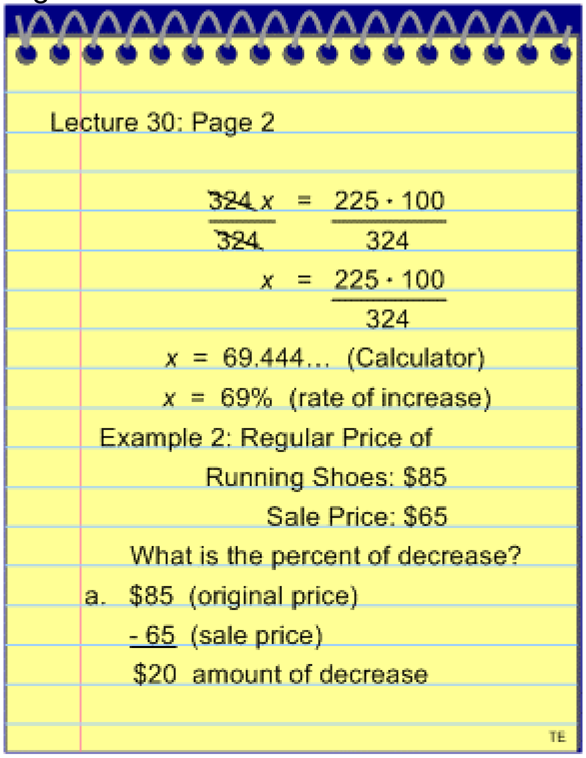
$\underline{-\ 324}$

225 more students

b. 225 is what % of 324

$\frac{\text{growth}}{\text{original}} \quad \frac{225}{324} = \frac{x}{100}$

Alg030-02



Lecture 30: Page 2

$$\frac{\cancel{324}\,x}{\cancel{324}} = \frac{225 \cdot 100}{324}$$

$$x = \frac{225 \cdot 100}{324}$$

$x = 69.444\ldots$ (Calculator)

$x = 69\%$ (rate of increase)

Example 2: Regular Price of Running Shoes: $85

Sale Price: $65

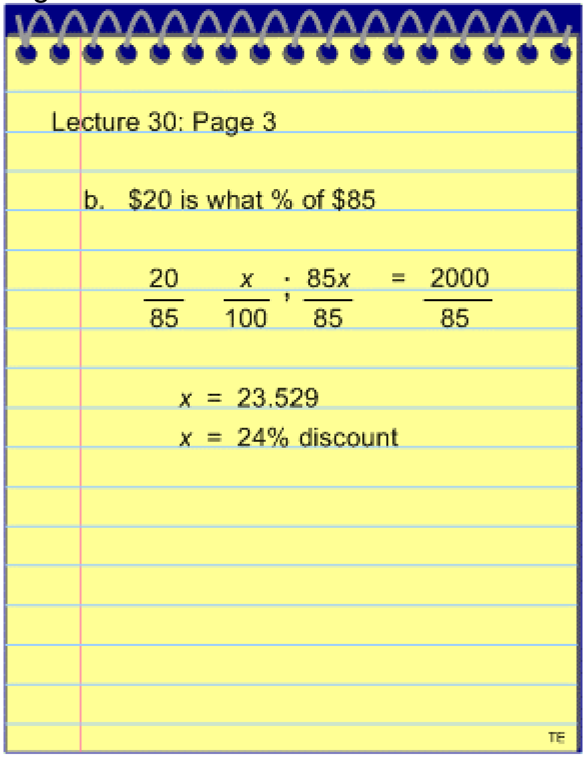
What is the percent of decrease?

a. $85 (original price)

$\underline{-\ 65}$ (sale price)

$20 amount of decrease

Alg030-03



Lecture 30: Page 3

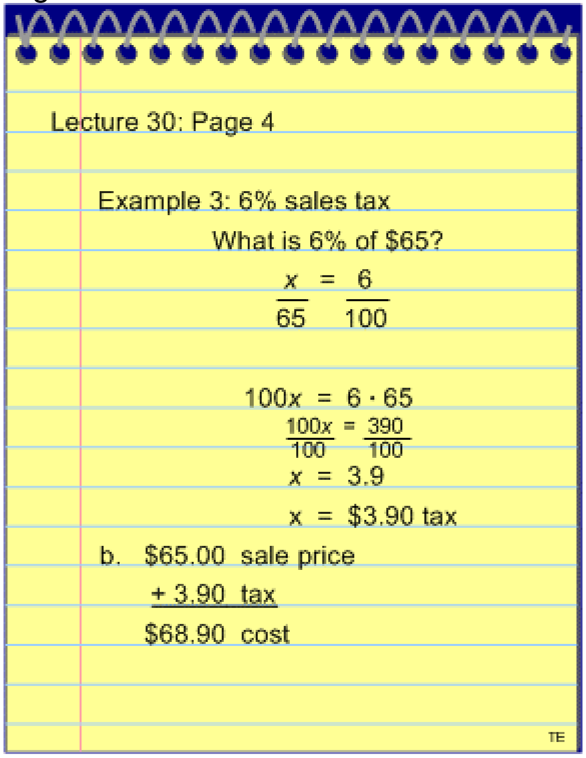
b. $20 is what % of $85

$$\frac{20}{85} \quad \frac{x}{100} \;;\; \frac{85x}{85} = \frac{2000}{85}$$

$x = 23.529$

$x = 24\%$ discount

Alg030-04



Lecture 30: Page 4

Example 3: 6% sales tax

What is 6% of $65?

$$\frac{x}{65} = \frac{6}{100}$$

$$100x = 6 \cdot 65$$

$$\frac{100x}{100} = \frac{390}{100}$$

$$x = 3.9$$

x = $3.90 tax

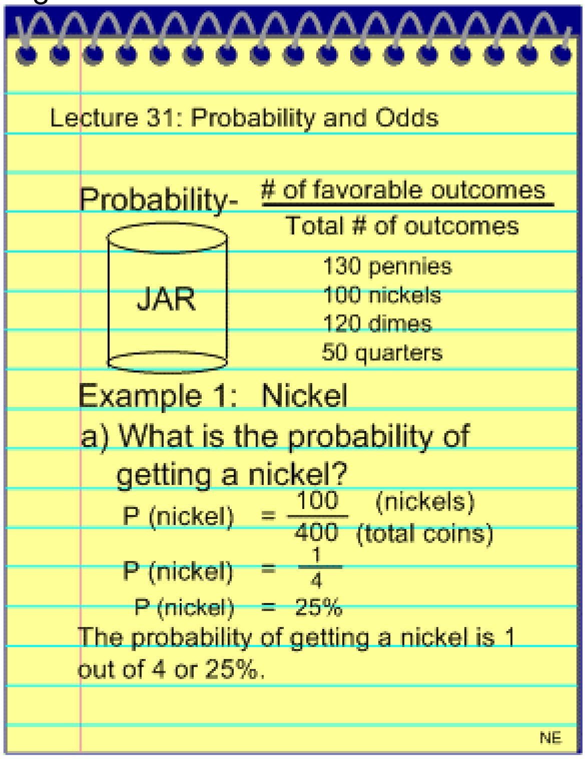
b. $65.00 sale price

$\underline{+\ 3.90}$ tax

$68.90 cost

Lecture 31 Notes

Alg031-01



Lecture 31: Probability and Odds

Probability- $\frac{\text{\# of favorable outcomes}}{\text{Total \# of outcomes}}$

JAR

130 pennies
100 nickels
120 dimes
50 quarters

Example 1: Nickel

a) What is the probability of getting a nickel?

$P\text{ (nickel)} = \frac{100}{400}\ \frac{\text{(nickels)}}{\text{(total coins)}}$

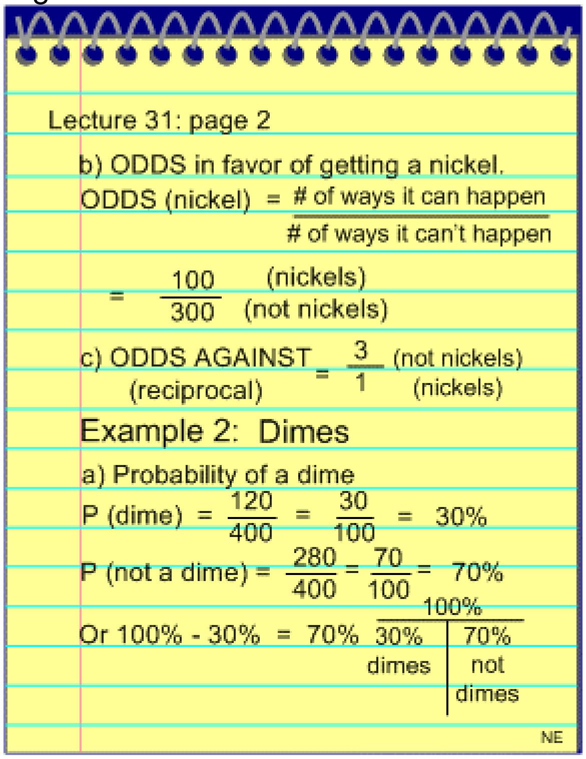
$P\text{ (nickel)} = \frac{1}{4}$

$P\text{ (nickel)} = 25\%$

The probability of getting a nickel is 1 out of 4 or 25%.

NE

Alg031-02



Lecture 31: page 2

b) ODDS in favor of getting a nickel.

$\text{ODDS (nickel)} = \frac{\text{\# of ways it can happen}}{\text{\# of ways it can't happen}}$

$= \frac{100}{300}\ \frac{\text{(nickels)}}{\text{(not nickels)}}$

c) ODDS AGAINST (reciprocal) $= \frac{3}{1}\ \frac{\text{(not nickels)}}{\text{(nickels)}}$

Example 2: Dimes

a) Probability of a dime

$P\text{ (dime)} = \frac{120}{400} = \frac{30}{100} = 30\%$

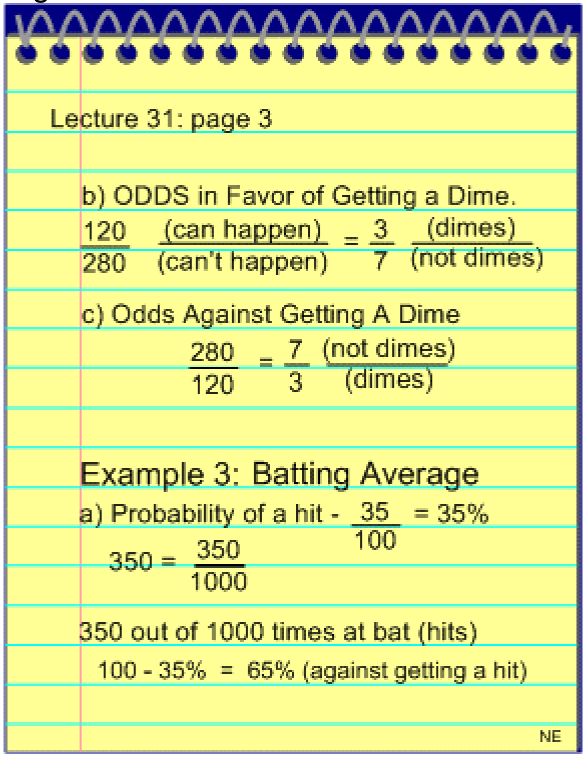
$P\text{ (not a dime)} = \frac{280}{400} = \frac{70}{100} = 70\%$

Or 100% - 30% = 70%

100%
30% | 70%
dimes | not dimes

NE

Alg031-03



Lecture 31: page 3

b) ODDS in Favor of Getting a Dime.

$\frac{120}{280}\ \frac{\text{(can happen)}}{\text{(can't happen)}} = \frac{3}{7}\ \frac{\text{(dimes)}}{\text{(not dimes)}}$

c) Odds Against Getting A Dime

$\frac{280}{120} = \frac{7}{3}\ \frac{\text{(not dimes)}}{\text{(dimes)}}$

Example 3: Batting Average

a) Probability of a hit - $\frac{35}{100} = 35\%$

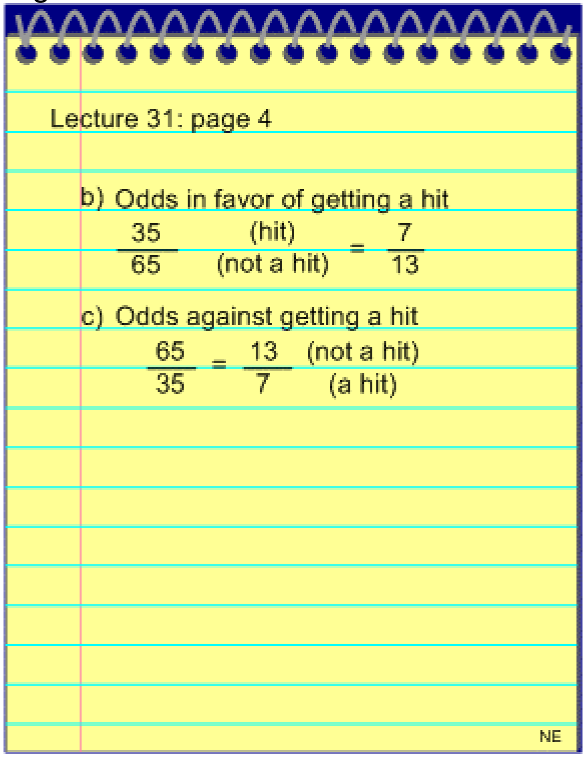
$350 = \frac{350}{1000}$

350 out of 1000 times at bat (hits)

100 - 35% = 65% (against getting a hit)

NE

Alg031-04



Lecture 31: page 4

b) Odds in favor of getting a hit

$\frac{35}{65}\ \frac{\text{(hit)}}{\text{(not a hit)}} = \frac{7}{13}$

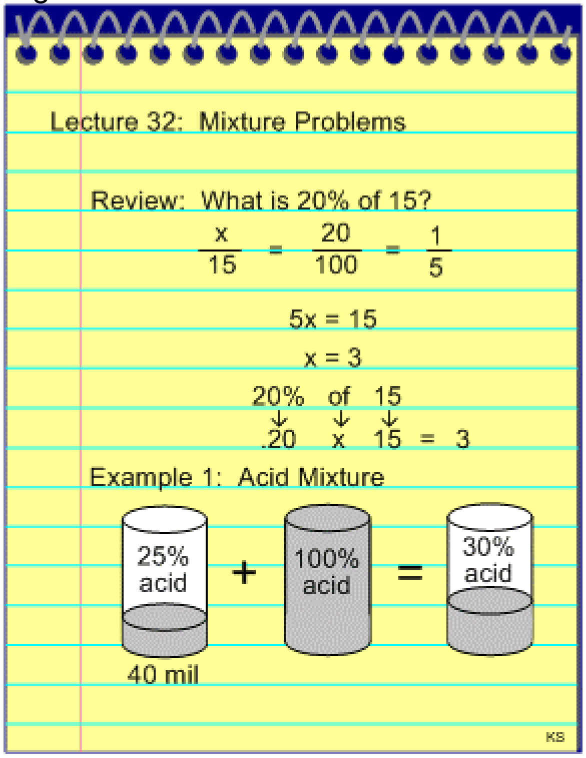
c) Odds against getting a hit

$\frac{65}{35} = \frac{13}{7}\ \frac{\text{(not a hit)}}{\text{(a hit)}}$

NE

Lecture 32 Notes

Alg032-01



Lecture 32: Mixture Problems

Review: What is 20% of 15?

$$\frac{x}{15} = \frac{20}{100} = \frac{1}{5}$$

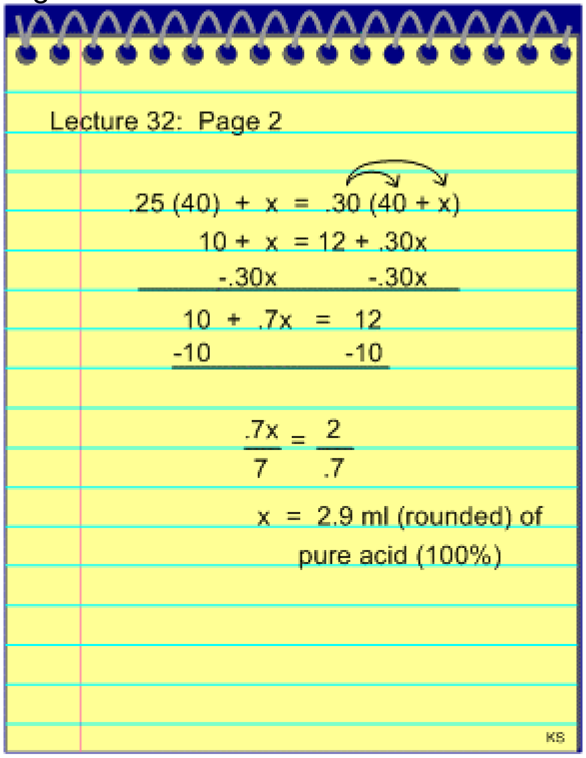
$$5x = 15$$

$$x = 3$$

$$\begin{array}{ccc} 20\% & \text{of} & 15 \\ \downarrow & \downarrow & \downarrow \\ .20 & x & 15 \end{array} = 3$$

Example 1: Acid Mixture

Alg032-02



Lecture 32: Page 2

$$.25\,(40) + x = .30\,(40 + x)$$

$$10 + x = 12 + .30x$$

$$\underline{\quad -.30x \qquad\quad -.30x \quad}$$

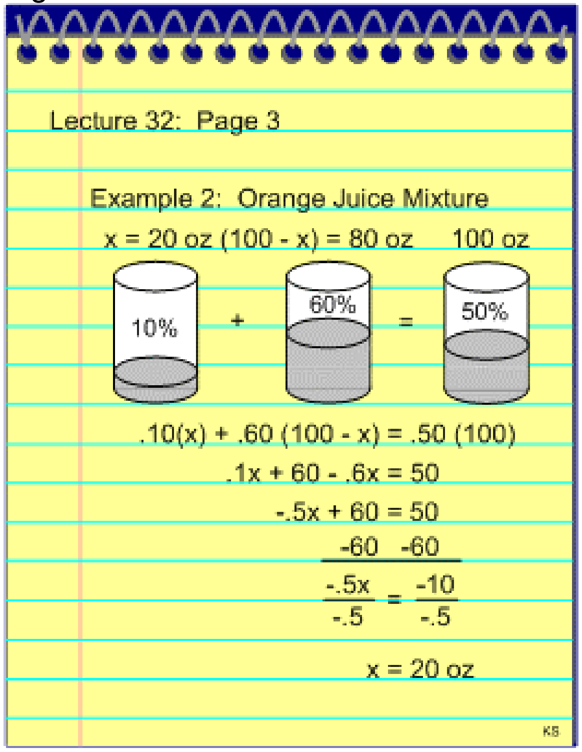
$$10 + .7x = 12$$

$$\underline{-10 \qquad\qquad -10}$$

$$\frac{.7x}{7} = \frac{2}{.7}$$

x = 2.9 ml (rounded) of pure acid (100%)

Alg032-03



Lecture 32: Page 3

Example 2: Orange Juice Mixture

$$.10(x) + .60\,(100 - x) = .50\,(100)$$

$$.1x + 60 - .6x = 50$$

$$-.5x + 60 = 50$$

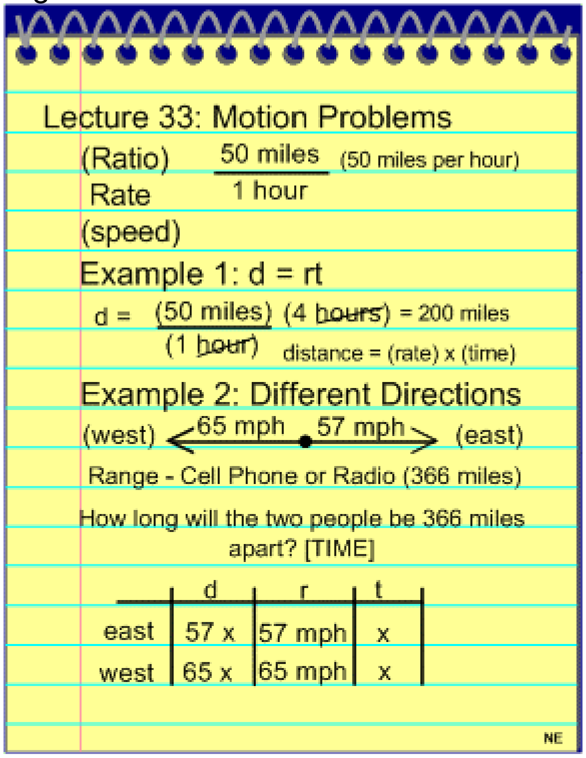
$$\underline{\quad -60 \quad -60 \quad}$$

$$\frac{-.5x}{-.5} = \frac{-10}{-.5}$$

$$x = 20 \text{ oz}$$

Lecture 33 Notes

Alg033-01



Lecture 33: Motion Problems

(Ratio) Rate (speed) $\frac{50 \text{ miles}}{1 \text{ hour}}$ (50 miles per hour)

Example 1: d = rt

d = $\frac{(50 \text{ miles})}{(1 \cancel{\text{hour}})}$ $(4 \cancel{\text{hours}})$ = 200 miles

distance = (rate) x (time)

Example 2: Different Directions

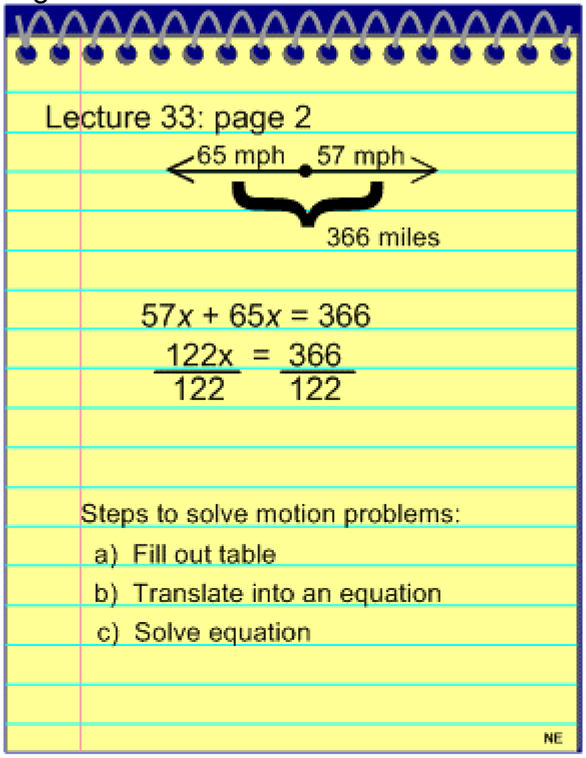
(west) $\xleftarrow{65 \text{ mph}} \bullet \xrightarrow{57 \text{ mph}}$ (east)

Range - Cell Phone or Radio (366 miles)

How long will the two people be 366 miles apart? [TIME]

	d	r	t
east	57 x	57 mph	x
west	65 x	65 mph	x

Alg033-02



Lecture 33: page 2

$\xleftarrow{65 \text{ mph}} \bullet \xrightarrow{57 \text{ mph}}$

366 miles

$57x + 65x = 366$

$\frac{122x}{122} = \frac{366}{122}$

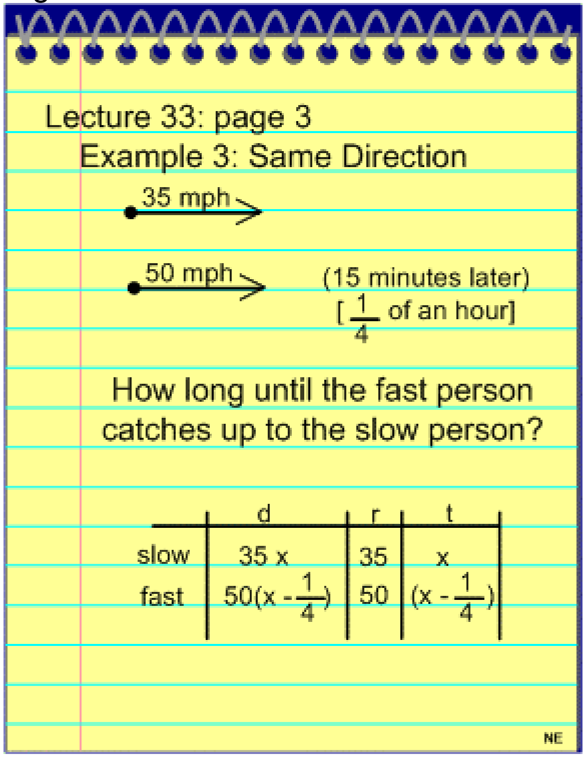
Steps to solve motion problems:

a) Fill out table

b) Translate into an equation

c) Solve equation

Alg033-03



Lecture 33: page 3

Example 3: Same Direction

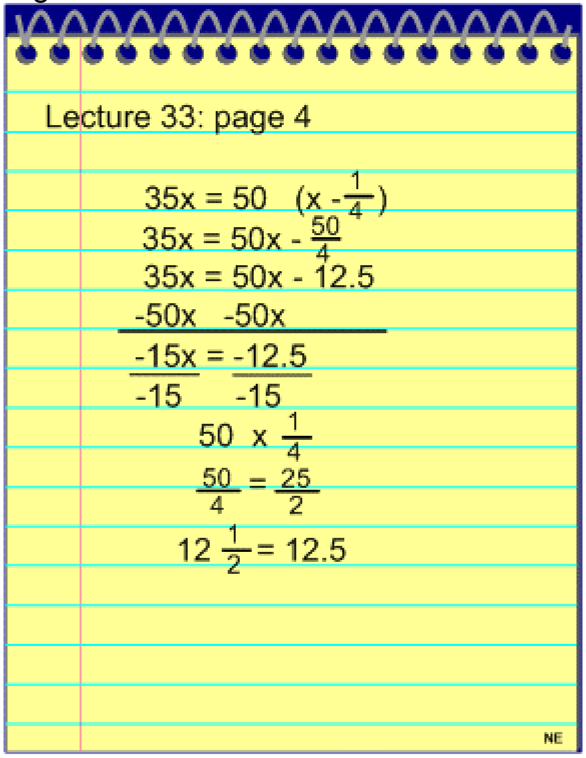
$\bullet \xrightarrow{35 \text{ mph}}$

$\bullet \xrightarrow{50 \text{ mph}}$ (15 minutes later) [$\frac{1}{4}$ of an hour]

How long until the fast person catches up to the slow person?

	d	r	t
slow	35 x	35	x
fast	$50(x - \frac{1}{4})$	50	$(x - \frac{1}{4})$

Alg033-04



Lecture 33: page 4

$35x = 50 \ (x - \frac{1}{4})$

$35x = 50x - \frac{50}{4}$

$35x = 50x - 12.5$

$-50x \quad -50x$

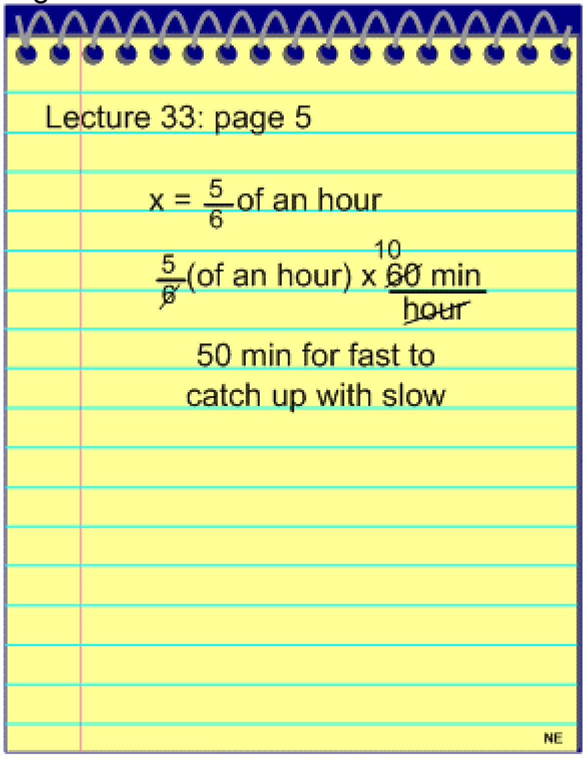
$\frac{-15x}{-15} = \frac{-12.5}{-15}$

$50 \times \frac{1}{4}$

$\frac{50}{4} = \frac{25}{2}$

$12 \frac{1}{2} = 12.5$

Alg033-05



Lecture 33: page 5

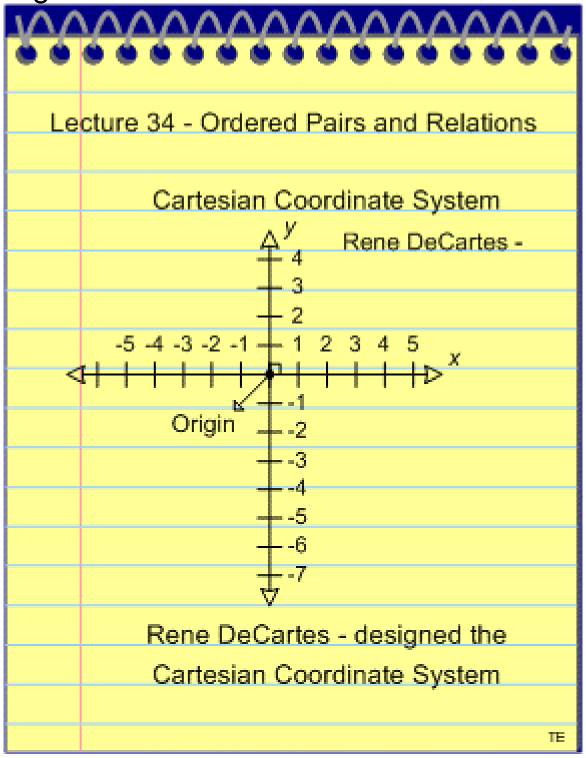
$x = \frac{5}{6}$ of an hour

$\frac{5}{\cancel{6}}$(of an hour) x $\frac{\overset{10}{\cancel{60}}\text{ min}}{\cancel{\text{hour}}}$

50 min for fast to
catch up with slow

Lecture 34 Notes

Alg034-01



Lecture 34 - Ordered Pairs and Relations

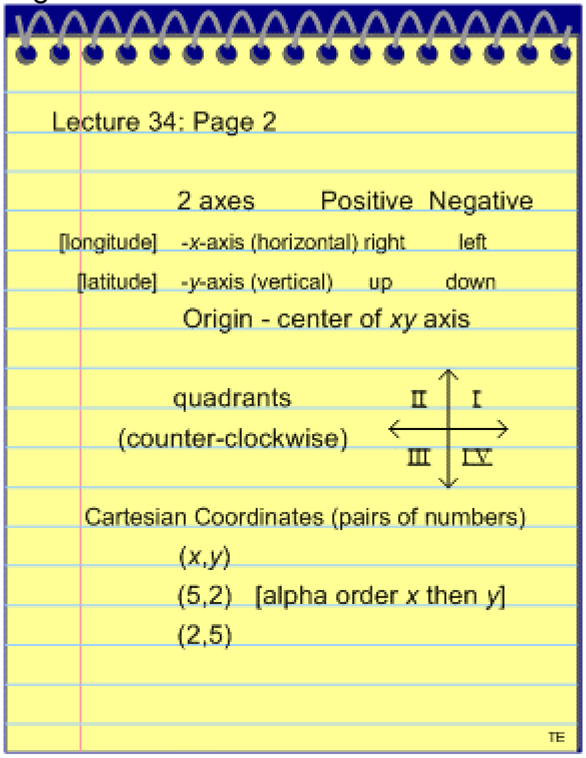
Cartesian Coordinate System

Rene DeCartes -

Rene DeCartes - designed the Cartesian Coordinate System

TE

Alg034-02



Lecture 34: Page 2

	2 axes	Positive	Negative
[longitude]	-*x*-axis (horizontal)	right	left
[latitude]	-*y*-axis (vertical)	up	down

Origin - center of *xy* axis

quadrants (counter-clockwise)

Cartesian Coordinates (pairs of numbers)

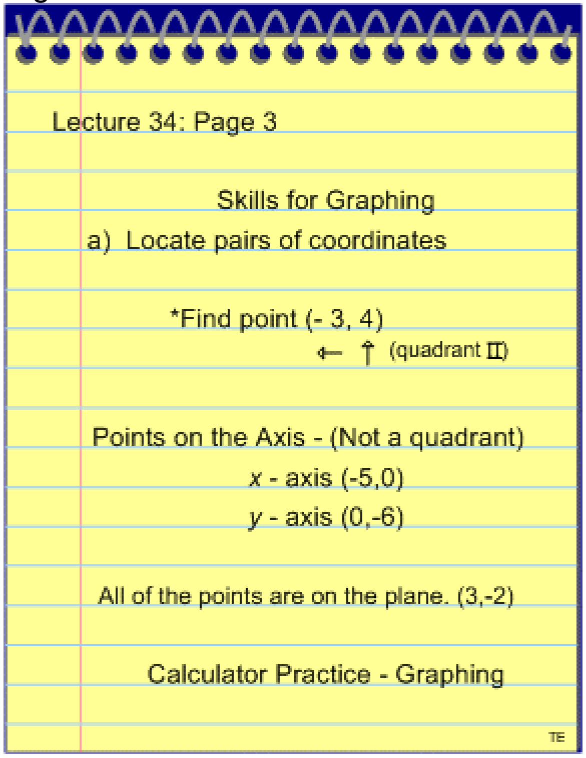
(*x*,*y*)

(5,2) [alpha order *x* then *y*]

(2,5)

TE

Alg034-03



Lecture 34: Page 3

Skills for Graphing

a) Locate pairs of coordinates

*Find point (- 3, 4)

← ↑ (quadrant II)

Points on the Axis - (Not a quadrant)

x - axis (-5,0)

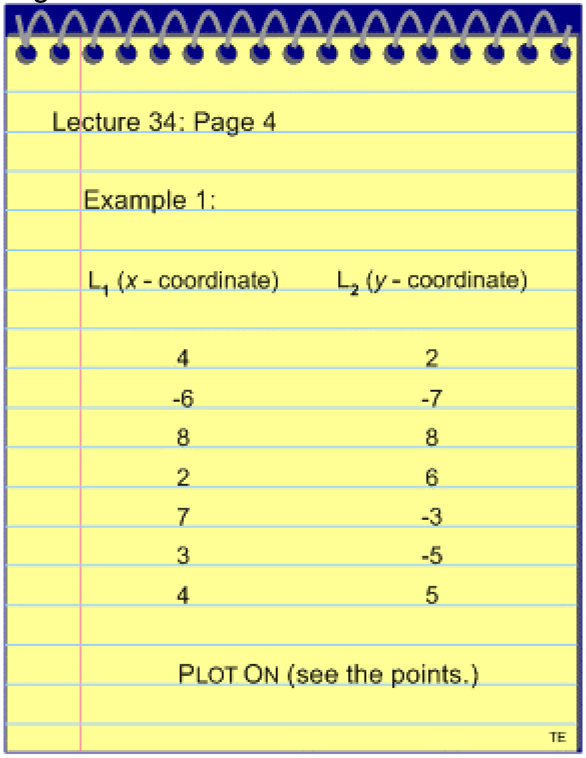
y - axis (0,-6)

All of the points are on the plane. (3,-2)

Calculator Practice - Graphing

TE

Alg034-04



Lecture 34: Page 4

Example 1:

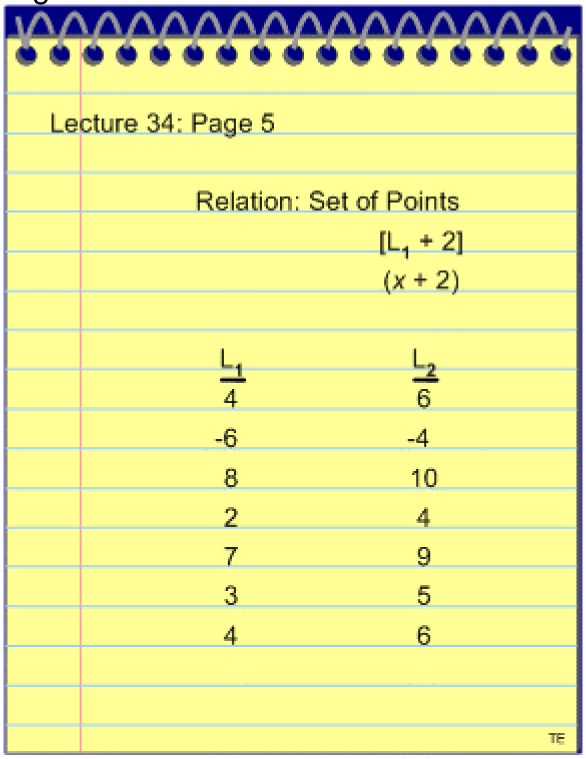
L_1 (*x* - coordinate)	L_2 (*y* - coordinate)
4	2
-6	-7
8	8
2	6
7	-3
3	-5
4	5

PLOT ON (see the points.)

TE

Lecture 34 Notes, Continued

Alg034-05



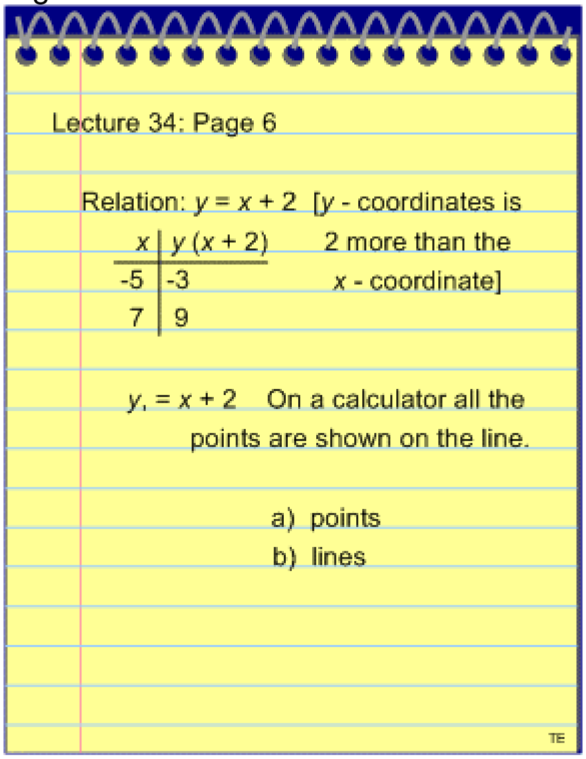
Lecture 34: Page 5

Relation: Set of Points

[$L_1 + 2$]
($x + 2$)

L_1	L_2
4	6
-6	-4
8	10
2	4
7	9
3	5
4	6

Alg034-06



Lecture 34: Page 6

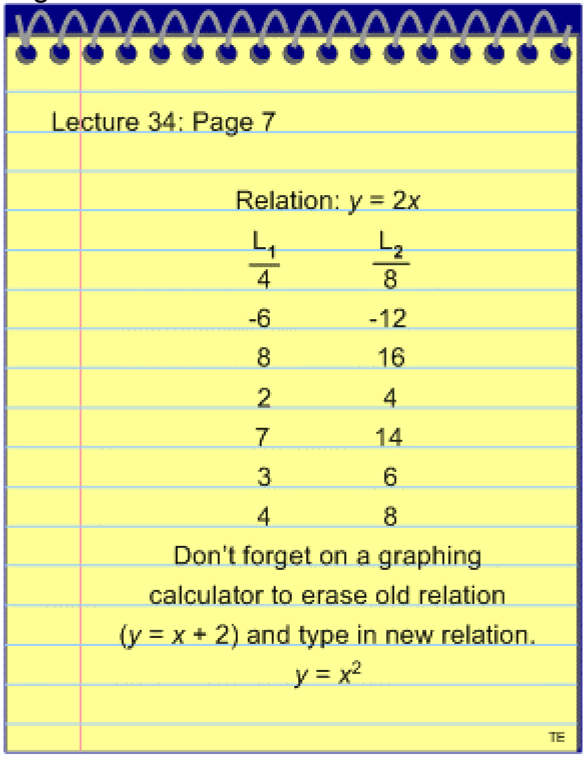
Relation: $y = x + 2$ [y - coordinates is 2 more than the x - coordinate]

x	y ($x + 2$)
-5	-3
7	9

$y, = x + 2$ On a calculator all the points are shown on the line.

a) points
b) lines

Alg034-07



Lecture 34: Page 7

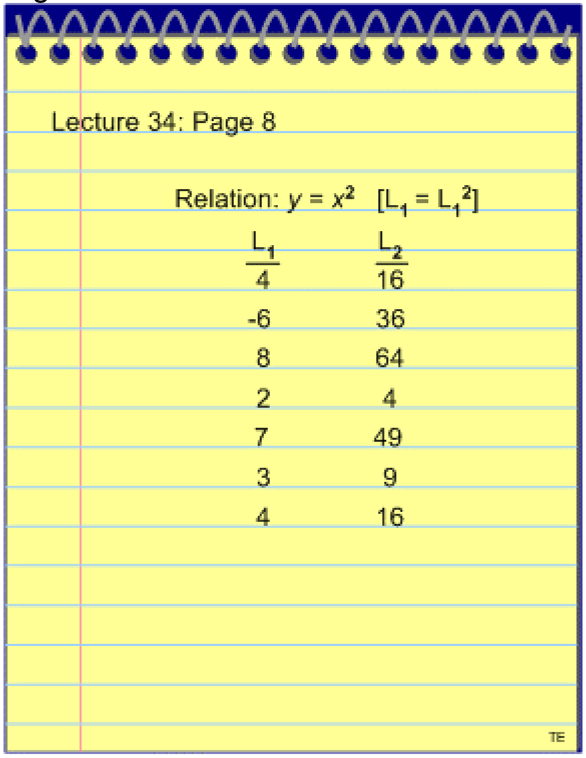
Relation: $y = 2x$

L_1	L_2
4	8
-6	-12
8	16
2	4
7	14
3	6
4	8

Don't forget on a graphing calculator to erase old relation ($y = x + 2$) and type in new relation.

$y = x^2$

Alg034-08

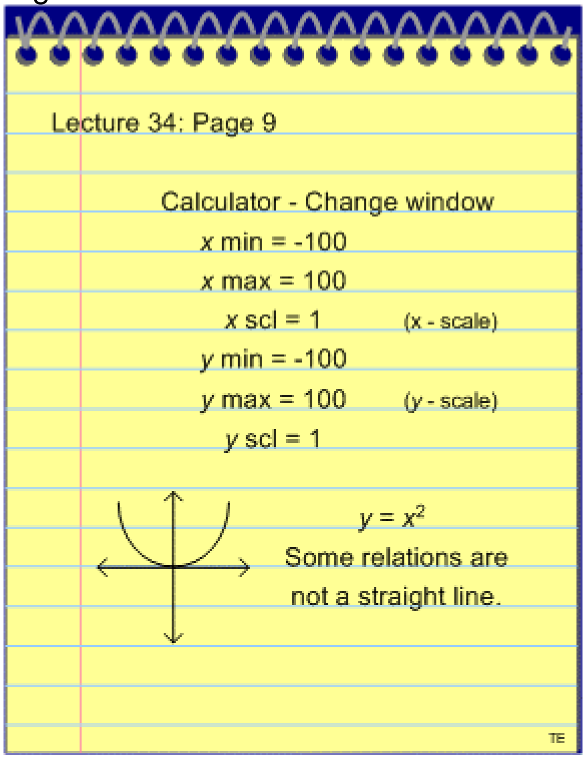


Lecture 34: Page 8

Relation: $y = x^2$ [$L_1 = L_1^2$]

L_1	L_2
4	16
-6	36
8	64
2	4
7	49
3	9
4	16

Alg034-09



Lecture 34: Page 9

Calculator - Change window

x min = -100

x max = 100

x scl = 1 (x - scale)

y min = -100

y max = 100 (y - scale)

y scl = 1

$y = x^2$

Some relations are not a straight line.

TE

Lecture 35 Notes

Alg035-01

Lecture 35: Graphing Linear Relations

Equation Pattern

y = __?__ times "x" + __?__

Linear Equations graph as a straight line.

Example 1: $y = 2x + 1$

x	y
2	5
-3	-5
-1	-1
0	1
3	7

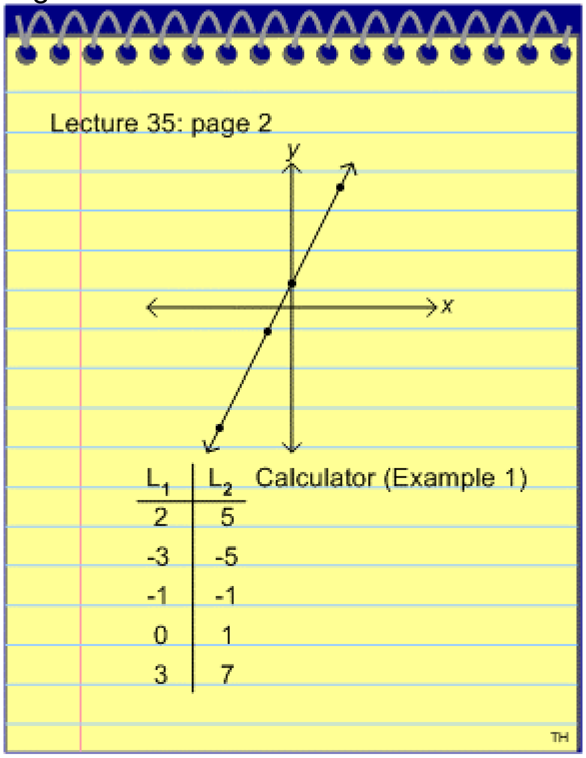
$y = 2(-3) + 1$

$= -6 + 1$

$= -5$

TH

Alg035-02



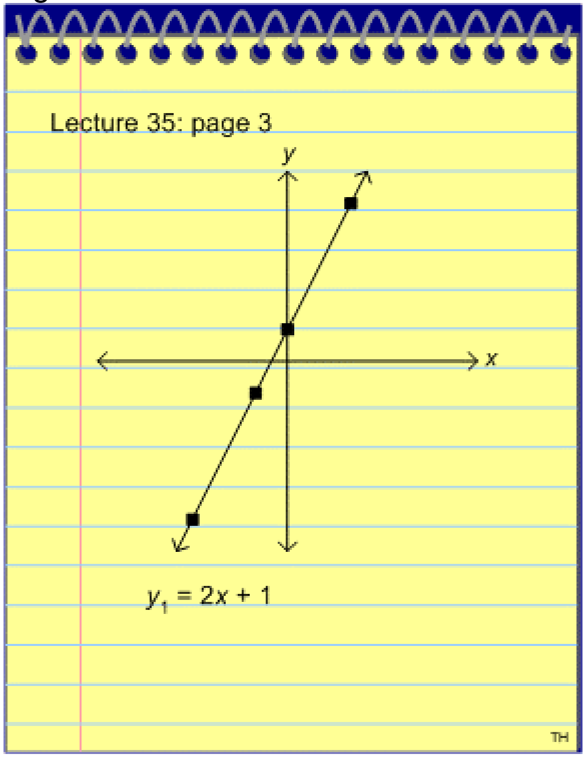
Lecture 35: page 2

L_1	L_2
2	5
-3	-5
-1	-1
0	1
3	7

Calculator (Example 1)

TH

Alg035-03

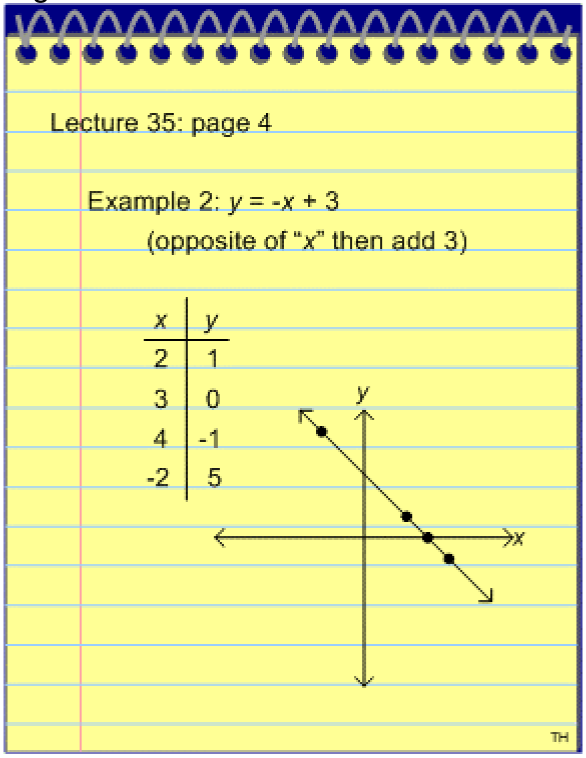


Lecture 35: page 3

$y_1 = 2x + 1$

TH

Alg035-04



Lecture 35: page 4

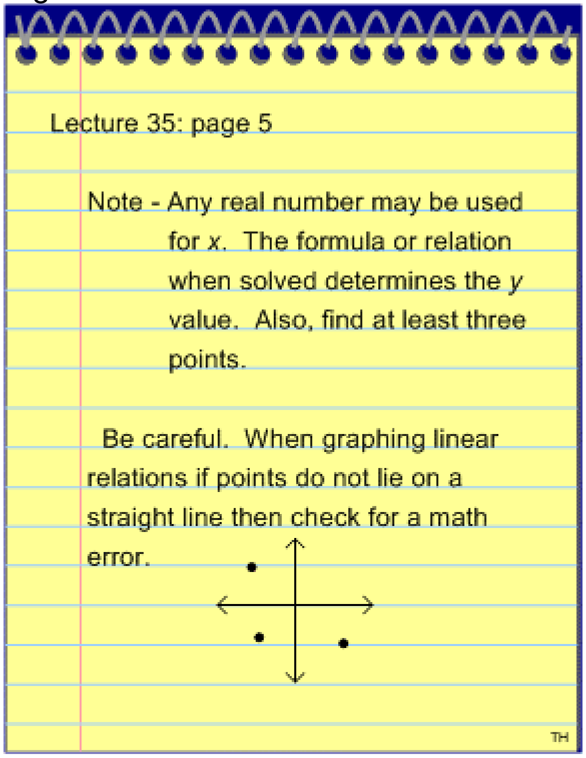
Example 2: $y = -x + 3$

(opposite of "x" then add 3)

x	y
2	1
3	0
4	-1
-2	5

TH

Alg035-05



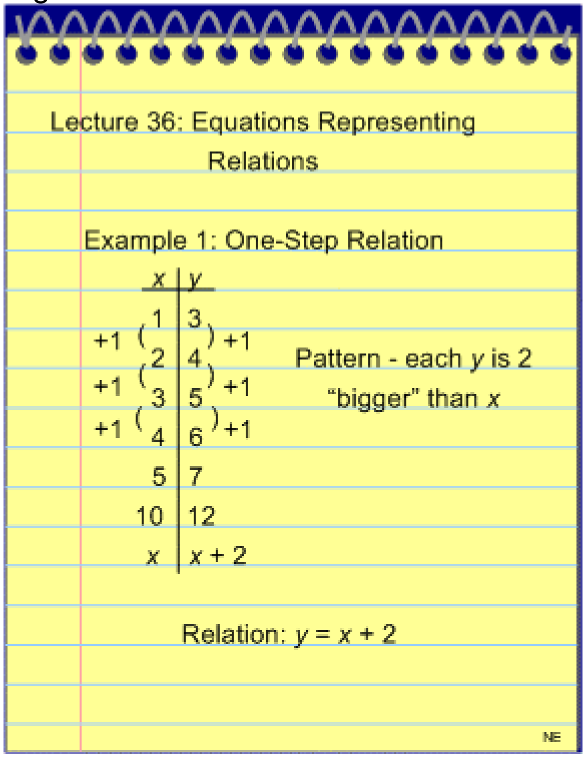
Lecture 35: page 5

Note - Any real number may be used for *x*. The formula or relation when solved determines the *y* value. Also, find at least three points.

Be careful. When graphing linear relations if points do not lie on a straight line then check for a math error.

Lecture 36 Notes

Alg036-01



Lecture 36: Equations Representing Relations

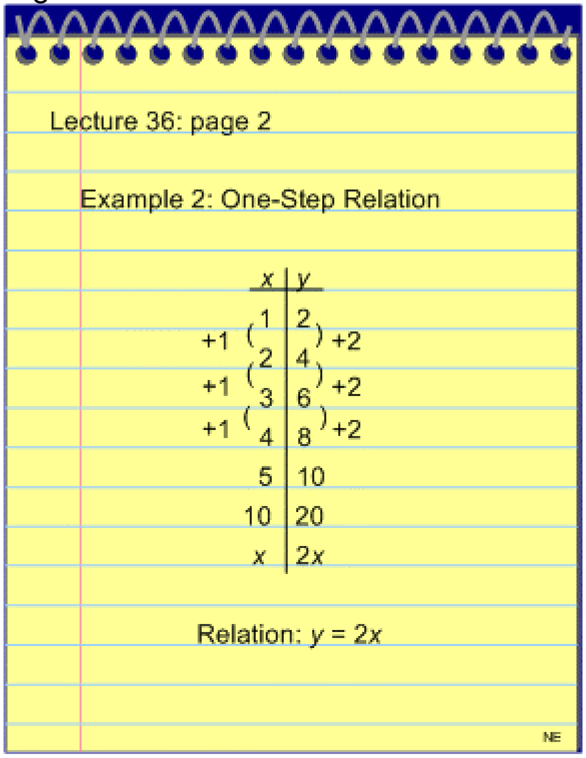
Example 1: One-Step Relation

	x	y	
	1	3	
+1			+1
	2	4	
+1			+1
	3	5	
+1			+1
	4	6	
	5	7	
	10	12	
	x	$x + 2$	

Pattern - each y is 2 "bigger" than x

Relation: $y = x + 2$

Alg036-02



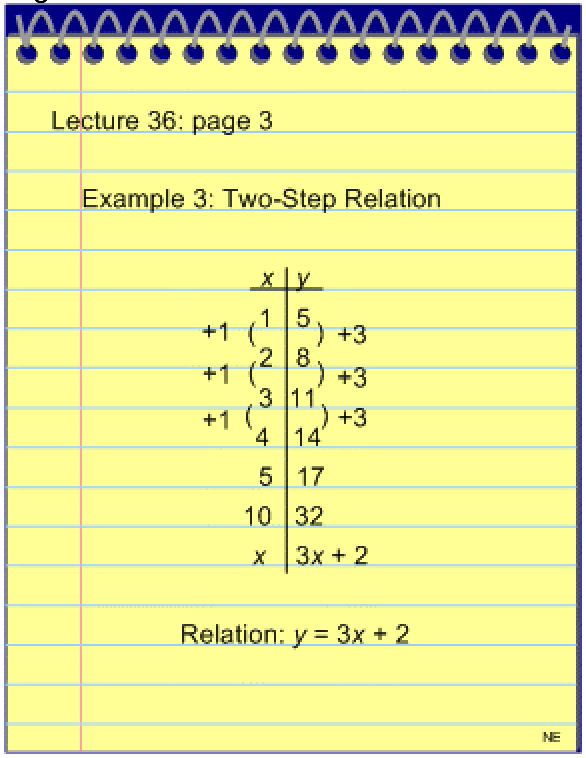
Lecture 36: page 2

Example 2: One-Step Relation

	x	y	
	1	2	
+1			+2
	2	4	
+1			+2
	3	6	
+1			+2
	4	8	
	5	10	
	10	20	
	x	$2x$	

Relation: $y = 2x$

Alg036-03



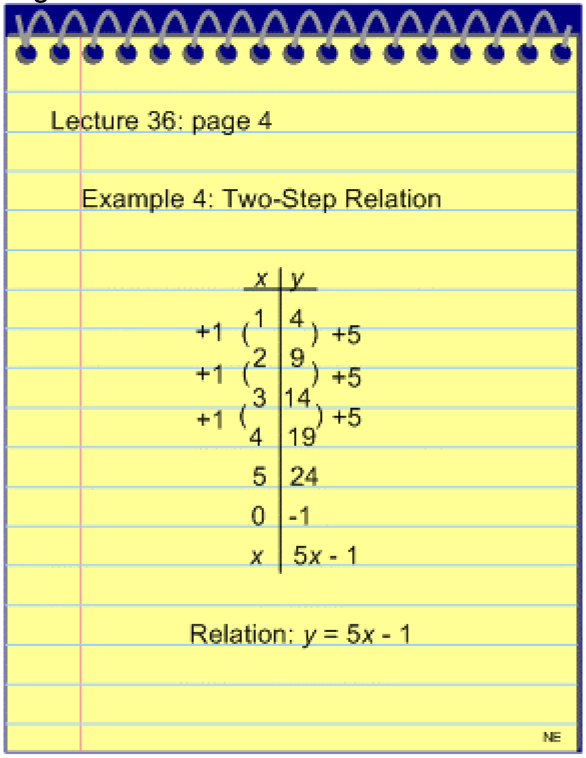
Lecture 36: page 3

Example 3: Two-Step Relation

	x	y	
	1	5	
+1			+3
	2	8	
+1			+3
	3	11	
+1			+3
	4	14	
	5	17	
	10	32	
	x	$3x + 2$	

Relation: $y = 3x + 2$

Alg036-04



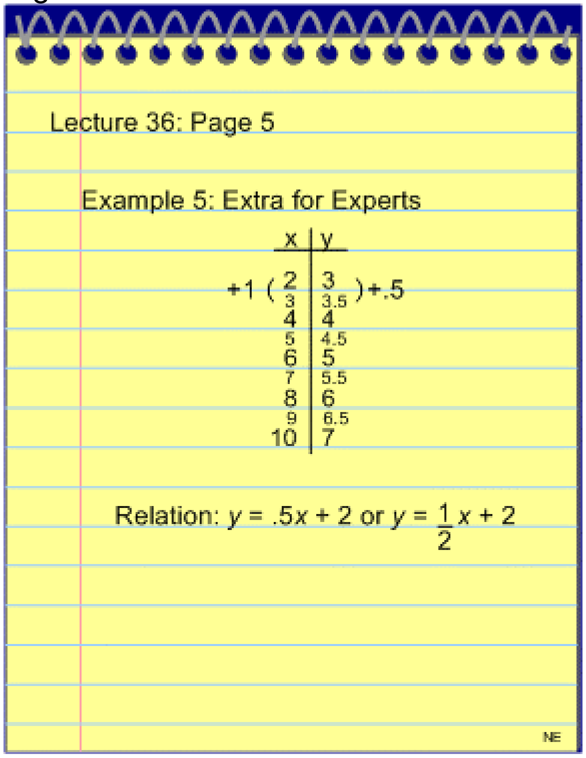
Lecture 36: page 4

Example 4: Two-Step Relation

	x	y	
	1	4	
+1			+5
	2	9	
+1			+5
	3	14	
+1			+5
	4	19	
	5	24	
	0	-1	
	x	$5x - 1$	

Relation: $y = 5x - 1$

Alg036-05



Lecture 36: Page 5

Example 5: Extra for Experts

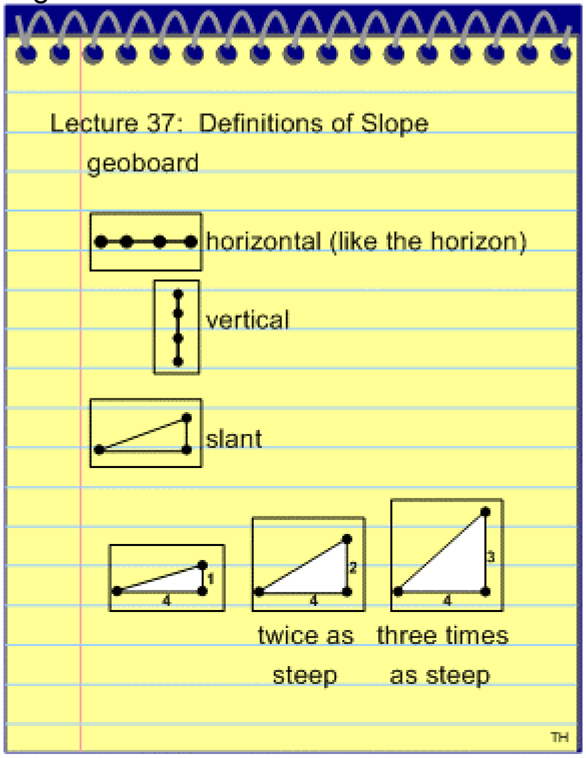
	x	y	
+1 (	2	3	)+.5
	3	3.5	
	4	4	
	5	4.5	
	6	5	
	7	5.5	
	8	6	
	9	6.5	
	10	7	

Relation: $y = .5x + 2$ or $y = \frac{1}{2}x + 2$

NE

Lecture 37 Notes

Alg037-01



Lecture 37: Definitions of Slope

geoboard

horizontal (like the horizon)

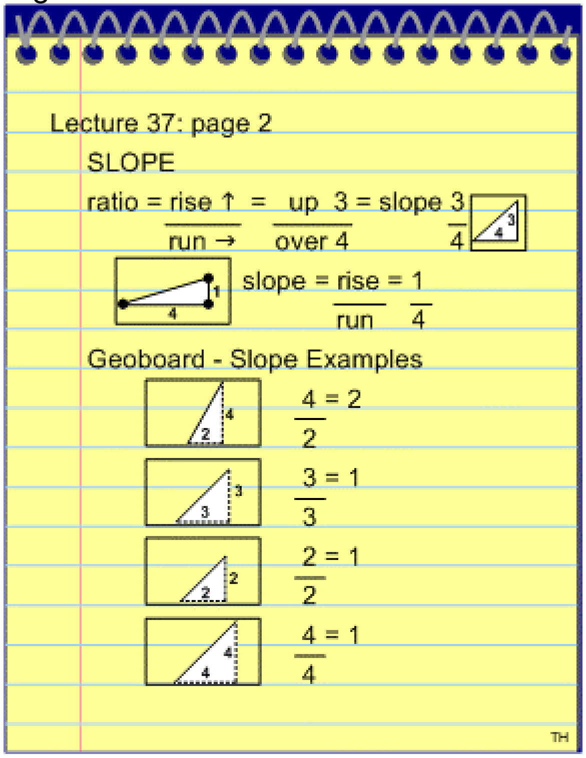
vertical

slant

twice as steep

three times as steep

Alg037-02



Lecture 37: page 2

SLOPE

ratio = $\frac{\text{rise} \uparrow}{\text{run} \rightarrow} = \frac{\text{up } 3}{\text{over } 4}$ = slope $\frac{3}{4}$

slope = $\frac{\text{rise}}{\text{run}} = \frac{1}{4}$

Geoboard - Slope Examples

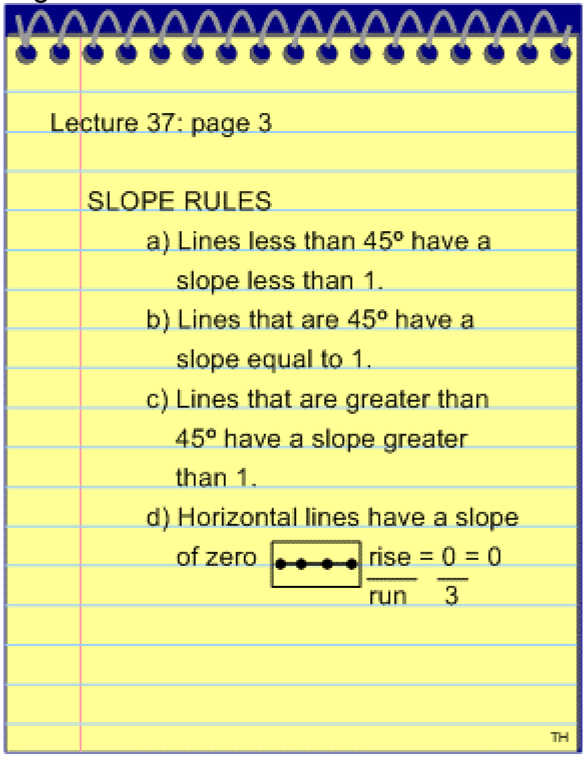
$\frac{4}{2} = 2$

$\frac{3}{3} = 1$

$\frac{2}{2} = 1$

$\frac{4}{4} = 1$

Alg037-03



Lecture 37: page 3

SLOPE RULES

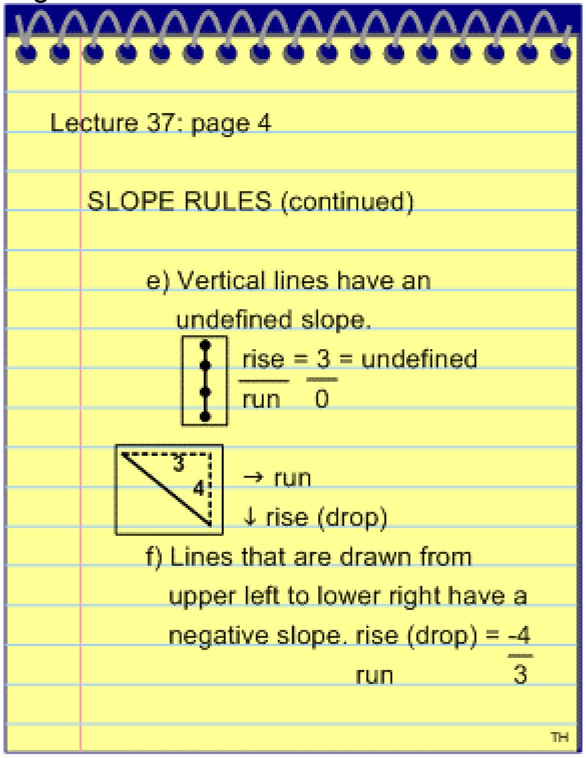
a) Lines less than 45° have a slope less than 1.

b) Lines that are 45° have a slope equal to 1.

c) Lines that are greater than 45° have a slope greater than 1.

d) Horizontal lines have a slope of zero $\frac{\text{rise}}{\text{run}} = \frac{0}{3} = 0$

Alg037-04



Lecture 37: page 4

SLOPE RULES (continued)

e) Vertical lines have an undefined slope.

$\frac{\text{rise}}{\text{run}} = \frac{3}{0}$ = undefined

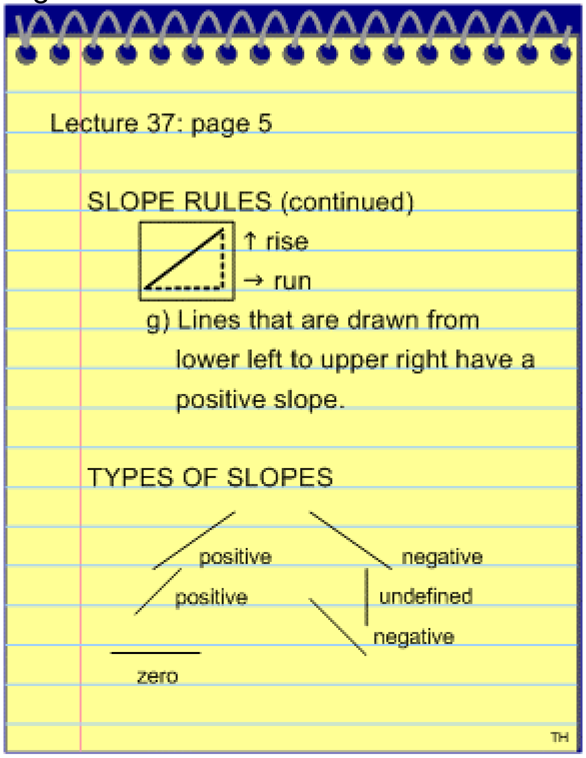
→ run

↓ rise (drop)

f) Lines that are drawn from upper left to lower right have a negative slope. $\frac{\text{rise (drop)}}{\text{run}} = \frac{-4}{3}$

Lecture 37 Notes, Continued

Alg037-05



Lecture 37: page 5

SLOPE RULES (continued)

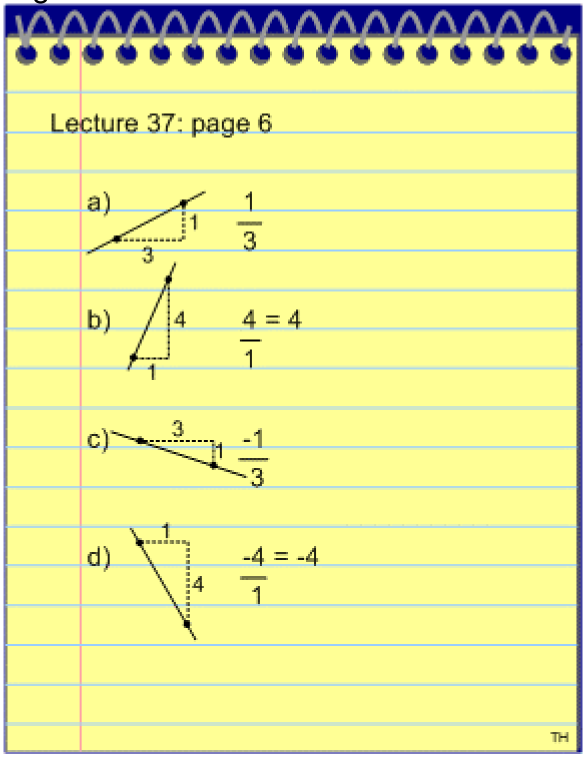
↑ rise

→ run

g) Lines that are drawn from lower left to upper right have a positive slope.

TYPES OF SLOPES

Alg037-06



Lecture 37: page 6

a) $\frac{1}{3}$

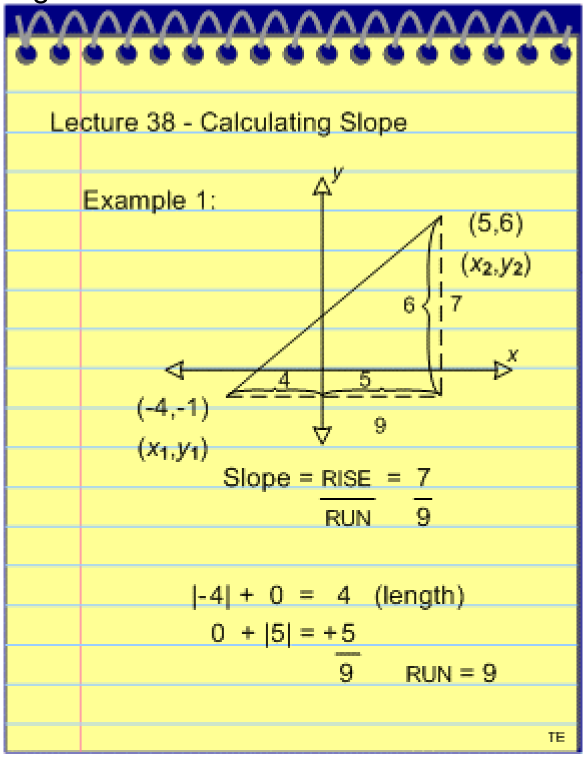
b) $\frac{4}{1} = 4$

c) $\frac{-1}{3}$

d) $\frac{-4}{1} = -4$

Lecture 38 Notes

Alg038-01



Lecture 38 - Calculating Slope

Example 1:

$$\text{Slope} = \frac{\text{RISE}}{\text{RUN}} = \frac{7}{9}$$

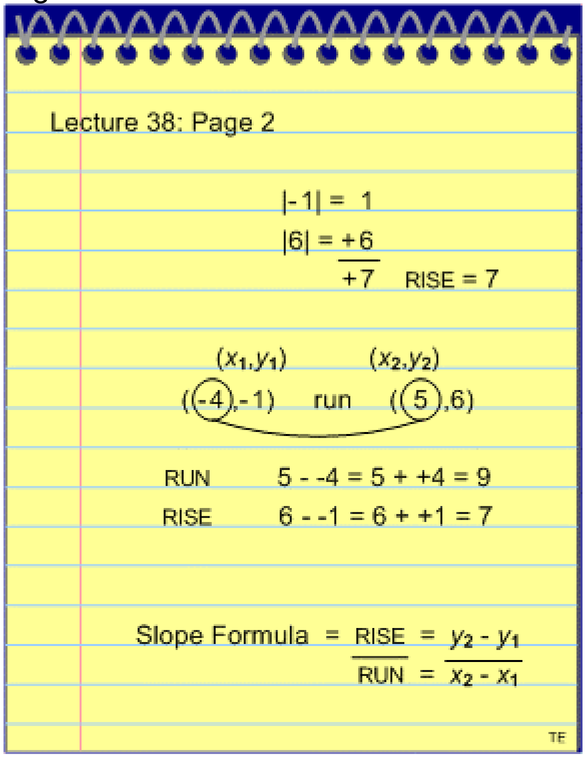
$|-4| + 0 = 4$ (length)

$0 + |5| = +5$

9 RUN = 9

TE

Alg038-02



Lecture 38: Page 2

$|-1| = 1$

$|6| = +6$

$+7$ RISE = 7

(x_1,y_1) (x_2,y_2)

$(\textcircled{-4},-1)$ run $(\textcircled{5},6)$

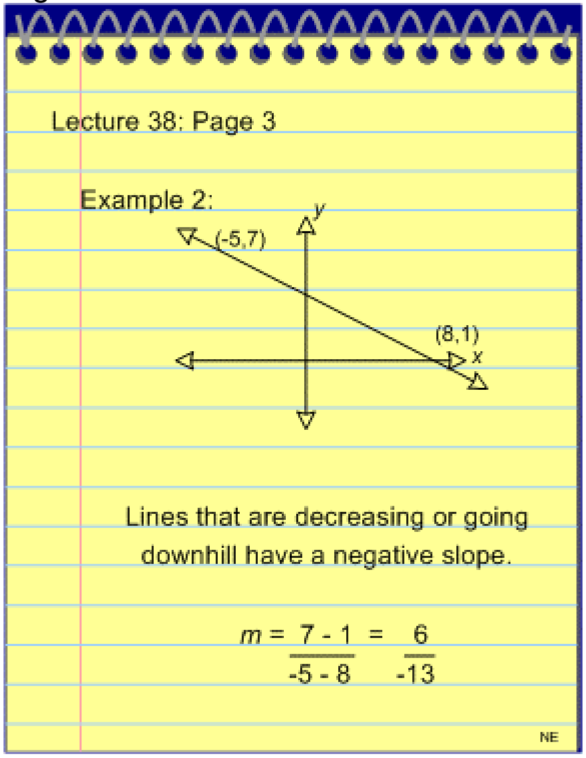
RUN $5 - -4 = 5 + +4 = 9$

RISE $6 - -1 = 6 + +1 = 7$

$$\text{Slope Formula} = \frac{\text{RISE}}{\text{RUN}} = \frac{y_2 - y_1}{x_2 - x_1}$$

TE

Alg038-03



Lecture 38: Page 3

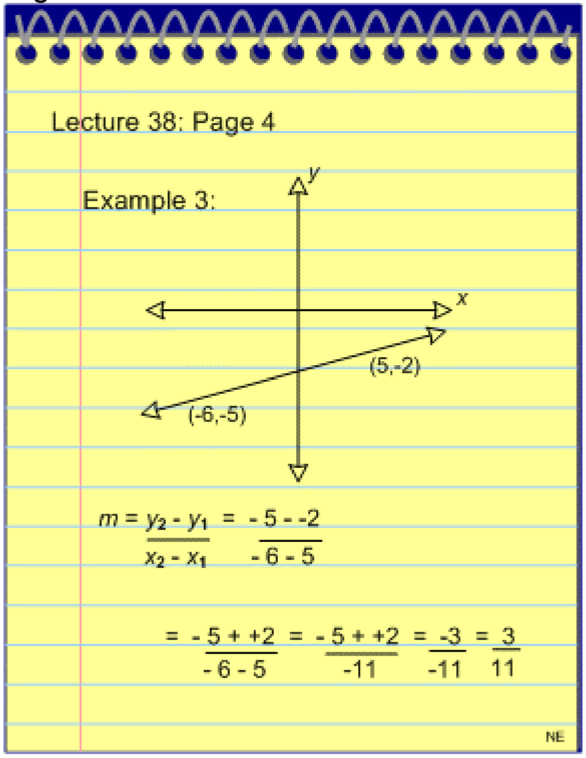
Example 2:

Lines that are decreasing or going downhill have a negative slope.

$$m = \frac{7 - 1}{-5 - 8} = \frac{6}{-13}$$

NE

Alg038-04



Lecture 38: Page 4

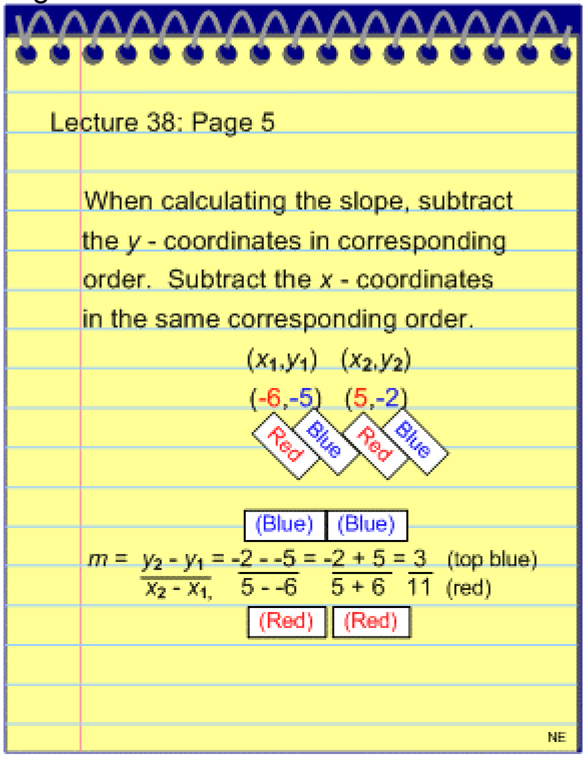
Example 3:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - -2}{-6 - 5}$$

$$= \frac{-5 + +2}{-6 - 5} = \frac{-5 + +2}{-11} = \frac{-3}{-11} = \frac{3}{11}$$

NE

Alg038-05



Lecture 38: Page 5

When calculating the slope, subtract the *y* - coordinates in corresponding order. Subtract the *x* - coordinates in the same corresponding order.

$(x_1,y_1) \quad (x_2,y_2)$

$(-6,-5) \quad (5,-2)$

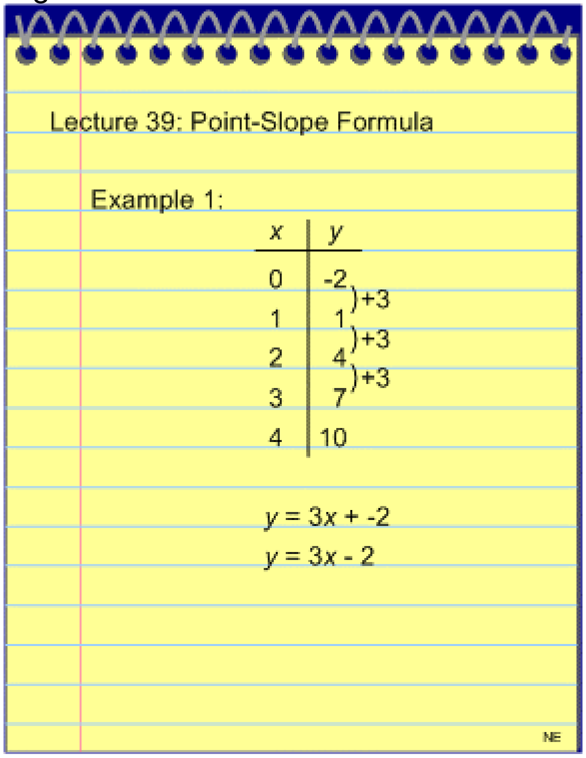
(Blue) (Blue)

$m = \frac{y_2 - y_1}{x_2 - x_1,} = \frac{-2 - -5}{5 - -6} = \frac{-2 + 5}{5 + 6} = \frac{3}{11}$ (top blue) (red)

(Red) (Red)

Lecture 39 Notes

Alg039-01



Lecture 39: Point-Slope Formula

Example 1:

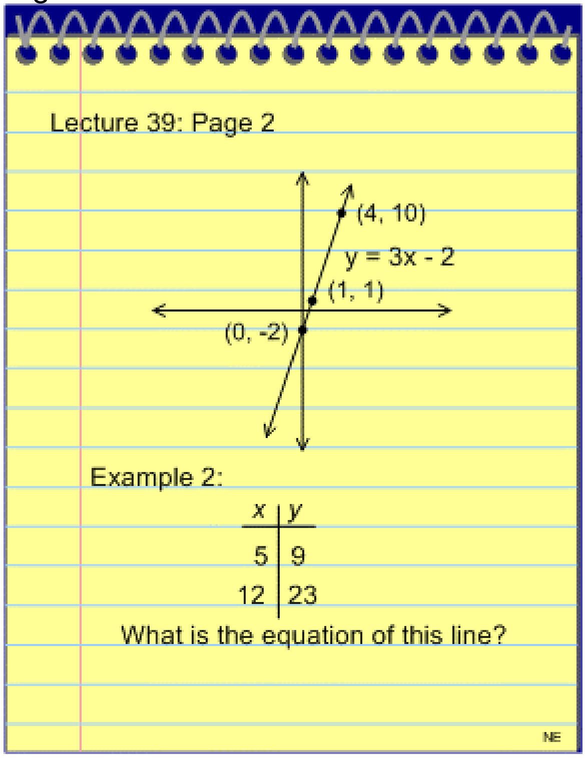
x	y	
0	-2	)+3
1	1	)+3
2	4	)+3
3	7	
4	10	

$y = 3x + -2$

$y = 3x - 2$

NE

Alg039-02



Lecture 39: Page 2

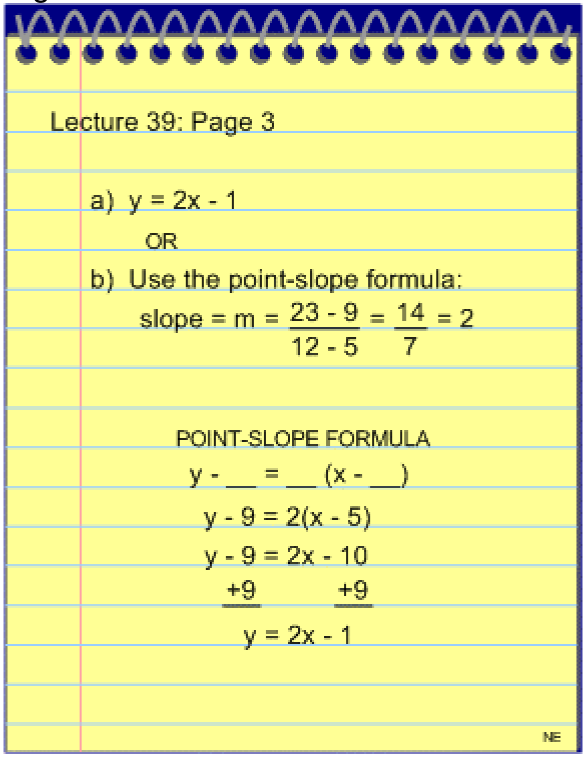
Example 2:

x	y
5	9
12	23

What is the equation of this line?

NE

Alg039-03



Lecture 39: Page 3

a) $y = 2x - 1$

OR

b) Use the point-slope formula:

$$\text{slope} = m = \frac{23 - 9}{12 - 5} = \frac{14}{7} = 2$$

POINT-SLOPE FORMULA

$y - \underline{\quad} = \underline{\quad}(x - \underline{\quad})$

$y - 9 = 2(x - 5)$

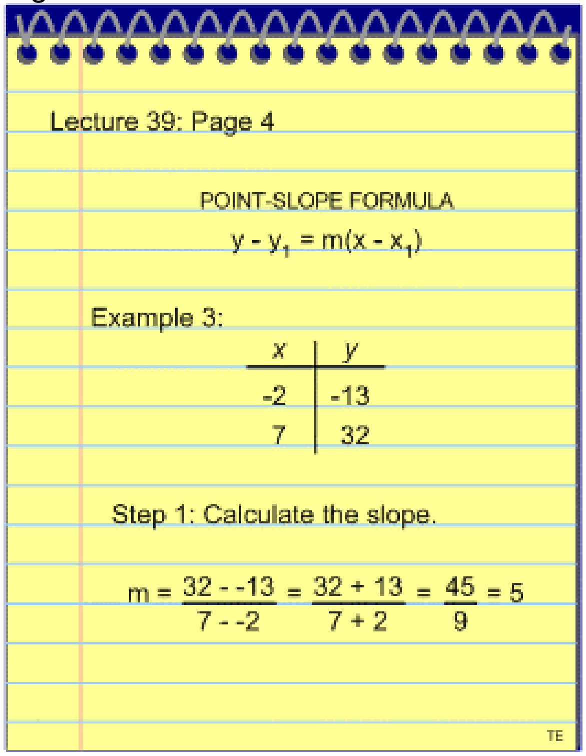
$y - 9 = 2x - 10$

$\underline{+9} \qquad \underline{+9}$

$y = 2x - 1$

NE

Alg039-04



Lecture 39: Page 4

POINT-SLOPE FORMULA

$y - y_1 = m(x - x_1)$

Example 3:

x	y
-2	-13
7	32

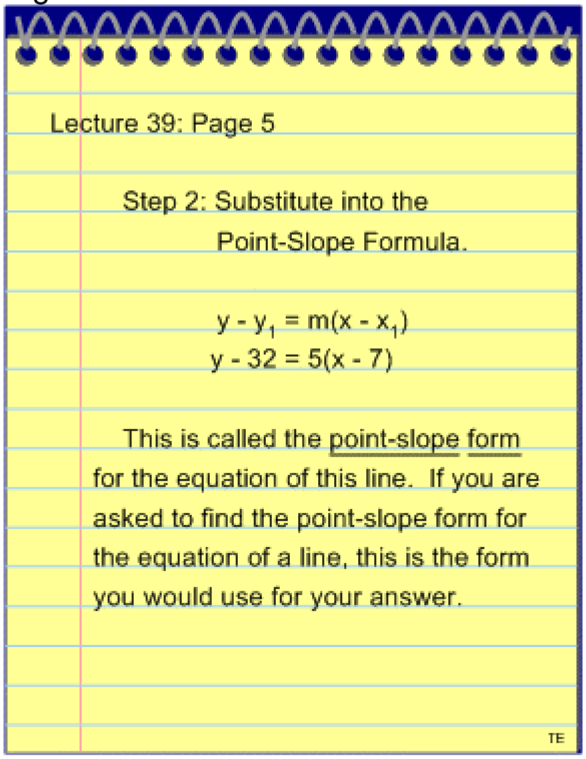
Step 1: Calculate the slope.

$$m = \frac{32 - -13}{7 - -2} = \frac{32 + 13}{7 + 2} = \frac{45}{9} = 5$$

TE

Lecture 39 Notes, Continued

Alg039-05



Lecture 39: Page 5

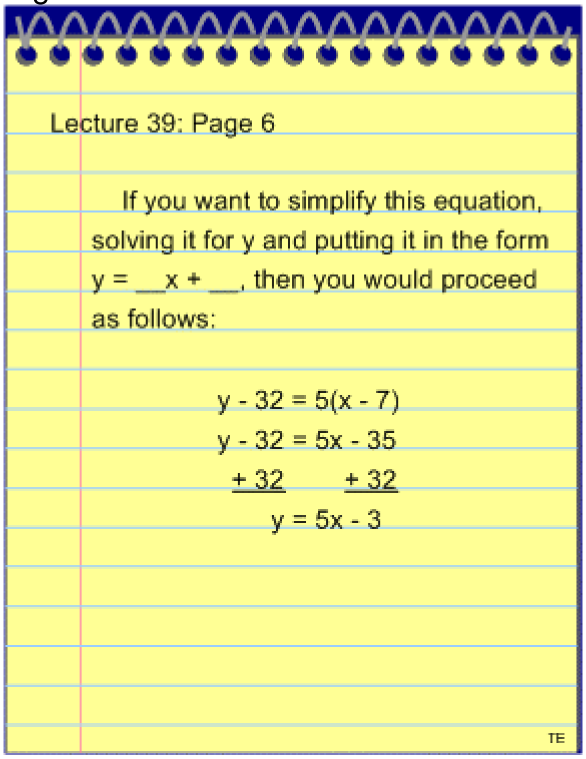
Step 2: Substitute into the Point-Slope Formula.

$y - y_1 = m(x - x_1)$

$y - 32 = 5(x - 7)$

This is called the point-slope form for the equation of this line. If you are asked to find the point-slope form for the equation of a line, this is the form you would use for your answer.

Alg039-06

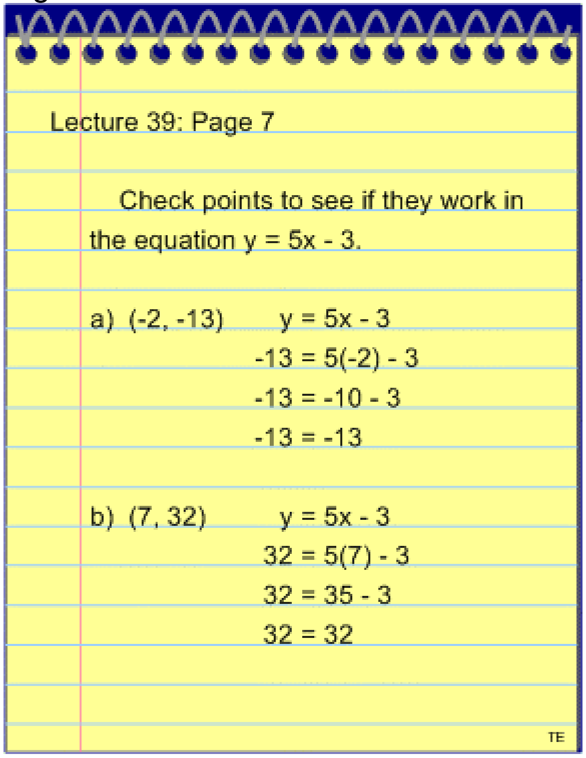


Lecture 39: Page 6

If you want to simplify this equation, solving it for y and putting it in the form y = __x + __, then you would proceed as follows:

$$\begin{aligned} y - 32 &= 5(x - 7) \\ y - 32 &= 5x - 35 \\ \underline{+\,32} &\quad \underline{+\,32} \\ y &= 5x - 3 \end{aligned}$$

Alg039-07



Lecture 39: Page 7

Check points to see if they work in the equation $y = 5x - 3$.

a) (-2, -13)

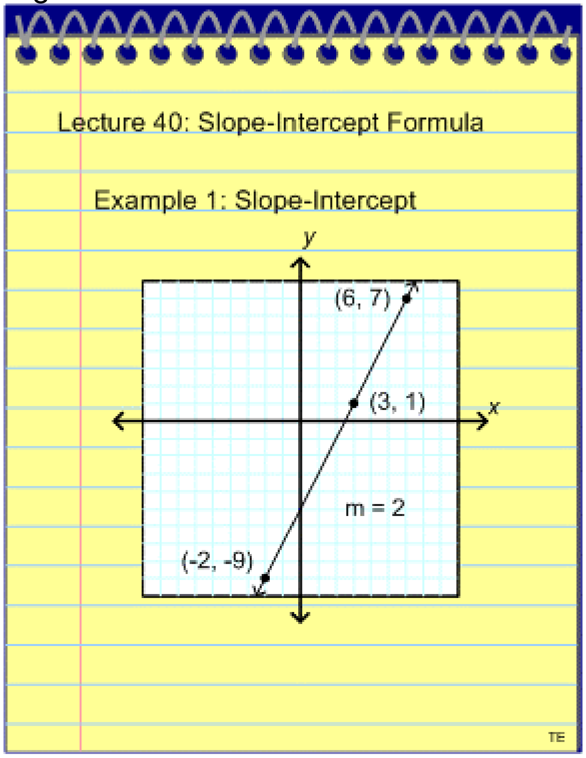
$$\begin{aligned} y &= 5x - 3 \\ -13 &= 5(-2) - 3 \\ -13 &= -10 - 3 \\ -13 &= -13 \end{aligned}$$

b) (7, 32)

$$\begin{aligned} y &= 5x - 3 \\ 32 &= 5(7) - 3 \\ 32 &= 35 - 3 \\ 32 &= 32 \end{aligned}$$

Lecture 40 Notes

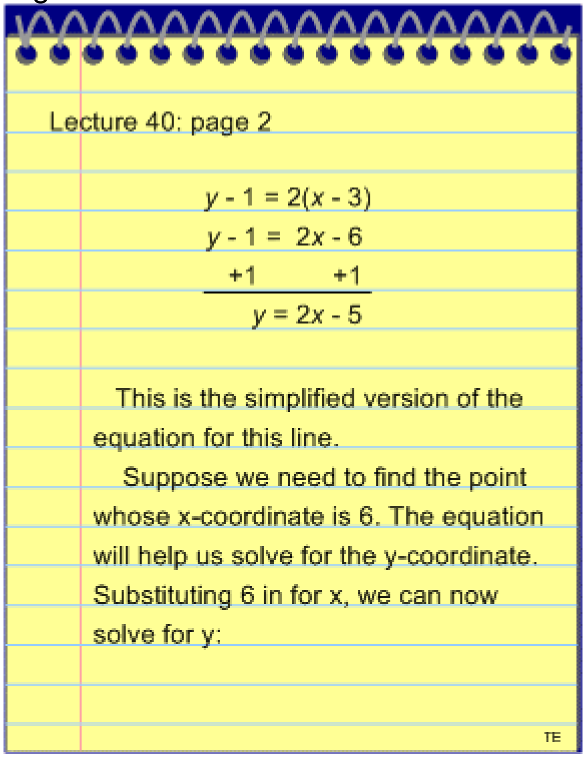
Alg040-01



Lecture 40: Slope-Intercept Formula

Example 1: Slope-Intercept

Alg040-02



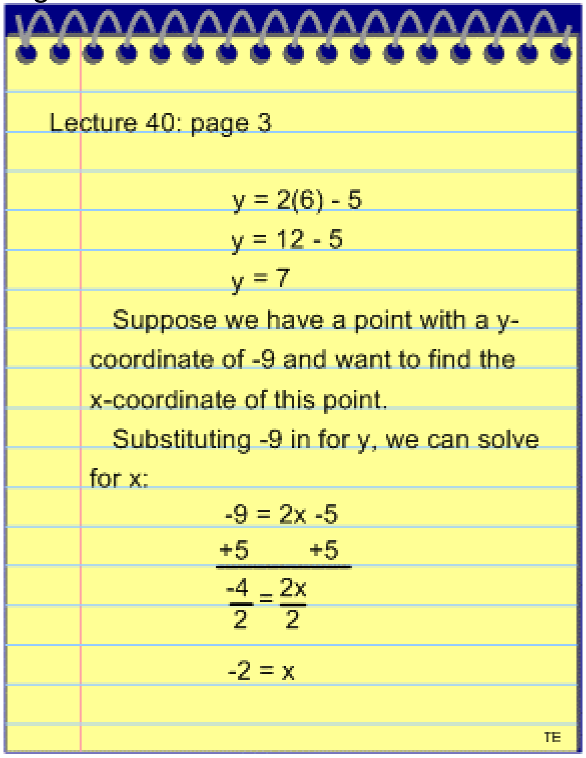
Lecture 40: page 2

$$\begin{aligned} y - 1 &= 2(x - 3) \\ y - 1 &= 2x - 6 \\ +1 \quad & \quad\ +1 \\ \hline y &= 2x - 5 \end{aligned}$$

This is the simplified version of the equation for this line.

Suppose we need to find the point whose x-coordinate is 6. The equation will help us solve for the y-coordinate. Substituting 6 in for x, we can now solve for y:

Alg040-03



Lecture 40: page 3

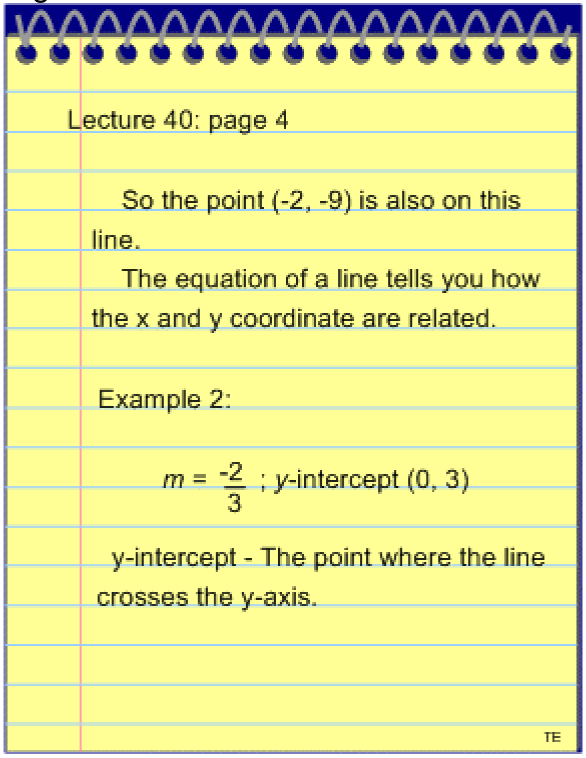
$$\begin{aligned} y &= 2(6) - 5 \\ y &= 12 - 5 \\ y &= 7 \end{aligned}$$

Suppose we have a point with a y-coordinate of -9 and want to find the x-coordinate of this point.

Substituting -9 in for y, we can solve for x:

$$\begin{aligned} -9 &= 2x - 5 \\ +5 \quad & \quad +5 \\ \hline \frac{-4}{2} &= \frac{2x}{2} \\ -2 &= x \end{aligned}$$

Alg040-04



Lecture 40: page 4

So the point (-2, -9) is also on this line.

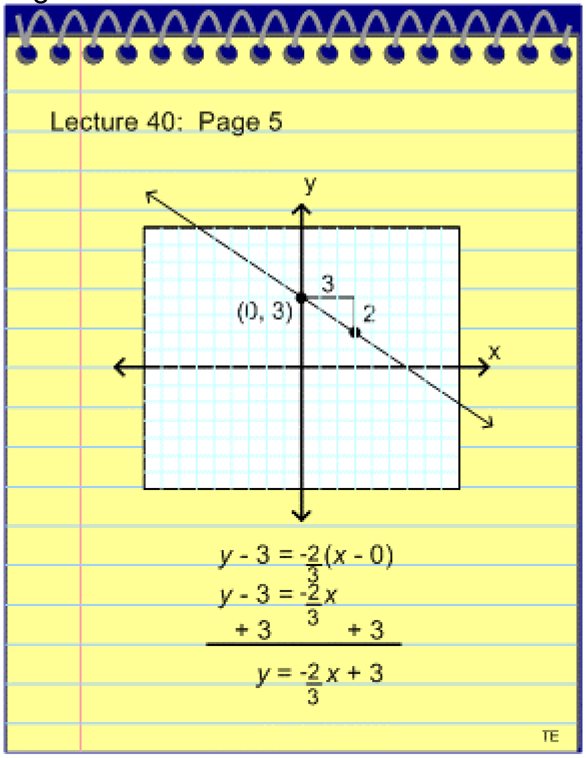
The equation of a line tells you how the x and y coordinate are related.

Example 2:

$$m = \frac{-2}{3}\ ;\ y\text{-intercept } (0, 3)$$

y-intercept - The point where the line crosses the y-axis.

Alg040-05



Lecture 40: Page 5

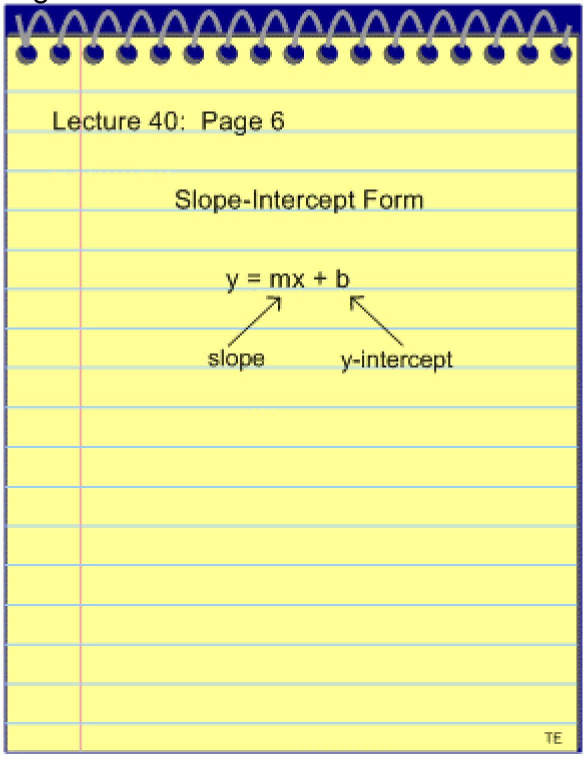
$$y - 3 = -\frac{2}{3}(x - 0)$$

$$y - 3 = -\frac{2}{3}x$$

$$\underline{+3 \qquad\quad +3}$$

$$y = -\frac{2}{3}x + 3$$

Alg040-06

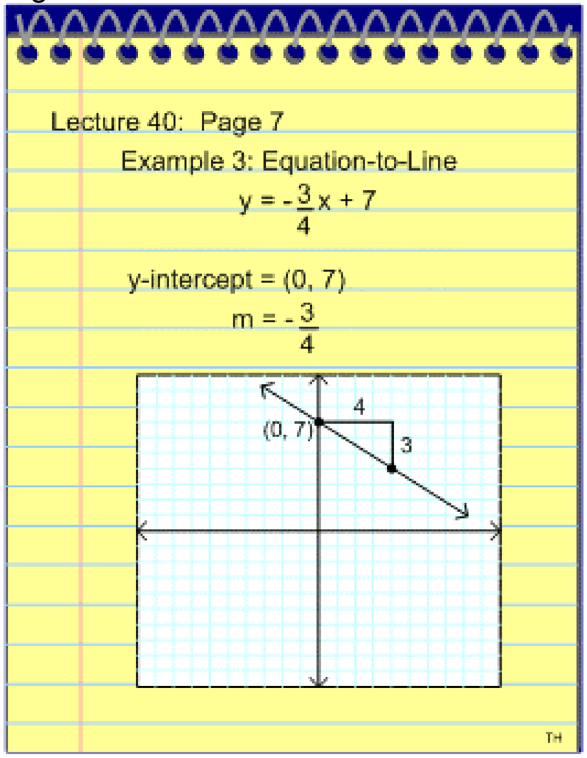


Lecture 40: Page 6

Slope-Intercept Form

$$y = \underset{\text{slope}}{m}x + \underset{\text{y-intercept}}{b}$$

Alg040-07



Lecture 40: Page 7

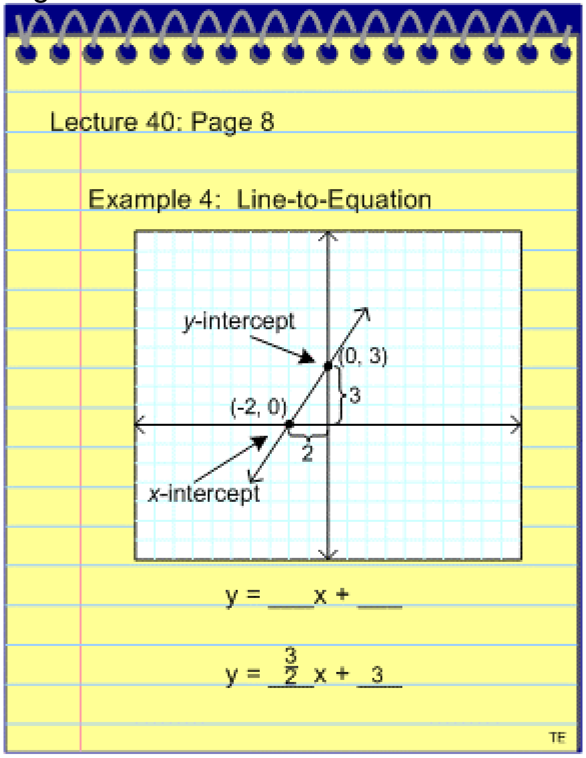
Example 3: Equation-to-Line

$$y = -\frac{3}{4}x + 7$$

y-intercept = (0, 7)

$$m = -\frac{3}{4}$$

Alg040-08



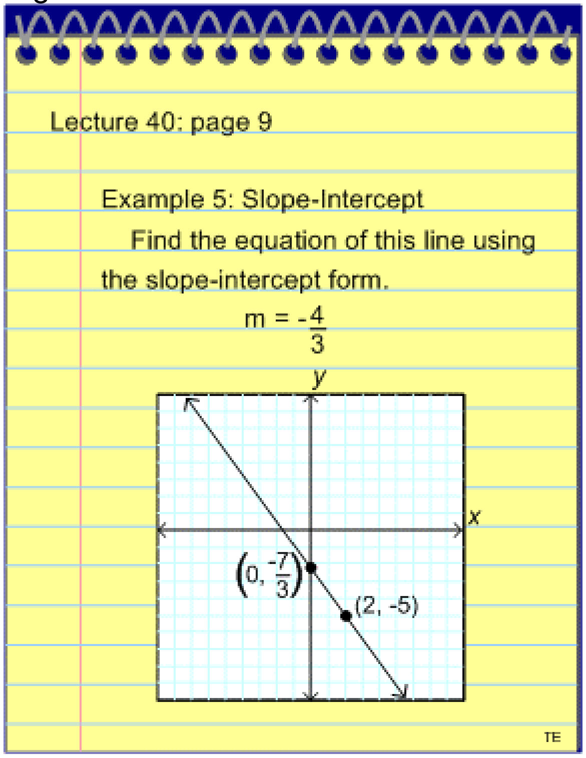
Lecture 40: Page 8

Example 4: Line-to-Equation

y = ____x + ____

$$y = \underline{\ \tfrac{3}{2}\ }\ x + \underline{\ 3\ }$$

Alg040-09



Lecture 40: page 9

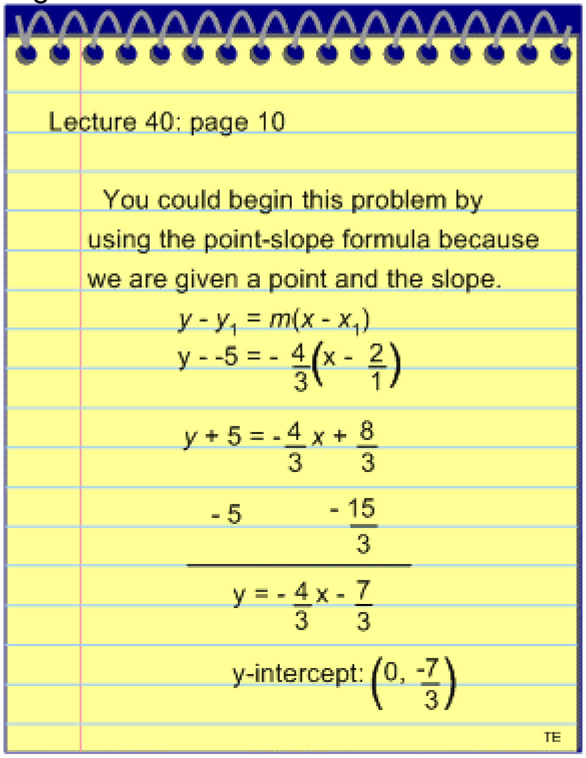
Example 5: Slope-Intercept

Find the equation of this line using the slope-intercept form.

$m = -\frac{4}{3}$

TE

Alg040-10



Lecture 40: page 10

You could begin this problem by using the point-slope formula because we are given a point and the slope.

$$y - y_1 = m(x - x_1)$$

$$y - -5 = -\frac{4}{3}\left(x - \frac{2}{1}\right)$$

$$y + 5 = -\frac{4}{3}x + \frac{8}{3}$$

$$-5 \qquad -\frac{15}{3}$$

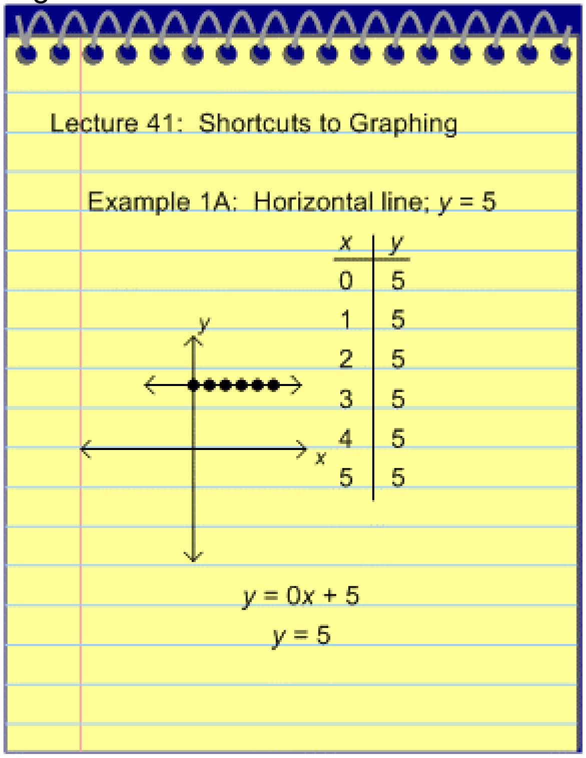
$$y = -\frac{4}{3}x - \frac{7}{3}$$

y-intercept: $\left(0, \frac{-7}{3}\right)$

TE

Lecture 41 Notes

Alg041-01



Lecture 41: Shortcuts to Graphing

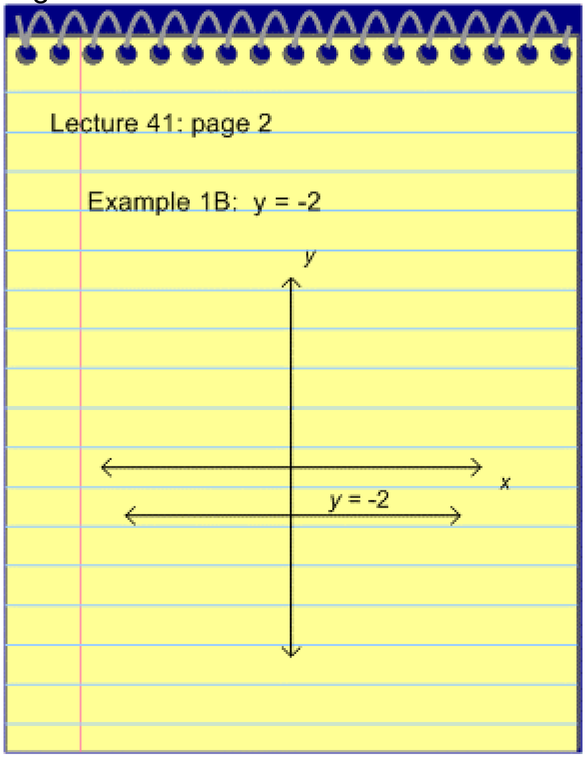
Example 1A: Horizontal line; $y = 5$

x	y
0	5
1	5
2	5
3	5
4	5
5	5

$y = 0x + 5$

$y = 5$

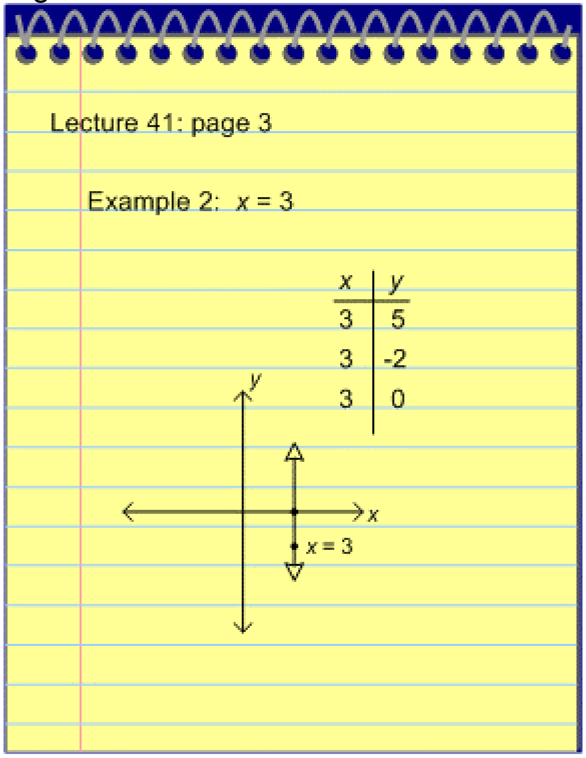
Alg041-02



Lecture 41: page 2

Example 1B: $y = -2$

Alg041-03

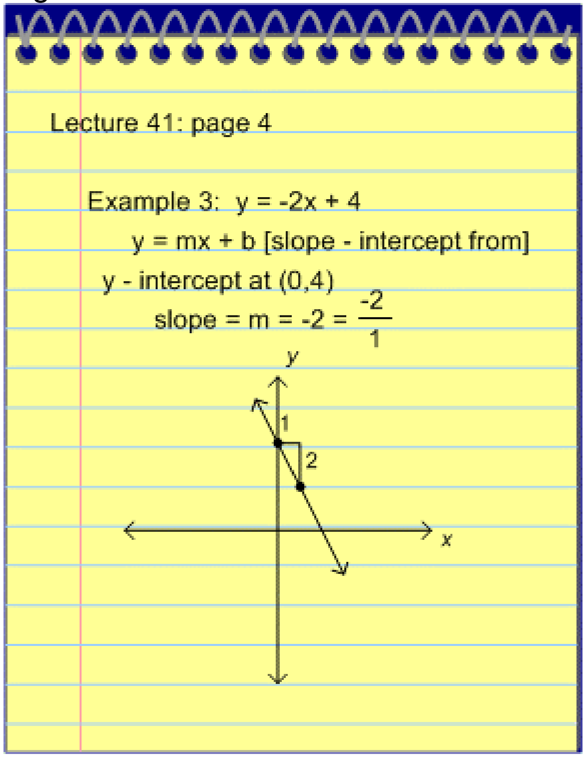


Lecture 41: page 3

Example 2: $x = 3$

x	y
3	5
3	-2
3	0

Alg041-04



Lecture 41: page 4

Example 3: $y = -2x + 4$

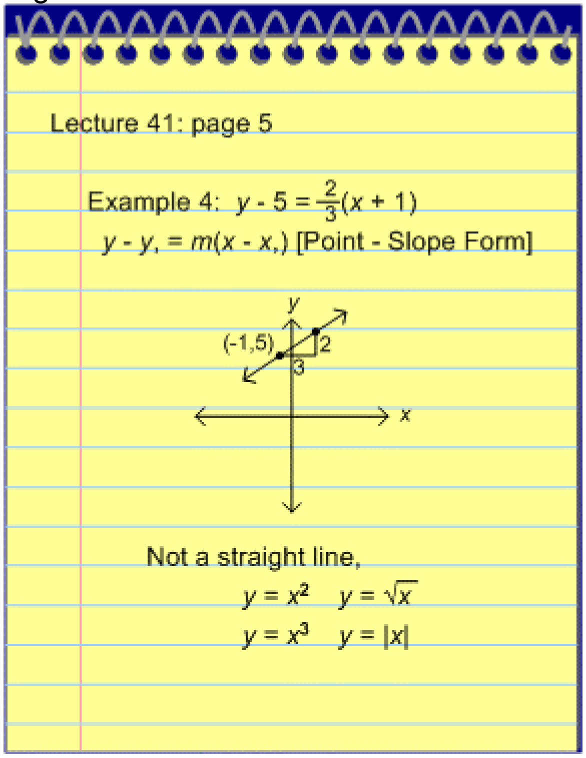
$y = mx + b$ [slope - intercept from]

y - intercept at (0,4)

slope $= m = -2 = \frac{-2}{1}$

Lecture 41 Notes, Continued

Alg041-05



Lecture 41: page 5

Example 4: $y - 5 = \frac{2}{3}(x + 1)$

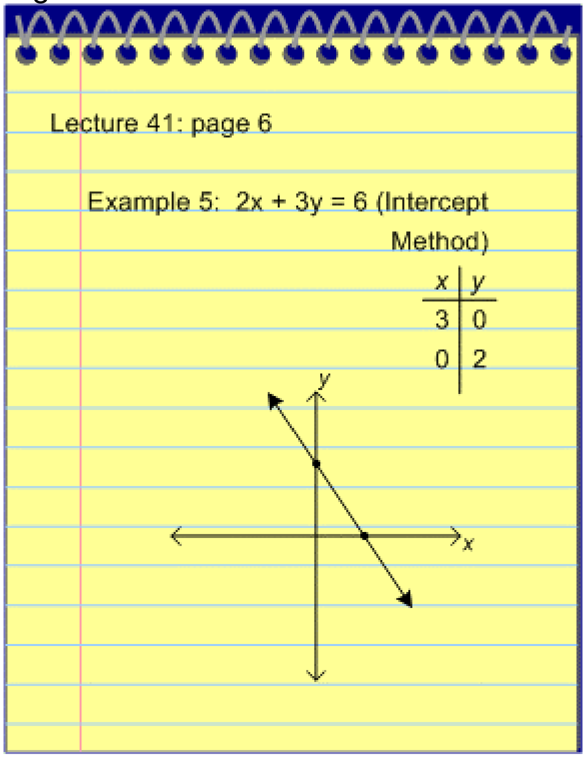
$y - y_1 = m(x - x_1)$ [Point - Slope Form]

Not a straight line,

$y = x^2 \quad y = \sqrt{x}$

$y = x^3 \quad y = |x|$

Alg041-06

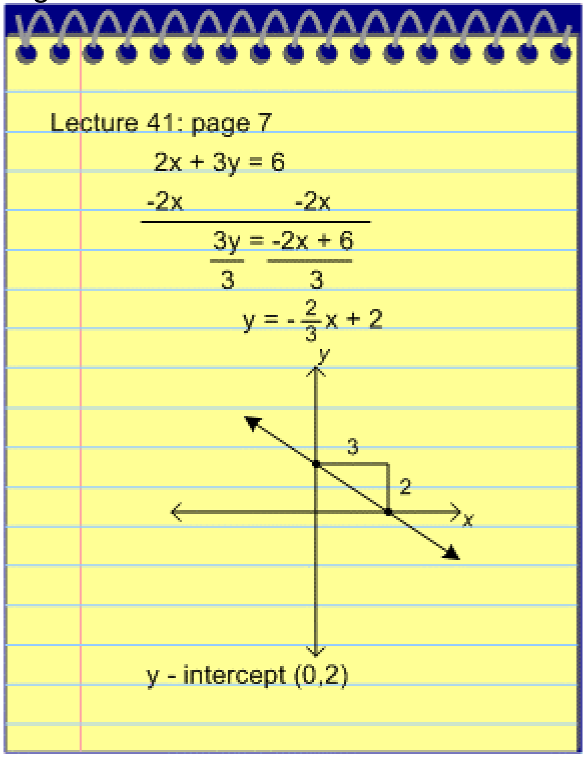


Lecture 41: page 6

Example 5: 2x + 3y = 6 (Intercept Method)

x	y
3	0
0	2

Alg041-07



Lecture 41: page 7

$$2x + 3y = 6$$

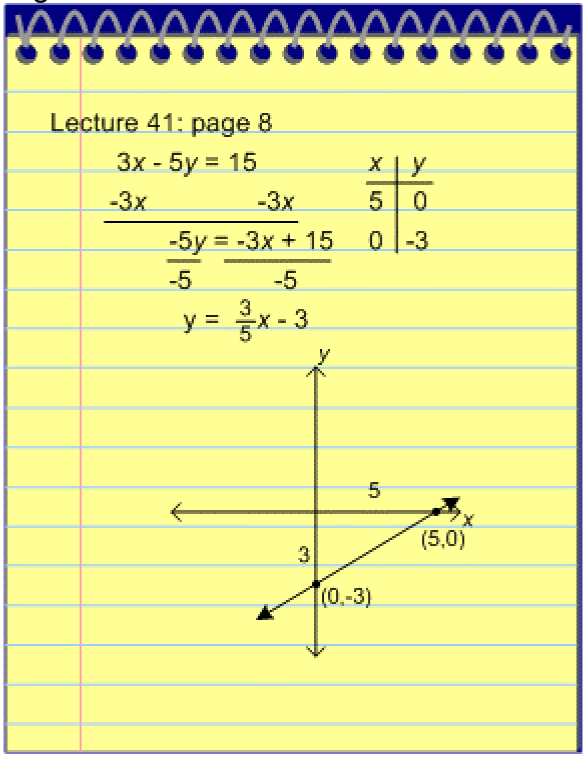
$$-2x \qquad -2x$$

$$\frac{3y}{3} = \frac{-2x + 6}{3}$$

$$y = -\frac{2}{3}x + 2$$

y - intercept (0,2)

Alg041-08



Lecture 41: page 8

$$3x - 5y = 15$$

$$-3x \qquad -3x$$

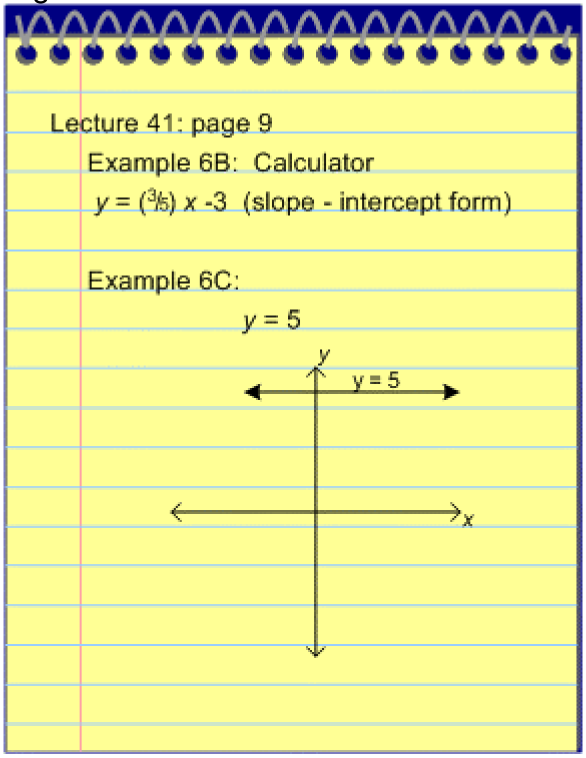
$$\frac{-5y}{-5} = \frac{-3x + 15}{-5}$$

$$y = \frac{3}{5}x - 3$$

x	y
5	0
0	-3

Lecture 41 Notes, Continued

Alg041-09



Lecture 41: page 9

Example 6B: Calculator

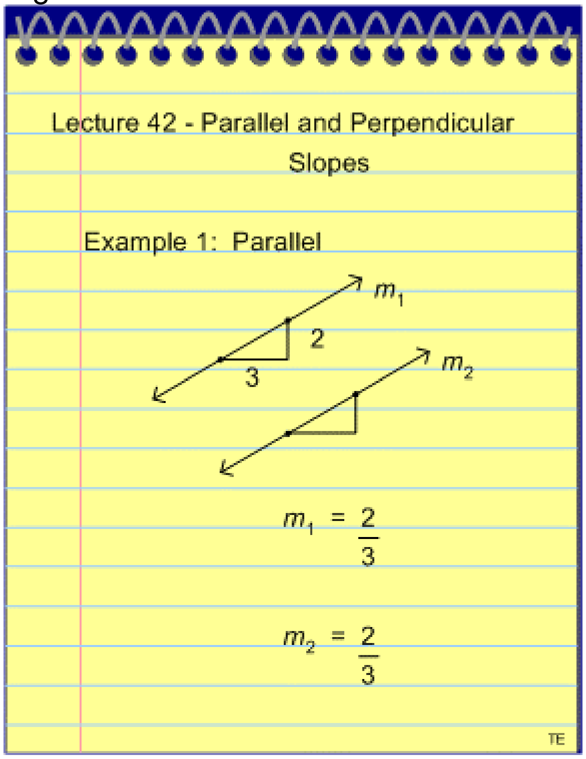
$y = (^{3}/_{5})\ x\ -3$ (slope - intercept form)

Example 6C:

$y = 5$

Lecture 42 Notes

Alg042-01



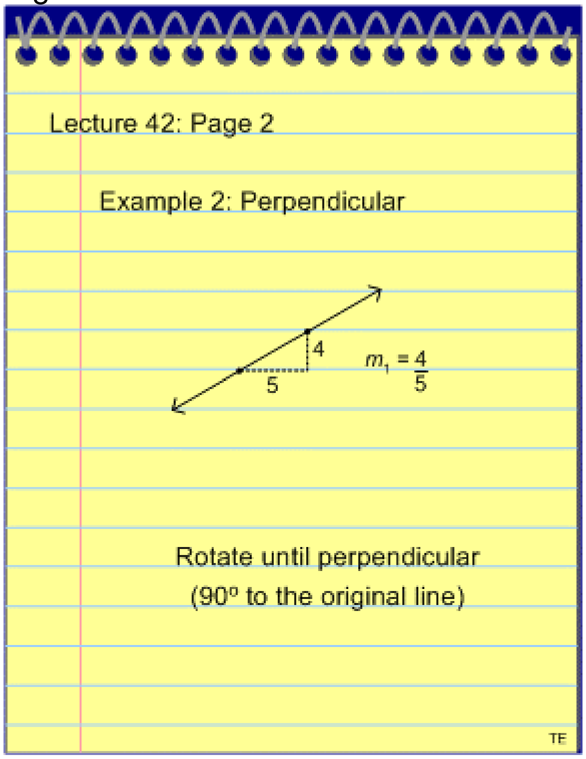
Lecture 42 - Parallel and Perpendicular Slopes

Example 1: Parallel

$m_1 = \frac{2}{3}$

$m_2 = \frac{2}{3}$

Alg042-02



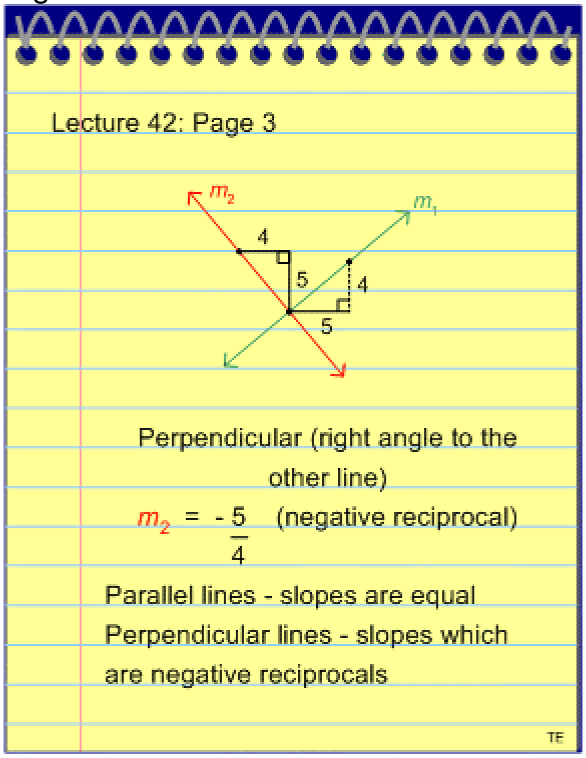
Lecture 42: Page 2

Example 2: Perpendicular

$m_1 = \frac{4}{5}$

Rotate until perpendicular (90º to the original line)

Alg042-03



Lecture 42: Page 3

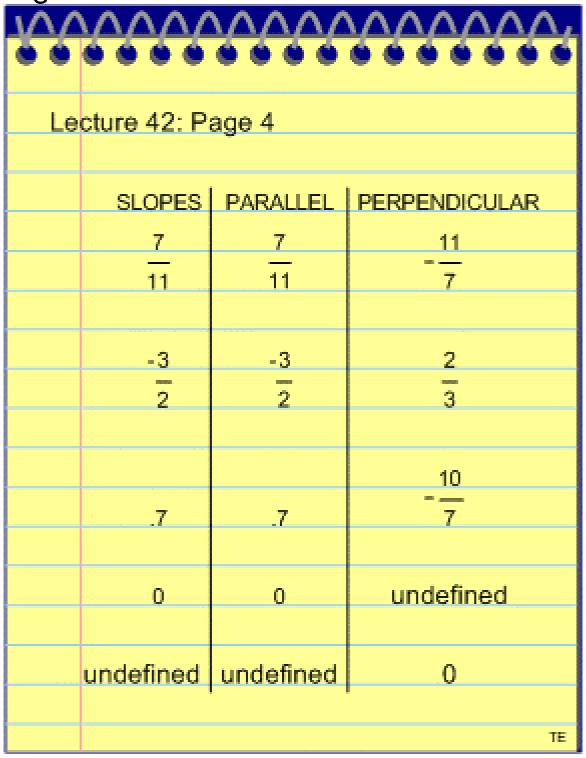
Perpendicular (right angle to the other line)

$m_2 = -\frac{5}{4}$ (negative reciprocal)

Parallel lines - slopes are equal

Perpendicular lines - slopes which are negative reciprocals

Alg042-04

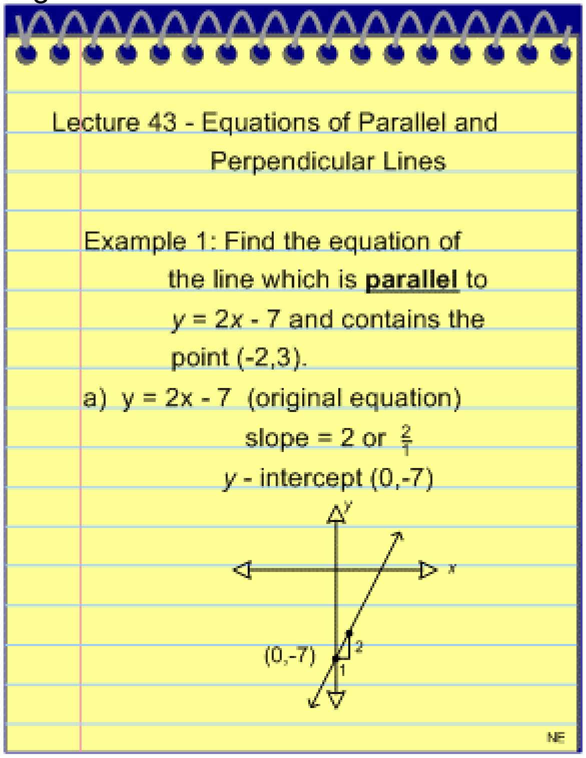


Lecture 42: Page 4

SLOPES	PARALLEL	PERPENDICULAR
$\frac{7}{11}$	$\frac{7}{11}$	$-\frac{11}{7}$
$\frac{-3}{2}$	$\frac{-3}{2}$	$\frac{2}{3}$
.7	.7	$-\frac{10}{7}$
0	0	undefined
undefined	undefined	0

Lecture 43 Notes

Alg043-01



Lecture 43 - Equations of Parallel and Perpendicular Lines

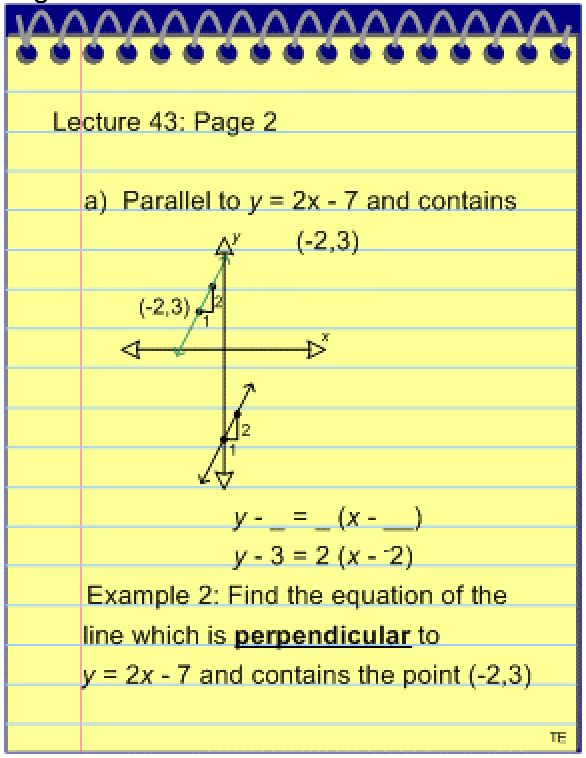
Example 1: Find the equation of the line which is **parallel** to $y = 2x - 7$ and contains the point (-2,3).

a) $y = 2x - 7$ (original equation)

slope = 2 or $\frac{2}{1}$

y - intercept (0,-7)

Alg043-02



Lecture 43: Page 2

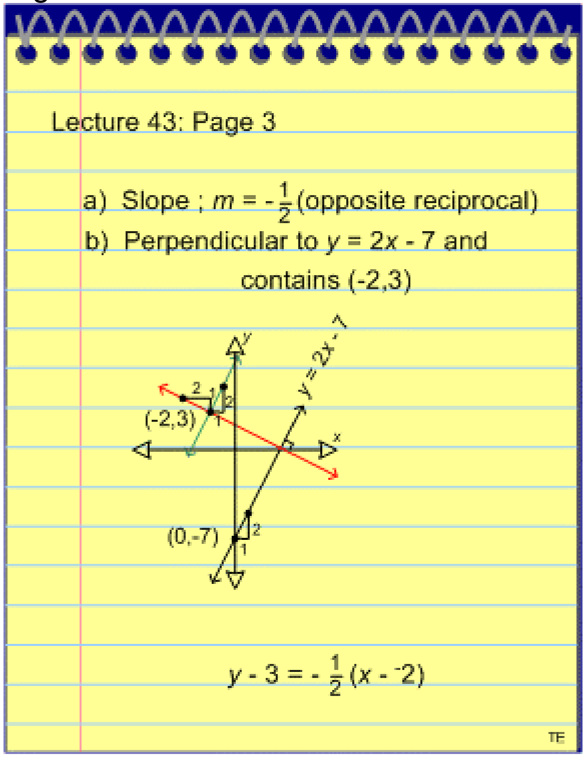
a) Parallel to $y = 2x - 7$ and contains (-2,3)

y - _ = _ (x - __)

$y - 3 = 2\,(x - {}^-2)$

Example 2: Find the equation of the line which is **perpendicular** to $y = 2x - 7$ and contains the point (-2,3)

Alg043-03



Lecture 43: Page 3

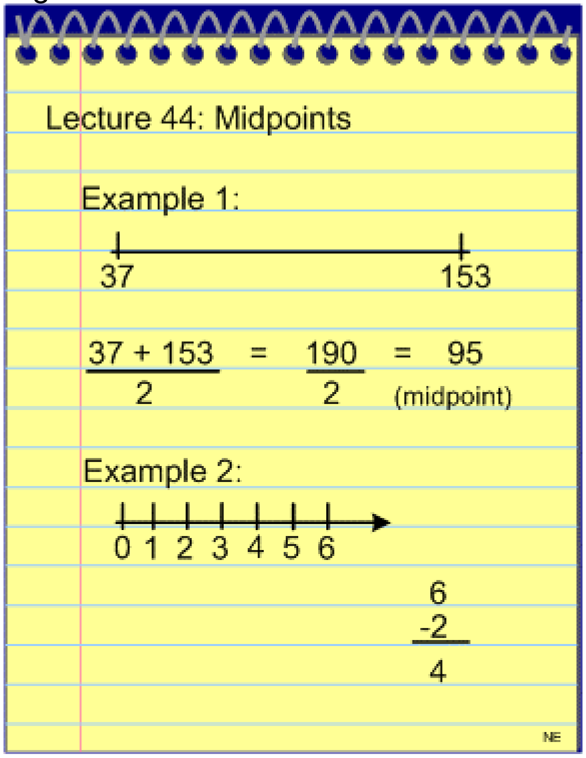
a) Slope ; $m = -\frac{1}{2}$ (opposite reciprocal)

b) Perpendicular to $y = 2x - 7$ and contains (-2,3)

$y - 3 = -\frac{1}{2}(x - {}^-2)$

Lecture 44 Notes

Alg044-01



Lecture 44: Midpoints

Example 1:

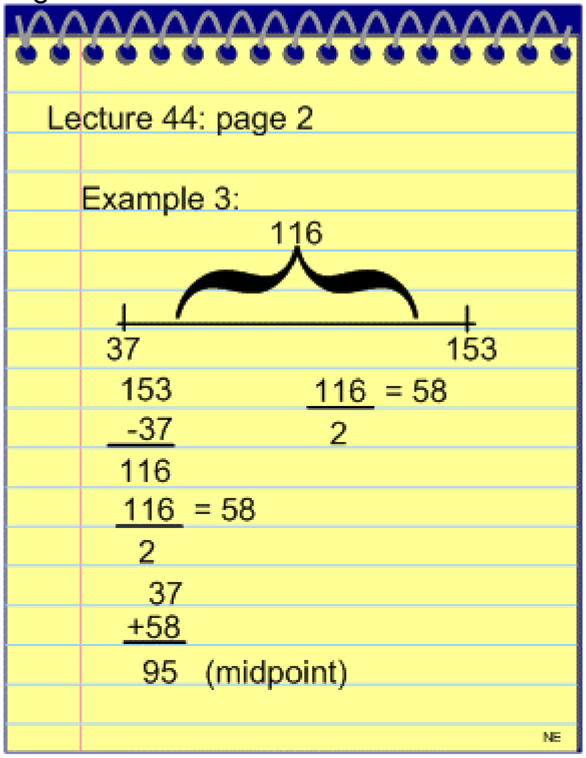
$$\frac{37 + 153}{2} = \frac{190}{2} = 95$$

(midpoint)

Example 2:

$$\begin{array}{r} 6 \\ \underline{-2} \\ 4 \end{array}$$

Alg044-02



Lecture 44: page 2

Example 3:

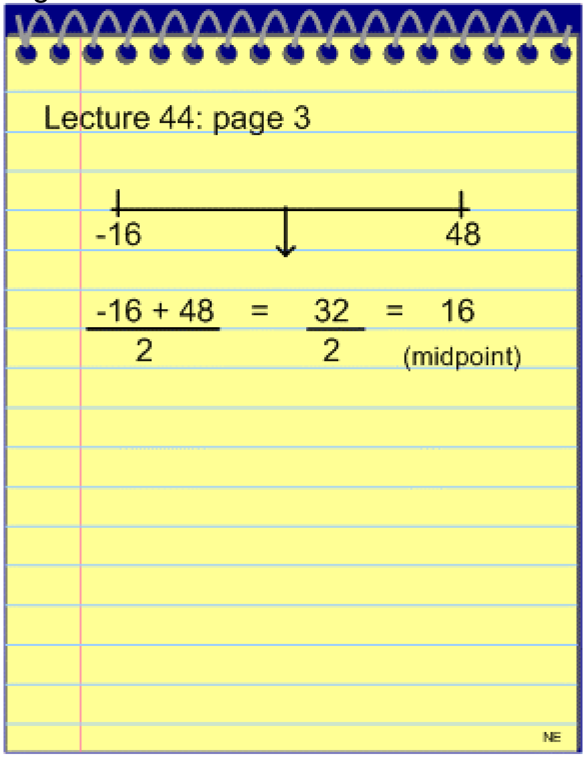
$$\begin{array}{r} 153 \\ \underline{-37} \\ 116 \end{array} \qquad \frac{116}{2} = 58$$

$$\frac{116}{2} = 58$$

$$\begin{array}{r} 37 \\ \underline{+58} \\ 95 \end{array}$$

(midpoint)

Alg044-03

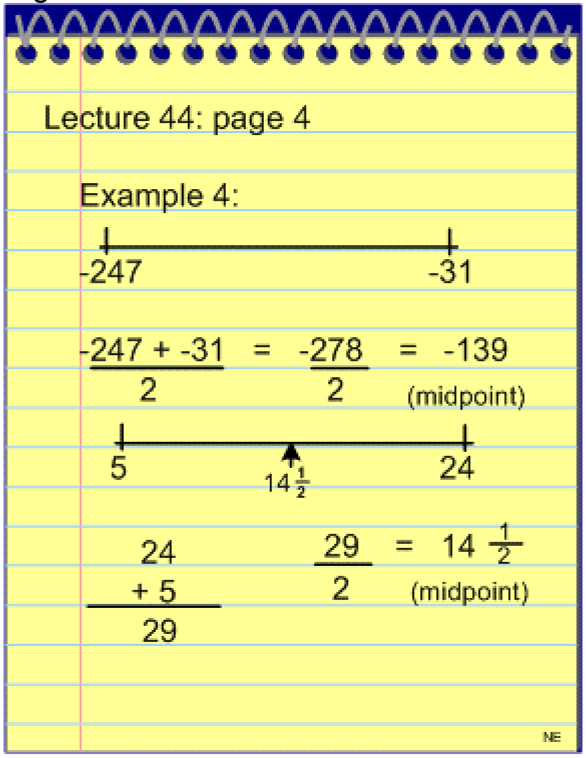


Lecture 44: page 3

$$\frac{-16 + 48}{2} = \frac{32}{2} = 16$$

(midpoint)

Alg044-04



Lecture 44: page 4

Example 4:

$$\frac{-247 + -31}{2} = \frac{-278}{2} = -139$$

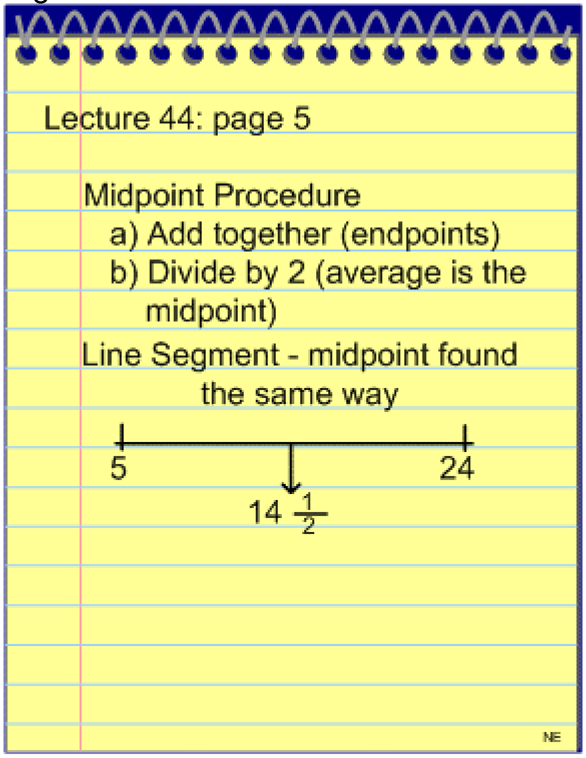
(midpoint)

$$\begin{array}{r} 24 \\ \underline{+\ 5} \\ 29 \end{array} \qquad \frac{29}{2} = 14\frac{1}{2}$$

(midpoint)

Lecture 44 Notes, Continued

Alg044-05

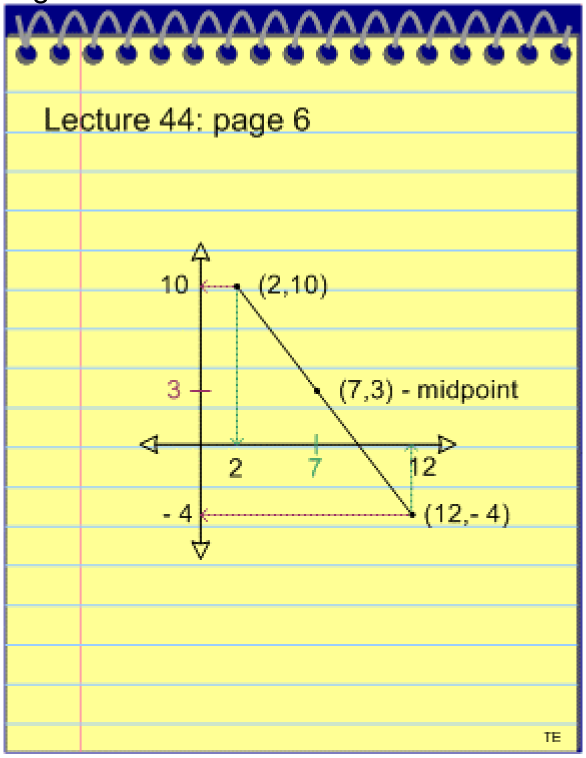


Lecture 44: page 5

Midpoint Procedure
- a) Add together (endpoints)
- b) Divide by 2 (average is the midpoint)

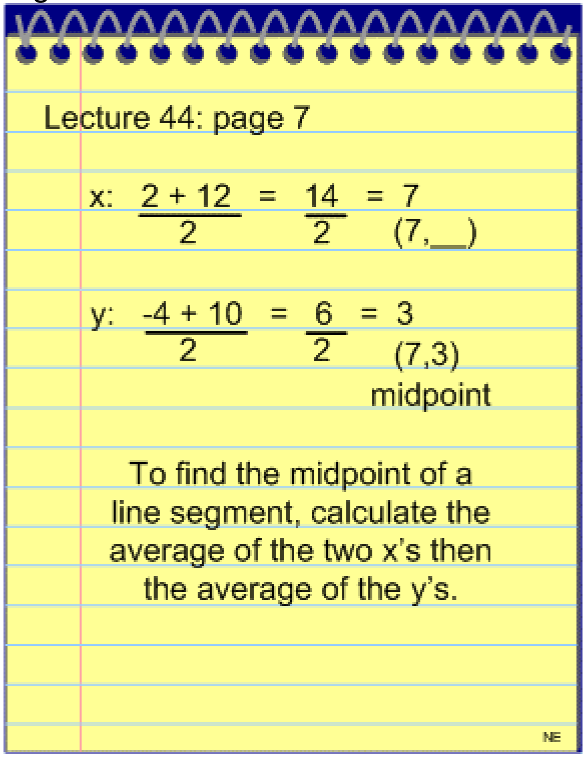
Line Segment - midpoint found the same way

Alg044-06



Lecture 44: page 6

Alg044-07



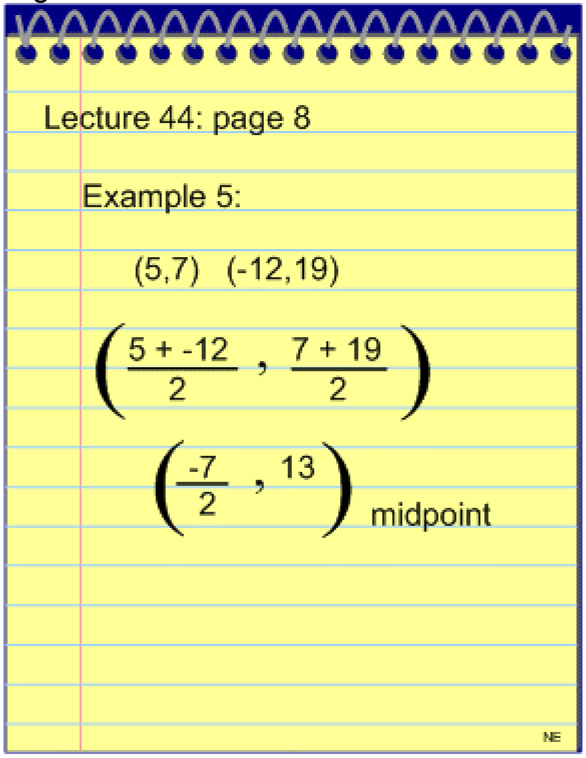
Lecture 44: page 7

x: $\frac{2 + 12}{2} = \frac{14}{2} = 7$ (7,__)

y: $\frac{-4 + 10}{2} = \frac{6}{2} = 3$ (7,3) midpoint

To find the midpoint of a line segment, calculate the average of the two x's then the average of the y's.

Alg044-08



Lecture 44: page 8

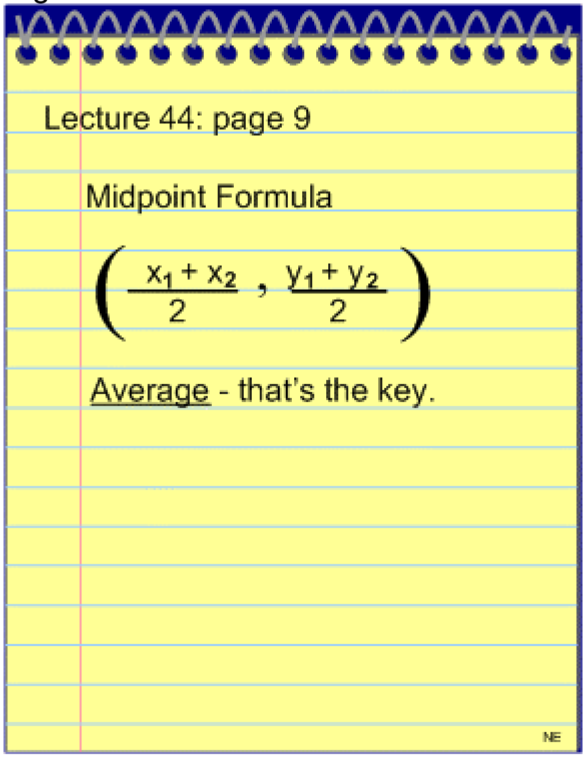
Example 5:

(5,7) (-12,19)

$$\left(\frac{5 + -12}{2}, \frac{7 + 19}{2}\right)$$

$$\left(\frac{-7}{2}, 13\right)$$ midpoint

Alg044-09



Lecture 44: page 9

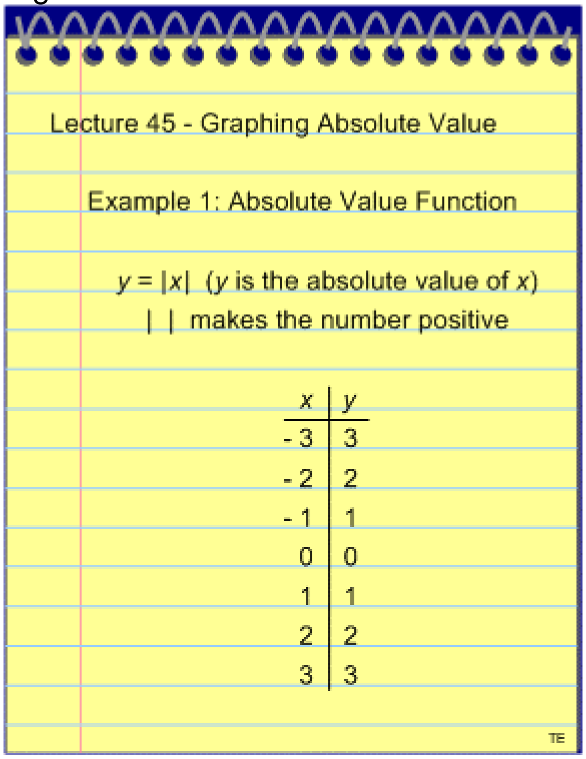
Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

<ins>Average</ins> - that's the key.

Lecture 45 Notes

Alg045-01



Lecture 45 - Graphing Absolute Value

Example 1: Absolute Value Function

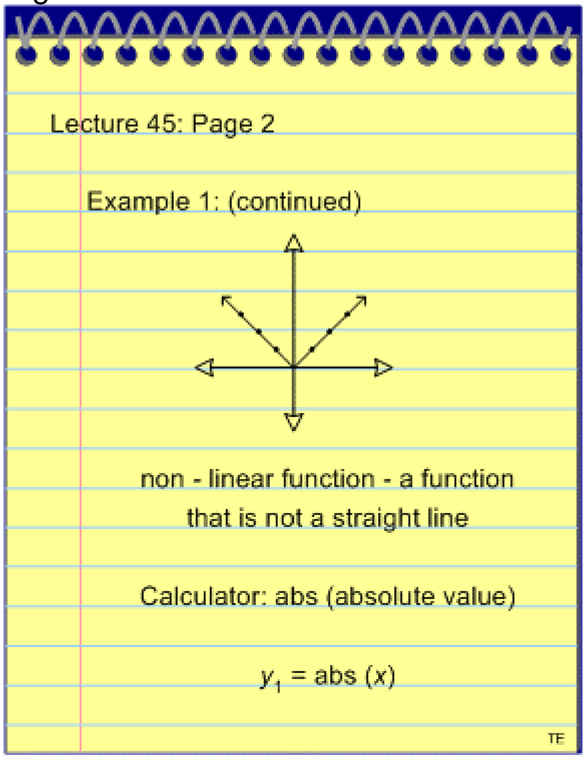
$y = |x|$ (y is the absolute value of x)

| | makes the number positive

x	y
- 3	3
- 2	2
- 1	1
0	0
1	1
2	2
3	3

TE

Alg045-02



Lecture 45: Page 2

Example 1: (continued)

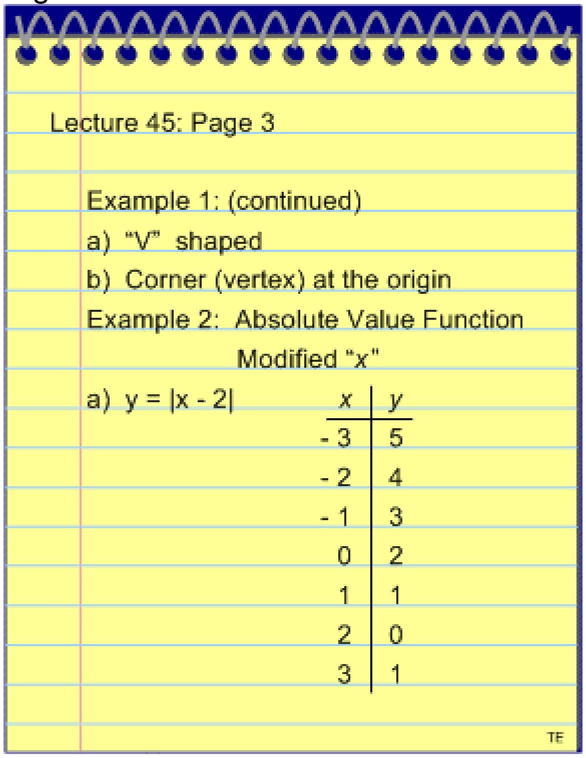
non - linear function - a function that is not a straight line

Calculator: abs (absolute value)

$y_1 = \text{abs}(x)$

TE

Alg045-03



Lecture 45: Page 3

Example 1: (continued)

a) "V" shaped

b) Corner (vertex) at the origin

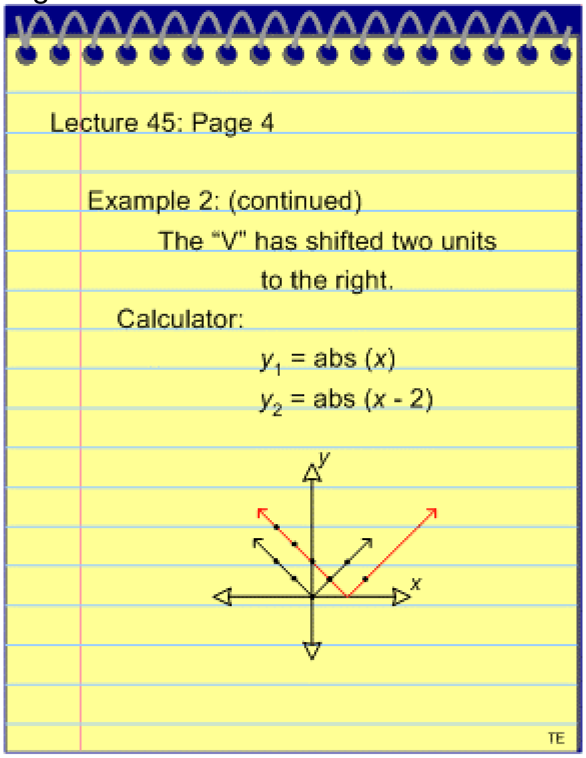
Example 2: Absolute Value Function Modified "x"

a) $y = |x - 2|$

x	y
- 3	5
- 2	4
- 1	3
0	2
1	1
2	0
3	1

TE

Alg045-04



Lecture 45: Page 4

Example 2: (continued)

The "V" has shifted two units to the right.

Calculator:

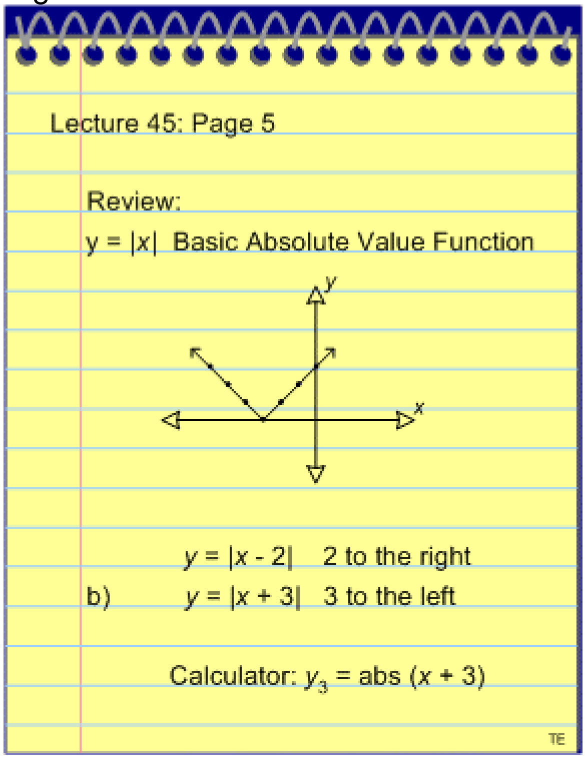
$y_1 = \text{abs}(x)$

$y_2 = \text{abs}(x - 2)$

TE

Lecture 45 Notes, Continued

Alg045-05



Lecture 45: Page 5

Review:

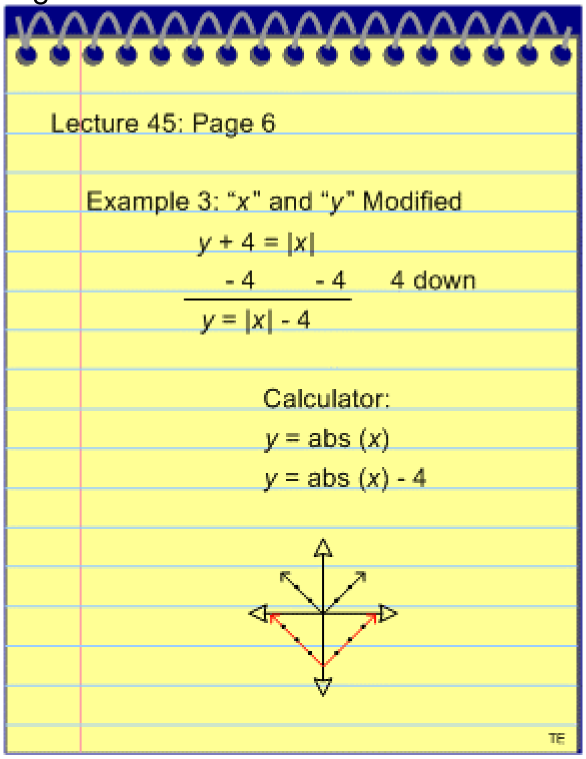
$y = |x|$ Basic Absolute Value Function

$y = |x - 2|$ 2 to the right

b) $y = |x + 3|$ 3 to the left

Calculator: $y_3 = \text{abs}(x + 3)$

Alg045-06



Lecture 45: Page 6

Example 3: "x" and "y" Modified

$y + 4 = |x|$

$\underline{\;-4 \qquad -4\;}$ 4 down

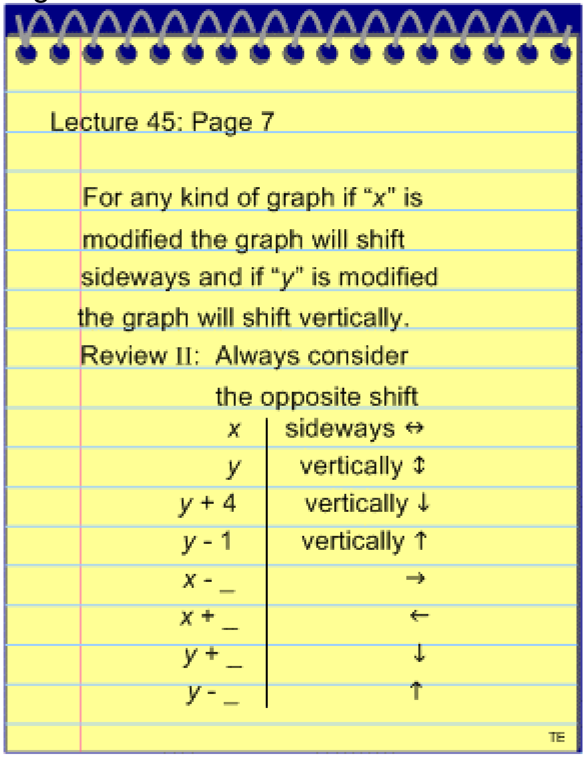
$y = |x| - 4$

Calculator:

$y = \text{abs}(x)$

$y = \text{abs}(x) - 4$

Alg045-07



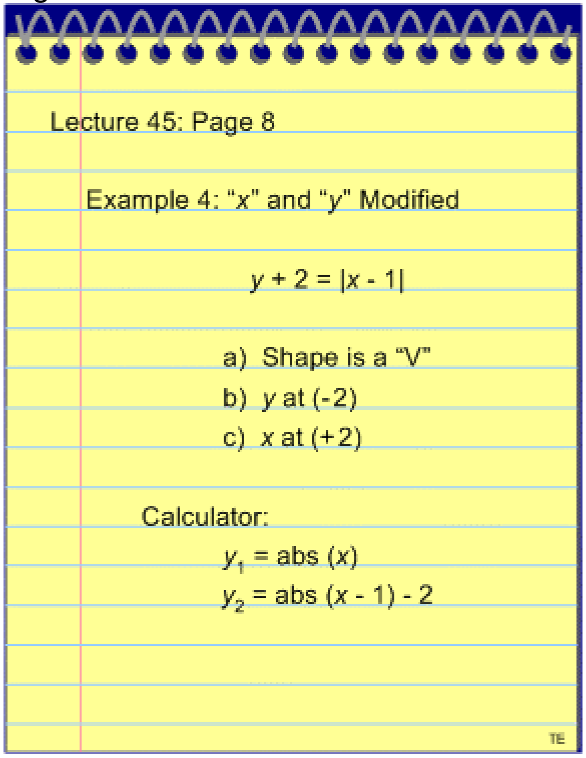
Lecture 45: Page 7

For any kind of graph if "x" is modified the graph will shift sideways and if "y" is modified the graph will shift vertically.

Review II: Always consider the opposite shift

x	sideways ↔
y	vertically ↕
$y + 4$	vertically ↓
$y - 1$	vertically ↑
x - _	→
x + _	←
y + _	↓
y - _	↑

Alg045-08



Lecture 45: Page 8

Example 4: "x" and "y" Modified

$y + 2 = |x - 1|$

a) Shape is a "V"

b) y at (-2)

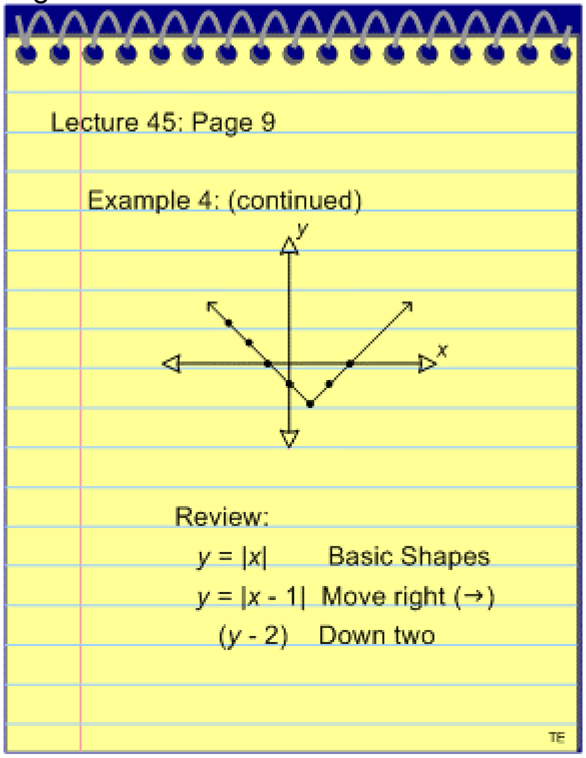
c) x at (+2)

Calculator:

$y_1 = \text{abs}(x)$

$y_2 = \text{abs}(x - 1) - 2$

Alg045-09



Lecture 45: Page 9

Example 4: (continued)

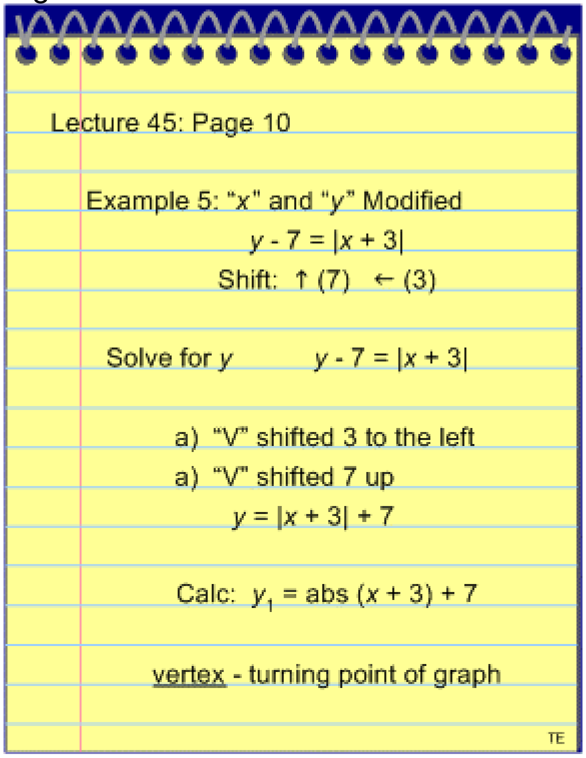
Review:

$y = |x|$ Basic Shapes

$y = |x - 1|$ Move right (→)

$(y - 2)$ Down two

Alg045-10



Lecture 45: Page 10

Example 5: "*x*" and "*y*" Modified

$y - 7 = |x + 3|$

Shift: ↑ (7) ← (3)

Solve for *y* $y - 7 = |x + 3|$

a) "V" shifted 3 to the left

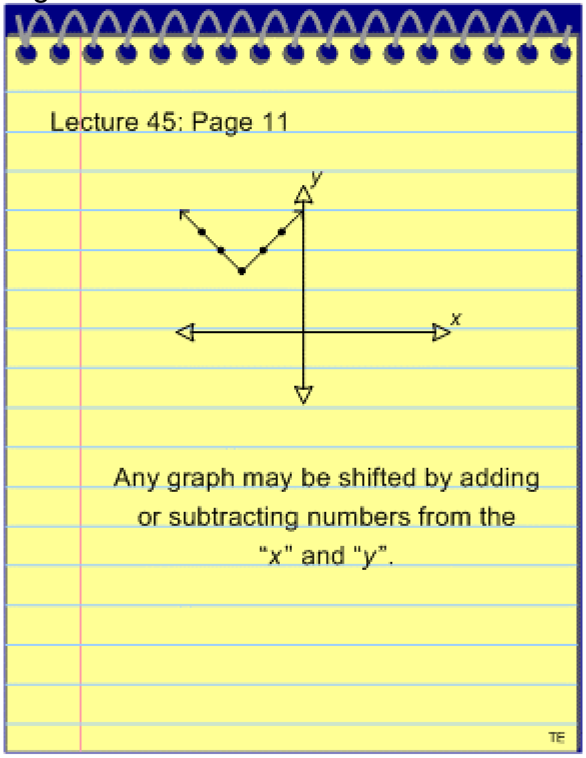
a) "V" shifted 7 up

$y = |x + 3| + 7$

Calc: y_1 = abs $(x + 3) + 7$

vertex - turning point of graph

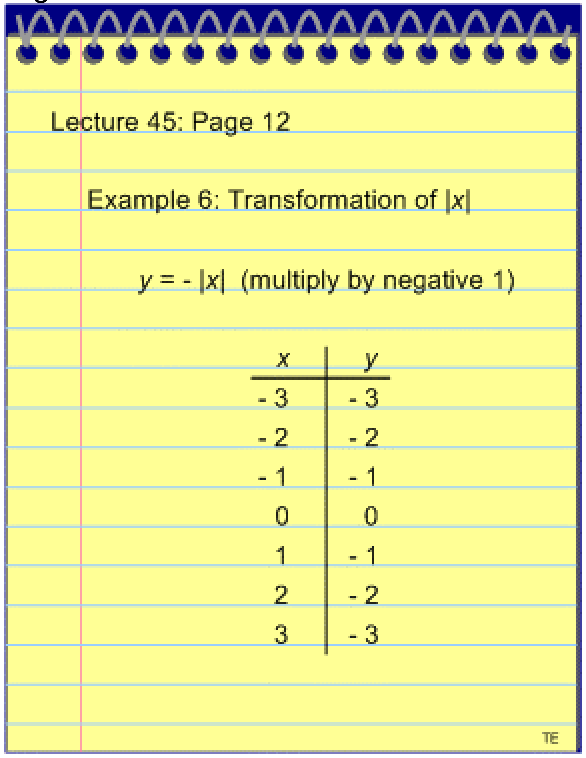
Alg045-11



Lecture 45: Page 11

Any graph may be shifted by adding or subtracting numbers from the "*x*" and "*y*".

Alg045-12



Lecture 45: Page 12

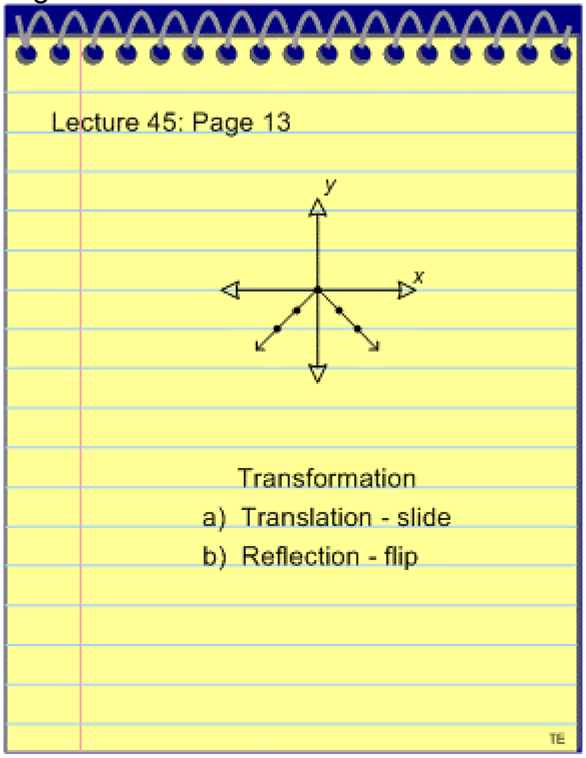
Example 6: Transformation of $|x|$

$y = - |x|$ (multiply by negative 1)

x	y
- 3	- 3
- 2	- 2
- 1	- 1
0	0
1	- 1
2	- 2
3	- 3

Lecture 45 Notes, Continued

Alg045-13



Lecture 45: Page 13

Transformation

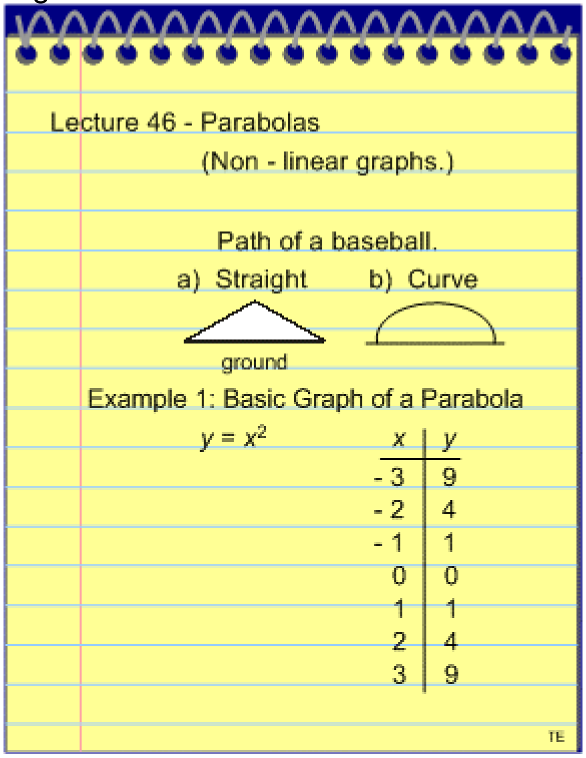
a) Translation - slide

b) Reflection - flip

TE

Lecture 46 Notes

Alg046-01



Lecture 46 - Parabolas

(Non - linear graphs.)

Path of a baseball.

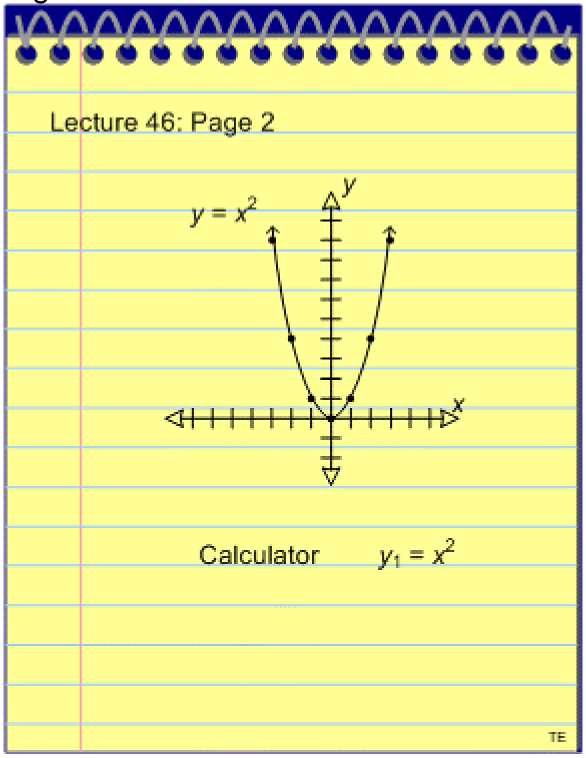
a) Straight b) Curve

Example 1: Basic Graph of a Parabola

$y = x^2$

x	y
- 3	9
- 2	4
- 1	1
0	0
1	1
2	4
3	9

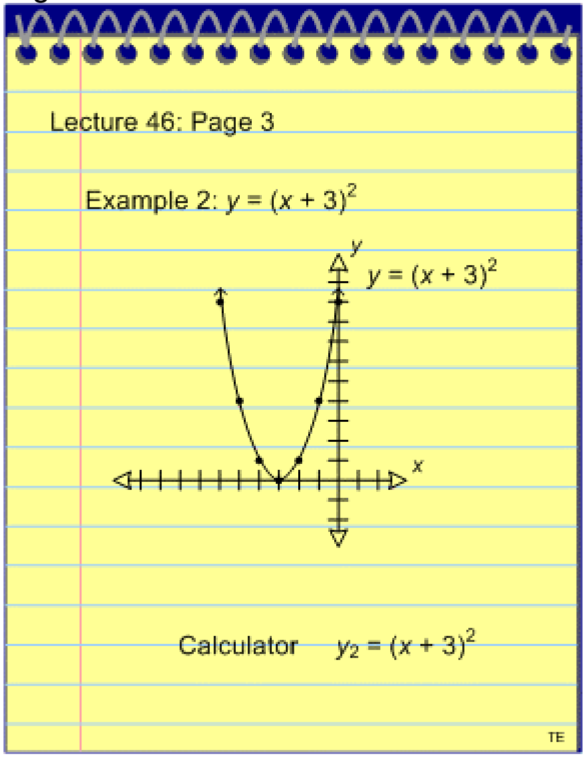
Alg046-02



Lecture 46: Page 2

Calculator $y_1 = x^2$

Alg046-03

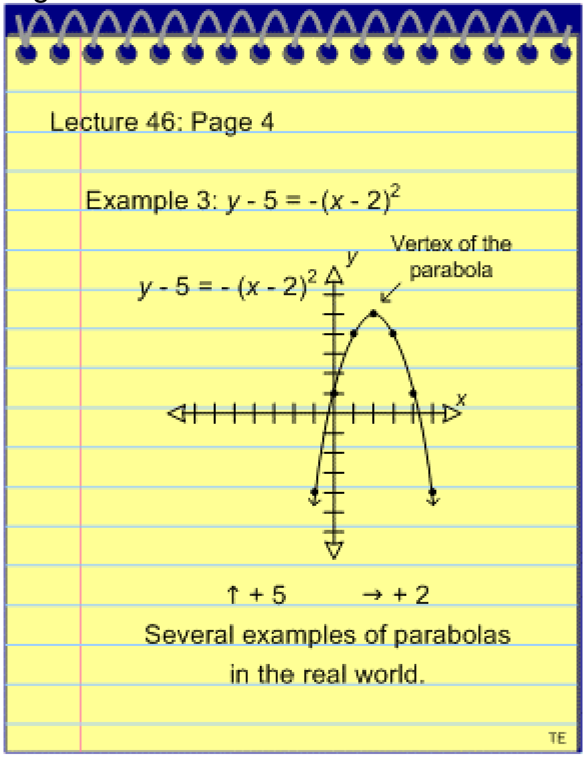


Lecture 46: Page 3

Example 2: $y = (x + 3)^2$

Calculator $y_2 = (x + 3)^2$

Alg046-04



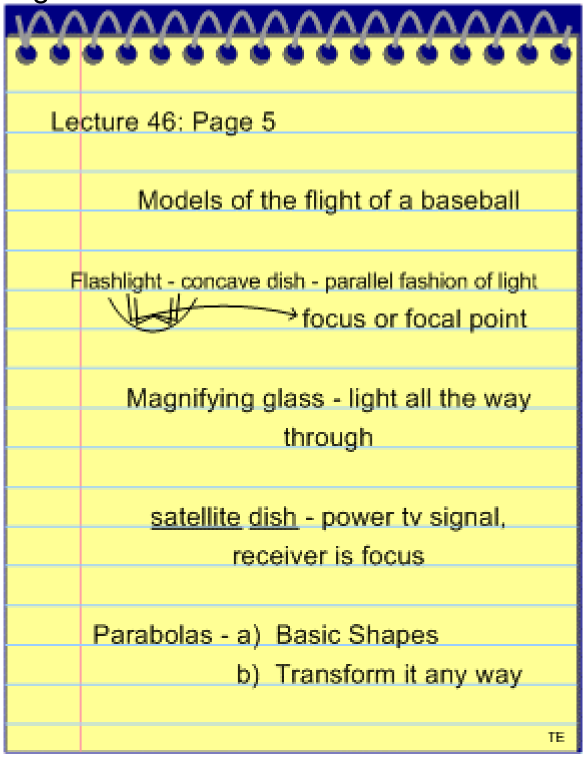
Lecture 46: Page 4

Example 3: $y - 5 = -(x - 2)^2$

↑ + 5 → + 2

Several examples of parabolas in the real world.

Alg046-05



Lecture 46: Page 5

Models of the flight of a baseball

Flashlight - concave dish - parallel fashion of light

focus or focal point

Magnifying glass - light all the way through

satellite dish - power tv signal, receiver is focus

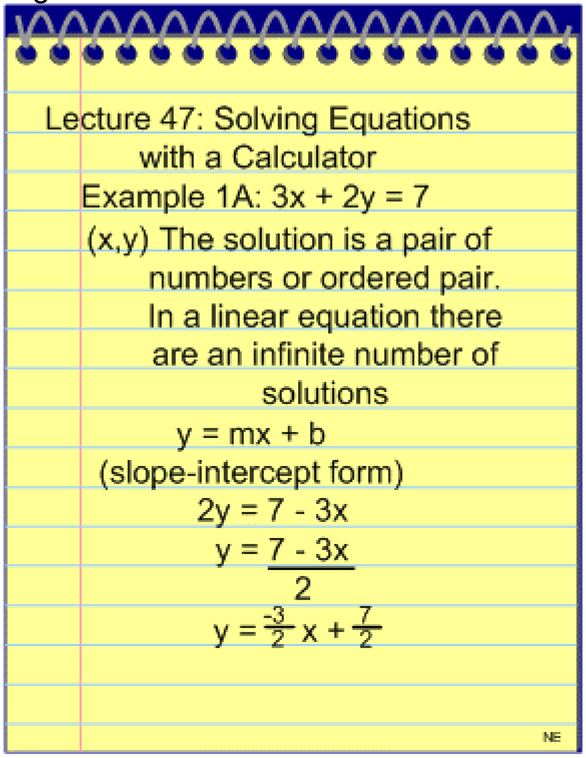
Parabolas - a) Basic Shapes

b) Transform it any way

TE

Lecture 47 Notes

Alg047-01



Lecture 47: Solving Equations
with a Calculator

Example 1A: $3x + 2y = 7$

(x,y) The solution is a pair of numbers or ordered pair. In a linear equation there are an infinite number of solutions

$y = mx + b$

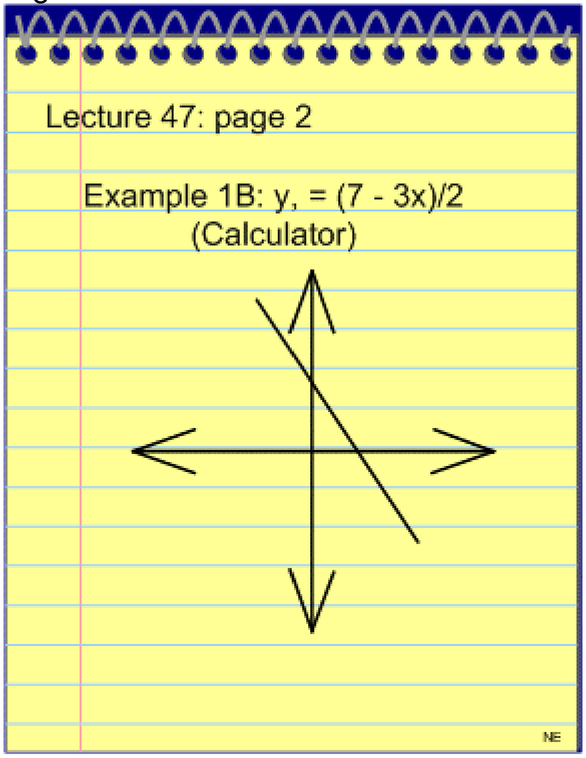
(slope-intercept form)

$2y = 7 - 3x$

$$y = \frac{7 - 3x}{2}$$

$$y = \frac{-3}{2}x + \frac{7}{2}$$

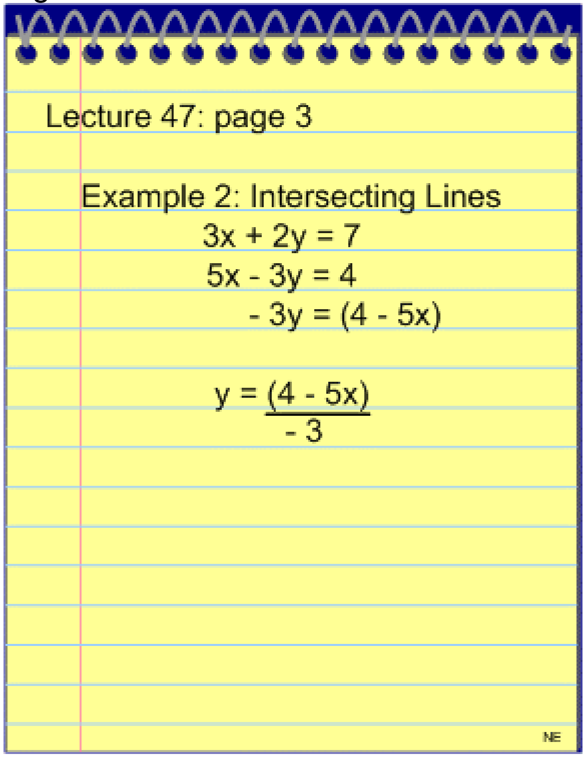
Alg047-02



Lecture 47: page 2

Example 1B: y, = (7 - 3x)/2
(Calculator)

Alg047-03



Lecture 47: page 3

Example 2: Intersecting Lines

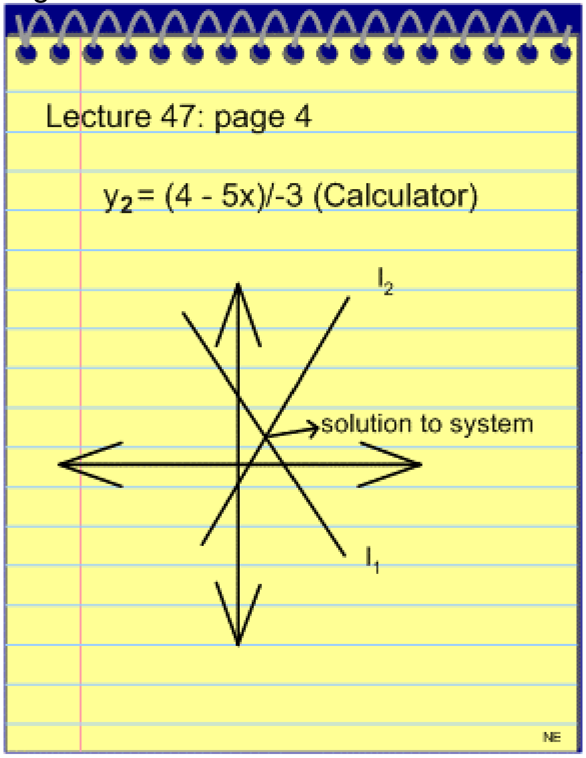
$3x + 2y = 7$

$5x - 3y = 4$

$- 3y = (4 - 5x)$

$$y = \frac{(4 - 5x)}{- 3}$$

Alg047-04

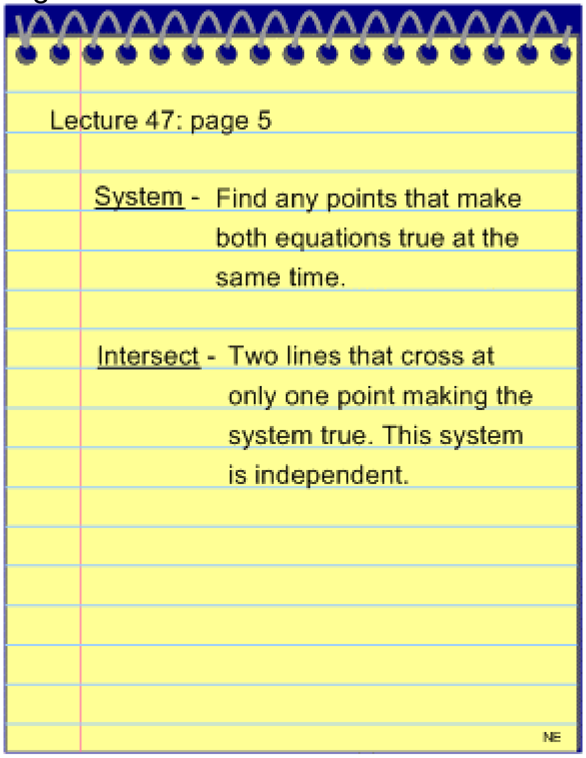


Lecture 47: page 4

$y_2 = (4 - 5x)/-3$ (Calculator)

Lecture 47 Notes, continued

Alg047-05



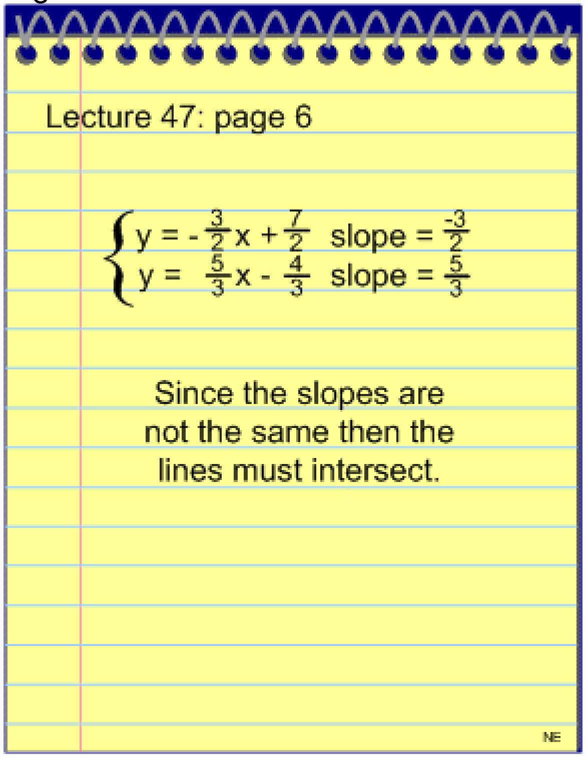
Lecture 47: page 5

<u>System</u> - Find any points that make both equations true at the same time.

<u>Intersect</u> - Two lines that cross at only one point making the system true. This system is independent.

NE

Alg047-06



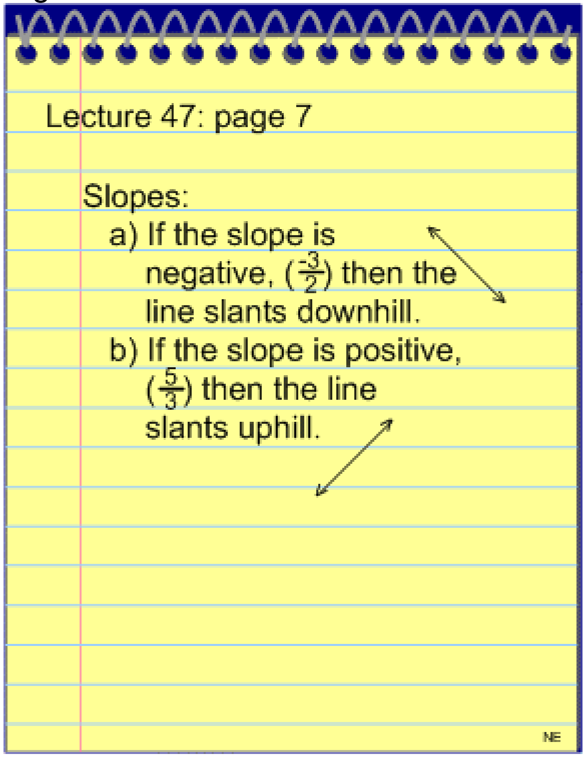
Lecture 47: page 6

$$\begin{cases} y = -\frac{3}{2}x + \frac{7}{2} & \text{slope} = \frac{-3}{2} \\ y = \frac{5}{3}x - \frac{4}{3} & \text{slope} = \frac{5}{3} \end{cases}$$

Since the slopes are not the same then the lines must intersect.

NE

Alg047-07



Lecture 47: page 7

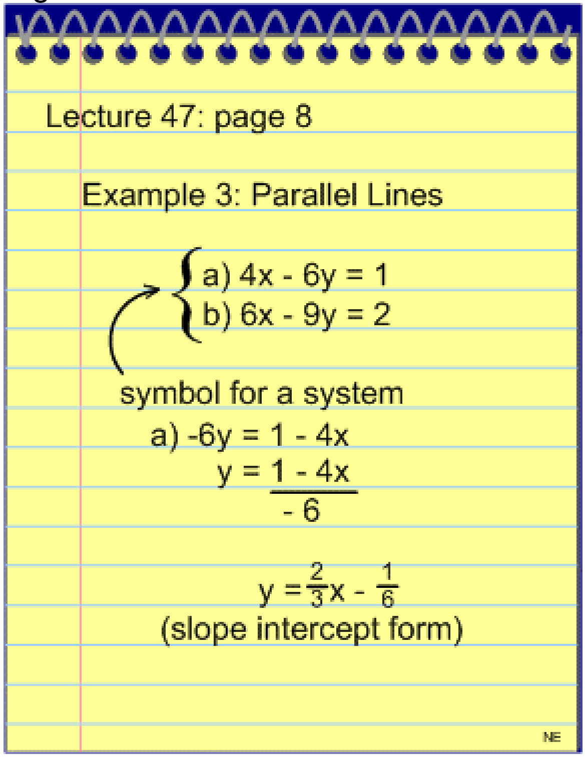
Slopes:

a) If the slope is negative, $(\frac{-3}{2})$ then the line slants downhill.

b) If the slope is positive, $(\frac{5}{3})$ then the line slants uphill.

NE

Alg047-08



Lecture 47: page 8

Example 3: Parallel Lines

$$\begin{cases} \text{a) } 4x - 6y = 1 \\ \text{b) } 6x - 9y = 2 \end{cases}$$

symbol for a system

a) $-6y = 1 - 4x$

$$y = \frac{1 - 4x}{-6}$$

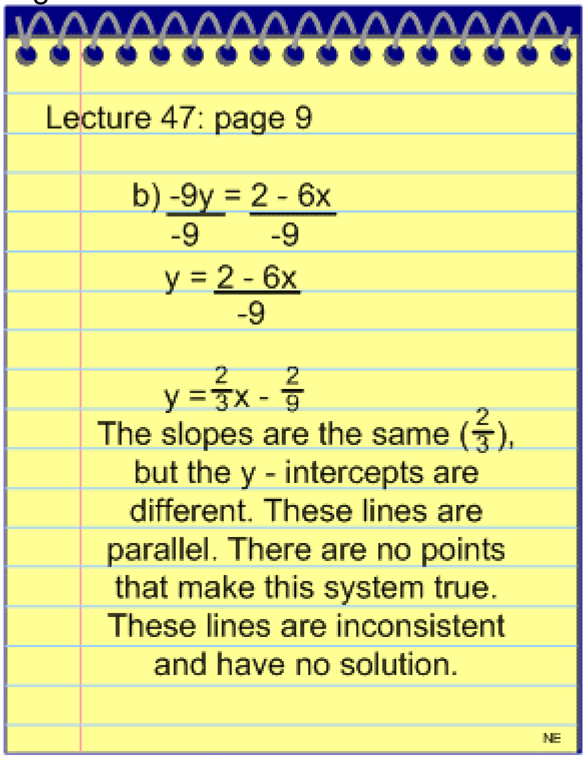
$$y = \frac{2}{3}x - \frac{1}{6}$$

(slope intercept form)

NE

Lecture 47 Notes, continued

Alg047-09



Lecture 47: page 9

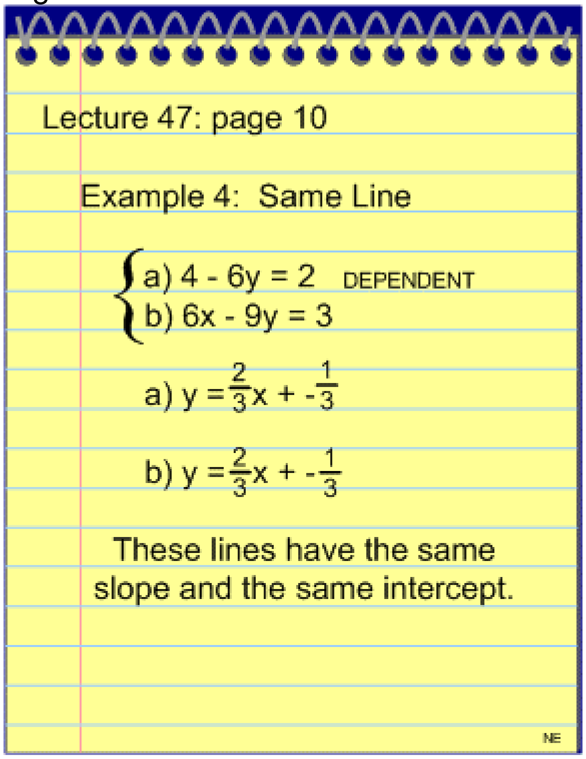
b) $\frac{-9y}{-9} = \frac{2 - 6x}{-9}$

$y = \frac{2 - 6x}{-9}$

$y = \frac{2}{3}x - \frac{2}{9}$

The slopes are the same ($\frac{2}{3}$), but the y - intercepts are different. These lines are parallel. There are no points that make this system true. These lines are inconsistent and have no solution.

Alg047-10



Lecture 47: page 10

Example 4: Same Line

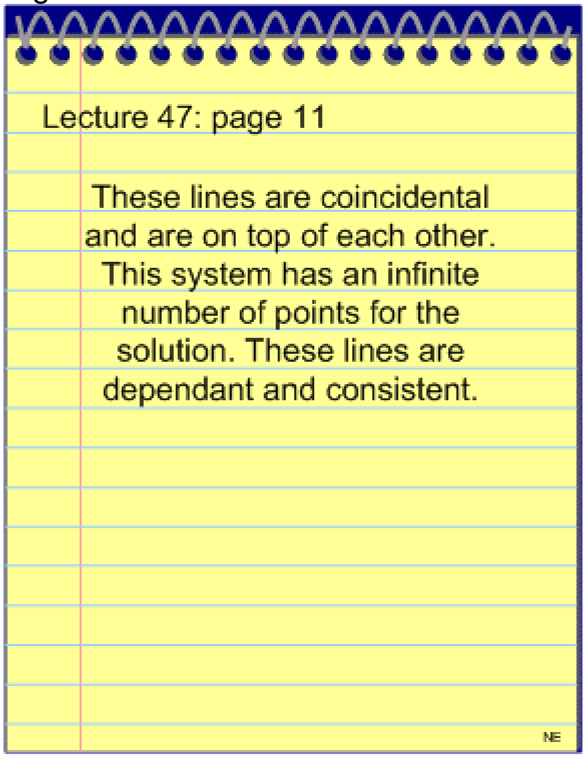
$\begin{cases} \text{a) } 4 - 6y = 2 & \text{DEPENDENT} \\ \text{b) } 6x - 9y = 3 & \end{cases}$

a) $y = \frac{2}{3}x + -\frac{1}{3}$

b) $y = \frac{2}{3}x + -\frac{1}{3}$

These lines have the same slope and the same intercept.

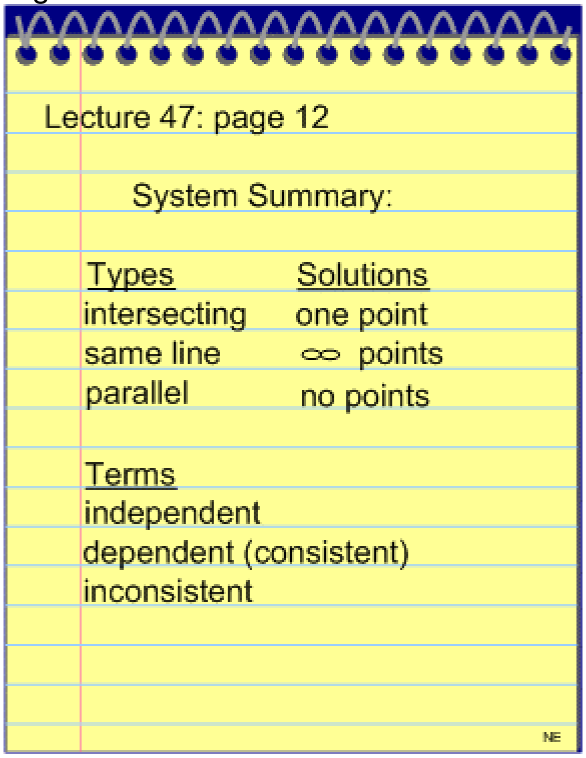
Alg047-11



Lecture 47: page 11

These lines are coincidental and are on top of each other. This system has an infinite number of points for the solution. These lines are dependant and consistent.

Alg047-12



Lecture 47: page 12

System Summary:

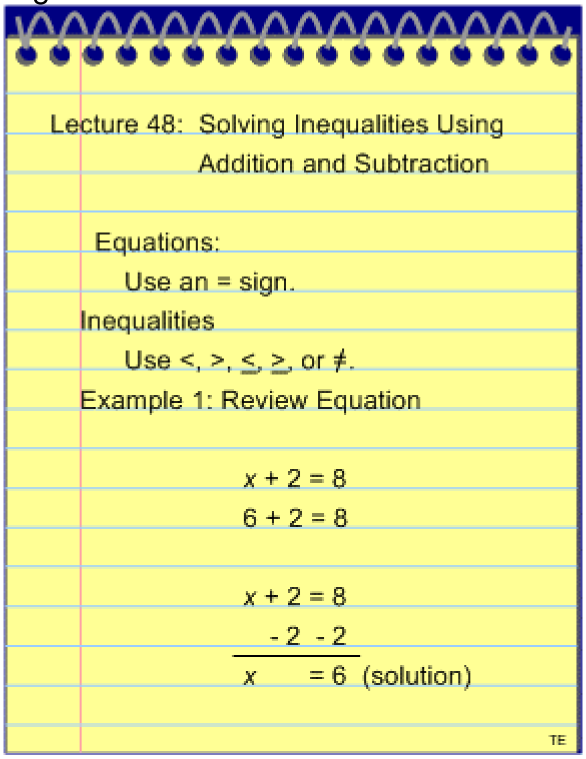
Types	Solutions
intersecting	one point
same line	∞ points
parallel	no points

Terms

independent
dependent (consistent)
inconsistent

Lecture 48 Notes

Alg048-01



Lecture 48: Solving Inequalities Using Addition and Subtraction

Equations:

Use an = sign.

Inequalities

Use $<$, $>$, $\leq$, $\geq$, or $\neq$.

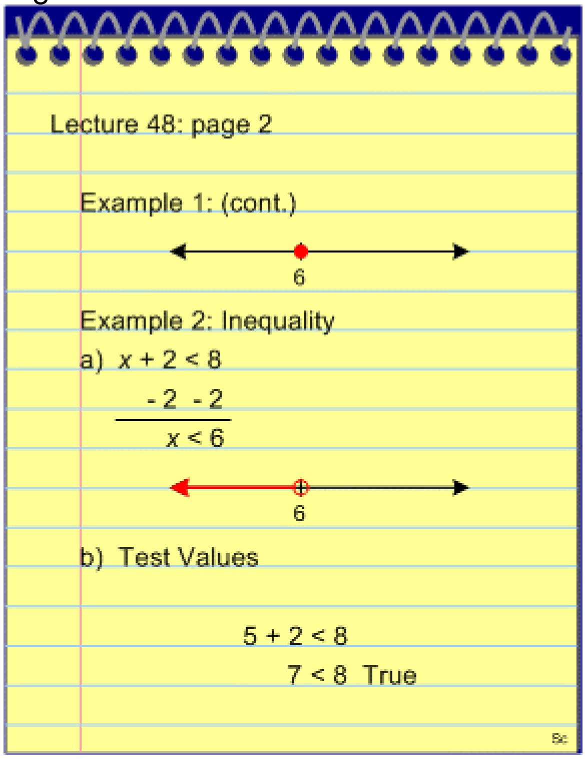
Example 1: Review Equation

$x + 2 = 8$

$6 + 2 = 8$

$$\begin{array}{r} x + 2 = 8 \\ \underline{\;-2\;\;-2} \\ x \quad\; = 6 \end{array}$$ (solution)

Alg048-02



Lecture 48: page 2

Example 1: (cont.)

Example 2: Inequality

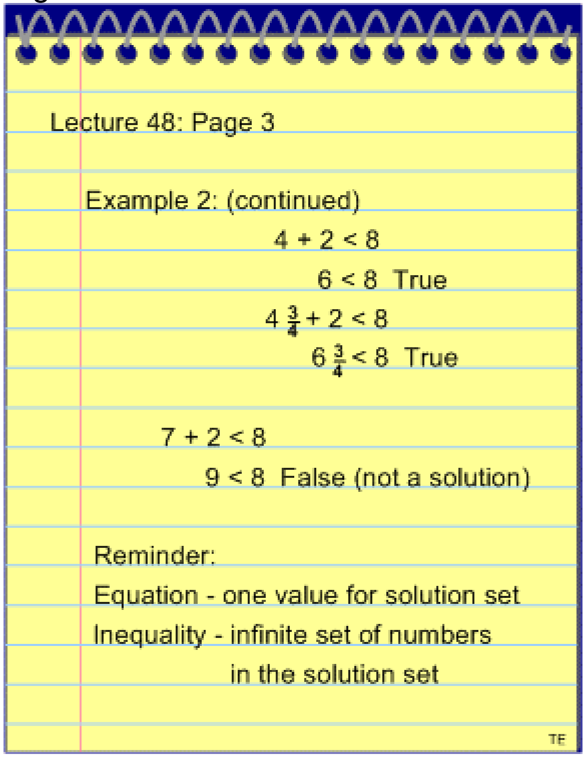
a) $$\begin{array}{r} x + 2 < 8 \\ \underline{\;-2\;\;-2} \\ x < 6 \end{array}$$

b) Test Values

$5 + 2 < 8$

$7 < 8$ True

Alg048-03



Lecture 48: Page 3

Example 2: (continued)

$4 + 2 < 8$

$6 < 8$ True

$4\frac{3}{4} + 2 < 8$

$6\frac{3}{4} < 8$ True

$7 + 2 < 8$

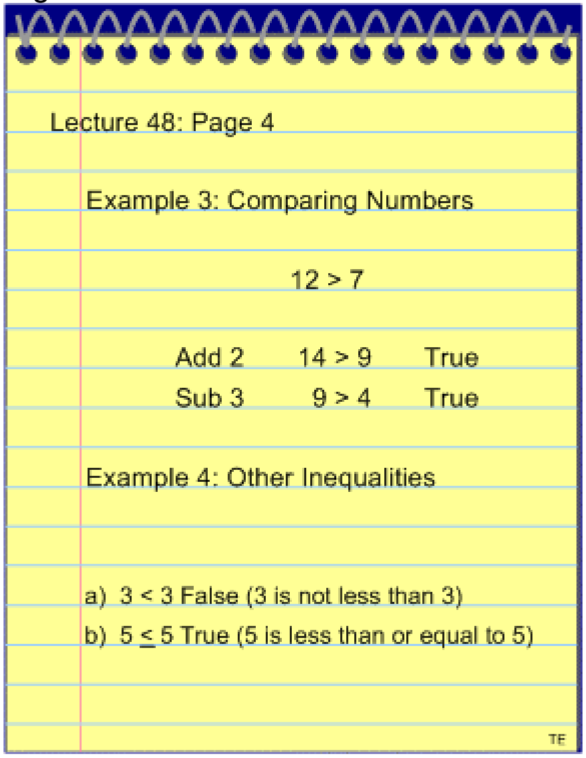
$9 < 8$ False (not a solution)

Reminder:

Equation - one value for solution set

Inequality - infinite set of numbers in the solution set

Alg048-04



Lecture 48: Page 4

Example 3: Comparing Numbers

$12 > 7$

Add 2	$14 > 9$	True
Sub 3	$9 > 4$	True

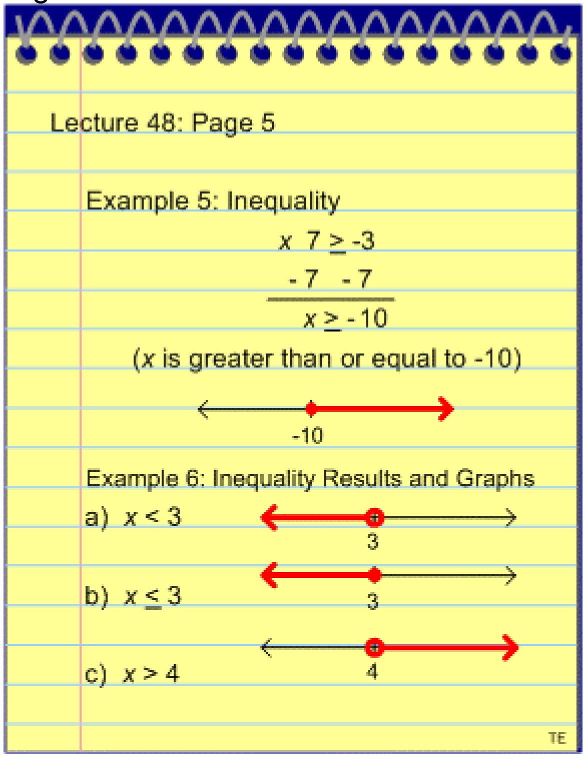
Example 4: Other Inequalities

a) $3 < 3$ False (3 is not less than 3)

b) $5 \leq 5$ True (5 is less than or equal to 5)

Lecture 48 Notes, continued

Alg048-05



Lecture 48: Page 5

Example 5: Inequality

$x \;\; 7 \geq -3$

$-7 \quad -7$

$x \geq -10$

(x is greater than or equal to -10)

-10

Example 6: Inequality Results and Graphs

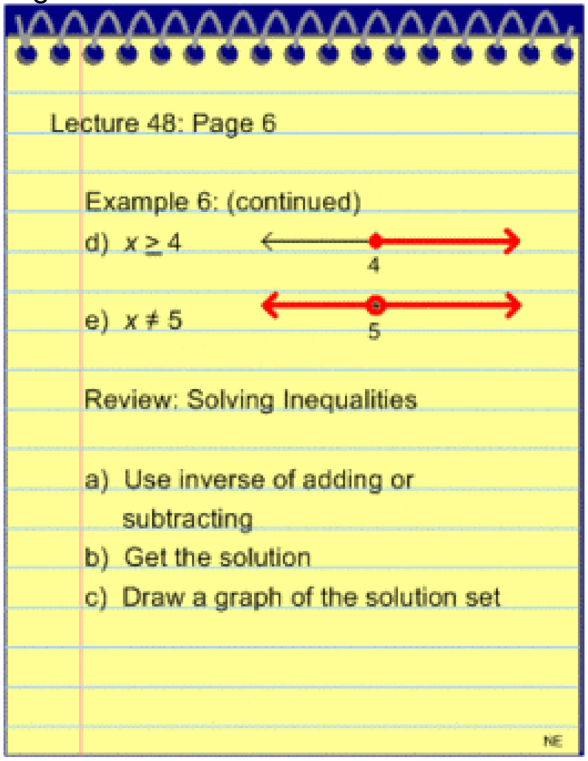
a) $x < 3$ 3

b) $x \leq 3$ 3

c) $x > 4$ 4

TE

Alg048-06



Lecture 48: Page 6

Example 6: (continued)

d) $x \geq 4$ 4

e) $x \neq 5$ 5

Review: Solving Inequalities

a) Use inverse of adding or subtracting

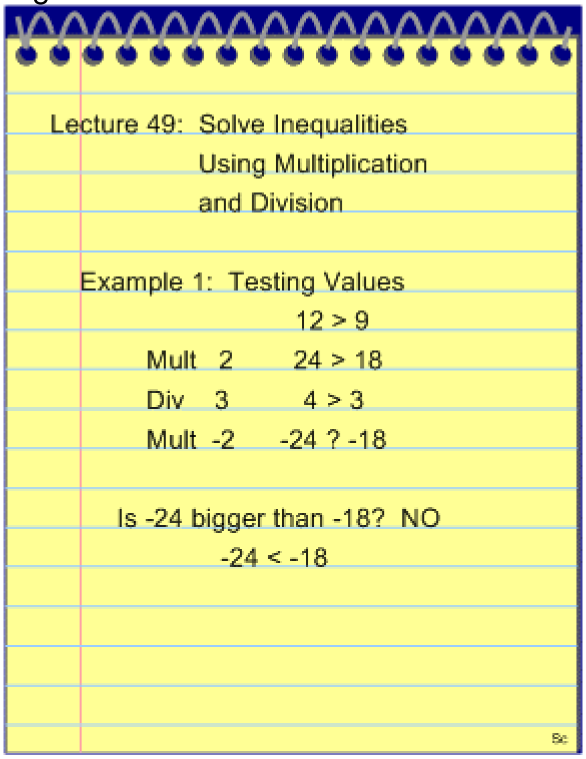
b) Get the solution

c) Draw a graph of the solution set

NE

Lecture 49 Notes

Alg049-01



Lecture 49: Solve Inequalities Using Multiplication and Division

Example 1: Testing Values

$12 > 9$

Mult 2 $24 > 18$

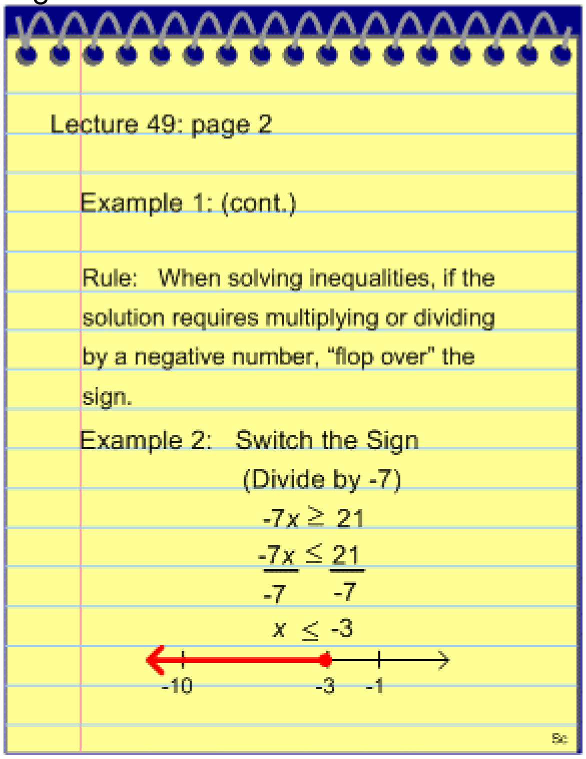
Div 3 $4 > 3$

Mult -2 $-24 \; ? \; -18$

Is -24 bigger than -18? NO

$-24 < -18$

Alg049-02



Lecture 49: page 2

Example 1: (cont.)

Rule: When solving inequalities, if the solution requires multiplying or dividing by a negative number, "flop over" the sign.

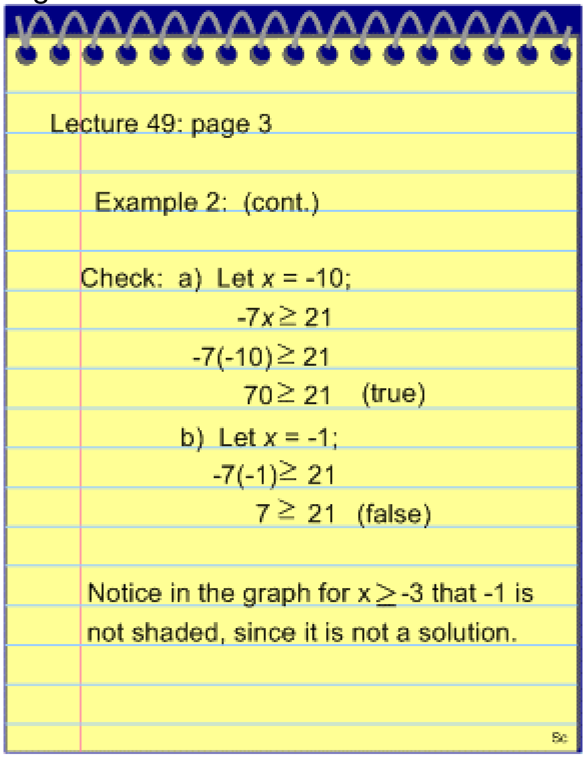
Example 2: Switch the Sign (Divide by -7)

$-7x \ge 21$

$\frac{-7x}{-7} \le \frac{21}{-7}$

$x \le -3$

Alg049-03



Lecture 49: page 3

Example 2: (cont.)

Check: a) Let $x = -10$;

$-7x \ge 21$

$-7(-10) \ge 21$

$70 \ge 21$ (true)

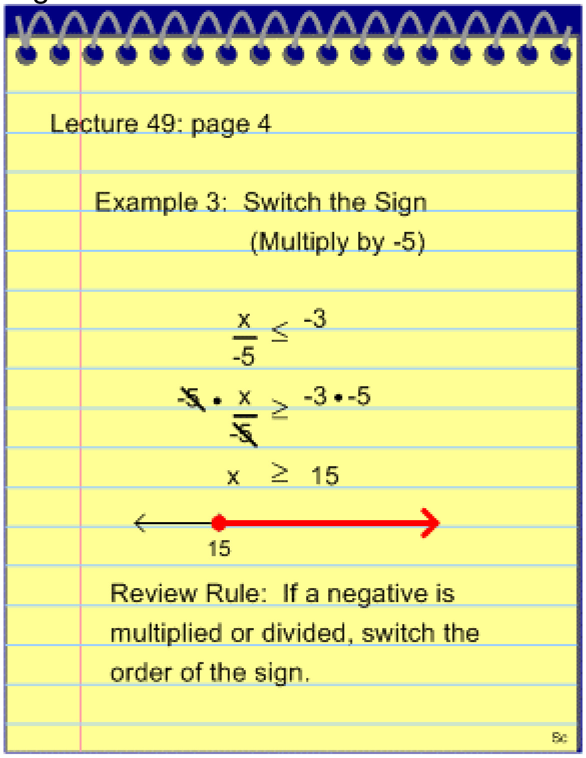
b) Let $x = -1$;

$-7(-1) \ge 21$

$7 \ge 21$ (false)

Notice in the graph for $x \ge -3$ that -1 is not shaded, since it is not a solution.

Alg049-04



Lecture 49: page 4

Example 3: Switch the Sign (Multiply by -5)

$\frac{x}{-5} \le -3$

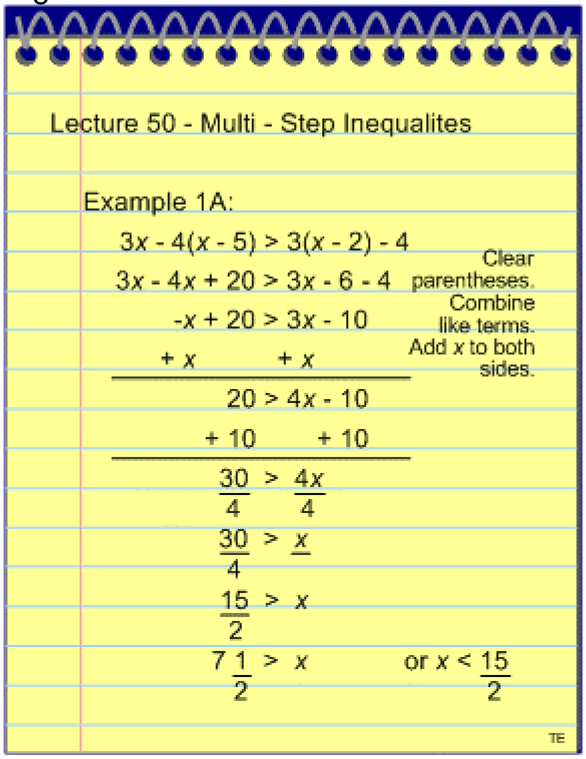
$\cancel{-5} \cdot \frac{x}{\cancel{-5}} \ge -3 \cdot -5$

$x \ge 15$

Review Rule: If a negative is multiplied or divided, switch the order of the sign.

Lecture 50 Notes

Alg050-01



Lecture 50 - Multi - Step Inequalites

Example 1A:

$3x - 4(x - 5) > 3(x - 2) - 4$

$3x - 4x + 20 > 3x - 6 - 4$ Clear parentheses.

$-x + 20 > 3x - 10$ Combine like terms.

$\underline{\quad + x \qquad + x \quad}$ Add x to both sides.

$20 > 4x - 10$

$\underline{\quad + 10 \qquad + 10 \quad}$

$\frac{30}{4} > \frac{4x}{4}$

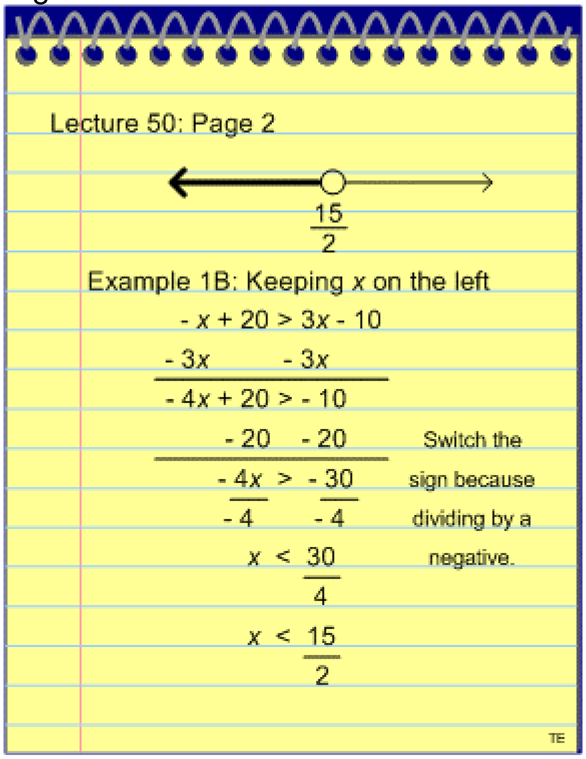
$\frac{30}{4} > \frac{x}{}$

$\frac{15}{2} > x$

$7\frac{1}{2} > x$ or $x < \frac{15}{2}$

TE

Alg050-02



Lecture 50: Page 2

Example 1B: Keeping x on the left

$-x + 20 > 3x - 10$

$\underline{\quad - 3x \qquad - 3x \quad}$

$-4x + 20 > -10$

$\underline{\quad - 20 \qquad - 20 \quad}$ Switch the sign because dividing by a negative.

$\frac{-4x}{-4} > \frac{-30}{-4}$

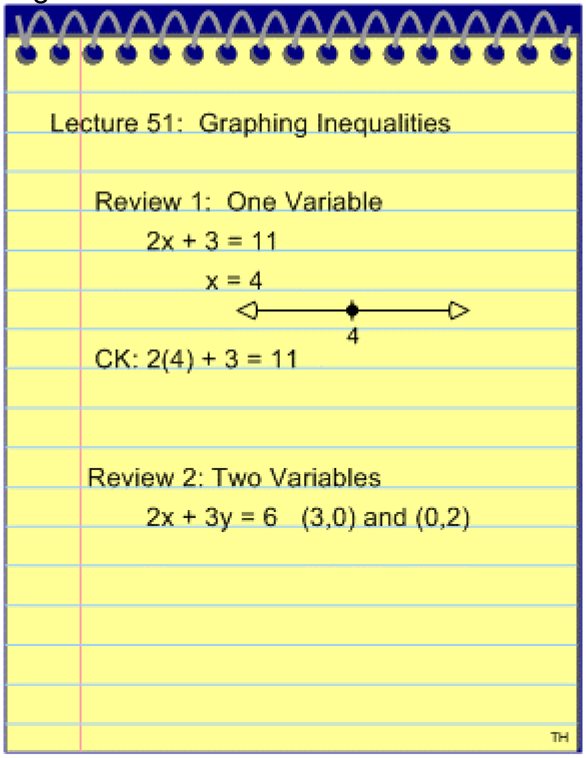
$x < \frac{30}{4}$

$x < \frac{15}{2}$

TE

Lecture 51 Notes

Alg051-01



Lecture 51: Graphing Inequalities

Review 1: One Variable

$2x + 3 = 11$

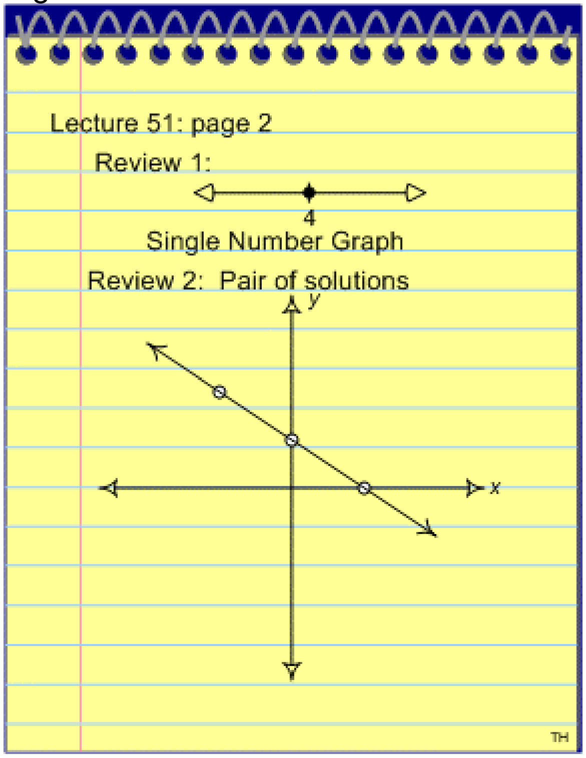
$x = 4$

CK: $2(4) + 3 = 11$

Review 2: Two Variables

$2x + 3y = 6$ (3,0) and (0,2)

Alg051-02



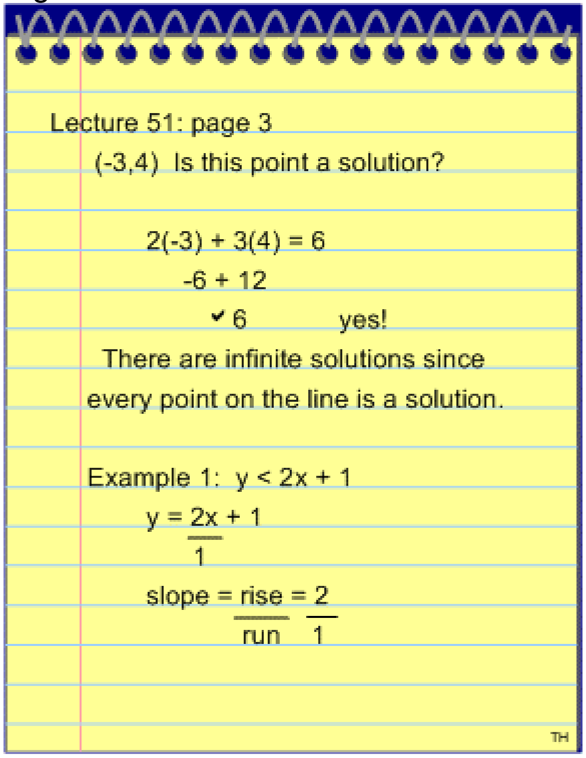
Lecture 51: page 2

Review 1:

Single Number Graph

Review 2: Pair of solutions

Alg051-03



Lecture 51: page 3

(-3,4) Is this point a solution?

$2(-3) + 3(4) = 6$

$-6 + 12$

$\checkmark 6$ yes!

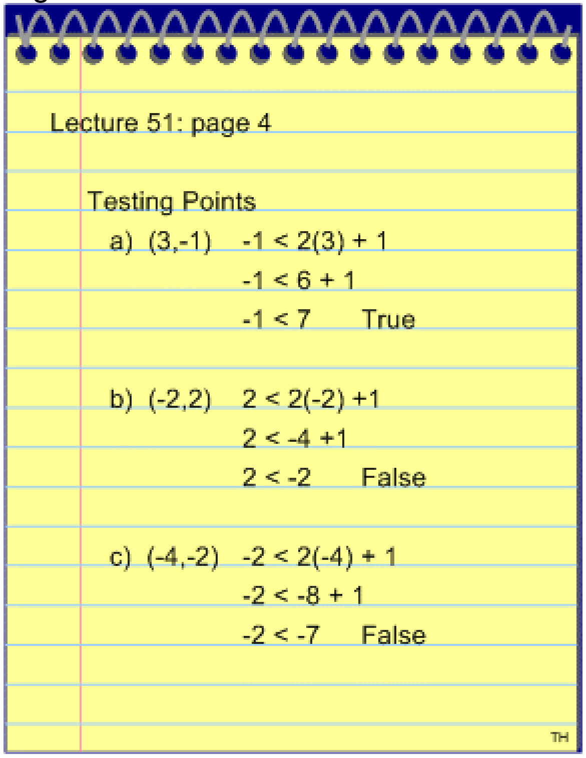
There are infinite solutions since every point on the line is a solution.

Example 1: $y < 2x + 1$

$y = \frac{2x}{1} + 1$

slope = $\frac{rise}{run} = \frac{2}{1}$

Alg051-04



Lecture 51: page 4

Testing Points

a) (3,-1) $-1 < 2(3) + 1$

$-1 < 6 + 1$

$-1 < 7$ True

b) (-2,2) $2 < 2(-2) + 1$

$2 < -4 + 1$

$2 < -2$ False

c) (-4,-2) $-2 < 2(-4) + 1$

$-2 < -8 + 1$

$-2 < -7$ False

Lecture 51 Notes, Continued

Alg051-05

Lecture 51: page 5

Testing Points (continued)

d) (-2,1) $1 < 2(-2) + 1$
$1 < -4 + 1$
$1 < -3$ False

e) (3,-3) $-3 < 2(3) + 1$
$-3 < 6 + 1$
$-3 < 7$ True

f) (-1,-9) $-9 < 2(-1) + 1$
$-9 < -2 + 1$
$-9 < -1$ True

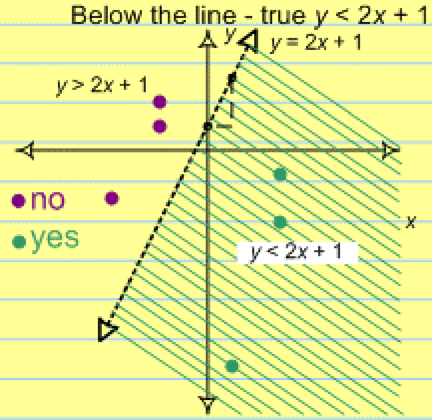
Alg051-06

Lecture 51: page 6

Find line; $y = 2x + 1$ - boundary but,
not a part of a solution

Above the line - false $y > 2x + 1$

Below the line - true $y < 2x + 1$

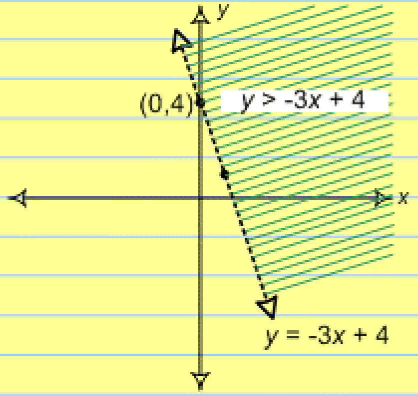


Alg051-07

Lecture 51: page 7

Example 2: $y > -3x + 4$

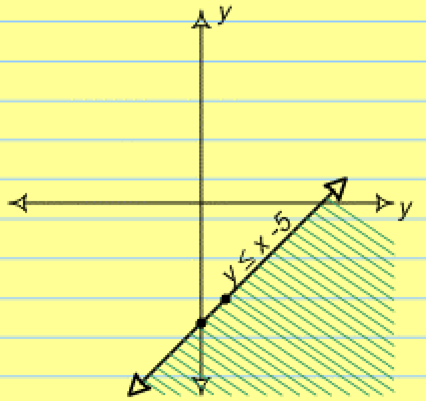
a) Line is $y = \frac{-3x}{1} + 4$



Alg051-08

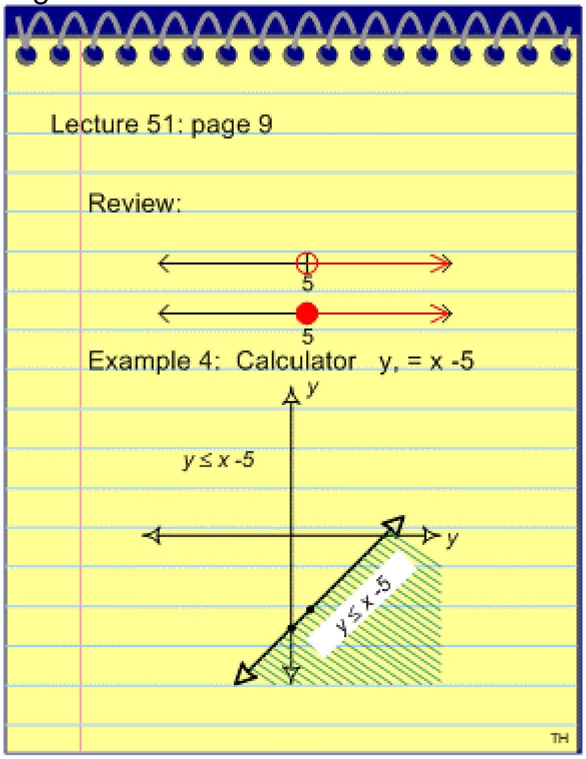
Lecture 51: page 8

Example 3: $y \leq \frac{1}{1}x - 5$



Lecture 51 Notes, Continued

Alg051-09



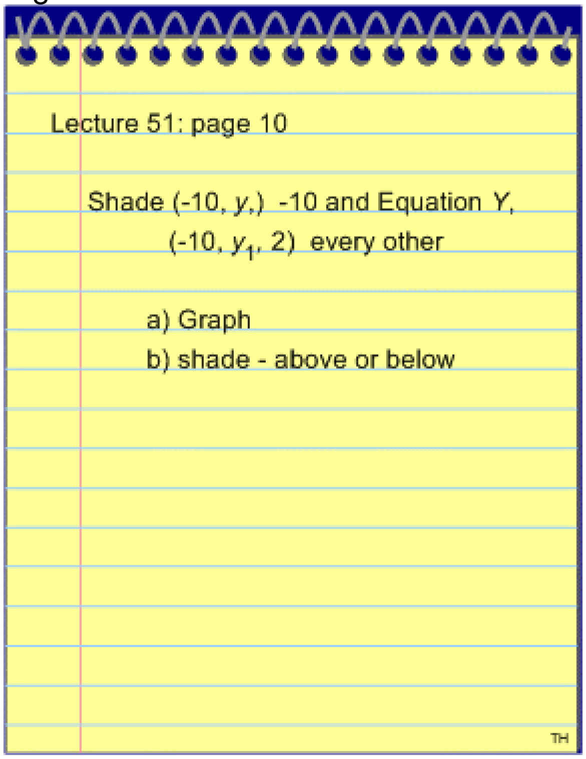
Lecture 51: page 9

Review:

Example 4: Calculator $y_1 = x -5$

TH

Alg051-10



Lecture 51: page 10

Shade (-10, y_1) -10 and Equation Y_1

(-10, y_1, 2) every other

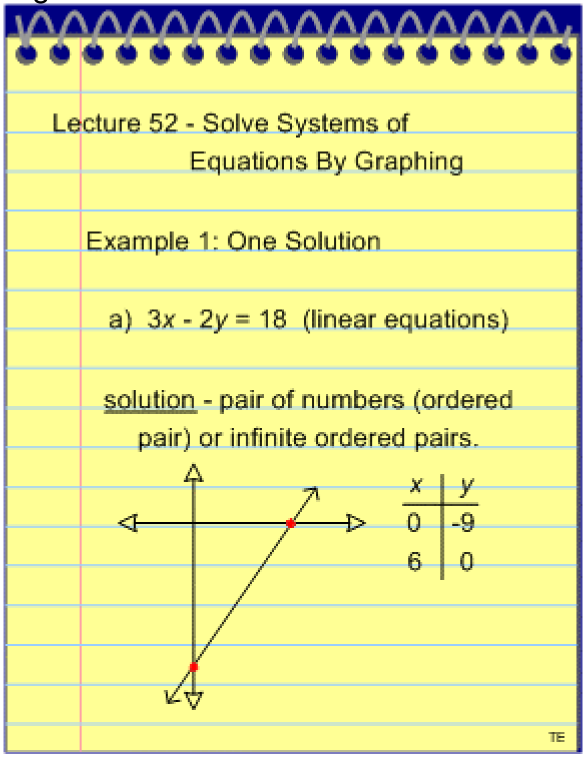
a) Graph

b) shade - above or below

TH

Lecture 52 Notes

Alg052-01



Lecture 52 - Solve Systems of Equations By Graphing

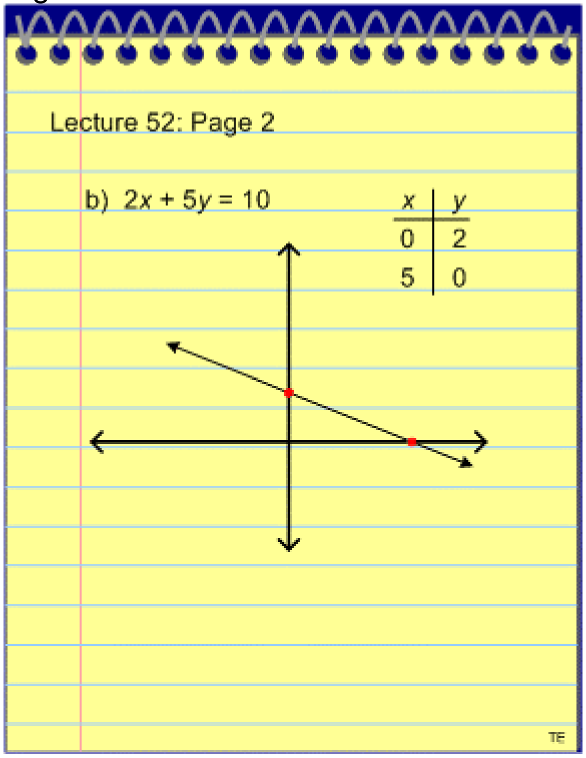
Example 1: One Solution

a) $3x - 2y = 18$ (linear equations)

solution - pair of numbers (ordered pair) or infinite ordered pairs.

x	y
0	-9
6	0

Alg052-02

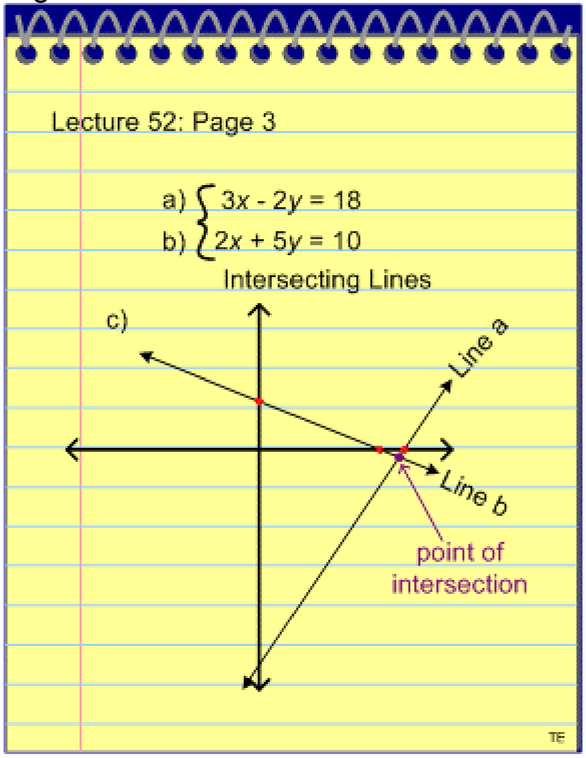


Lecture 52: Page 2

b) $2x + 5y = 10$

x	y
0	2
5	0

Alg052-03



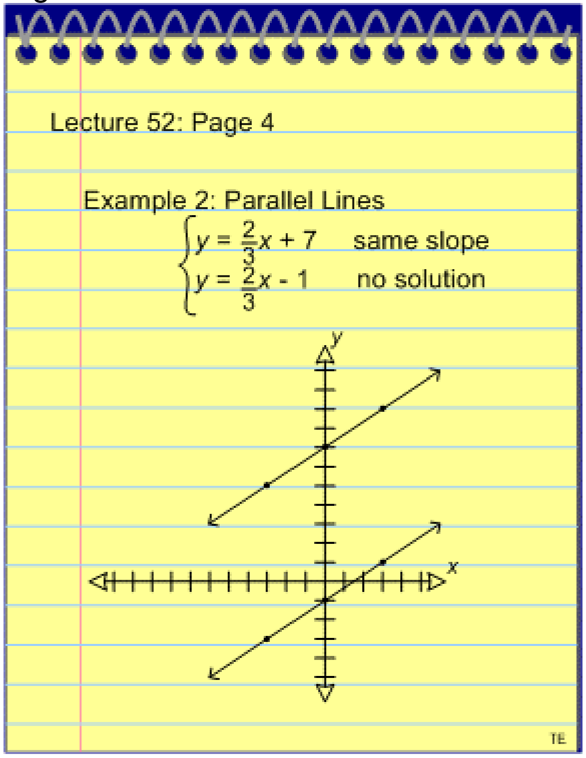
Lecture 52: Page 3

$$\begin{cases} 3x - 2y = 18 & \text{a)} \\ 2x + 5y = 10 & \text{b)} \end{cases}$$

Intersecting Lines

c)

Alg052-04



Lecture 52: Page 4

Example 2: Parallel Lines

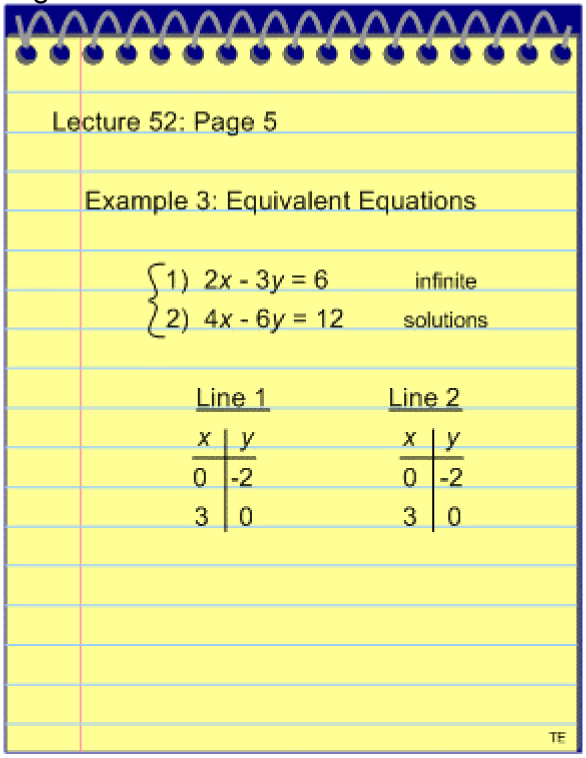
$$\begin{cases} y = \frac{2}{3}x + 7 \\ y = \frac{2}{3}x - 1 \end{cases}$$

same slope

no solution

Lecture 52 Notes, Continued

Alg052-05



Lecture 52: Page 5

Example 3: Equivalent Equations

$$\begin{cases} 1)\ 2x - 3y = 6 \\ 2)\ 4x - 6y = 12 \end{cases}$$

infinite solutions

Line 1

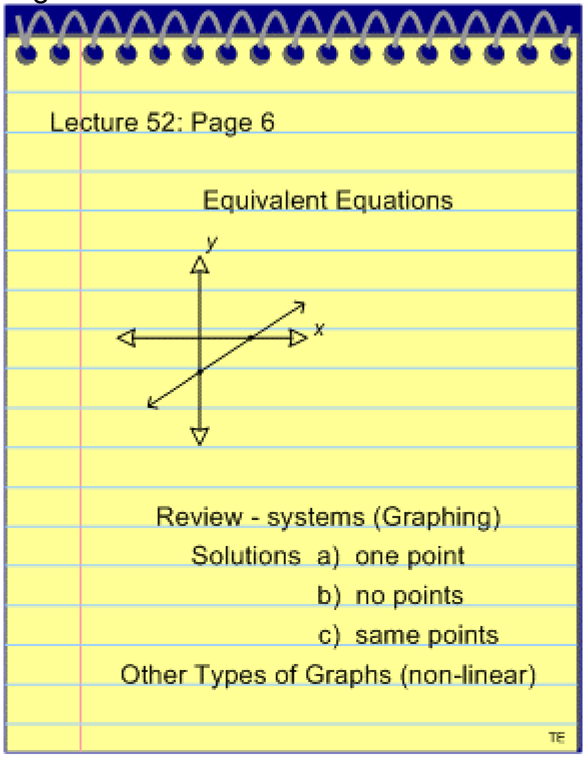
x	y
0	-2
3	0

Line 2

x	y
0	-2
3	0

TE

Alg052-06



Lecture 52: Page 6

Equivalent Equations

Review - systems (Graphing)

Solutions a) one point

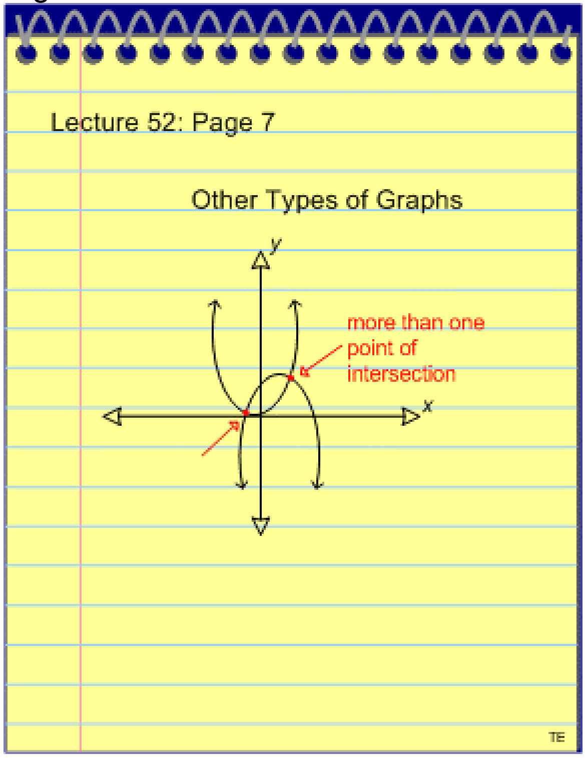
b) no points

c) same points

Other Types of Graphs (non-linear)

TE

Alg052-07



Lecture 52: Page 7

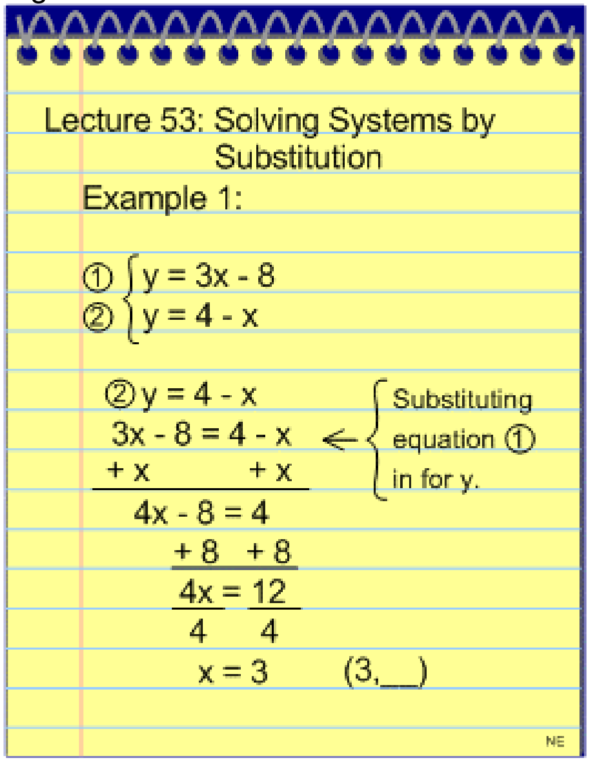
Other Types of Graphs

more than one point of intersection

TE

Lecture 53 Notes

Alg053-01



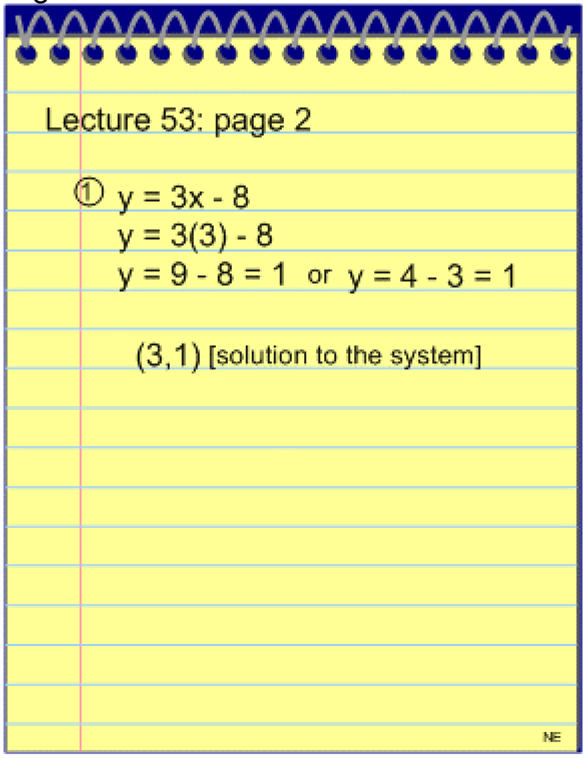
Lecture 53: Solving Systems by Substitution

Example 1:

① $y = 3x - 8$
② $y = 4 - x$

② $y = 4 - x$
$3x - 8 = 4 - x$ ← Substituting equation ① in for y.
$\underline{+x \qquad +x}$
$4x - 8 = 4$
$\underline{+8 \quad +8}$
$\frac{4x}{4} = \frac{12}{4}$
$x = 3$ (3,__)

Alg053-02

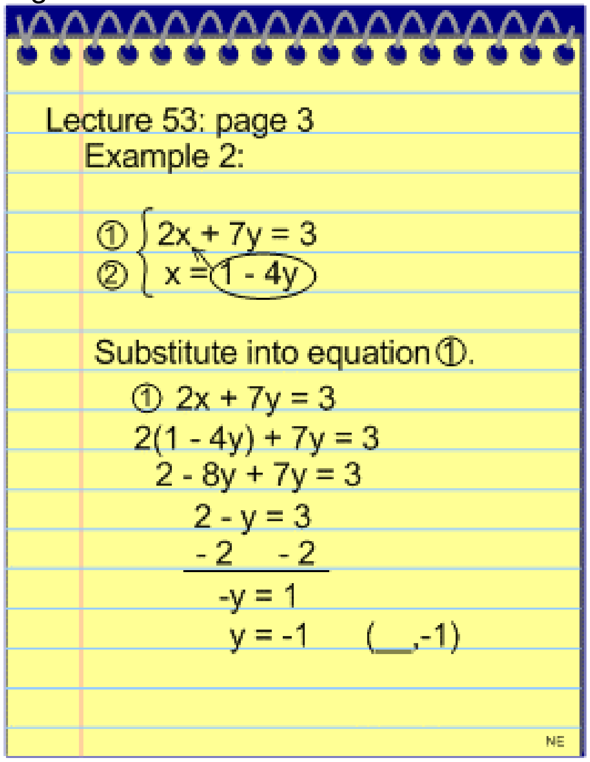


Lecture 53: page 2

① $y = 3x - 8$
$y = 3(3) - 8$
$y = 9 - 8 = 1$ or $y = 4 - 3 = 1$

(3,1) [solution to the system]

Alg053-03



Lecture 53: page 3

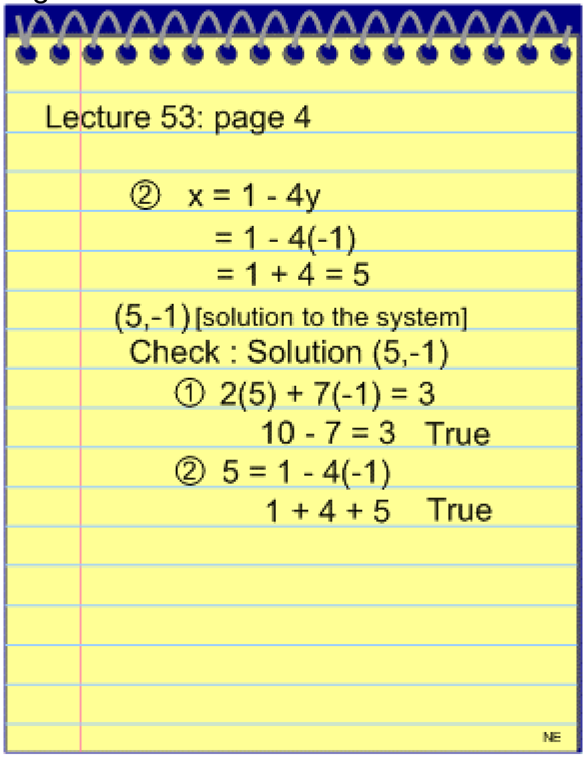
Example 2:

① $2x + 7y = 3$
② $x = 1 - 4y$

Substitute into equation ①.

① $2x + 7y = 3$
$2(1 - 4y) + 7y = 3$
$2 - 8y + 7y = 3$
$2 - y = 3$
$\underline{-2 \qquad -2}$
$-y = 1$
$y = -1$ (__,-1)

Alg053-04



Lecture 53: page 4

② $x = 1 - 4y$
$= 1 - 4(-1)$
$= 1 + 4 = 5$

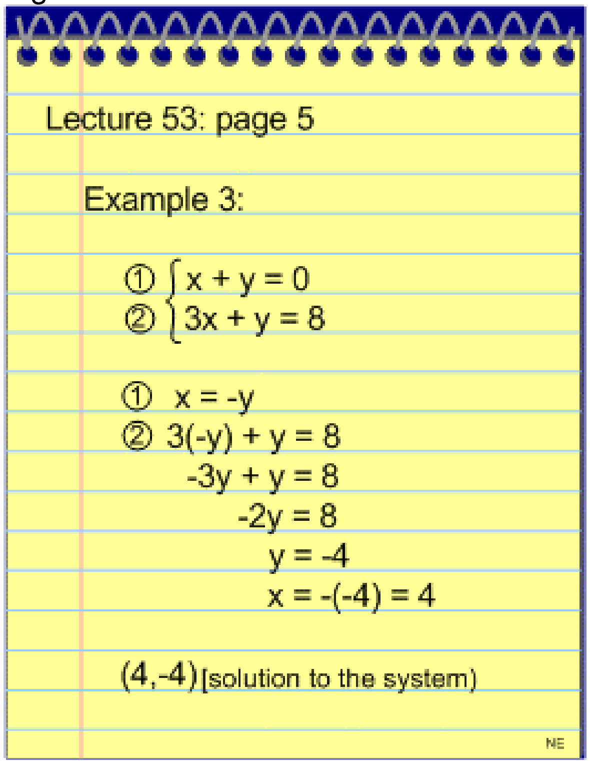
(5,-1) [solution to the system]

Check : Solution (5,-1)

① $2(5) + 7(-1) = 3$
$10 - 7 = 3$ True
② $5 = 1 - 4(-1)$
$1 + 4 + 5$ True

Lecture 53 Notes, Continued

Alg053-05



Lecture 53: page 5

Example 3:

$$\begin{matrix} \textcircled{1} \\ \textcircled{2} \end{matrix} \begin{cases} x + y = 0 \\ 3x + y = 8 \end{cases}$$

① $x = -y$

② $3(-y) + y = 8$

$-3y + y = 8$

$-2y = 8$

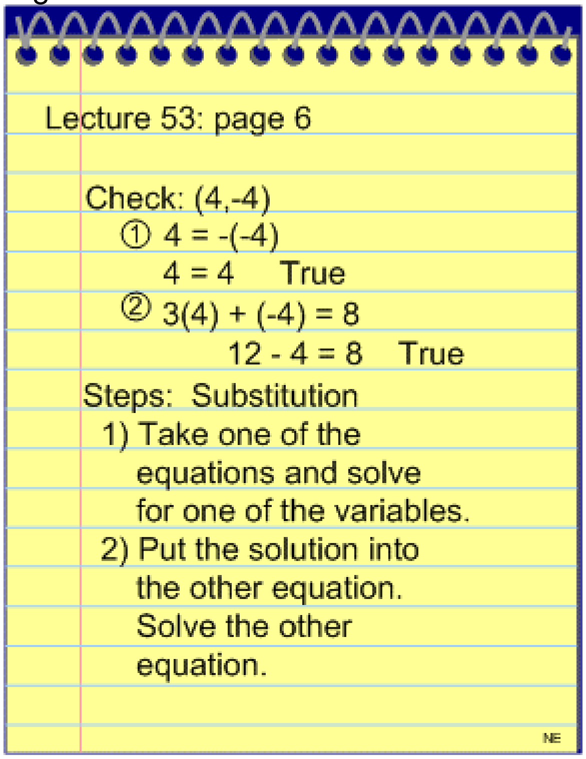
$y = -4$

$x = -(-4) = 4$

$(4,-4)$ [solution to the system)

NE

Alg053-06



Lecture 53: page 6

Check: $(4,-4)$

① $4 = -(-4)$

$4 = 4$ True

② $3(4) + (-4) = 8$

$12 - 4 = 8$ True

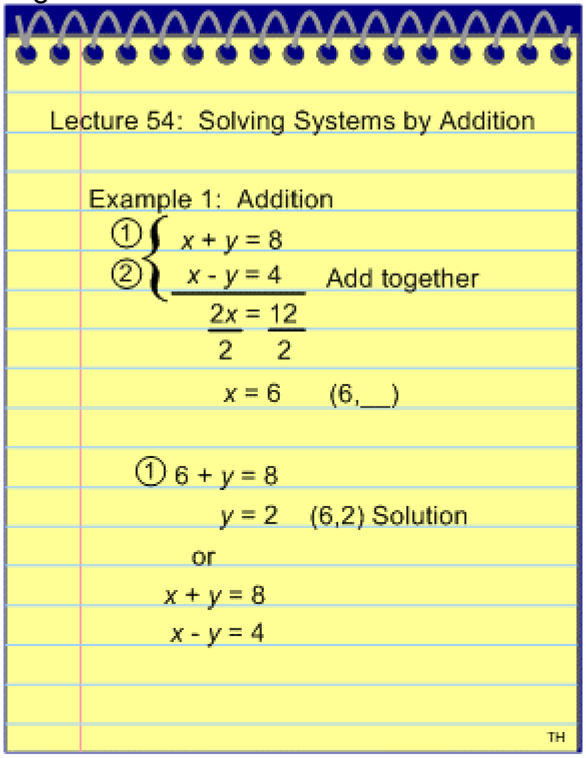
Steps: Substitution

1) Take one of the equations and solve for one of the variables.
2) Put the solution into the other equation. Solve the other equation.

NE

Lecture 54 Notes

Alg054-01



Lecture 54: Solving Systems by Addition

Example 1: Addition

$$\begin{array}{r l} ① & \begin{cases} x + y = 8 \\ \underline{x - y = 4} \end{cases} \\ ② & \end{array} \quad \text{Add together}$$

$$\frac{2x}{2} = \frac{12}{2}$$

$x = 6$ (6,__)

① $6 + y = 8$

$y = 2$ (6,2) Solution

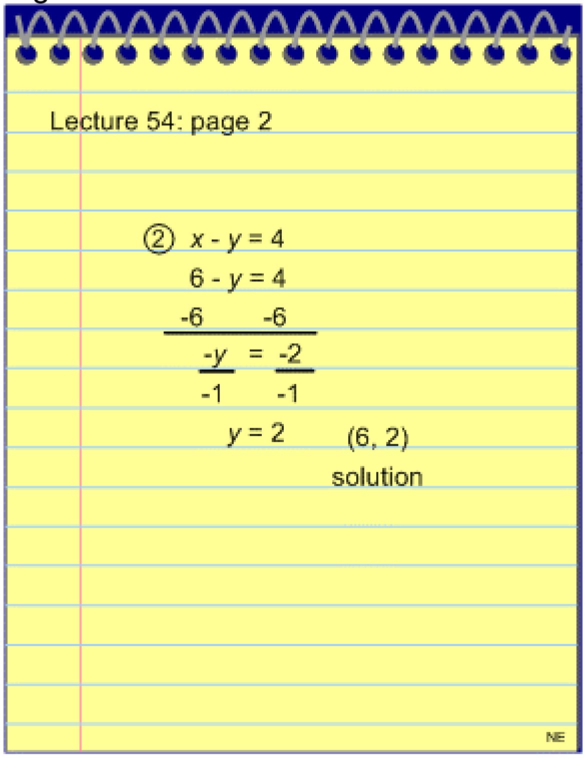
or

$x + y = 8$

$x - y = 4$

TH

Alg054-02



Lecture 54: page 2

② $x - y = 4$

$6 - y = 4$

$\underline{-6 \qquad -6}$

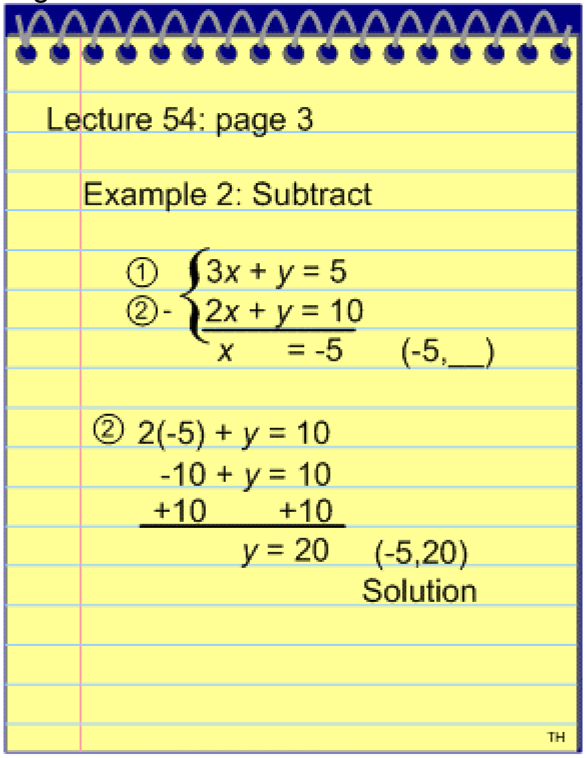
$$\frac{-y}{-1} = \frac{-2}{-1}$$

$y = 2$ (6, 2)

solution

NE

Alg054-03



Lecture 54: page 3

Example 2: Subtract

$$\begin{array}{r l} ① & \begin{cases} 3x + y = 5 \\ \underline{2x + y = 10} \end{cases} \\ ② - & \end{array}$$

$x = -5$ (-5,__)

② $2(-5) + y = 10$

$-10 + y = 10$

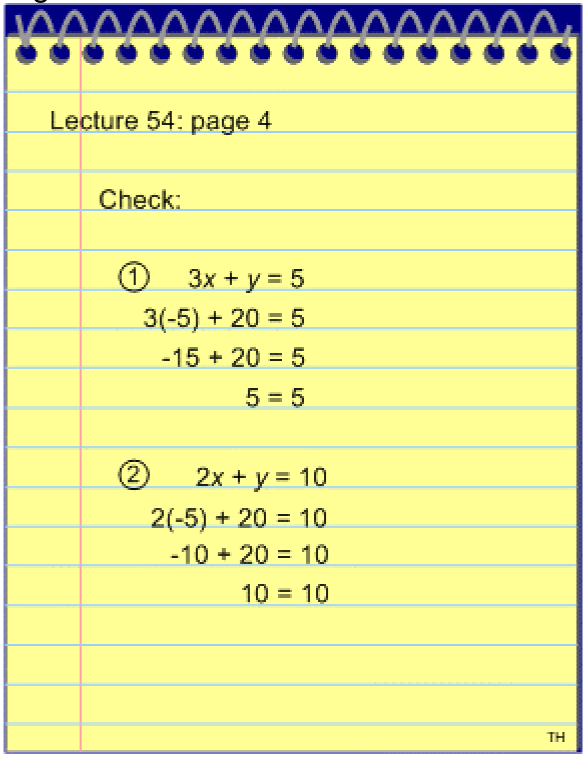
$\underline{+10 \qquad +10}$

$y = 20$ (-5,20)

Solution

TH

Alg054-04



Lecture 54: page 4

Check:

① $3x + y = 5$

$3(-5) + 20 = 5$

$-15 + 20 = 5$

$5 = 5$

② $2x + y = 10$

$2(-5) + 20 = 10$

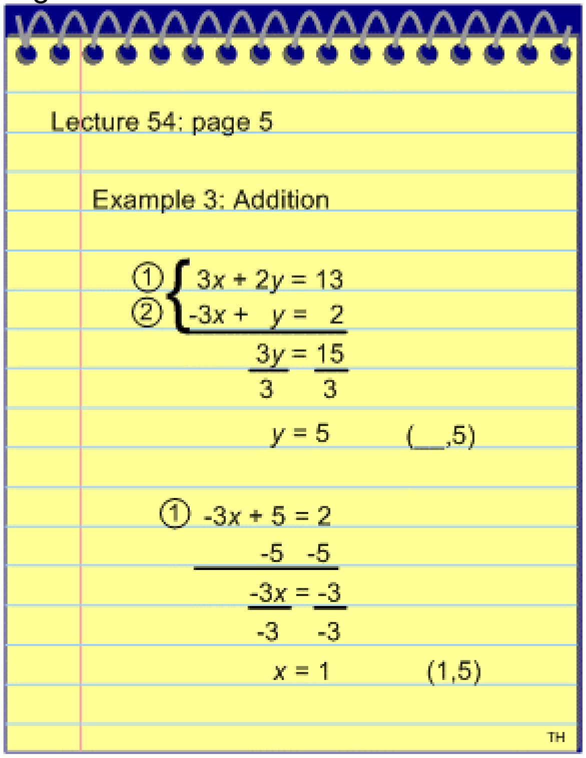
$-10 + 20 = 10$

$10 = 10$

TH

Lecture 54 Notes

Alg054-05



Lecture 54: page 5

Example 3: Addition

$$\begin{array}{rl} ① & \left\{\begin{array}{r} 3x + 2y = 13 \\ -3x + \quad y = \quad 2 \end{array}\right. \\ ② & \end{array}$$

$$\underline{3y} = \underline{15}$$
$$3 \qquad 3$$

$y = 5$ (__,5)

① $-3x + 5 = 2$

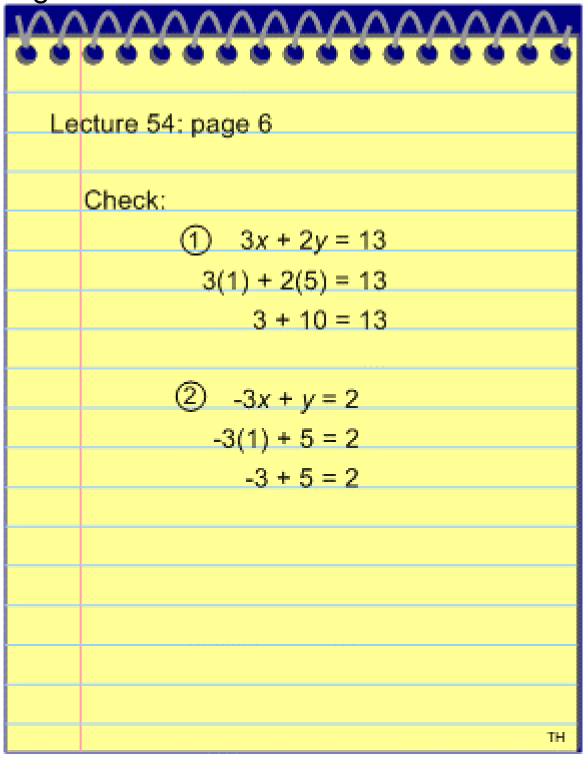
$$\underline{\quad -5 \quad -5}$$

$$\underline{-3x} = \underline{-3}$$
$$-3 \qquad -3$$

$x = 1$ (1,5)

TH

Alg054-06



Lecture 54: page 6

Check:

① $3x + 2y = 13$

$3(1) + 2(5) = 13$

$3 + 10 = 13$

② $-3x + y = 2$

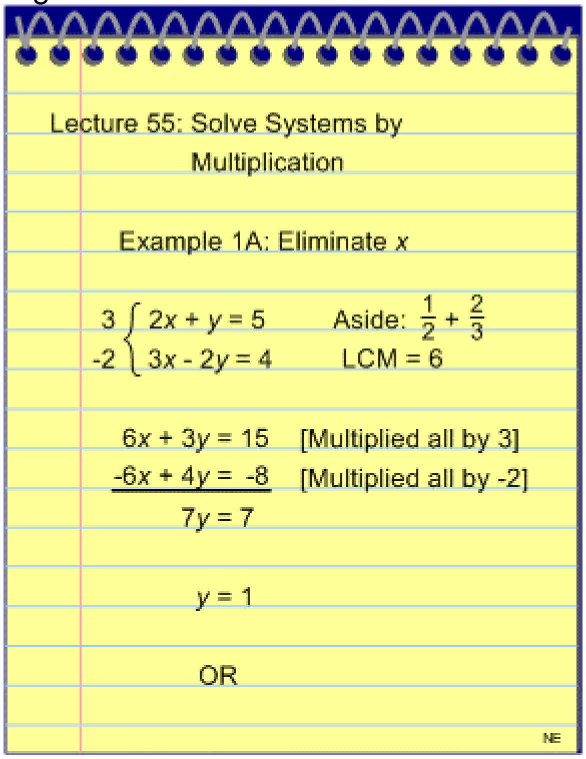
$-3(1) + 5 = 2$

$-3 + 5 = 2$

TH

Lecture 55 Notes

Alg055-01



Lecture 55: Solve Systems by Multiplication

Example 1A: Eliminate x

$$\begin{matrix}3\\-2\end{matrix}\begin{cases}2x+y=5\\3x-2y=4\end{cases}$$

Aside: $\frac{1}{2}+\frac{2}{3}$

LCM = 6

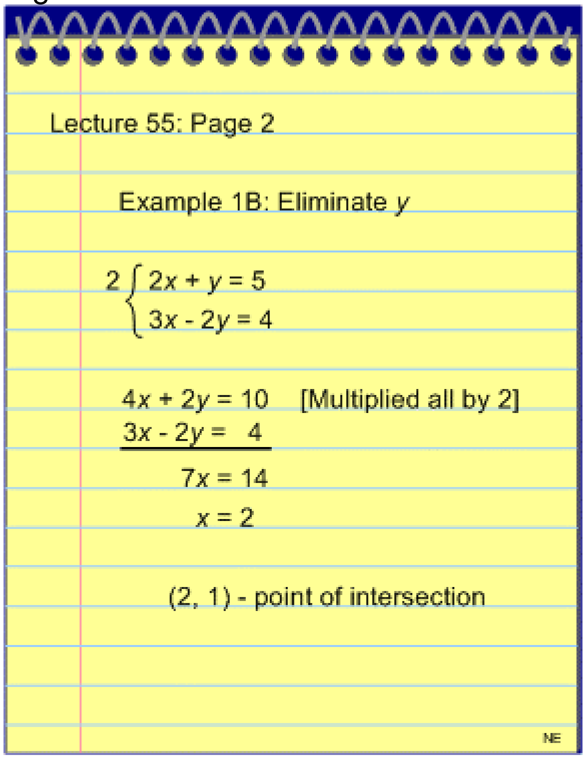
$$\begin{array}{rl} 6x+3y=15 & \text{[Multiplied all by 3]}\\ \underline{-6x+4y=\ -8} & \text{[Multiplied all by -2]}\\ 7y=7 & \end{array}$$

$y=1$

OR

NE

Alg055-02



Lecture 55: Page 2

Example 1B: Eliminate y

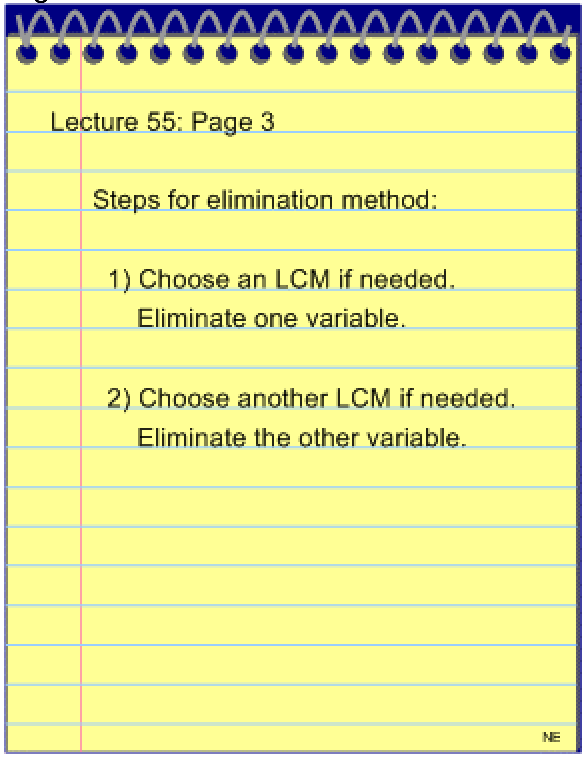
$$2\begin{cases}2x+y=5\\3x-2y=4\end{cases}$$

$$\begin{array}{rl} 4x+2y=10 & \text{[Multiplied all by 2]}\\ \underline{3x-2y=\ \ 4} & \\ 7x=14 & \\ x=2 & \end{array}$$

(2, 1) - point of intersection

NE

Alg055-03



Lecture 55: Page 3

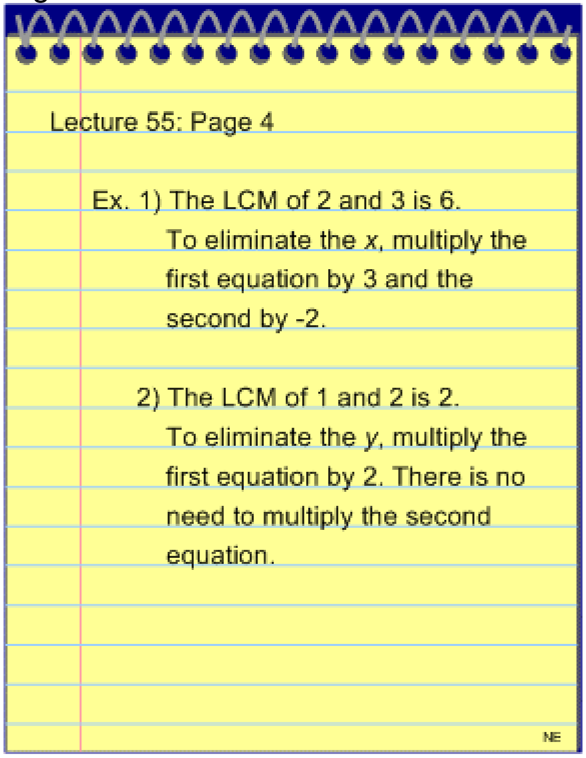
Steps for elimination method:

1) Choose an LCM if needed. Eliminate one variable.

2) Choose another LCM if needed. Eliminate the other variable.

NE

Alg055-04



Lecture 55: Page 4

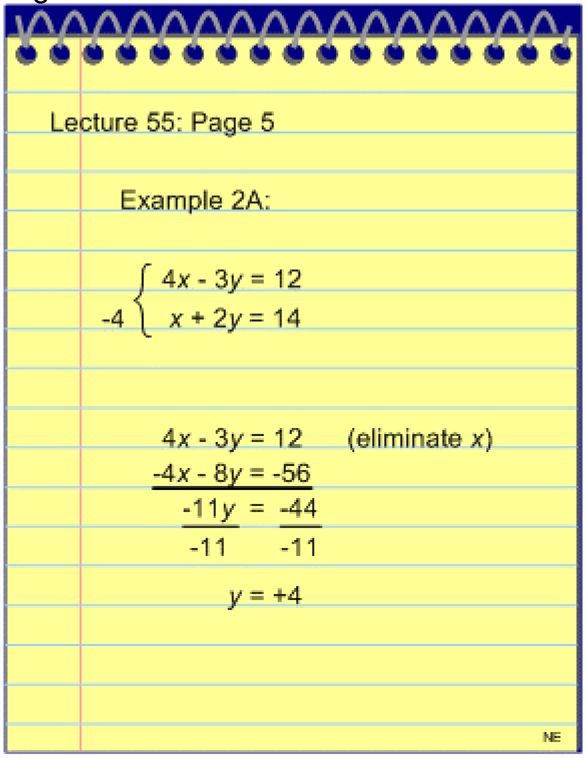
Ex. 1) The LCM of 2 and 3 is 6. To eliminate the x, multiply the first equation by 3 and the second by -2.

2) The LCM of 1 and 2 is 2. To eliminate the y, multiply the first equation by 2. There is no need to multiply the second equation.

NE

Lecture 55 Notes, Continued

Alg055-05



Lecture 55: Page 5

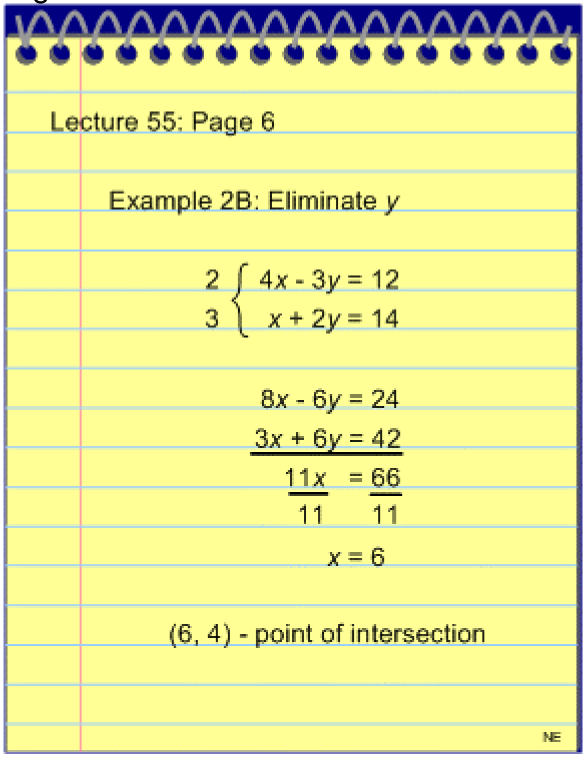
Example 2A:

$$\begin{array}{r} \\ -4 \end{array}\left\{\begin{array}{l} 4x - 3y = 12 \\ x + 2y = 14 \end{array}\right.$$

$$\begin{array}{r} 4x - 3y = 12 \\ \underline{-4x - 8y = -56} \\ \frac{-11y}{-11} = \frac{-44}{-11} \\ y = +4 \end{array}$$

(eliminate x)

Alg055-06



Lecture 55: Page 6

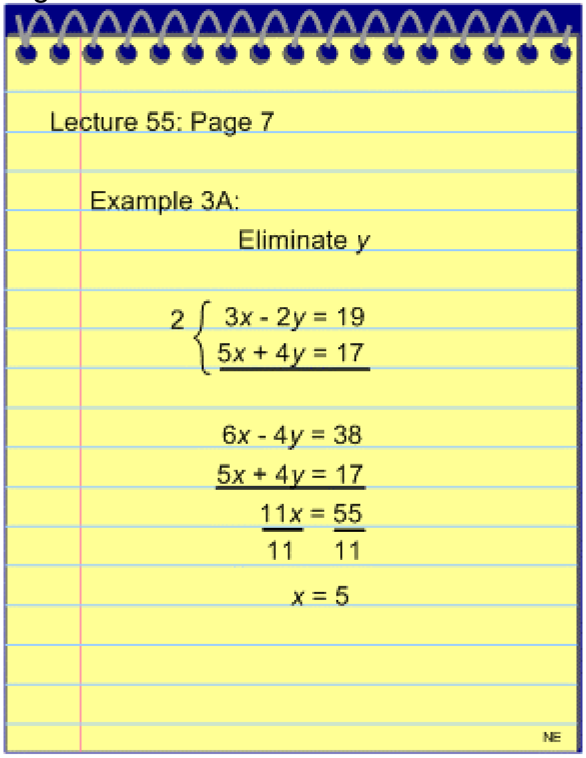
Example 2B: Eliminate y

$$\begin{array}{r} 2 \\ 3 \end{array}\left\{\begin{array}{l} 4x - 3y = 12 \\ x + 2y = 14 \end{array}\right.$$

$$\begin{array}{r} 8x - 6y = 24 \\ \underline{3x + 6y = 42} \\ \frac{11x}{11} = \frac{66}{11} \\ x = 6 \end{array}$$

(6, 4) - point of intersection

Alg055-07



Lecture 55: Page 7

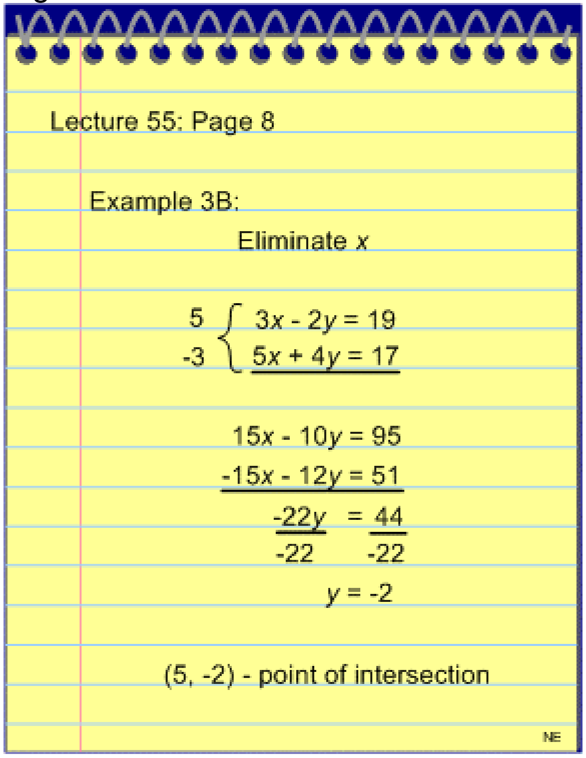
Example 3A:

Eliminate y

$$\begin{array}{r} 2 \\ \\ \end{array}\left\{\begin{array}{l} 3x - 2y = 19 \\ \underline{5x + 4y = 17} \end{array}\right.$$

$$\begin{array}{r} 6x - 4y = 38 \\ \underline{5x + 4y = 17} \\ \frac{11x}{11} = \frac{55}{11} \\ x = 5 \end{array}$$

Alg055-08



Lecture 55: Page 8

Example 3B:

Eliminate x

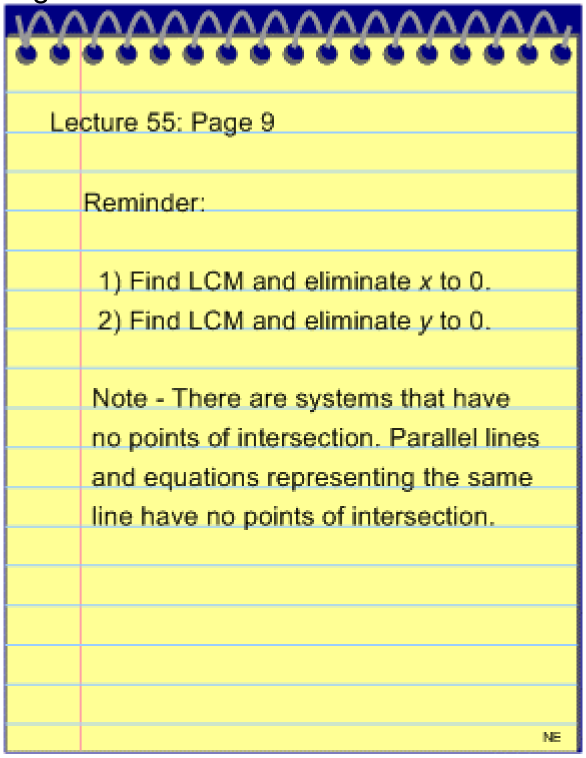
$$\begin{array}{r} 5 \\ -3 \end{array}\left\{\begin{array}{l} 3x - 2y = 19 \\ \underline{5x + 4y = 17} \end{array}\right.$$

$$\begin{array}{r} 15x - 10y = 95 \\ \underline{-15x - 12y = 51} \\ \frac{-22y}{-22} = \frac{44}{-22} \\ y = -2 \end{array}$$

(5, -2) - point of intersection

Lecture 55 Notes, Continued

Alg055-09



Lecture 55: Page 9

Reminder:

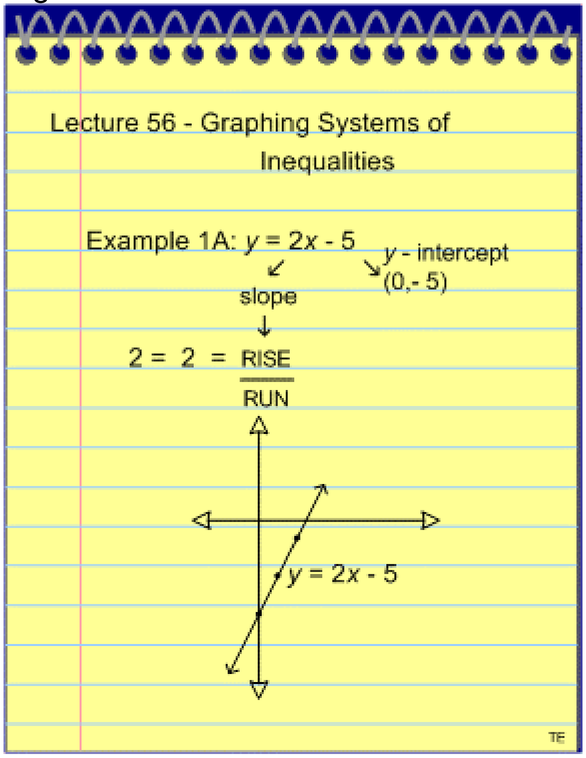
1) Find LCM and eliminate x to 0.
2) Find LCM and eliminate y to 0.

Note - There are systems that have no points of intersection. Parallel lines and equations representing the same line have no points of intersection.

NE

Lecture 56 Notes

Alg056-01



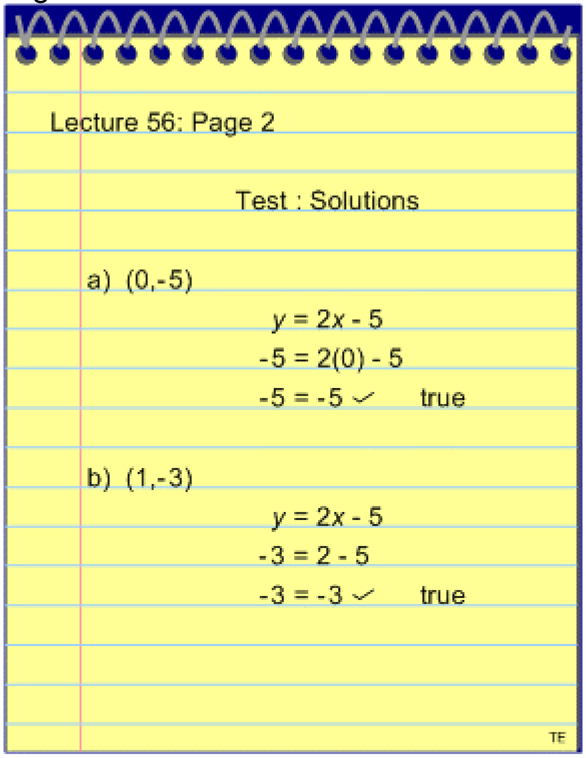
Lecture 56 - Graphing Systems of Inequalities

Example 1A: $y = 2x - 5$

slope ↙ ↘ y - intercept $(0,-5)$

$2 = 2 = \frac{\text{RISE}}{\text{RUN}}$

Alg056-02



Lecture 56: Page 2

Test : Solutions

a) $(0,-5)$

$y = 2x - 5$

$-5 = 2(0) - 5$

$-5 = -5$ ✓ true

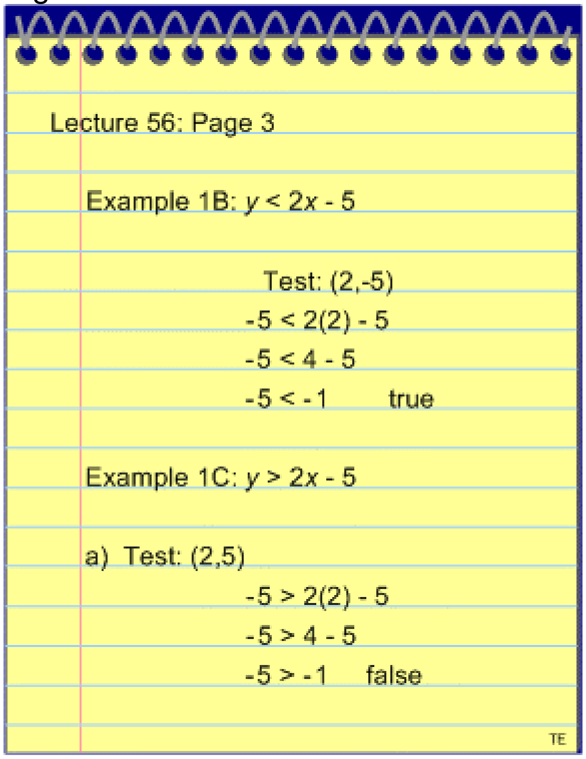
b) $(1,-3)$

$y = 2x - 5$

$-3 = 2 - 5$

$-3 = -3$ ✓ true

Alg056-03



Lecture 56: Page 3

Example 1B: $y < 2x - 5$

Test: $(2,-5)$

$-5 < 2(2) - 5$

$-5 < 4 - 5$

$-5 < -1$ true

Example 1C: $y > 2x - 5$

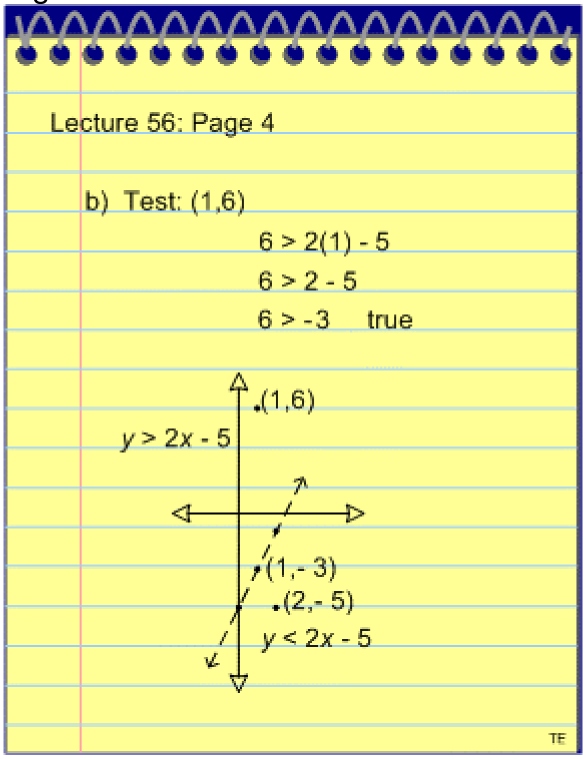
a) Test: $(2,5)$

$-5 > 2(2) - 5$

$-5 > 4 - 5$

$-5 > -1$ false

Alg056-04



Lecture 56: Page 4

b) Test: $(1,6)$

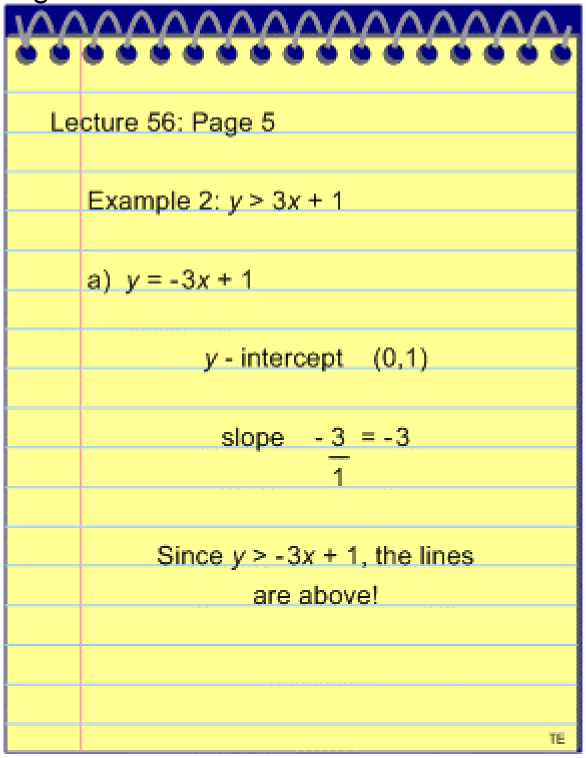
$6 > 2(1) - 5$

$6 > 2 - 5$

$6 > -3$ true

Lecture 56 Notes, Continued

Alg056-05



Lecture 56: Page 5

Example 2: $y > 3x + 1$

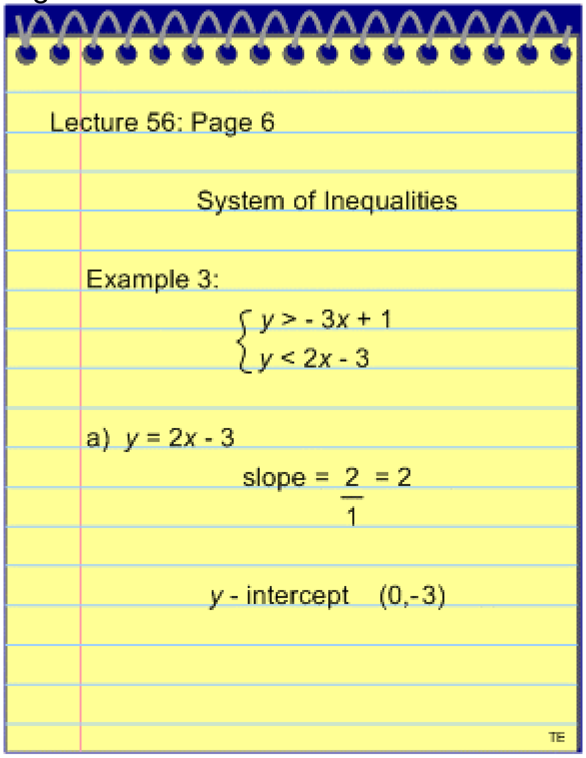
a) $y = -3x + 1$

y - intercept (0,1)

slope $\frac{-3}{1} = -3$

Since $y > -3x + 1$, the lines are above!

Alg056-06



Lecture 56: Page 6

System of Inequalities

Example 3:

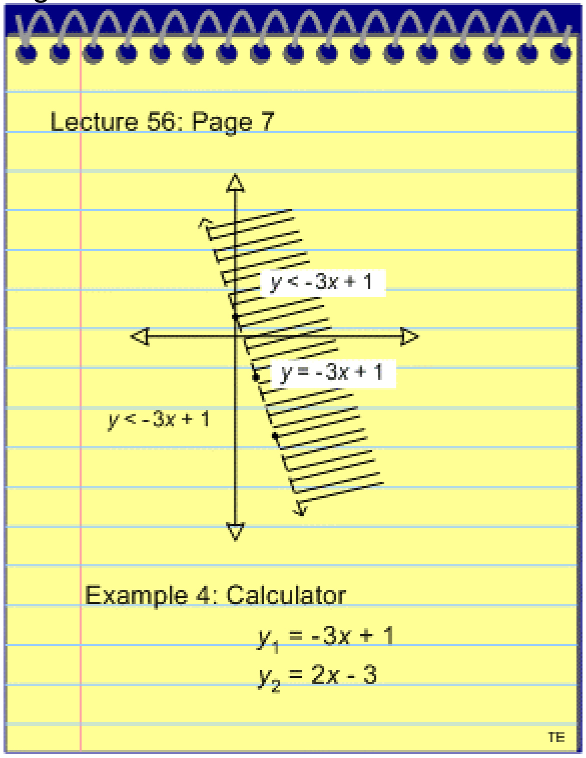
$$\begin{cases} y > -3x + 1 \\ y < 2x - 3 \end{cases}$$

a) $y = 2x - 3$

slope = $\frac{2}{1} = 2$

y - intercept (0,-3)

Alg056-07



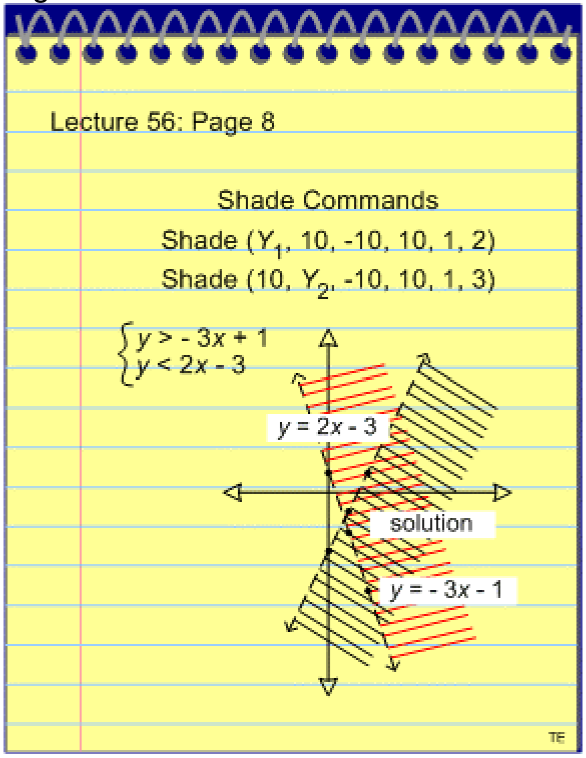
Lecture 56: Page 7

Example 4: Calculator

$y_1 = -3x + 1$

$y_2 = 2x - 3$

Alg056-08



Lecture 56: Page 8

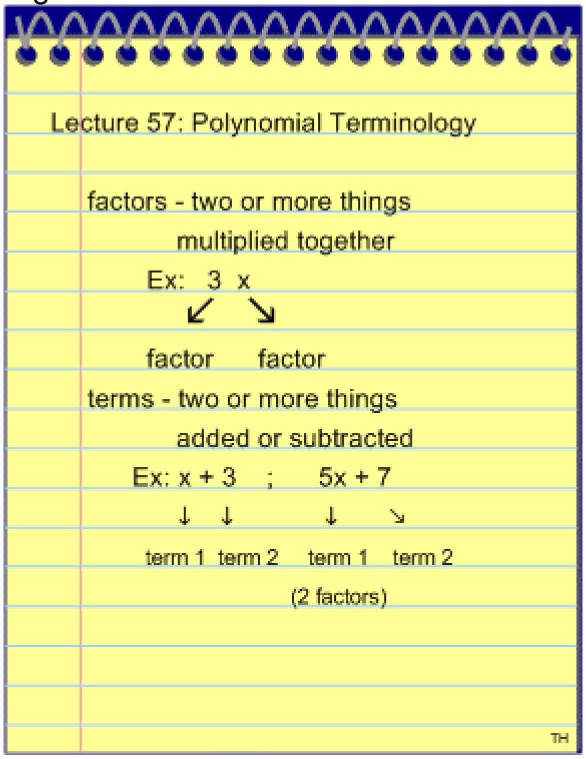
Shade Commands

Shade (Y_1, 10, -10, 10, 1, 2)

Shade (10, Y_2, -10, 10, 1, 3)

Lecture 57 Notes

Alg057-01



Lecture 57: Polynomial Terminology

factors - two or more things multiplied together

Ex: $3\ x$

↙ ↘

factor factor

terms - two or more things added or subtracted

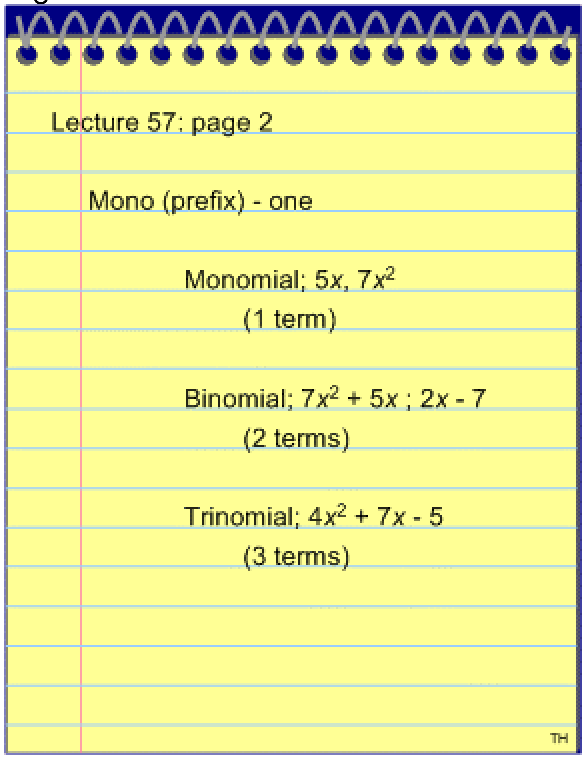
Ex: $x + 3$; $5x + 7$

↓ ↓ ↓ ↘

term 1 term 2 term 1 term 2

(2 factors)

Alg057-02



Lecture 57: page 2

Mono (prefix) - one

Monomial; $5x$, $7x^2$

(1 term)

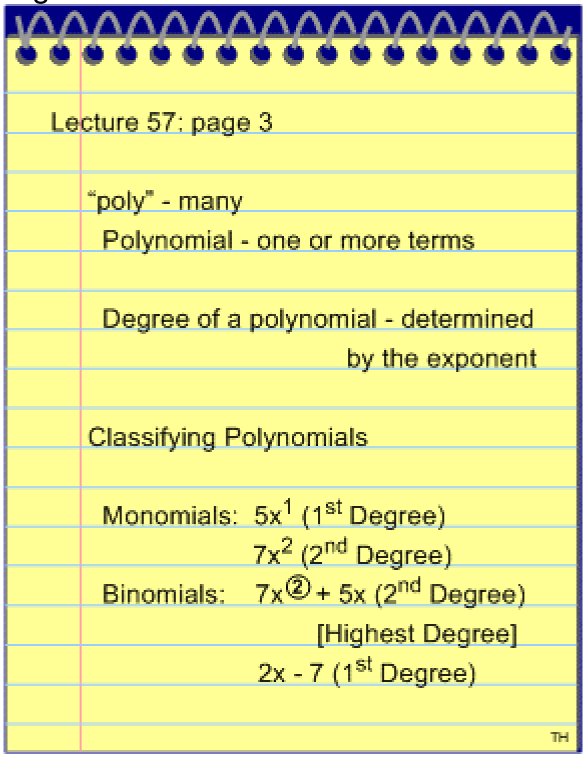
Binomial; $7x^2 + 5x$; $2x - 7$

(2 terms)

Trinomial; $4x^2 + 7x - 5$

(3 terms)

Alg057-03



Lecture 57: page 3

"poly" - many

Polynomial - one or more terms

Degree of a polynomial - determined by the exponent

Classifying Polynomials

Monomials: $5x^1$ (1st Degree)

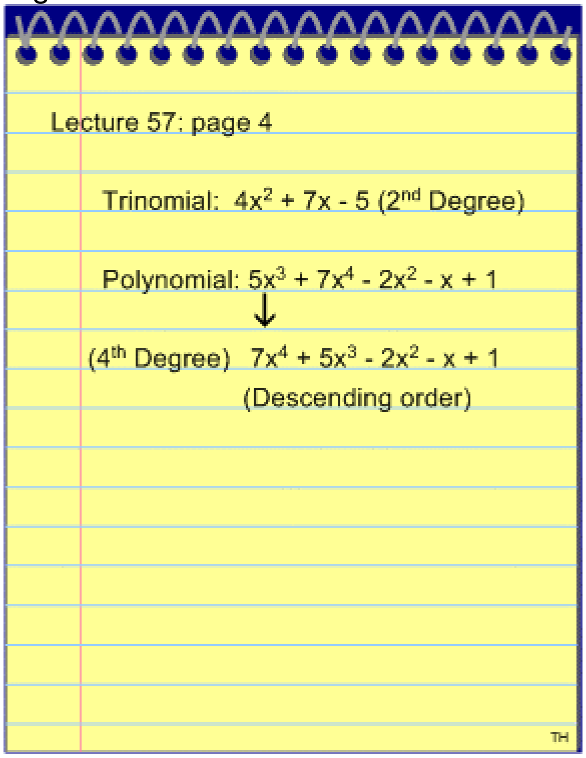
$7x^2$ (2nd Degree)

Binomials: $7x^{\textcircled{2}} + 5x$ (2nd Degree)

[Highest Degree]

$2x - 7$ (1st Degree)

Alg057-04



Lecture 57: page 4

Trinomial: $4x^2 + 7x - 5$ (2nd Degree)

Polynomial: $5x^3 + 7x^4 - 2x^2 - x + 1$

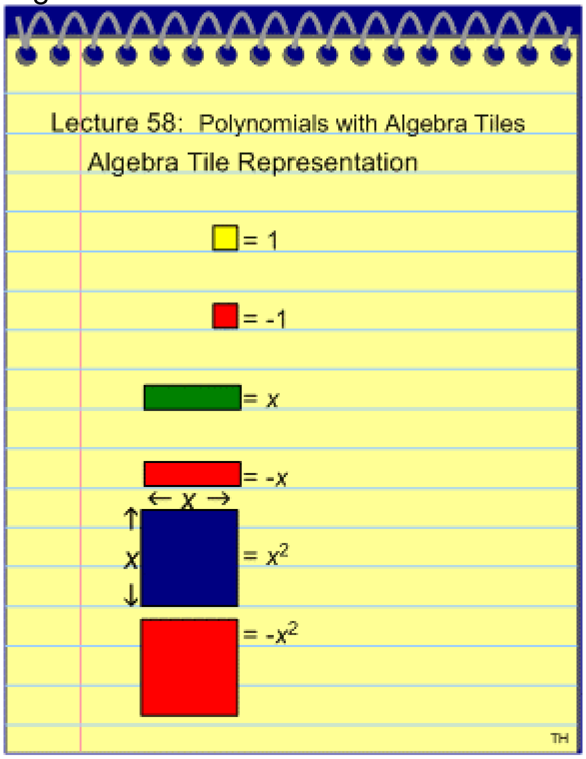
↓

(4th Degree) $7x^4 + 5x^3 - 2x^2 - x + 1$

(Descending order)

Lecture 58 Notes

Alg058-01



Lecture 58: Polynomials with Algebra Tiles

Algebra Tile Representation

🟨 = 1

🟥 = -1

🟩 = x

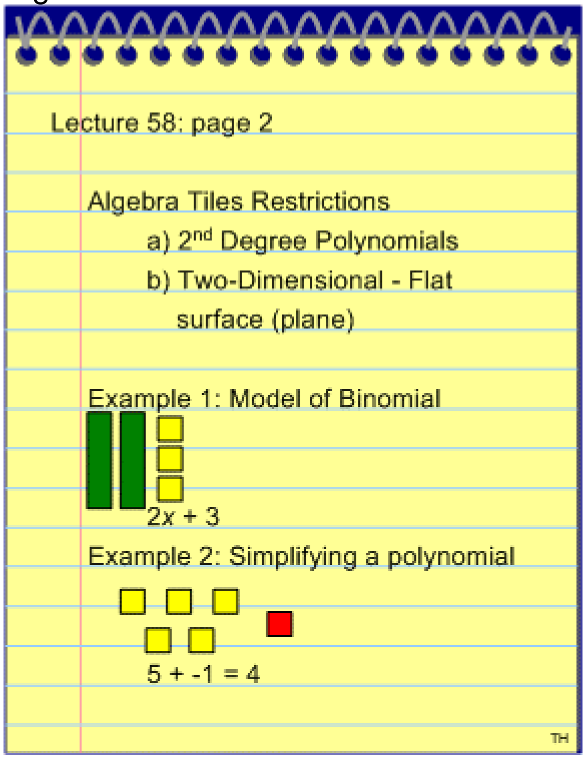
🟥 = $-x$

$\leftarrow x \rightarrow$

$\uparrow x \downarrow$ 🟦 = x^2

🟥 = $-x^2$

Alg058-02



Lecture 58: page 2

Algebra Tiles Restrictions

a) 2nd Degree Polynomials

b) Two-Dimensional - Flat surface (plane)

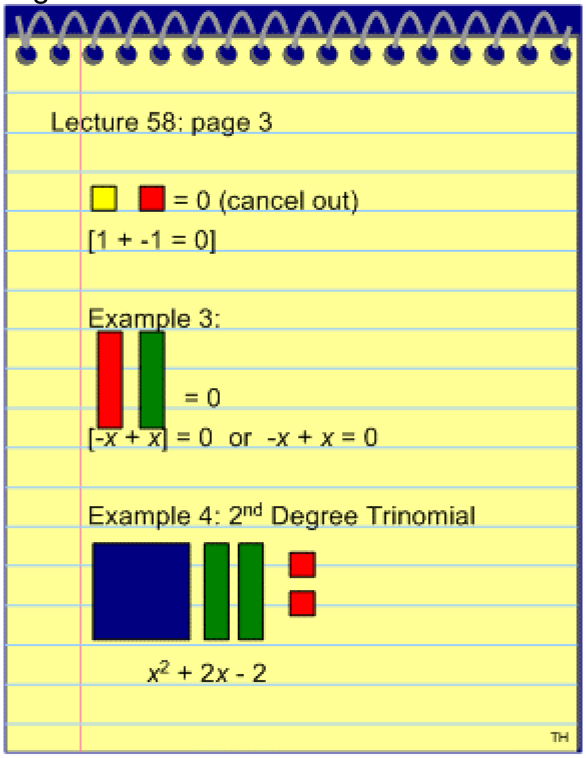
Example 1: Model of Binomial

$2x + 3$

Example 2: Simplifying a polynomial

$5 + -1 = 4$

Alg058-03



Lecture 58: page 3

🟨 🟥 = 0 (cancel out)

[1 + -1 = 0]

Example 3:

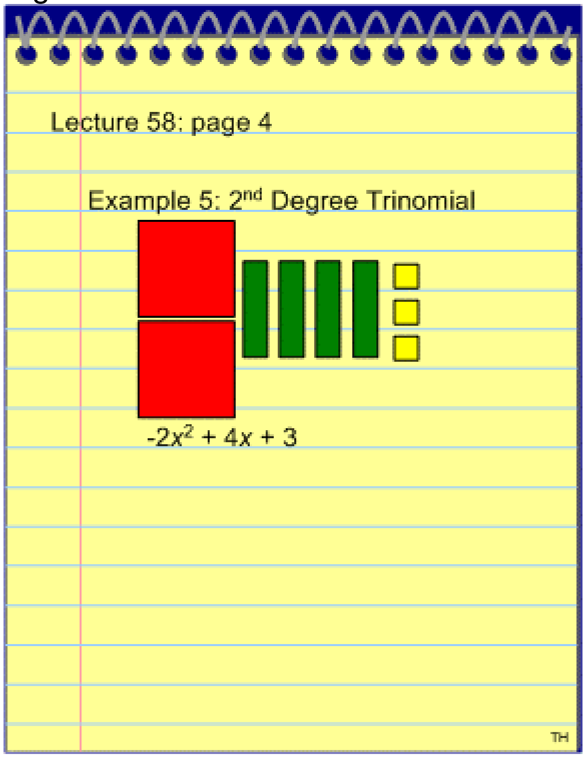
= 0

$[-x + x] = 0$ or $-x + x = 0$

Example 4: 2nd Degree Trinomial

$x^2 + 2x - 2$

Alg058-04



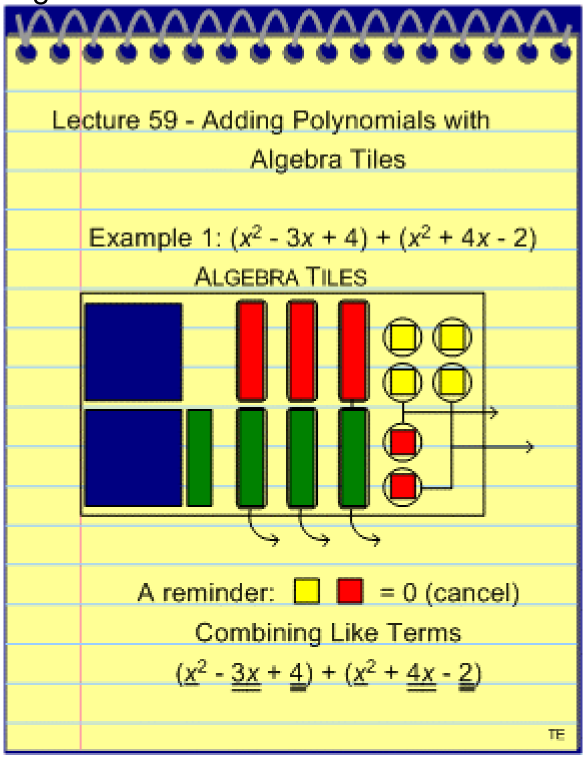
Lecture 58: page 4

Example 5: 2nd Degree Trinomial

$-2x^2 + 4x + 3$

Lecture 59 Notes

Alg059-01



Lecture 59 - Adding Polynomials with Algebra Tiles

Example 1: $(x^2 - 3x + 4) + (x^2 + 4x - 2)$

ALGEBRA TILES

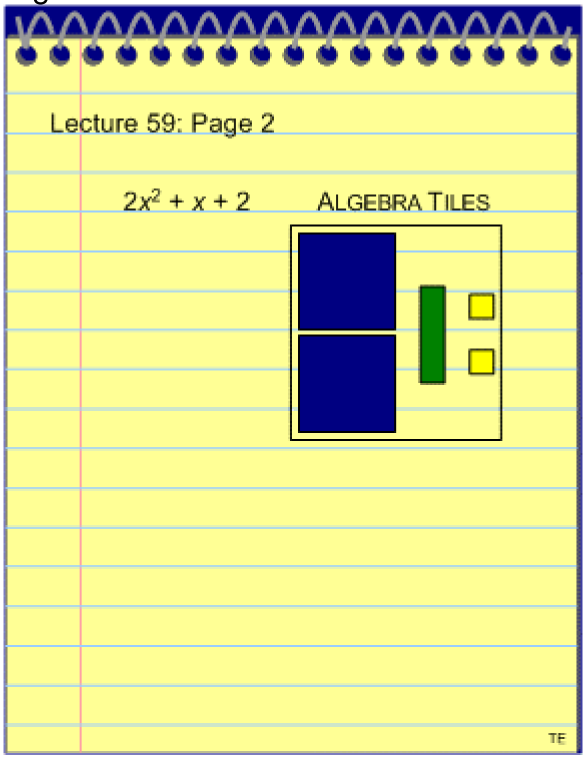
A reminder: 🟨 🟥 = 0 (cancel)

Combining Like Terms

$(\underline{x^2} - \underline{\underline{3x}} + \underline{\underline{\underline{4}}}) + (\underline{x^2} + \underline{\underline{4x}} - \underline{\underline{\underline{2}}})$

TE

Alg059-02



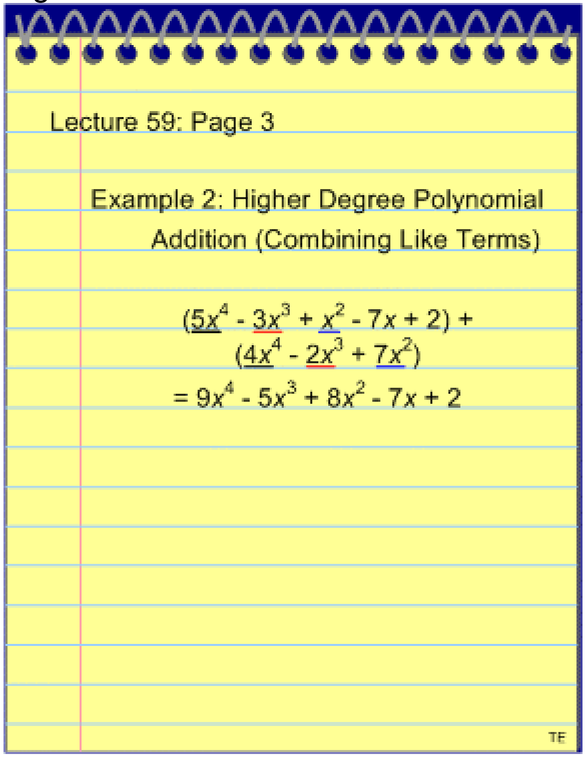
Lecture 59: Page 2

$2x^2 + x + 2$

ALGEBRA TILES

TE

Alg059-03



Lecture 59: Page 3

Example 2: Higher Degree Polynomial Addition (Combining Like Terms)

$(\underline{5x^4} - \underline{3x^3} + \underline{x^2} - 7x + 2) +$

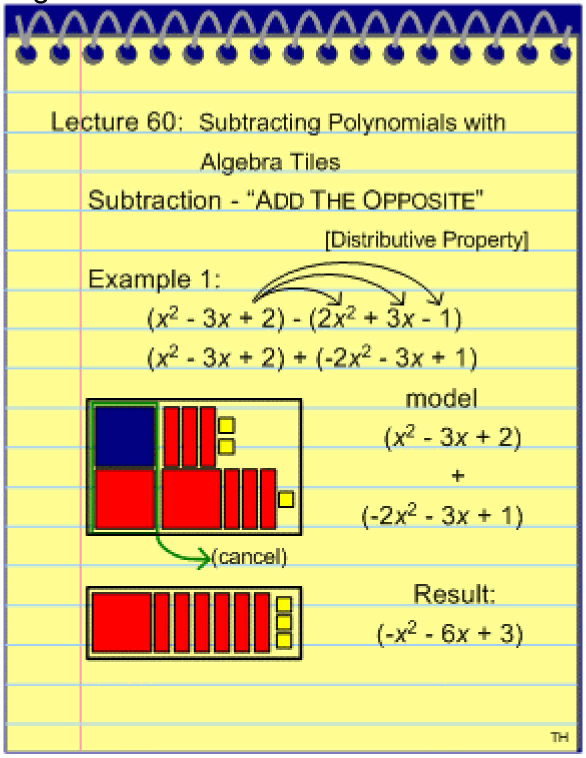
$(\underline{4x^4} - \underline{2x^3} + \underline{7x^2})$

$= 9x^4 - 5x^3 + 8x^2 - 7x + 2$

TE

Lecture 60 Notes

Alg060-01



Lecture 60: Subtracting Polynomials with Algebra Tiles

Subtraction - "ADD THE OPPOSITE"

[Distributive Property]

Example 1:

$(x^2 - 3x + 2) - (2x^2 + 3x - 1)$

$(x^2 - 3x + 2) + (-2x^2 - 3x + 1)$

model

$(x^2 - 3x + 2)$

+

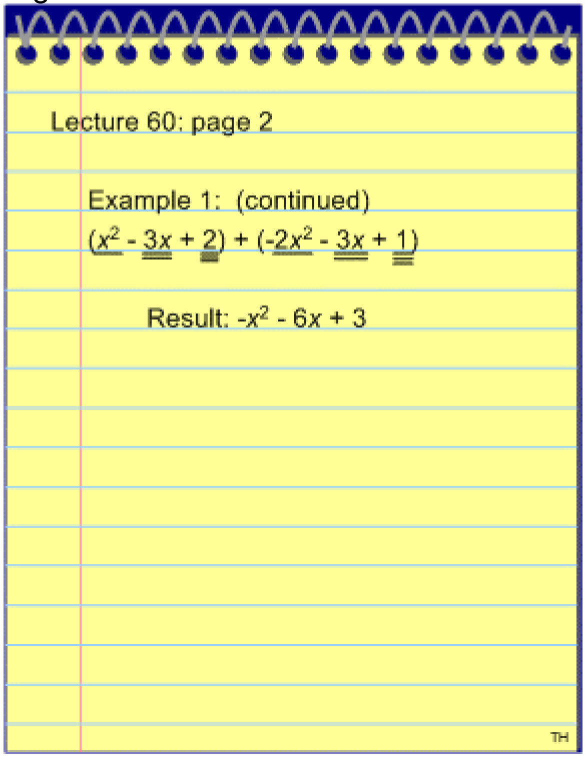
$(-2x^2 - 3x + 1)$

Result:

$(-x^2 - 6x + 3)$

TH

Alg060-02



Lecture 60: page 2

Example 1: (continued)

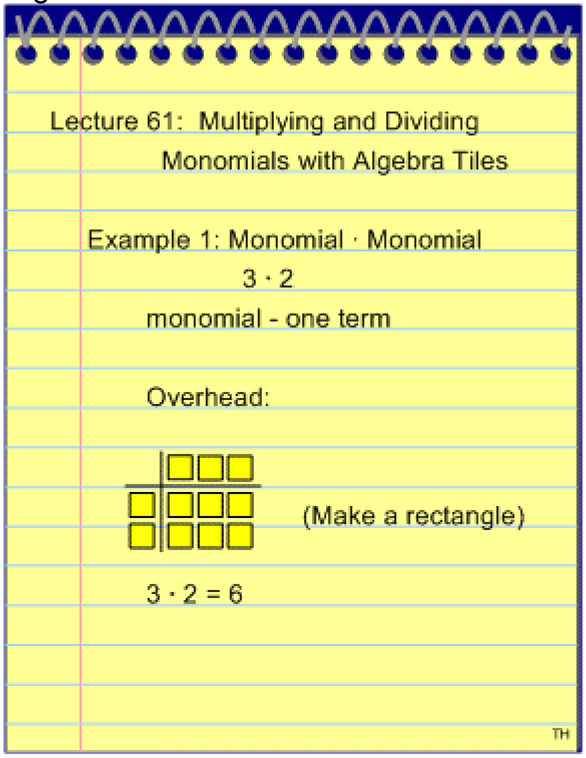
$(\underline{x^2} - \underline{\underline{3x}} + \underline{\underline{\underline{2}}}) + (-\underline{2x^2} - \underline{\underline{3x}} + \underline{\underline{\underline{1}}})$

Result: $-x^2 - 6x + 3$

TH

Lecture 61 Notes

Alg061-01



Lecture 61: Multiplying and Dividing Monomials with Algebra Tiles

Example 1: Monomial · Monomial

$3 \cdot 2$

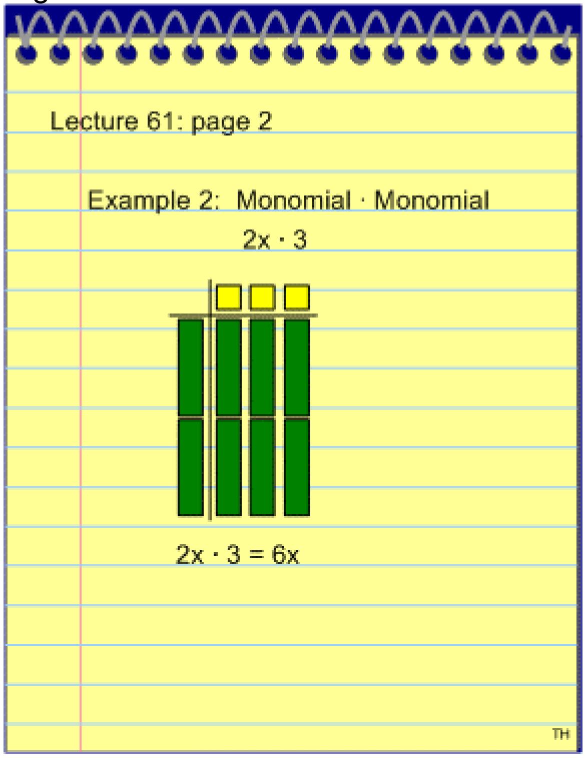
monomial - one term

Overhead:

(Make a rectangle)

$3 \cdot 2 = 6$

Alg061-02



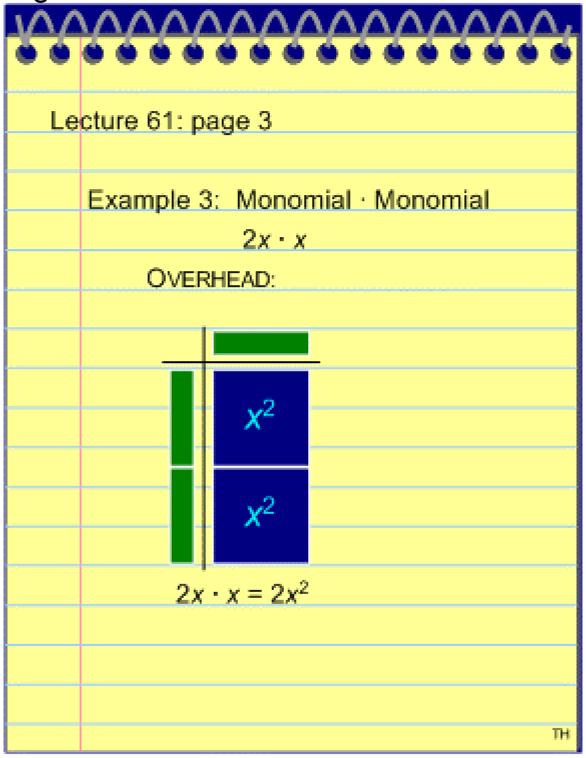
Lecture 61: page 2

Example 2: Monomial · Monomial

$2x \cdot 3$

$2x \cdot 3 = 6x$

Alg061-03



Lecture 61: page 3

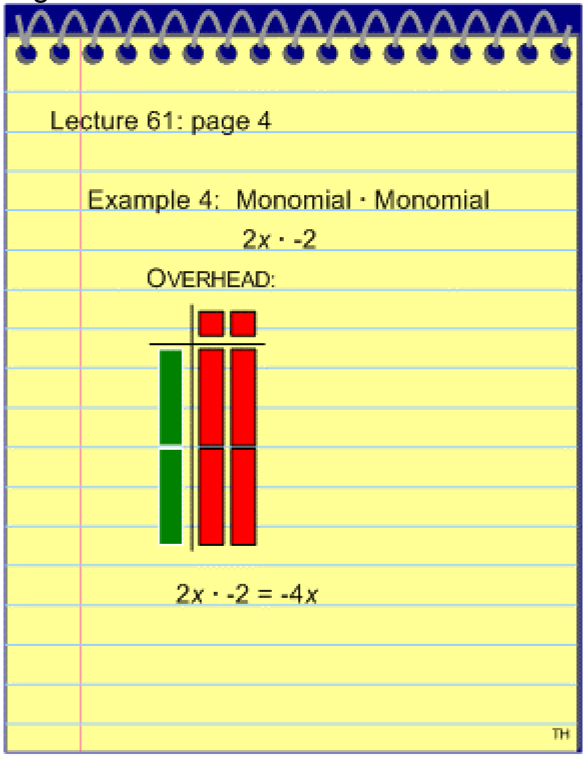
Example 3: Monomial · Monomial

$2x \cdot x$

OVERHEAD:

$2x \cdot x = 2x^2$

Alg061-04



Lecture 61: page 4

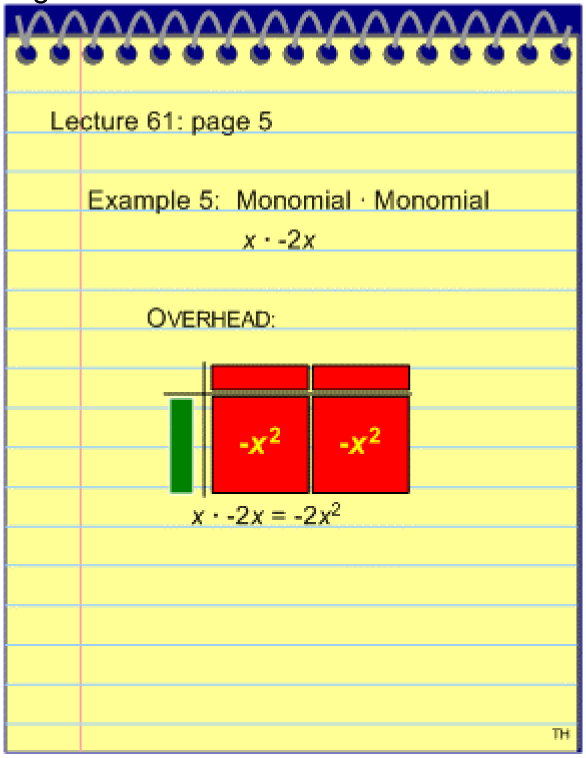
Example 4: Monomial · Monomial

$2x \cdot -2$

OVERHEAD:

$2x \cdot -2 = -4x$

Alg061-05



Lecture 61: page 5

Example 5: Monomial · Monomial

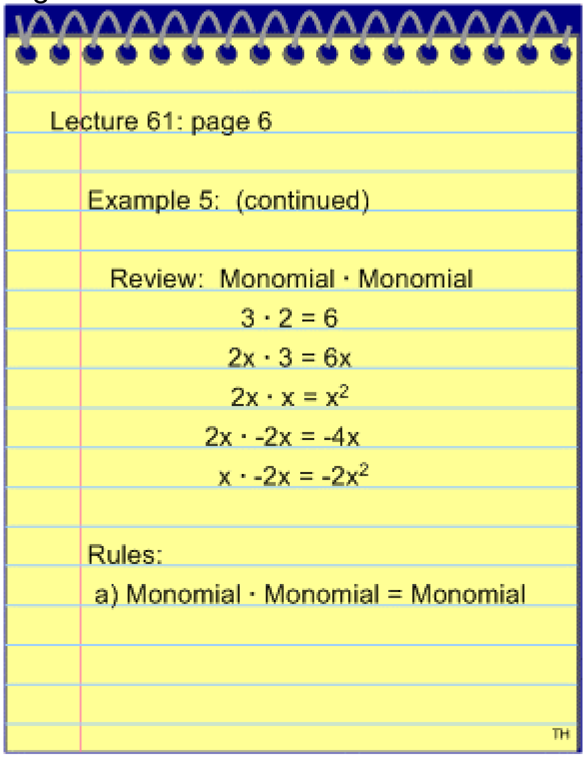
$x \cdot -2x$

OVERHEAD:

$x \cdot -2x = -2x^2$

TH

Alg061-06



Lecture 61: page 6

Example 5: (continued)

Review: Monomial · Monomial

$3 \cdot 2 = 6$

$2x \cdot 3 = 6x$

$2x \cdot x = x^2$

$2x \cdot -2x = -4x$

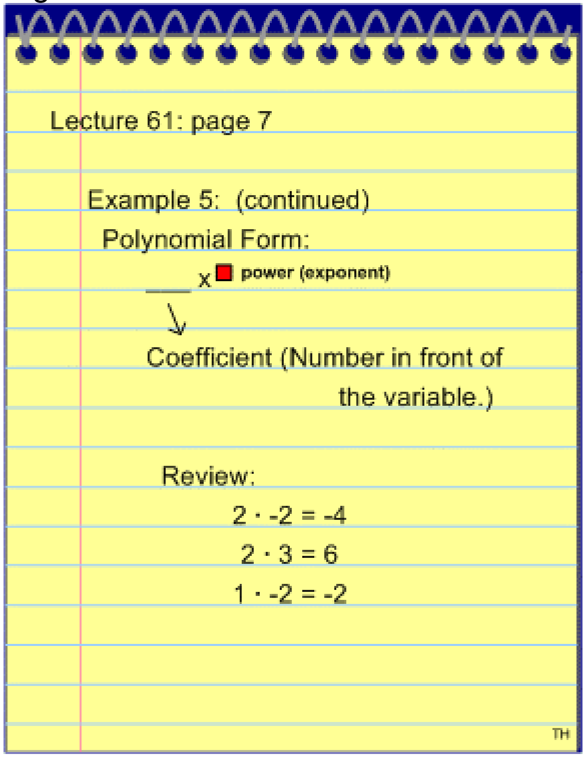
$x \cdot -2x = -2x^2$

Rules:

a) Monomial · Monomial = Monomial

TH

Alg061-07



Lecture 61: page 7

Example 5: (continued)

Polynomial Form:

___ x🟥 power (exponent)

↘

Coefficient (Number in front of the variable.)

Review:

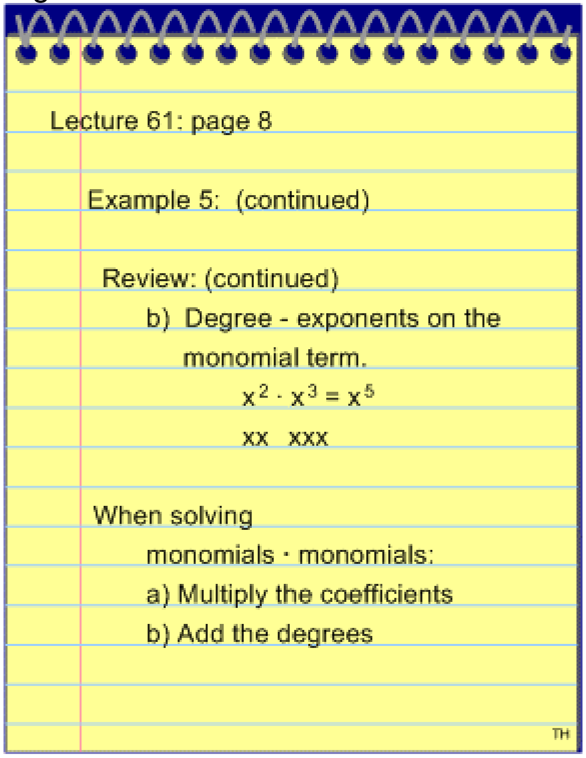
$2 \cdot -2 = -4$

$2 \cdot 3 = 6$

$1 \cdot -2 = -2$

TH

Alg061-08



Lecture 61: page 8

Example 5: (continued)

Review: (continued)

b) Degree - exponents on the monomial term.

$x^2 \cdot x^3 = x^5$

xx xxx

When solving monomials · monomials:

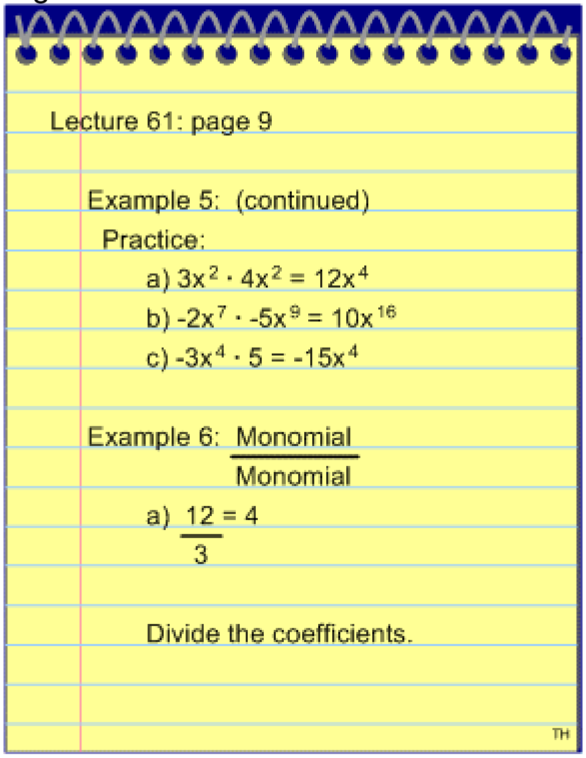
a) Multiply the coefficients

b) Add the degrees

TH

Lecture 61 Notes, Continued

Alg061-09



Lecture 61: page 9

Example 5: (continued)

Practice:

a) $3x^2 \cdot 4x^2 = 12x^4$

b) $-2x^7 \cdot -5x^9 = 10x^{16}$

c) $-3x^4 \cdot 5 = -15x^4$

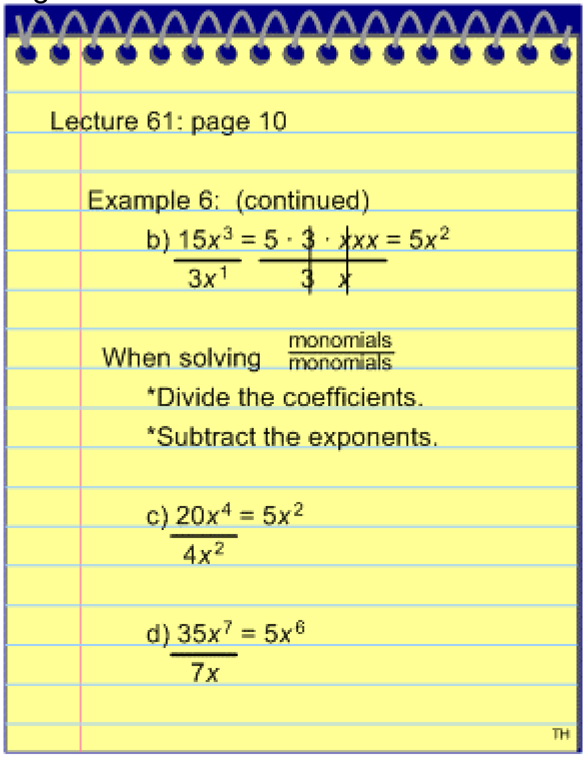
Example 6: $\frac{\text{Monomial}}{\text{Monomial}}$

a) $\frac{12}{3} = 4$

Divide the coefficients.

TH

Alg061-10



Lecture 61: page 10

Example 6: (continued)

b) $\frac{15x^3}{3x^1} = \frac{5 \cdot \cancel{3} \cdot \cancel{x}xx}{\cancel{3} \quad \cancel{x}} = 5x^2$

When solving $\frac{\text{monomials}}{\text{monomials}}$

*Divide the coefficients.

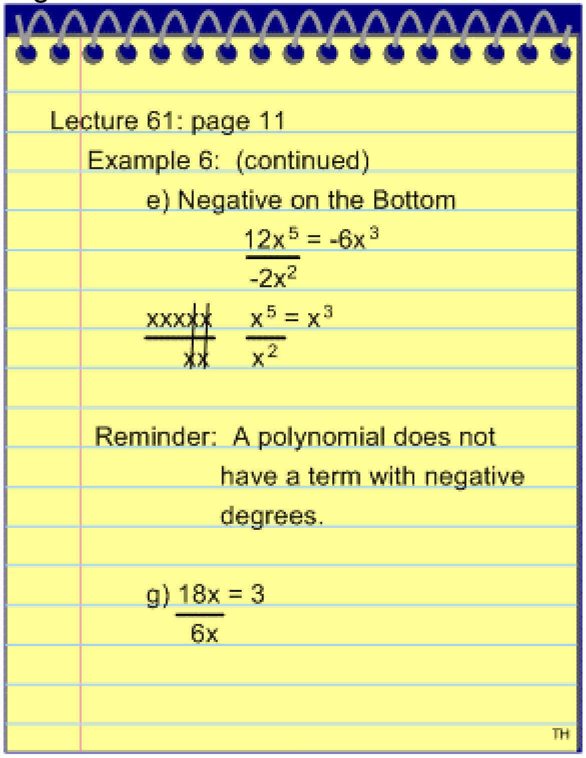
*Subtract the exponents.

c) $\frac{20x^4}{4x^2} = 5x^2$

d) $\frac{35x^7}{7x} = 5x^6$

TH

Alg061-11



Lecture 61: page 11

Example 6: (continued)

e) Negative on the Bottom

$\frac{12x^5}{-2x^2} = -6x^3$

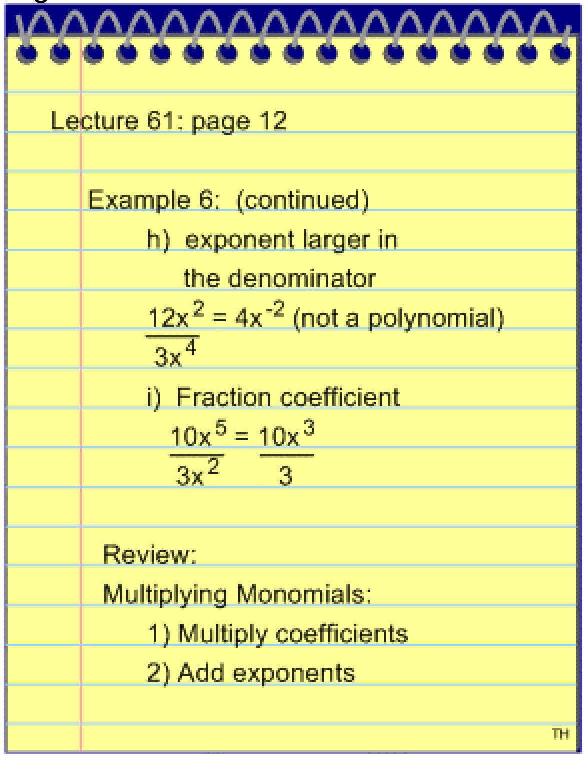
$\frac{xxx\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}}$ $\frac{x^5}{x^2} = x^3$

Reminder: A polynomial does not have a term with negative degrees.

g) $\frac{18x}{6x} = 3$

TH

Alg061-12



Lecture 61: page 12

Example 6: (continued)

h) exponent larger in the denominator

$\frac{12x^2}{3x^4} = 4x^{-2}$ (not a polynomial)

i) Fraction coefficient

$\frac{10x^5}{3x^2} = \frac{10x^3}{3}$

Review:

Multiplying Monomials:

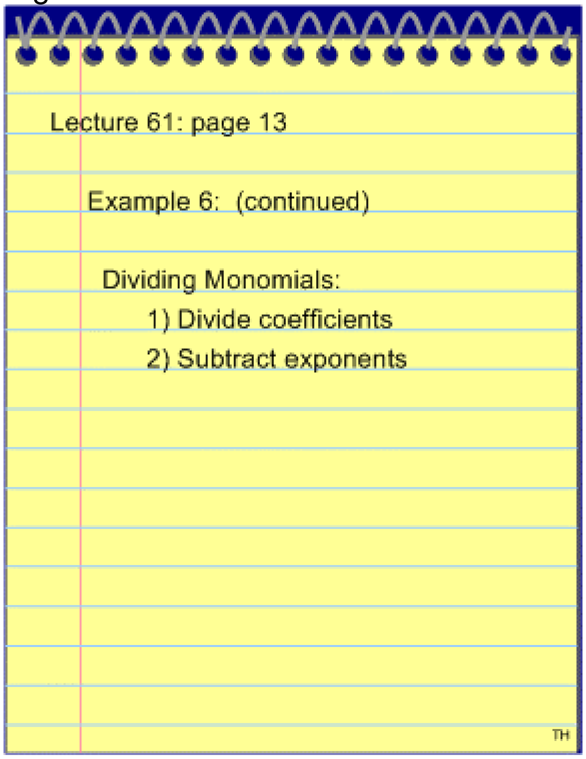
1) Multiply coefficients

2) Add exponents

TH

Lecture 61 Notes, Continued

Alg061-13



Lecture 61: page 13

Example 6: (continued)

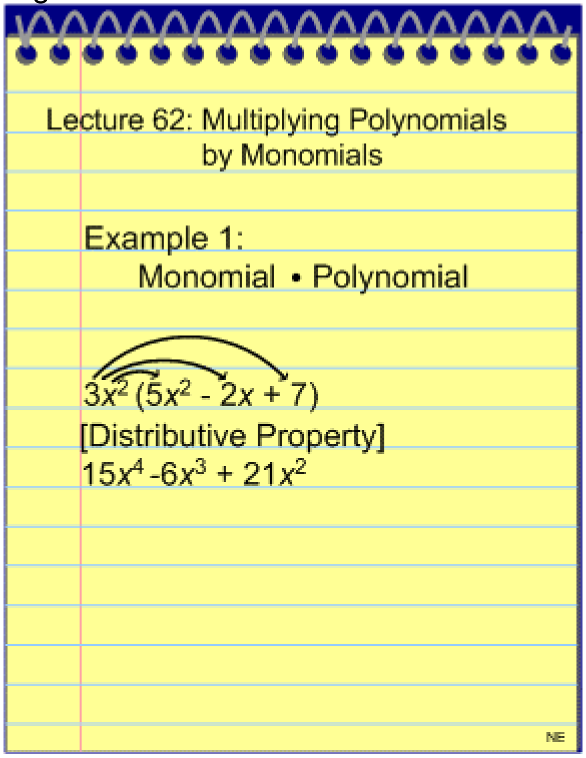
Dividing Monomials:

1) Divide coefficients
2) Subtract exponents

TH

Lecture 62 Notes

Alg062-01



Lecture 62: Multiplying Polynomials by Monomials

Example 1:

Monomial • Polynomial

$3x^2(5x^2 - 2x + 7)$

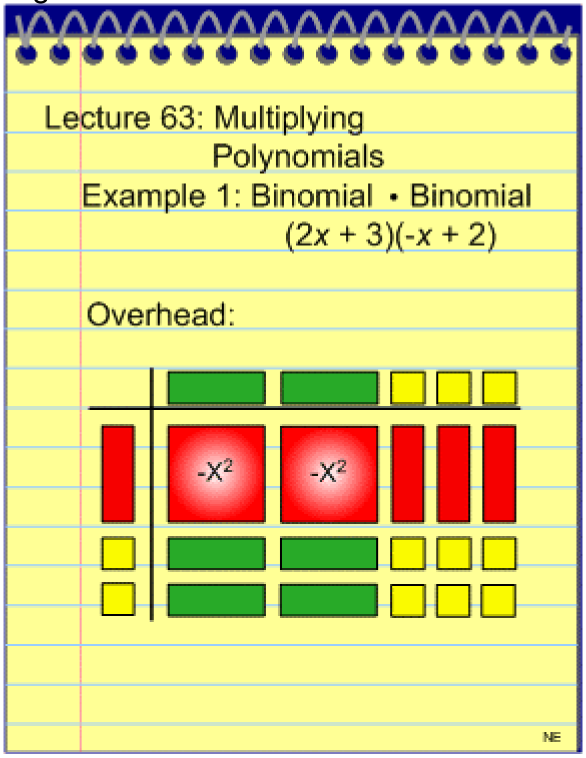
[Distributive Property]

$15x^4 -6x^3 + 21x^2$

NE

Lecture 63 Notes

Alg063-01



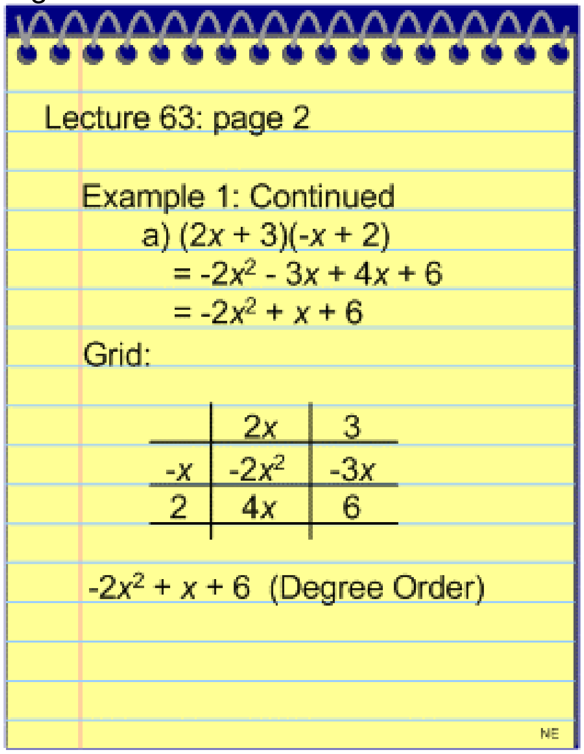
Lecture 63: Multiplying Polynomials

Example 1: Binomial • Binomial

$(2x + 3)(-x + 2)$

Overhead:

Alg063-02



Lecture 63: page 2

Example 1: Continued

a) $(2x + 3)(-x + 2)$

$= -2x^2 - 3x + 4x + 6$

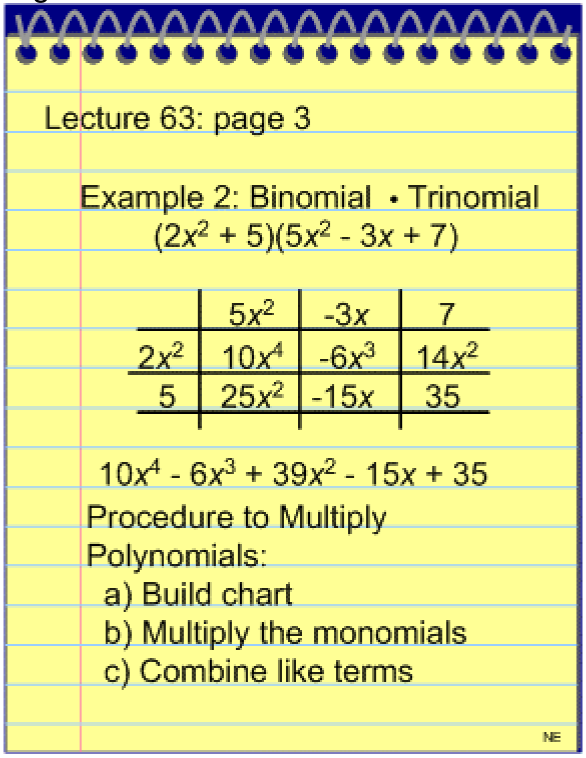
$= -2x^2 + x + 6$

Grid:

	$2x$	3
$-x$	$-2x^2$	$-3x$
2	$4x$	6

$-2x^2 + x + 6$ (Degree Order)

Alg063-03



Lecture 63: page 3

Example 2: Binomial • Trinomial

$(2x^2 + 5)(5x^2 - 3x + 7)$

	$5x^2$	$-3x$	7
$2x^2$	$10x^4$	$-6x^3$	$14x^2$
5	$25x^2$	$-15x$	35

$10x^4 - 6x^3 + 39x^2 - 15x + 35$

Procedure to Multiply Polynomials:

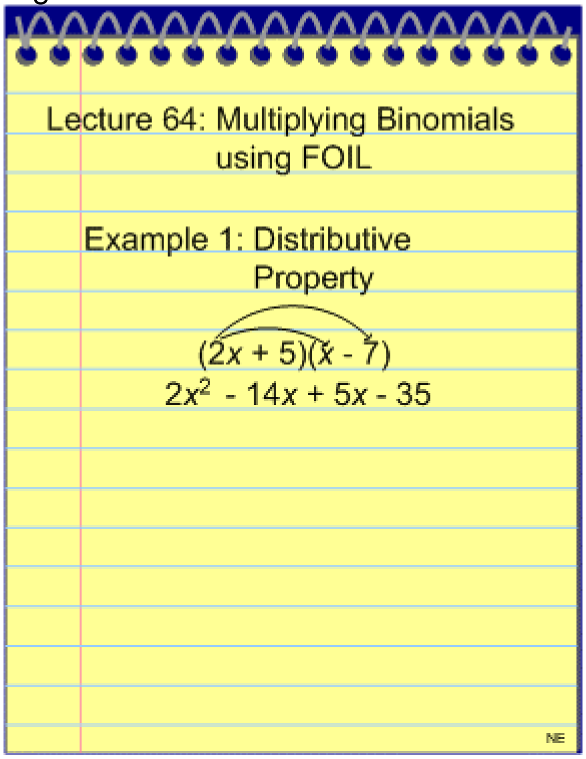
a) Build chart

b) Multiply the monomials

c) Combine like terms

Lecture 64 Notes

Alg064-01



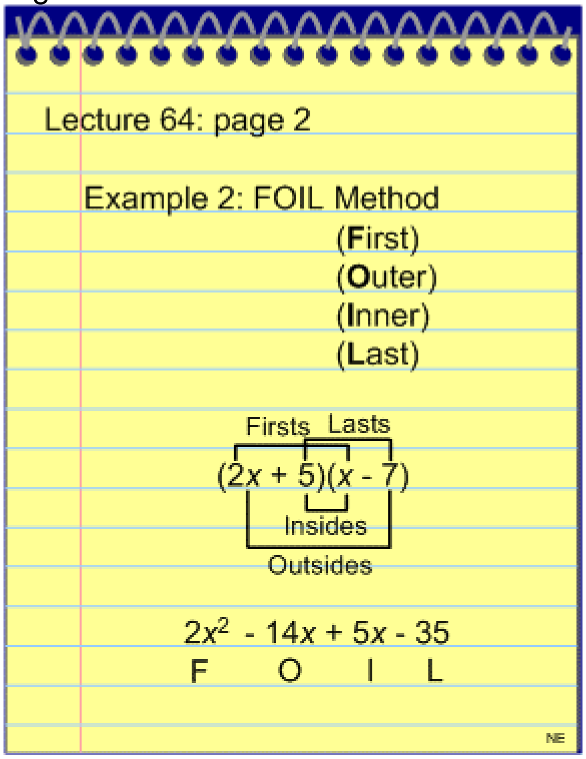
Lecture 64: Multiplying Binomials using FOIL

Example 1: Distributive Property

$(2x + 5)(x - 7)$

$2x^2 - 14x + 5x - 35$

Alg064-02



Lecture 64: page 2

Example 2: FOIL Method

(**F**irst)
(**O**uter)
(**I**nner)
(**L**ast)

Firsts Lasts

$(2x + 5)(x - 7)$

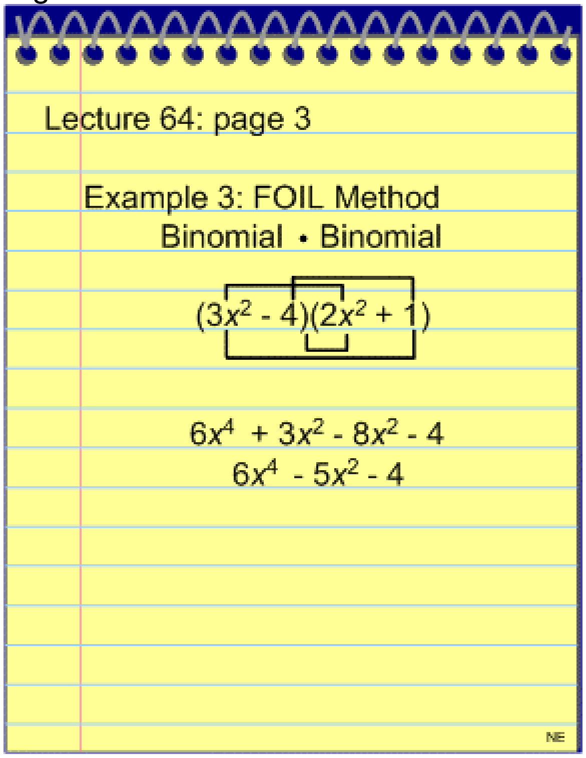
Insides

Outsides

$2x^2 - 14x + 5x - 35$

F O I L

Alg064-03



Lecture 64: page 3

Example 3: FOIL Method

Binomial • Binomial

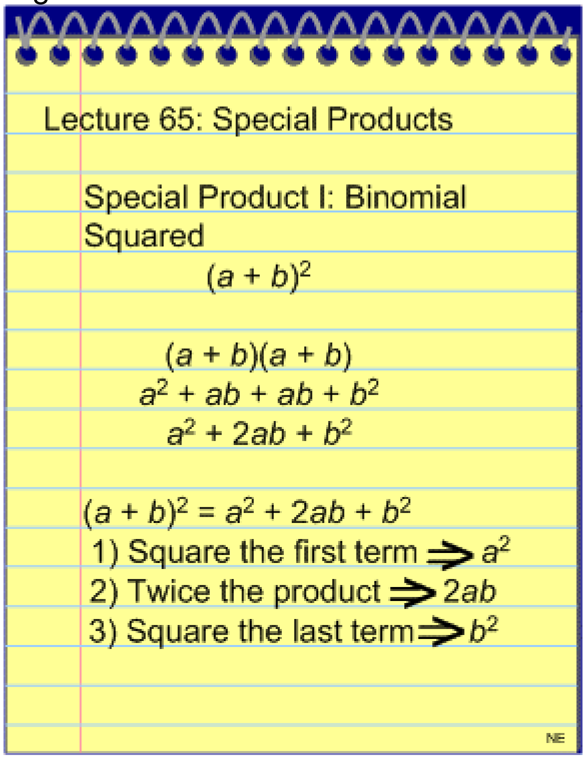
$(3x^2 - 4)(2x^2 + 1)$

$6x^4 + 3x^2 - 8x^2 - 4$

$6x^4 - 5x^2 - 4$

Lecture 65 Notes

Alg065-01



Lecture 65: Special Products

Special Product I: Binomial Squared

$(a + b)^2$

$(a + b)(a + b)$

$a^2 + ab + ab + b^2$

$a^2 + 2ab + b^2$

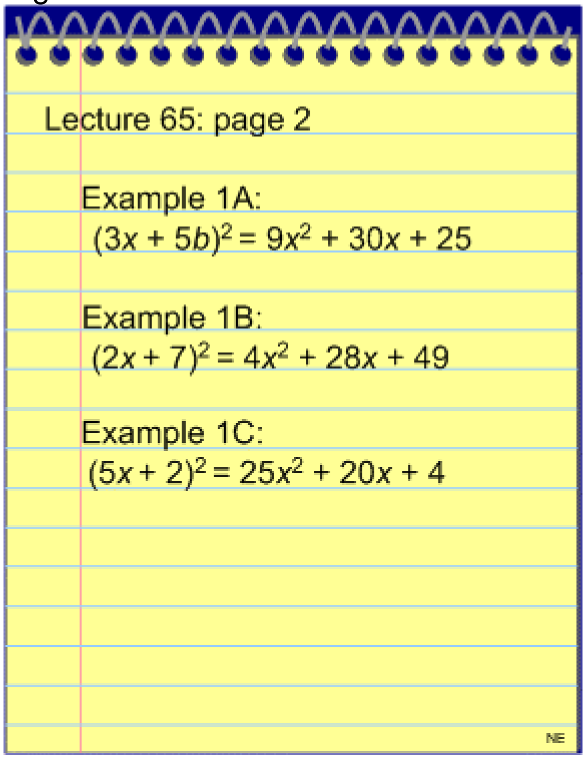
$(a + b)^2 = a^2 + 2ab + b^2$

1) Square the first term $\Rightarrow a^2$

2) Twice the product $\Rightarrow 2ab$

3) Square the last term $\Rightarrow b^2$

Alg065-02



Lecture 65: page 2

Example 1A:

$(3x + 5b)^2 = 9x^2 + 30x + 25$

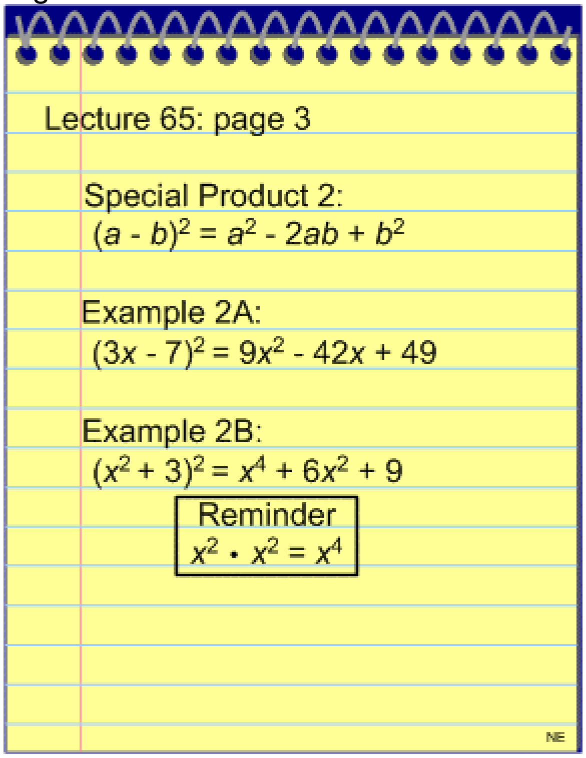
Example 1B:

$(2x + 7)^2 = 4x^2 + 28x + 49$

Example 1C:

$(5x + 2)^2 = 25x^2 + 20x + 4$

Alg065-03



Lecture 65: page 3

Special Product 2:

$(a - b)^2 = a^2 - 2ab + b^2$

Example 2A:

$(3x - 7)^2 = 9x^2 - 42x + 49$

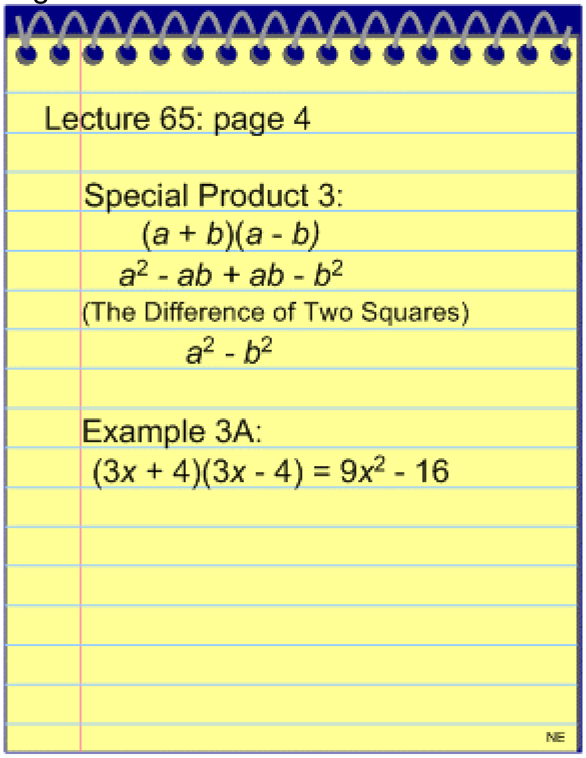
Example 2B:

$(x^2 + 3)^2 = x^4 + 6x^2 + 9$

Reminder

$x^2 \cdot x^2 = x^4$

Alg065-04



Lecture 65: page 4

Special Product 3:

$(a + b)(a - b)$

$a^2 - ab + ab - b^2$

(The Difference of Two Squares)

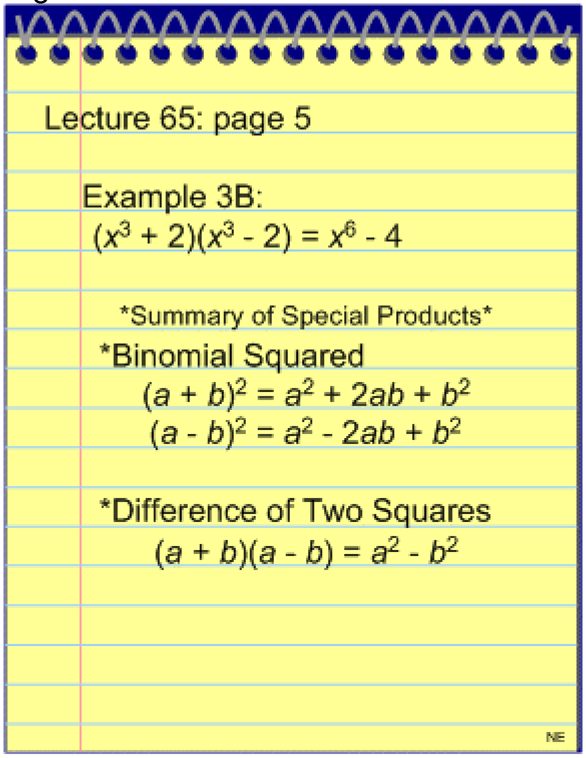
$a^2 - b^2$

Example 3A:

$(3x + 4)(3x - 4) = 9x^2 - 16$

Lecture 65 Notes, Continued

Alg065-05



Lecture 65: page 5

Example 3B:

$(x^3 + 2)(x^3 - 2) = x^6 - 4$

Summary of Special Products

*Binomial Squared

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

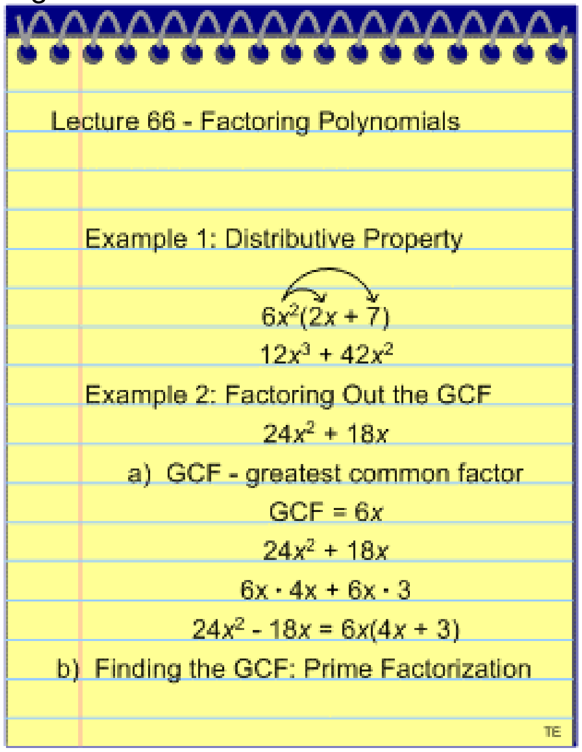
*Difference of Two Squares

$(a + b)(a - b) = a^2 - b^2$

NE

Lecture 66 Notes

Alg066-01



Lecture 66 - Factoring Polynomials

Example 1: Distributive Property

$$\overset{\curvearrowright}{6x^2(2x + 7)}$$

$$12x^3 + 42x^2$$

Example 2: Factoring Out the GCF

$$24x^2 + 18x$$

a) GCF - greatest common factor

$$\text{GCF} = 6x$$

$$24x^2 + 18x$$

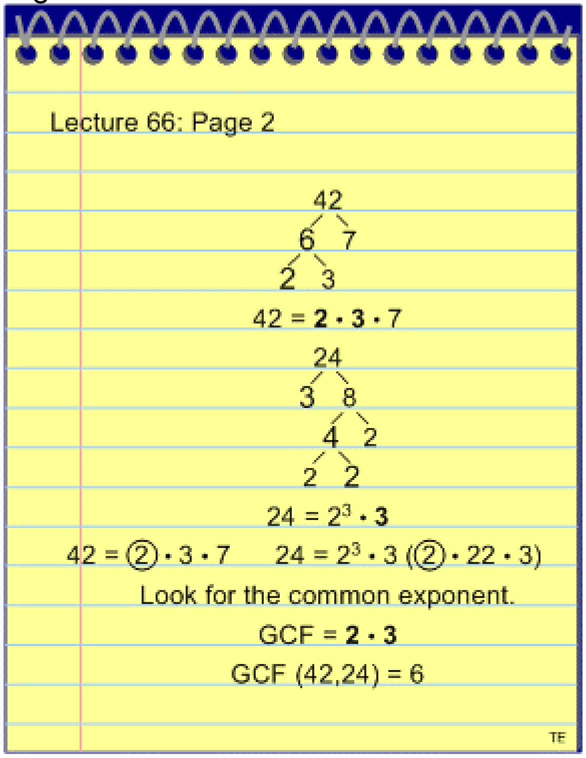
$$6x \cdot 4x + 6x \cdot 3$$

$$24x^2 - 18x = 6x(4x + 3)$$

b) Finding the GCF: Prime Factorization

TE

Alg066-02



Lecture 66: Page 2

42
6 7
2 3

$$42 = \mathbf{2} \cdot \mathbf{3} \cdot 7$$

24
3 8
4 2
2 2

$$24 = 2^3 \cdot \mathbf{3}$$

$$42 = \textcircled{2} \cdot 3 \cdot 7 \qquad 24 = 2^3 \cdot 3\ (\textcircled{2} \cdot 22 \cdot 3)$$

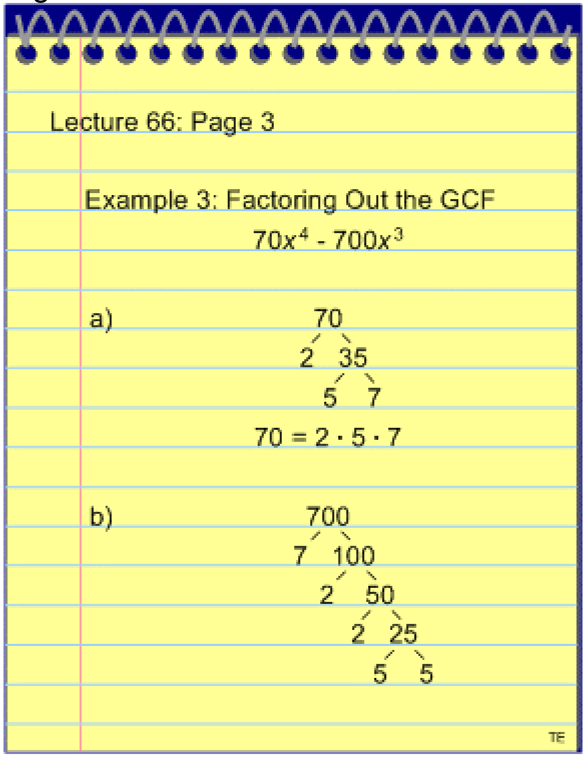
Look for the common exponent.

$$\text{GCF} = \mathbf{2} \cdot \mathbf{3}$$

$$\text{GCF }(42,24) = 6$$

TE

Alg066-03



Lecture 66: Page 3

Example 3: Factoring Out the GCF

$$70x^4 - 700x^3$$

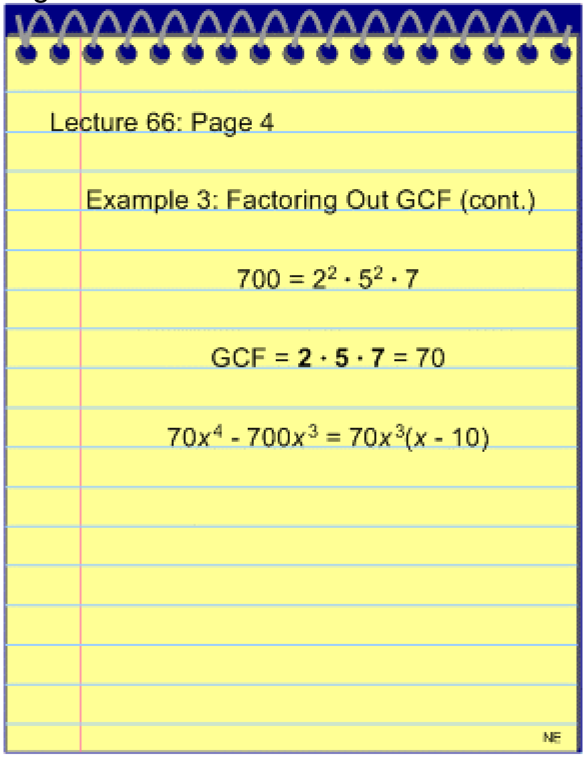
a)
70
2 35
5 7

$$70 = 2 \cdot 5 \cdot 7$$

b)
700
7 100
2 50
2 25
5 5

TE

Alg066-04



Lecture 66: Page 4

Example 3: Factoring Out GCF (cont.)

$$700 = 2^2 \cdot 5^2 \cdot 7$$

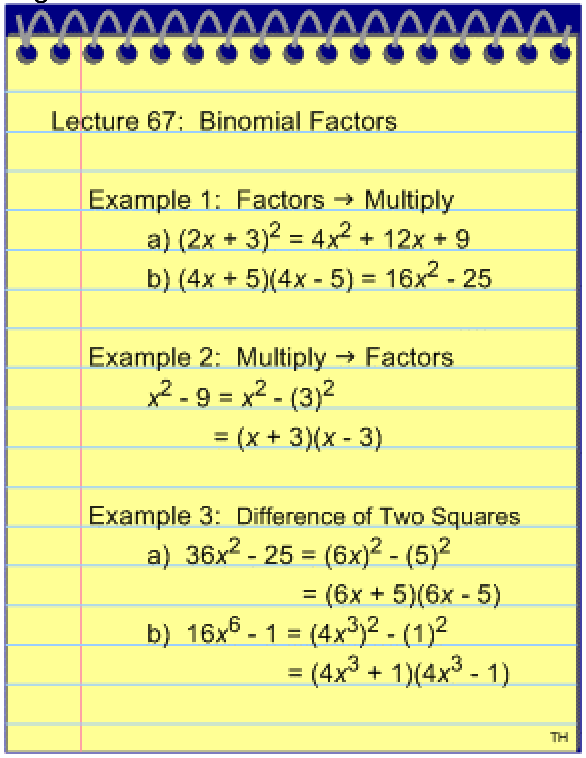
$$\text{GCF} = \mathbf{2} \cdot \mathbf{5} \cdot \mathbf{7} = 70$$

$$70x^4 - 700x^3 = 70x^3(x - 10)$$

NE

Lecture 67 Notes

Alg067-01



Lecture 67: Binomial Factors

Example 1: Factors → Multiply

a) $(2x + 3)^2 = 4x^2 + 12x + 9$

b) $(4x + 5)(4x - 5) = 16x^2 - 25$

Example 2: Multiply → Factors

$x^2 - 9 = x^2 - (3)^2$

$= (x + 3)(x - 3)$

Example 3: Difference of Two Squares

a) $36x^2 - 25 = (6x)^2 - (5)^2$

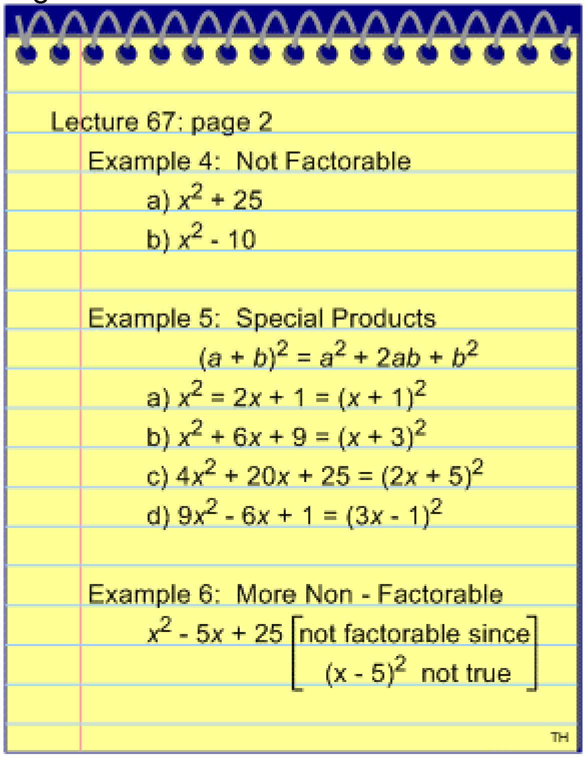
$= (6x + 5)(6x - 5)$

b) $16x^6 - 1 = (4x^3)^2 - (1)^2$

$= (4x^3 + 1)(4x^3 - 1)$

TH

Alg067-02



Lecture 67: page 2

Example 4: Not Factorable

a) $x^2 + 25$

b) $x^2 - 10$

Example 5: Special Products

$(a + b)^2 = a^2 + 2ab + b^2$

a) $x^2 = 2x + 1 = (x + 1)^2$

b) $x^2 + 6x + 9 = (x + 3)^2$

c) $4x^2 + 20x + 25 = (2x + 5)^2$

d) $9x^2 - 6x + 1 = (3x - 1)^2$

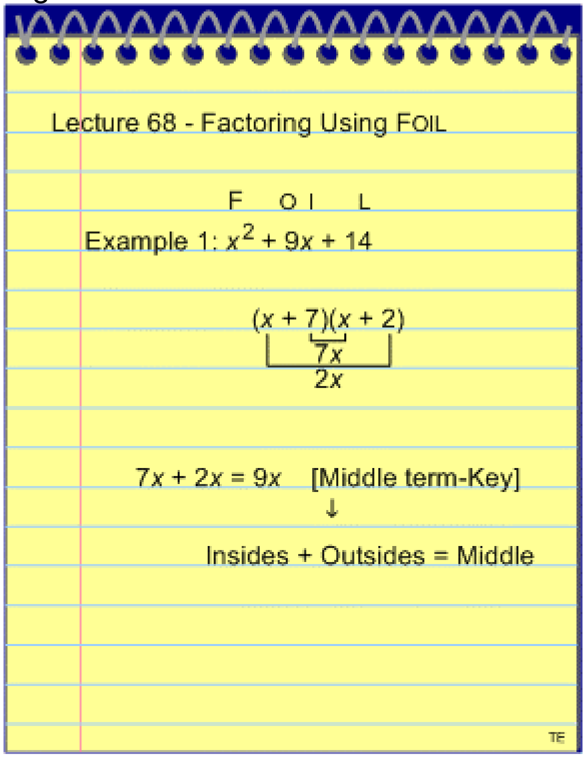
Example 6: More Non - Factorable

$x^2 - 5x + 25$ [not factorable since $(x - 5)^2$ not true]

TH

Lecture 68 Notes

Alg068-01



Lecture 68 - Factoring Using FOIL

F O I L

Example 1: $x^2 + 9x + 14$

$(x + 7)(x + 2)$

$7x$

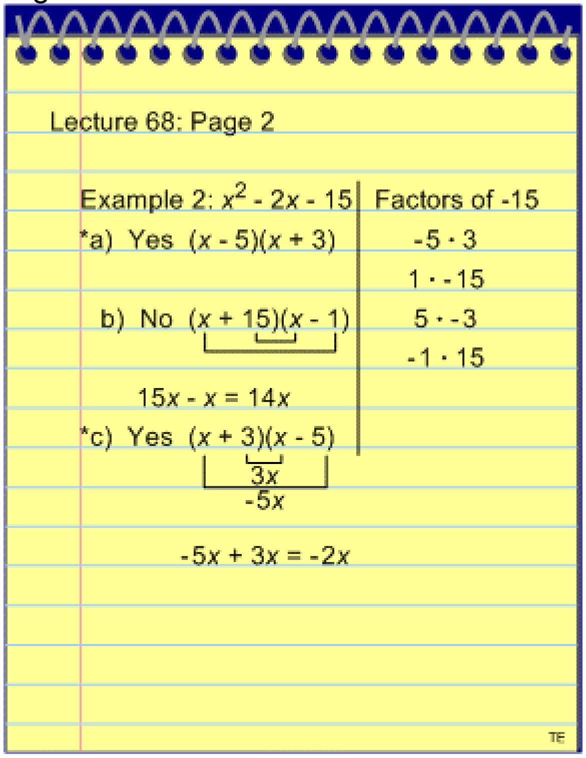
$2x$

$7x + 2x = 9x$ [Middle term-Key]

↓

Insides + Outsides = Middle

Alg068-02



Lecture 68: Page 2

Example 2: $x^2 - 2x - 15$ | Factors of -15

*a) Yes $(x - 5)(x + 3)$

b) No $(x + 15)(x - 1)$

$15x - x = 14x$

*c) Yes $(x + 3)(x - 5)$

$3x$

$-5x$

$-5x + 3x = -2x$

Factors of -15:

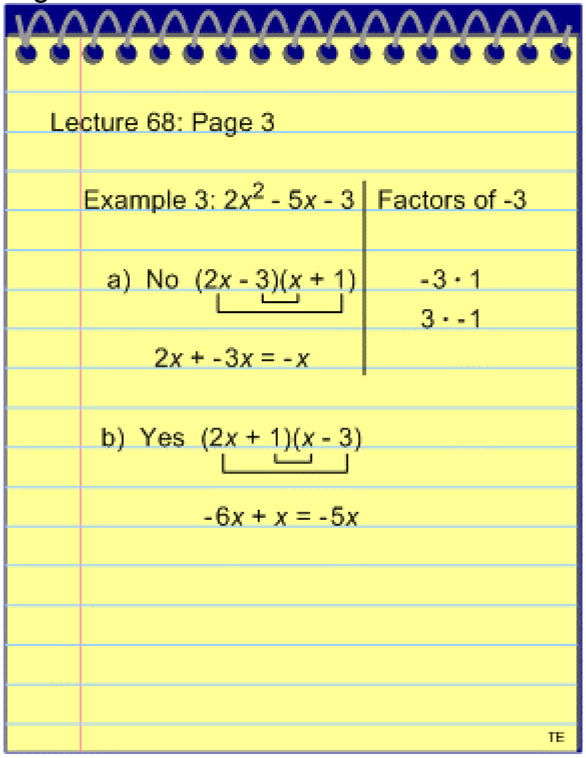
$-5 \cdot 3$

$1 \cdot -15$

$5 \cdot -3$

$-1 \cdot 15$

Alg068-03



Lecture 68: Page 3

Example 3: $2x^2 - 5x - 3$ | Factors of -3

a) No $(2x - 3)(x + 1)$

$2x + -3x = -x$

b) Yes $(2x + 1)(x - 3)$

$-6x + x = -5x$

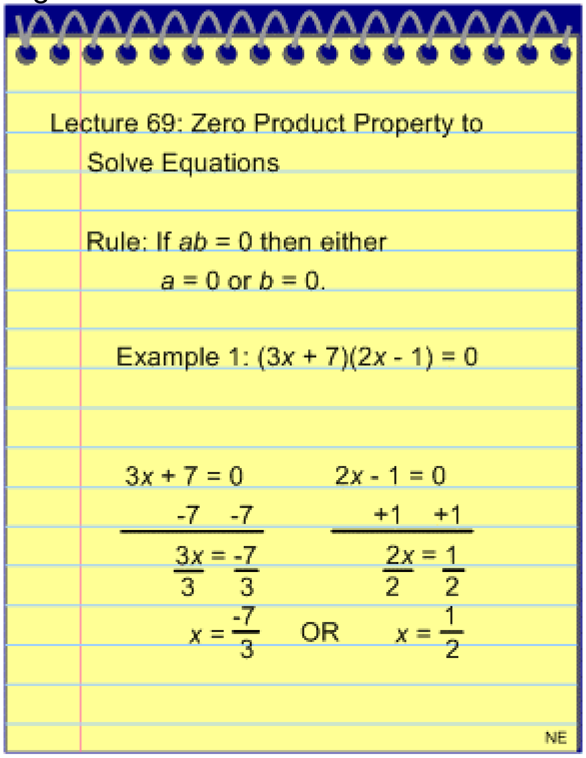
Factors of -3:

$-3 \cdot 1$

$3 \cdot -1$

Lecture 69 Notes

Alg069-01



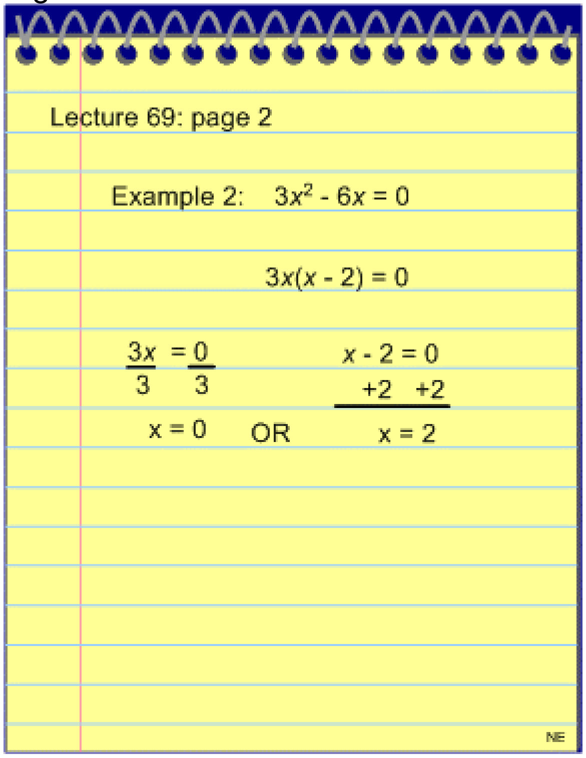
Lecture 69: Zero Product Property to Solve Equations

Rule: If $ab = 0$ then either $a = 0$ or $b = 0$.

Example 1: $(3x + 7)(2x - 1) = 0$

$$\begin{array}{ccc} 3x + 7 = 0 & & 2x - 1 = 0 \\ \underline{\quad -7 \quad -7} & & \underline{\quad +1 \quad +1} \\ \frac{3x}{3} = \frac{-7}{3} & & \frac{2x}{2} = \frac{1}{2} \\ x = \frac{-7}{3} & \text{OR} & x = \frac{1}{2} \end{array}$$

Alg069-02



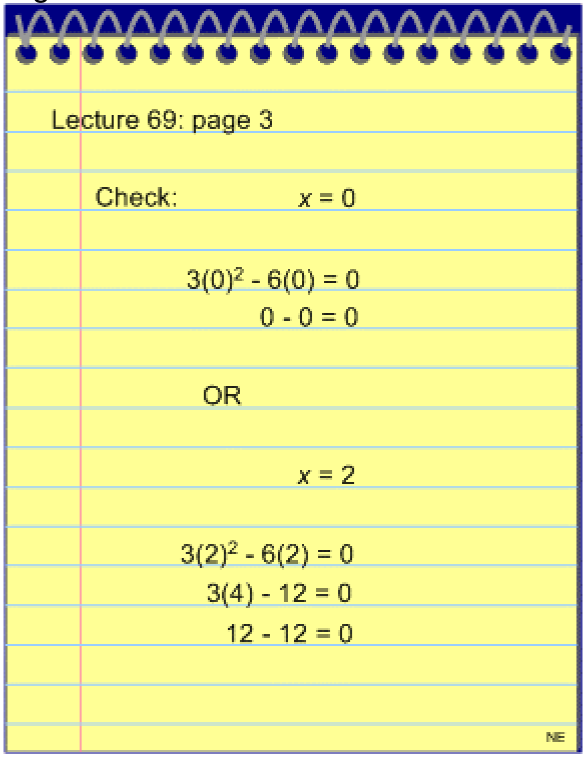
Lecture 69: page 2

Example 2: $3x^2 - 6x = 0$

$$3x(x - 2) = 0$$

$$\begin{array}{ccc} \frac{3x}{3} = \frac{0}{3} & & x - 2 = 0 \\ & & \underline{\quad +2 \quad +2} \\ x = 0 & \text{OR} & x = 2 \end{array}$$

Alg069-03



Lecture 69: page 3

Check: $x = 0$

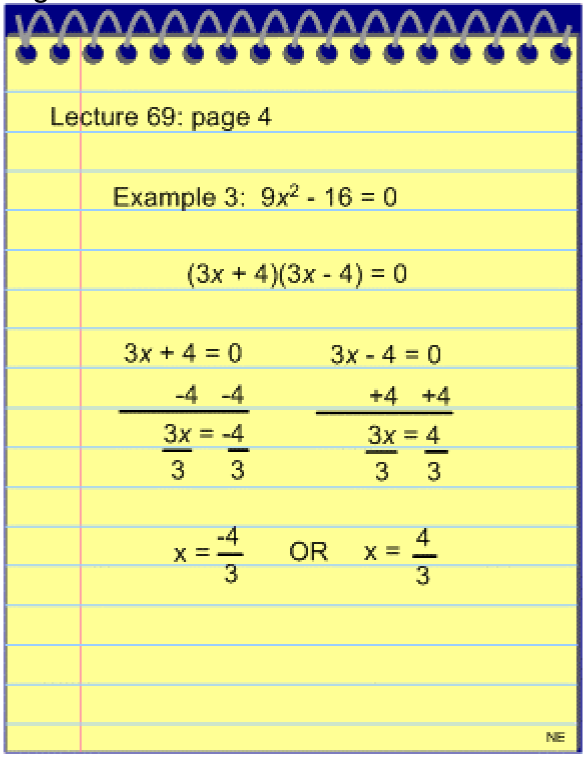
$$3(0)^2 - 6(0) = 0$$
$$0 - 0 = 0$$

OR

$x = 2$

$$3(2)^2 - 6(2) = 0$$
$$3(4) - 12 = 0$$
$$12 - 12 = 0$$

Alg069-04



Lecture 69: page 4

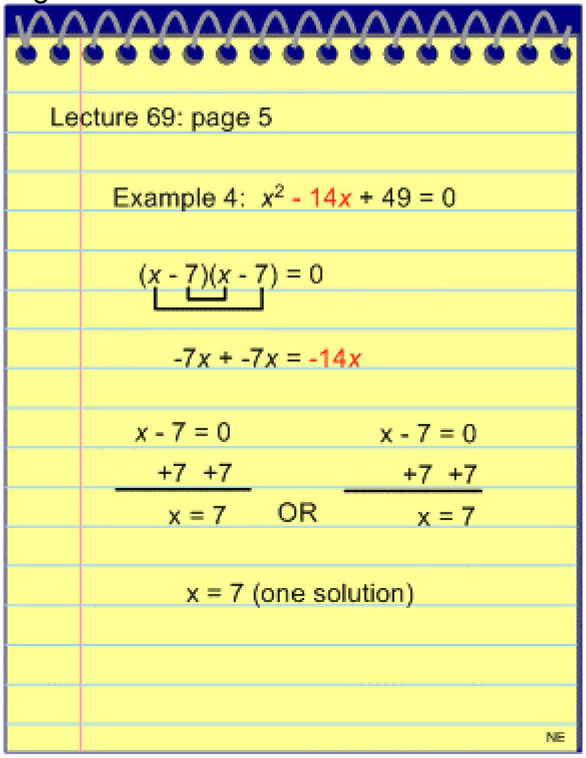
Example 3: $9x^2 - 16 = 0$

$$(3x + 4)(3x - 4) = 0$$

$$\begin{array}{ccc} 3x + 4 = 0 & & 3x - 4 = 0 \\ \underline{\quad -4 \quad -4} & & \underline{\quad +4 \quad +4} \\ \frac{3x}{3} = \frac{-4}{3} & & \frac{3x}{3} = \frac{4}{3} \\ x = \frac{-4}{3} & \text{OR} & x = \frac{4}{3} \end{array}$$

Lecture 69 Notes, Continued

Alg069-05



Lecture 69: page 5

Example 4: $x^2 - 14x + 49 = 0$

$(x - 7)(x - 7) = 0$

$-7x + -7x = -14x$

$x - 7 = 0$
$+7 \quad +7$
$x = 7$

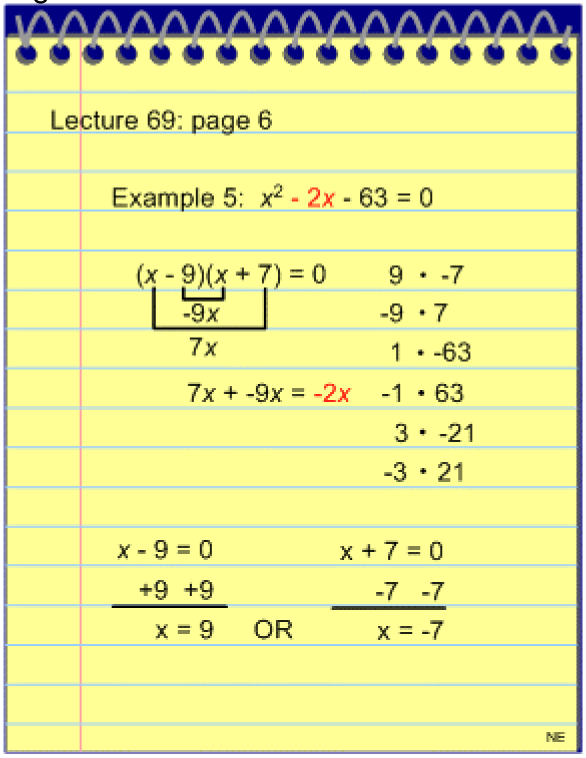
OR

$x - 7 = 0$
$+7 \quad +7$
$x = 7$

$x = 7$ (one solution)

NE

Alg069-06



Lecture 69: page 6

Example 5: $x^2 - 2x - 63 = 0$

$(x - 9)(x + 7) = 0$

$-9x$

$7x$

$7x + -9x = -2x$

9 • -7
-9 • 7
1 • -63
-1 • 63
3 • -21
-3 • 21

$x - 9 = 0$
$+9 \quad +9$
$x = 9$

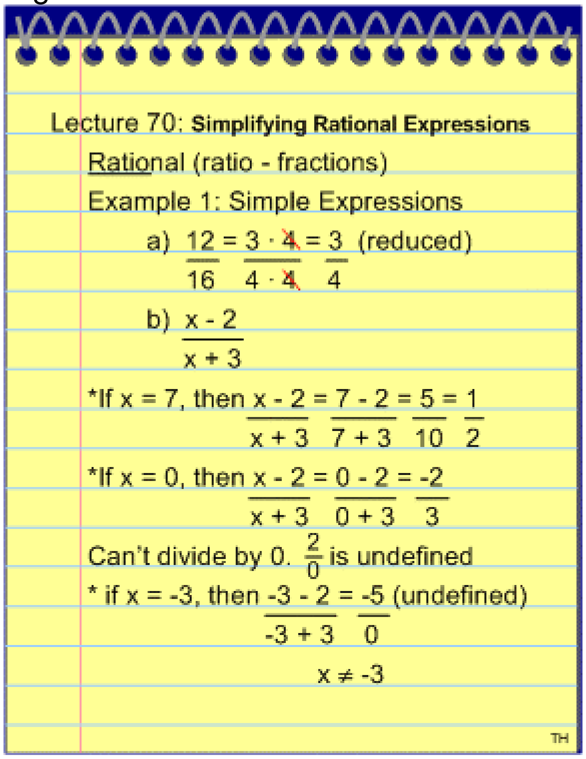
OR

$x + 7 = 0$
$-7 \quad -7$
$x = -7$

NE

Lecture 70 Notes

Alg070-01



Lecture 70: **Simplifying Rational Expressions**

Rational (ratio - fractions)

Example 1: Simple Expressions

a) $\frac{12}{16} = \frac{3 \cdot \cancel{4}}{4 \cdot \cancel{4}} = \frac{3}{4}$ (reduced)

b) $\frac{x - 2}{x + 3}$

*If x = 7, then $\frac{x - 2}{x + 3} = \frac{7 - 2}{7 + 3} = \frac{5}{10} = \frac{1}{2}$

*If x = 0, then $\frac{x - 2}{x + 3} = \frac{0 - 2}{0 + 3} = \frac{-2}{3}$

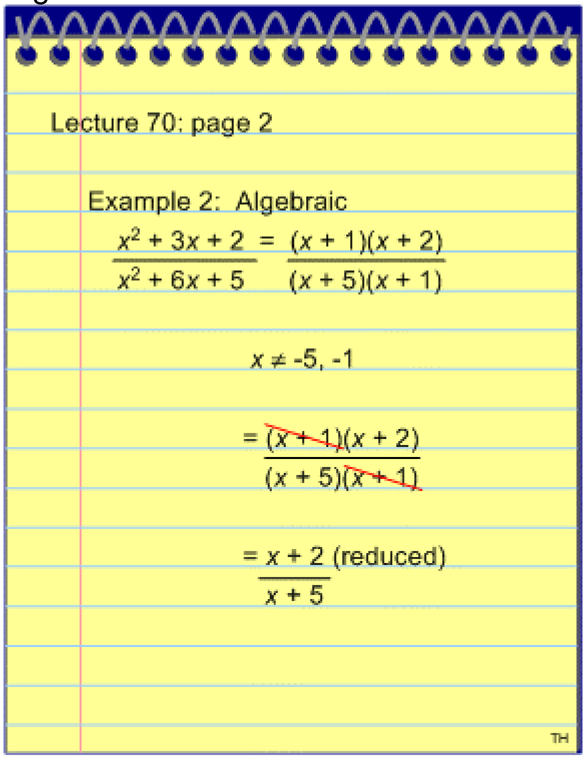
Can't divide by 0. $\frac{2}{0}$ is undefined

* if x = -3, then $\frac{-3 - 2}{-3 + 3} = \frac{-5}{0}$ (undefined)

$x \neq -3$

TH

Alg070-02



Lecture 70: page 2

Example 2: Algebraic

$$\frac{x^2 + 3x + 2}{x^2 + 6x + 5} = \frac{(x + 1)(x + 2)}{(x + 5)(x + 1)}$$

$x \neq -5, -1$

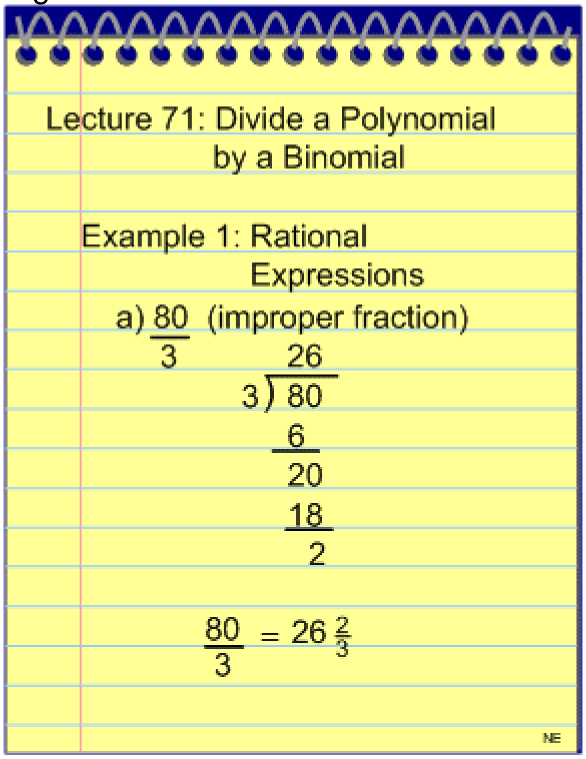
$$= \frac{\cancel{(x + 1)}(x + 2)}{(x + 5)\cancel{(x + 1)}}$$

$= \frac{x + 2}{x + 5}$ (reduced)

TH

Lecture 71 Notes

Alg071-01



Lecture 71: Divide a Polynomial by a Binomial

Example 1: Rational Expressions

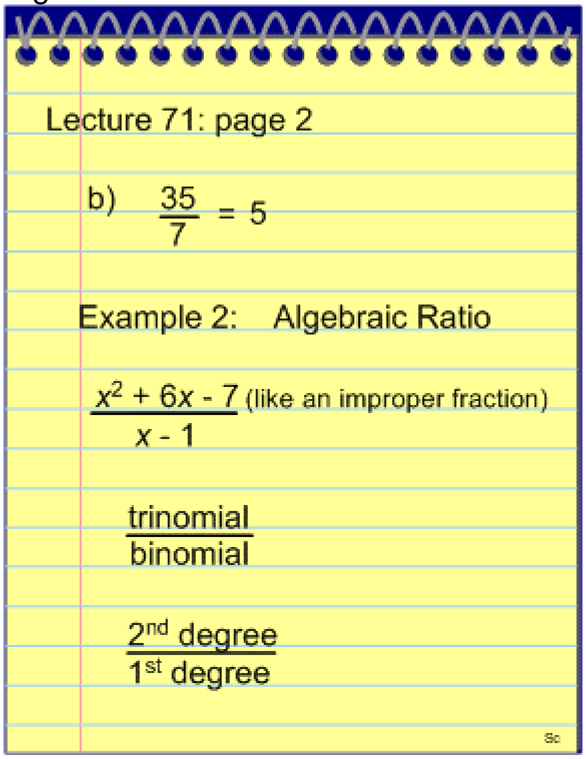
a) $\frac{80}{3}$ (improper fraction)

$$\begin{array}{r} 26 \\ 3\overline{)\,80} \\ \underline{6} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

$\frac{80}{3} = 26\frac{2}{3}$

NE

Alg071-02



Lecture 71: page 2

b) $\frac{35}{7} = 5$

Example 2: Algebraic Ratio

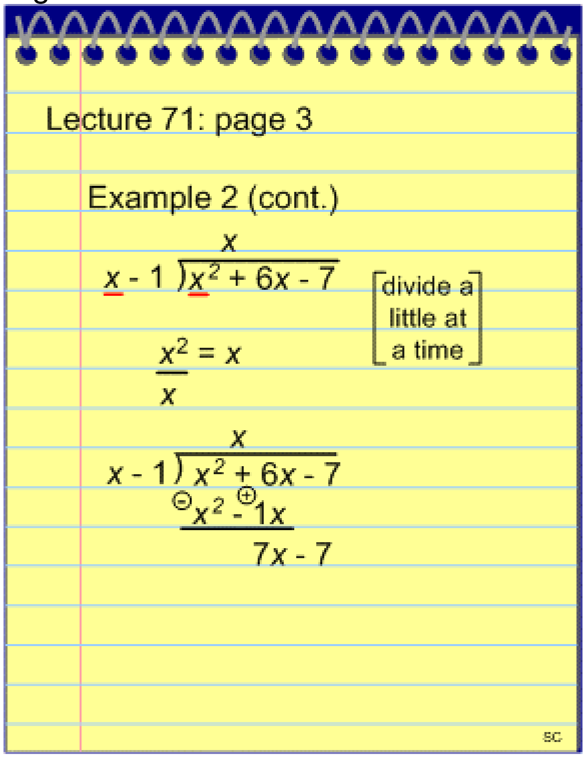
$\frac{x^2 + 6x - 7}{x - 1}$ (like an improper fraction)

$\frac{\text{trinomial}}{\text{binomial}}$

$\frac{2^{nd}\text{ degree}}{1^{st}\text{ degree}}$

Sc

Alg071-03



Lecture 71: page 3

Example 2 (cont.)

$$\begin{array}{r} x \\ \underline{x} - 1\overline{)\,\underline{x}^2 + 6x - 7} \end{array}$$

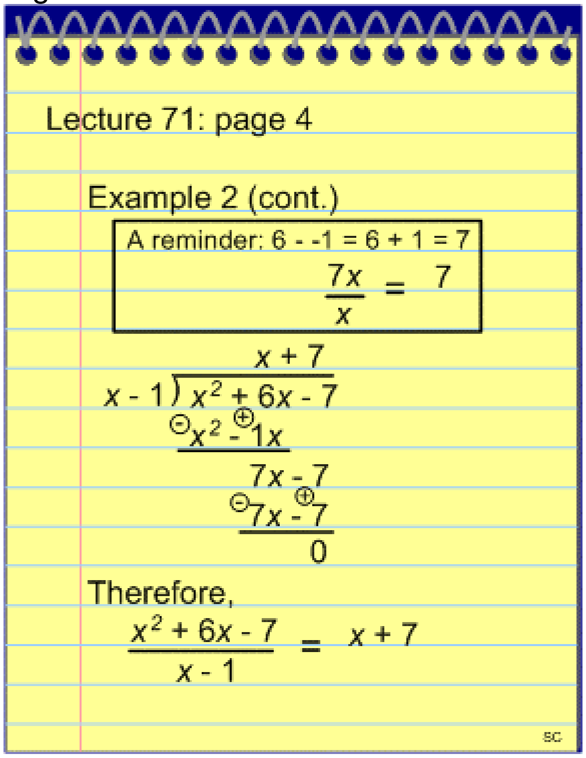
[divide a little at a time]

$\frac{x^2}{x} = x$

$$\begin{array}{r} x \\ x - 1\overline{)\,x^2 + 6x - 7} \\ \underline{{}^{\ominus}x^2\ {}^{\oplus}\!- 1x} \\ 7x - 7 \end{array}$$

SC

Alg-071-04



Lecture 71: page 4

Example 2 (cont.)

A reminder: 6 - -1 = 6 + 1 = 7

$\frac{7x}{x} = 7$

$$\begin{array}{r} x + 7 \\ x - 1\overline{)\,x^2 + 6x - 7} \\ \underline{{}^{\ominus}x^2\ {}^{\oplus}\!- 1x} \\ 7x - 7 \\ \underline{{}^{\ominus}7x\ {}^{\oplus}\!- 7} \\ 0 \end{array}$$

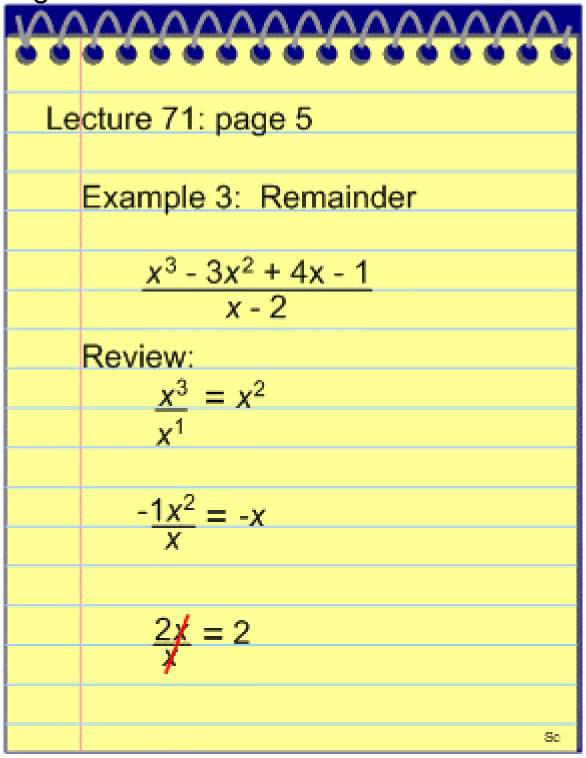
Therefore,

$\frac{x^2 + 6x - 7}{x - 1} = x + 7$

SC

Lecture 71 Notes, Continued

Alg071-05



Lecture 71: page 5

Example 3: Remainder

$$\frac{x^3 - 3x^2 + 4x - 1}{x - 2}$$

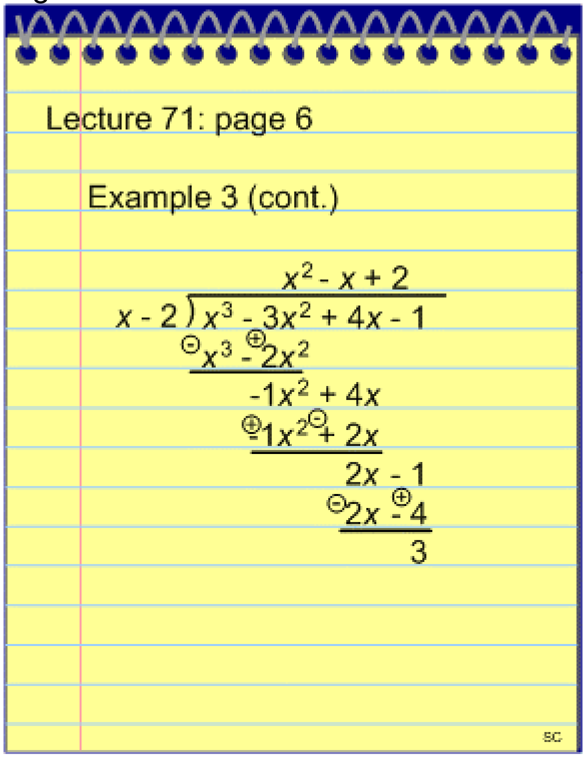
Review:

$$\frac{x^3}{x^1} = x^2$$

$$\frac{-1x^2}{x} = -x$$

$$\frac{2\cancel{x}}{\cancel{x}} = 2$$

Alg071-06

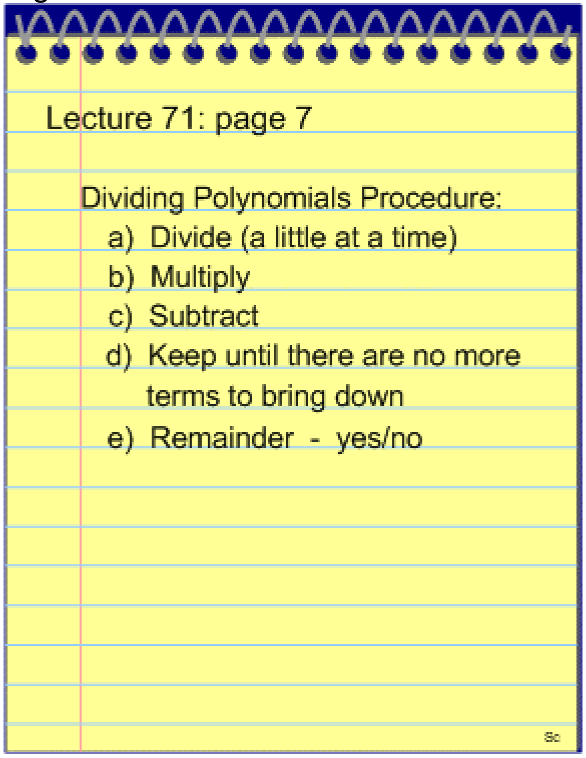


Lecture 71: page 6

Example 3 (cont.)

$$\begin{array}{r} x^2 - x + 2 \\ x - 2\,\overline{)\,x^3 - 3x^2 + 4x - 1} \\ \underline{\ominus x^3 \overset{\oplus}{-} 2x^2} \\ -1x^2 + 4x \\ \underline{\overset{\oplus}{-}1x^2 \overset{\ominus}{+} 2x} \\ 2x - 1 \\ \underline{\ominus 2x \overset{\oplus}{-} 4} \\ 3 \end{array}$$

Alg071-07



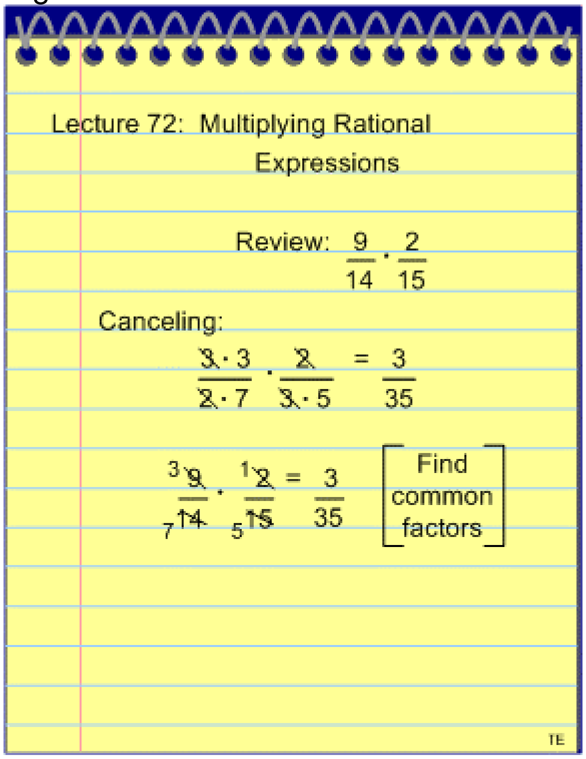
Lecture 71: page 7

Dividing Polynomials Procedure:

a) Divide (a little at a time)
b) Multiply
c) Subtract
d) Keep until there are no more terms to bring down
e) Remainder - yes/no

Lecture 72 Notes

Alg072-01



Lecture 72: Multiplying Rational Expressions

Review: $\frac{9}{14} \cdot \frac{2}{15}$

Canceling:

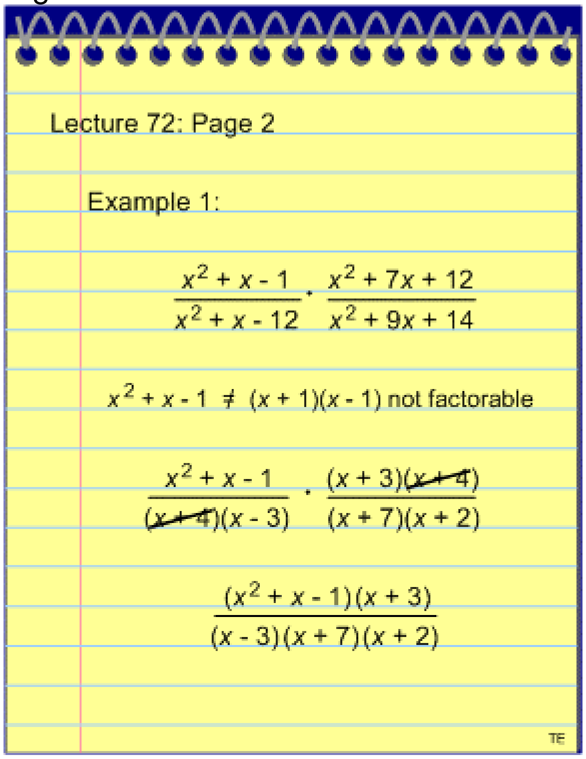
$$\frac{\cancel{3} \cdot 3}{\cancel{2} \cdot 7} \cdot \frac{\cancel{2}}{\cancel{3} \cdot 5} = \frac{3}{35}$$

$$\frac{\overset{3}{\cancel{9}}}{\underset{7}{\cancel{14}}} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{5}{\cancel{15}}} = \frac{3}{35}$$

Find
common
factors

TE

Alg072-02



Lecture 72: Page 2

Example 1:

$$\frac{x^2 + x - 1}{x^2 + x - 12} \cdot \frac{x^2 + 7x + 12}{x^2 + 9x + 14}$$

$x^2 + x - 1 \neq (x + 1)(x - 1)$ not factorable

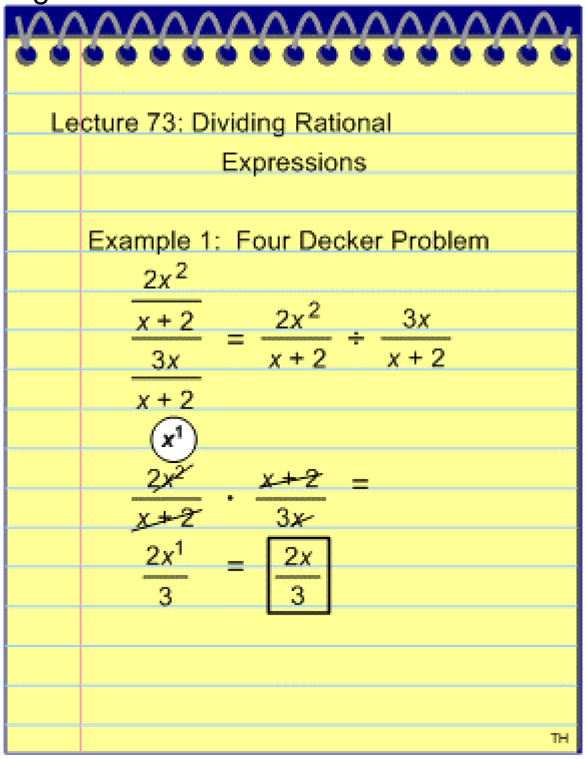
$$\frac{x^2 + x - 1}{\cancel{(x + 4)}(x - 3)} \cdot \frac{(x + 3)\cancel{(x + 4)}}{(x + 7)(x + 2)}$$

$$\frac{(x^2 + x - 1)(x + 3)}{(x - 3)(x + 7)(x + 2)}$$

TE

Lecture 73 Notes

Alg073-01



Lecture 73: Dividing Rational Expressions

Example 1: Four Decker Problem

$$\frac{\;\frac{2x^2}{x+2}\;}{\;\frac{3x}{x+2}\;} = \frac{2x^2}{x+2} \div \frac{3x}{x+2}$$

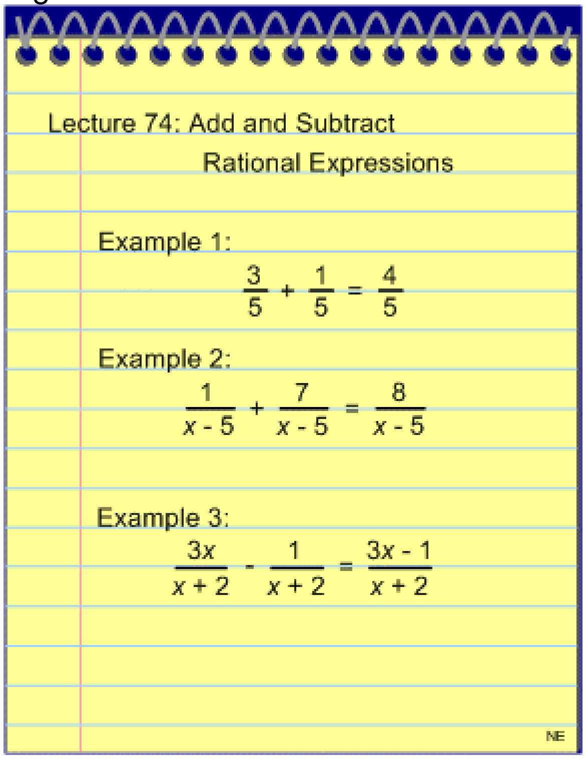
$$\frac{2\overset{x^1}{\cancel{x^2}}}{\cancel{x+2}} \cdot \frac{\cancel{x+2}}{3\cancel{x}} =$$

$$\frac{2x^1}{3} = \boxed{\frac{2x}{3}}$$

TH

Lecture 74 Notes

Alg074-01



Lecture 74: Add and Subtract Rational Expressions

Example 1:

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$$

Example 2:

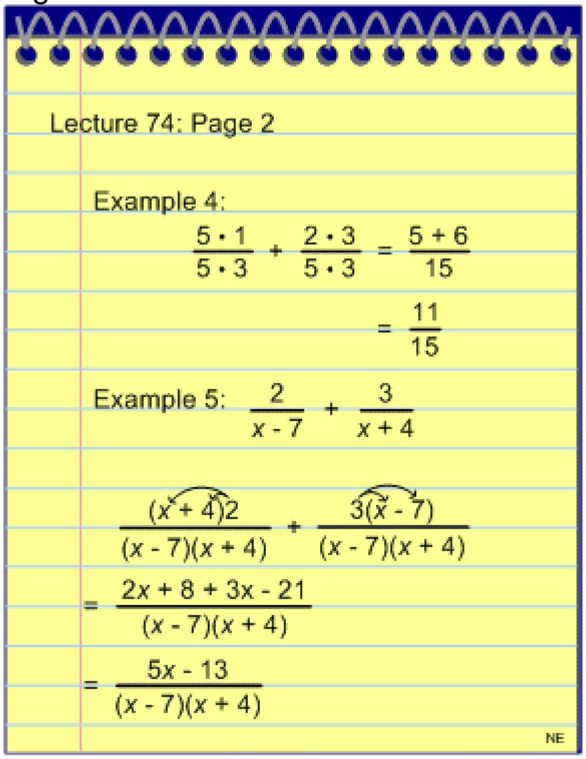
$$\frac{1}{x - 5} + \frac{7}{x - 5} = \frac{8}{x - 5}$$

Example 3:

$$\frac{3x}{x + 2} - \frac{1}{x + 2} = \frac{3x - 1}{x + 2}$$

NE

Alg074-02



Lecture 74: Page 2

Example 4:

$$\frac{5 \cdot 1}{5 \cdot 3} + \frac{2 \cdot 3}{5 \cdot 3} = \frac{5 + 6}{15}$$

$$= \frac{11}{15}$$

Example 5: $\frac{2}{x - 7} + \frac{3}{x + 4}$

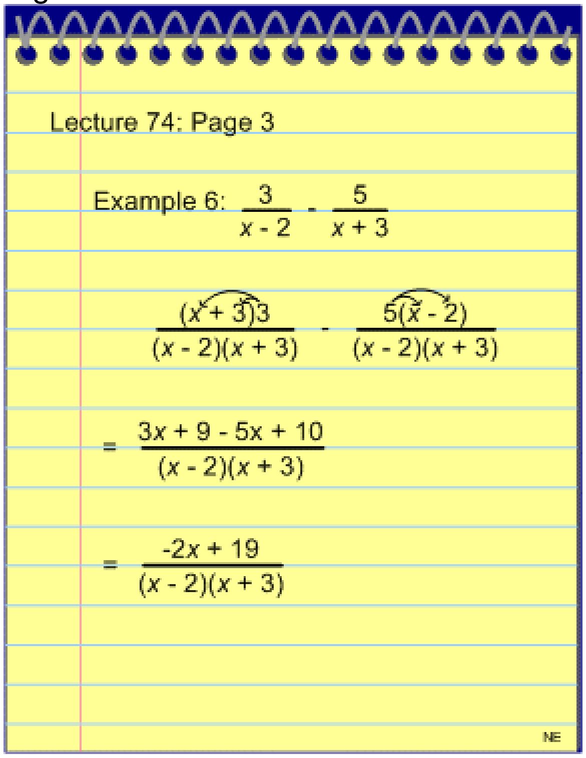
$$\frac{(x + 4)2}{(x - 7)(x + 4)} + \frac{3(x - 7)}{(x - 7)(x + 4)}$$

$$= \frac{2x + 8 + 3x - 21}{(x - 7)(x + 4)}$$

$$= \frac{5x - 13}{(x - 7)(x + 4)}$$

NE

Alg074-03



Lecture 74: Page 3

Example 6: $\frac{3}{x - 2} - \frac{5}{x + 3}$

$$\frac{(x + 3)3}{(x - 2)(x + 3)} - \frac{5(x - 2)}{(x - 2)(x + 3)}$$

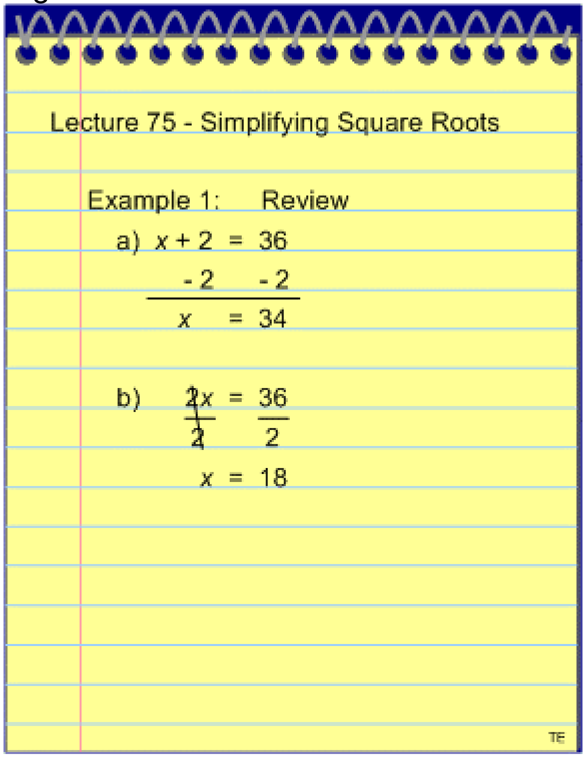
$$= \frac{3x + 9 - 5x + 10}{(x - 2)(x + 3)}$$

$$= \frac{-2x + 19}{(x - 2)(x + 3)}$$

NE

Lecture 75 Notes

Alg075-01



Lecture 75 - Simplifying Square Roots

Example 1: Review

a) $x + 2 = 36$

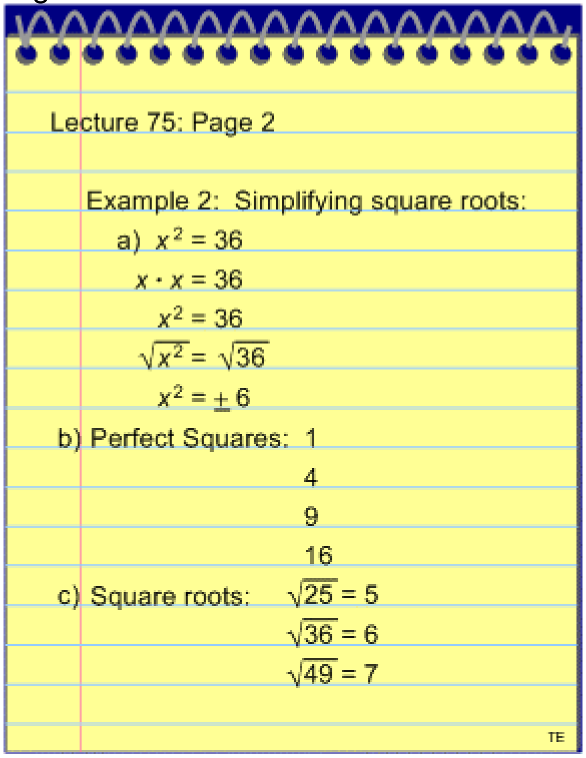
$$\begin{array}{rcl} x + 2 & = & 36 \\ -2 & & -2 \\ \hline x & = & 34 \end{array}$$

b) $\frac{\cancel{2}x}{\cancel{2}} = \frac{36}{2}$

$x = 18$

TE

Alg075-02



Lecture 75: Page 2

Example 2: Simplifying square roots:

a) $x^2 = 36$

$x \cdot x = 36$

$x^2 = 36$

$\sqrt{x^2} = \sqrt{36}$

$x^2 = \pm 6$

b) Perfect Squares: 1

4

9

16

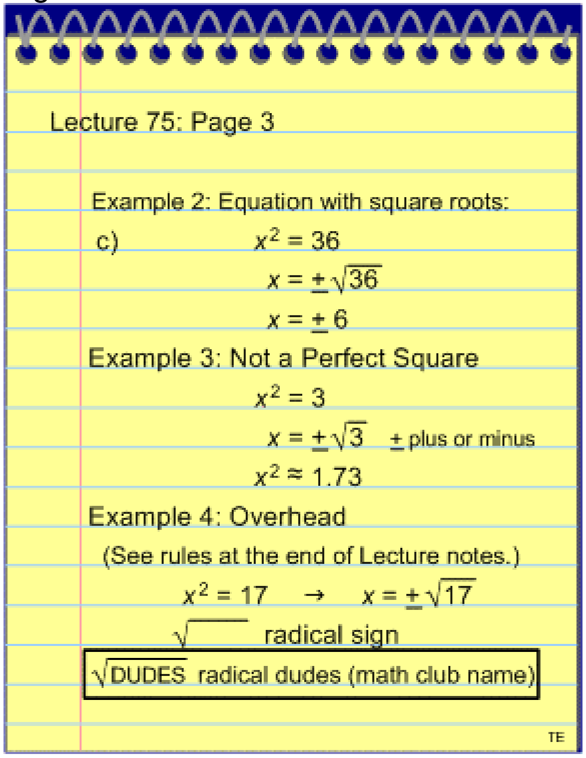
c) Square roots: $\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

TE

Alg075-03



Lecture 75: Page 3

Example 2: Equation with square roots:

c) $x^2 = 36$

$x = \pm\sqrt{36}$

$x = \pm 6$

Example 3: Not a Perfect Square

$x^2 = 3$

$x = \pm\sqrt{3}$ ± plus or minus

$x^2 \approx 1.73$

Example 4: Overhead

(See rules at the end of Lecture notes.)

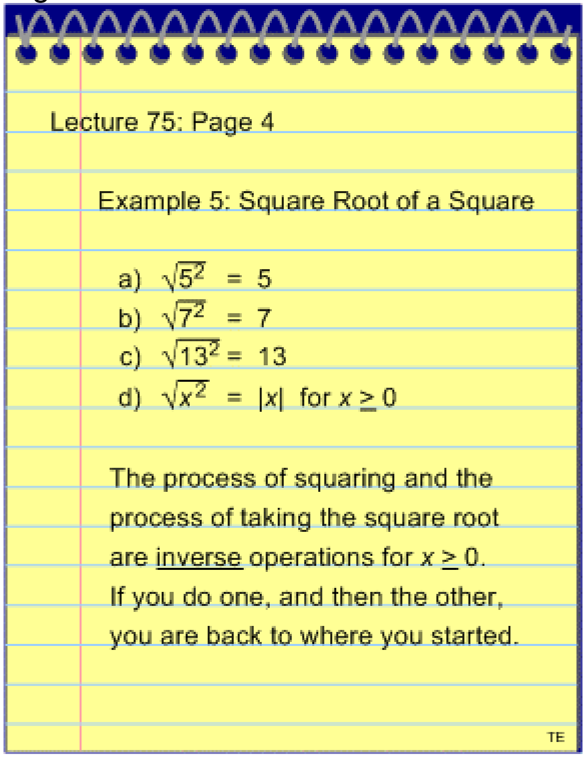
$x^2 = 17 \rightarrow x = \pm\sqrt{17}$

$\sqrt{}$ radical sign

$\boxed{\sqrt{\text{DUDES}}\ \text{radical dudes (math club name)}}$

TE

Alg075-04



Lecture 75: Page 4

Example 5: Square Root of a Square

a) $\sqrt{5^2} = 5$

b) $\sqrt{7^2} = 7$

c) $\sqrt{13^2} = 13$

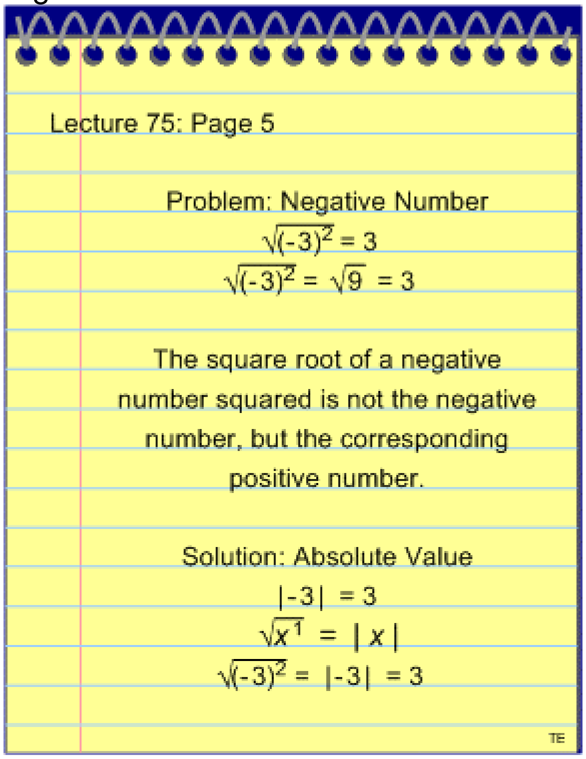
d) $\sqrt{x^2} = |x|$ for $x \geq 0$

The process of squaring and the process of taking the square root are inverse operations for $x \geq 0$. If you do one, and then the other, you are back to where you started.

TE

Lecture 75 Notes, Continued

Alg075-05



Lecture 75: Page 5

Problem: Negative Number

$\sqrt{(-3)^2} = 3$

$\sqrt{(-3)^2} = \sqrt{9} = 3$

The square root of a negative number squared is not the negative number, but the corresponding positive number.

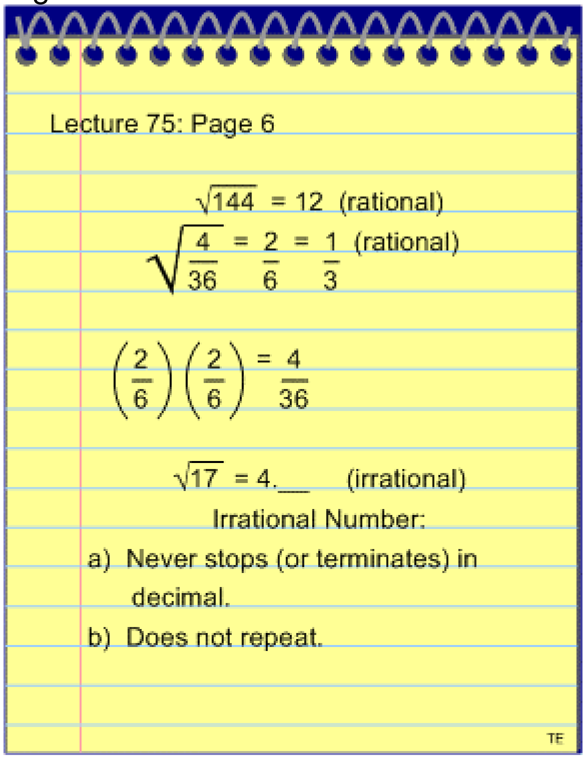
Solution: Absolute Value

$|-3| = 3$

$\sqrt{x^1} = |x|$

$\sqrt{(-3)^2} = |-3| = 3$

Alg075-06



Lecture 75: Page 6

$\sqrt{144} = 12$ (rational)

$\sqrt{\frac{4}{36}} = \frac{2}{6} = \frac{1}{3}$ (rational)

$\left(\frac{2}{6}\right)\left(\frac{2}{6}\right) = \frac{4}{36}$

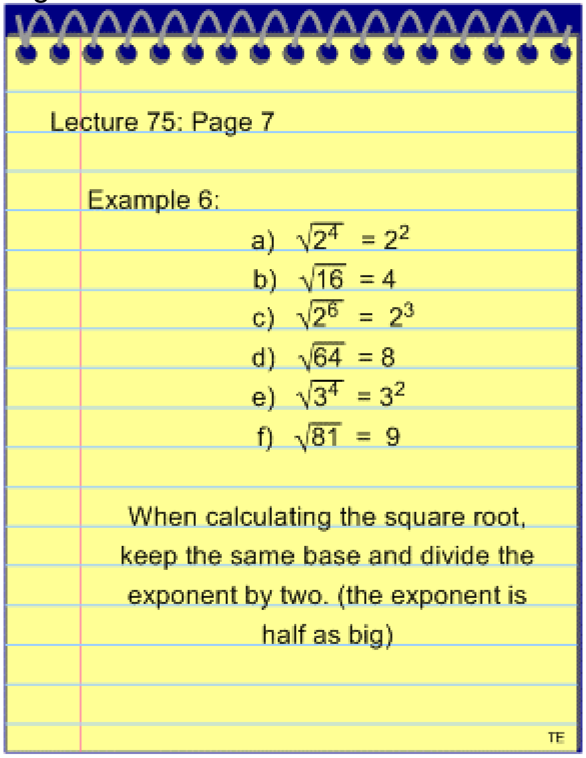
$\sqrt{17} = 4.$___ (irrational)

Irrational Number:

a) Never stops (or terminates) in decimal.

b) Does not repeat.

Alg075-07



Lecture 75: Page 7

Example 6:

a) $\sqrt{2^4} = 2^2$

b) $\sqrt{16} = 4$

c) $\sqrt{2^6} = 2^3$

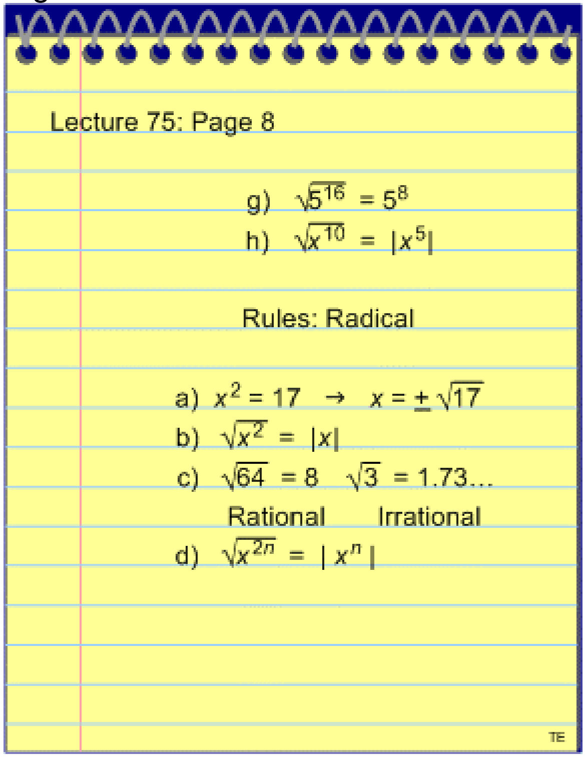
d) $\sqrt{64} = 8$

e) $\sqrt{3^4} = 3^2$

f) $\sqrt{81} = 9$

When calculating the square root, keep the same base and divide the exponent by two. (the exponent is half as big)

Alg075-08



Lecture 75: Page 8

g) $\sqrt{5^{16}} = 5^8$

h) $\sqrt{x^{10}} = |x^5|$

Rules: Radical

a) $x^2 = 17 \rightarrow x = \pm\sqrt{17}$

b) $\sqrt{x^2} = |x|$

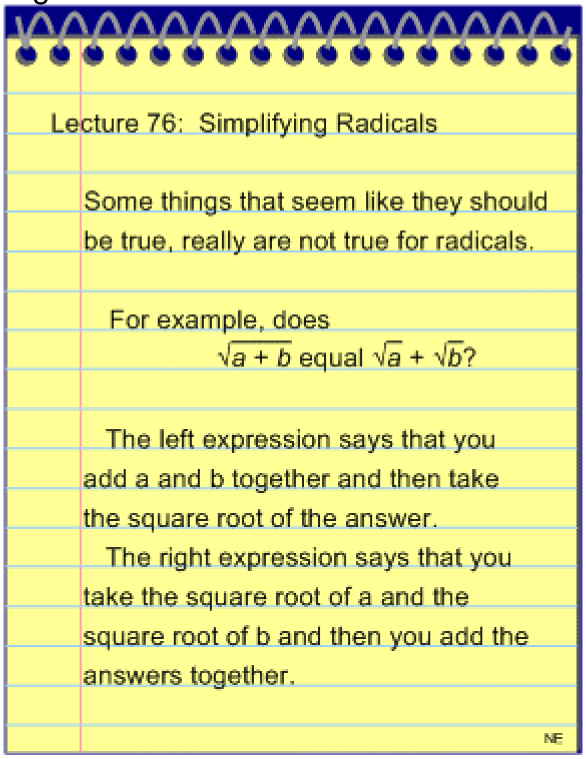
c) $\sqrt{64} = 8$ $\sqrt{3} = 1.73\ldots$

Rational Irrational

d) $\sqrt{x^{2n}} = |x^n|$

Lecture 76 Notes

Alg076-01



Lecture 76: Simplifying Radicals

Some things that seem like they should be true, really are not true for radicals.

For example, does

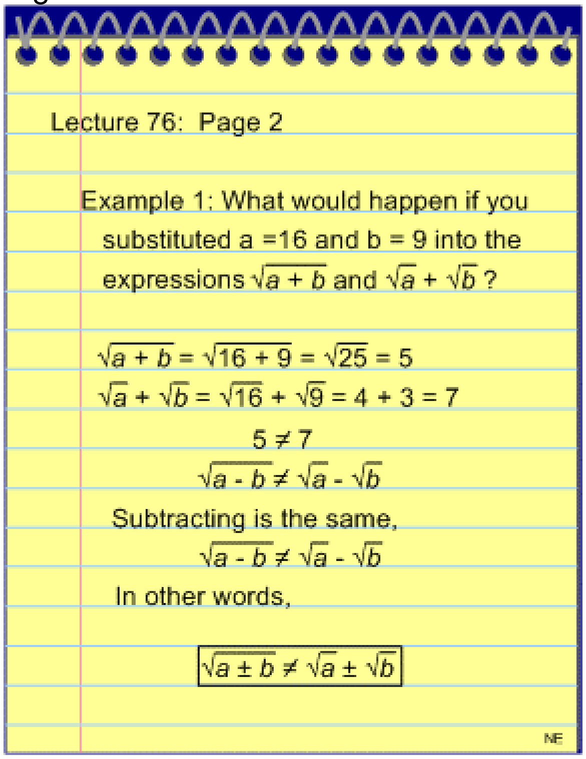
$\sqrt{a+b}$ equal $\sqrt{a} + \sqrt{b}$?

The left expression says that you add a and b together and then take the square root of the answer.

The right expression says that you take the square root of a and the square root of b and then you add the answers together.

NE

Alg076-02



Lecture 76: Page 2

Example 1: What would happen if you substituted a =16 and b = 9 into the expressions $\sqrt{a+b}$ and $\sqrt{a} + \sqrt{b}$?

$\sqrt{a+b} = \sqrt{16+9} = \sqrt{25} = 5$

$\sqrt{a} + \sqrt{b} = \sqrt{16} + \sqrt{9} = 4 + 3 = 7$

$5 \neq 7$

$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$

Subtracting is the same,

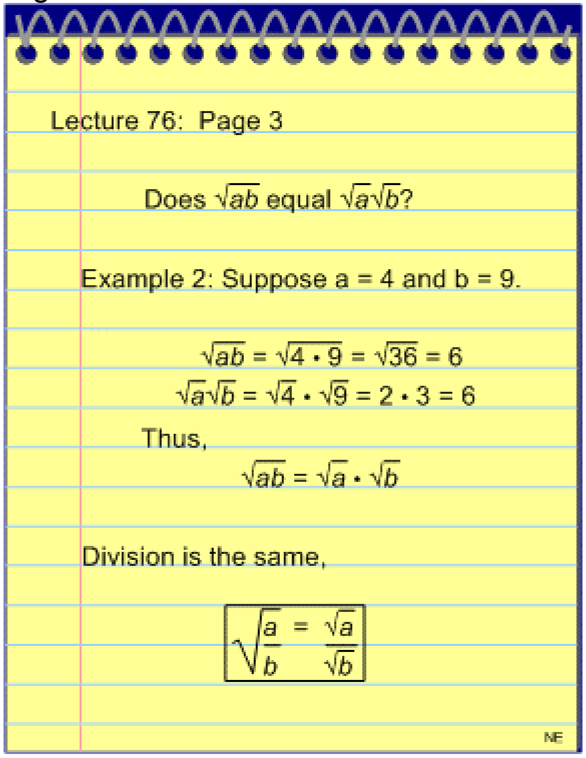
$\sqrt{a-b} \neq \sqrt{a} - \sqrt{b}$

In other words,

$\boxed{\sqrt{a \pm b} \neq \sqrt{a} \pm \sqrt{b}}$

NE

Alg076-03



Lecture 76: Page 3

Does $\sqrt{ab}$ equal $\sqrt{a}\sqrt{b}$?

Example 2: Suppose a = 4 and b = 9.

$\sqrt{ab} = \sqrt{4 \cdot 9} = \sqrt{36} = 6$

$\sqrt{a}\sqrt{b} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$

Thus,

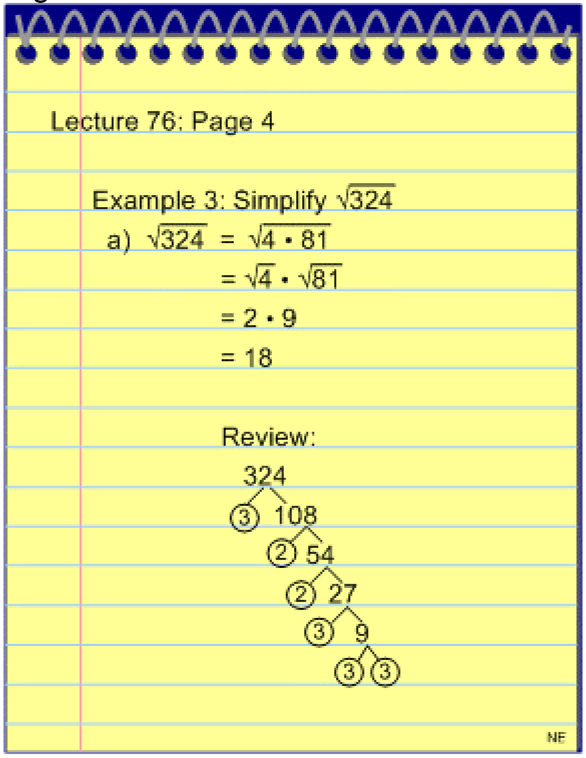
$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

Division is the same,

$\boxed{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}}$

NE

Alg076-04



Lecture 76: Page 4

Example 3: Simplify $\sqrt{324}$

a) $\sqrt{324} = \sqrt{4 \cdot 81}$

$= \sqrt{4} \cdot \sqrt{81}$

$= 2 \cdot 9$

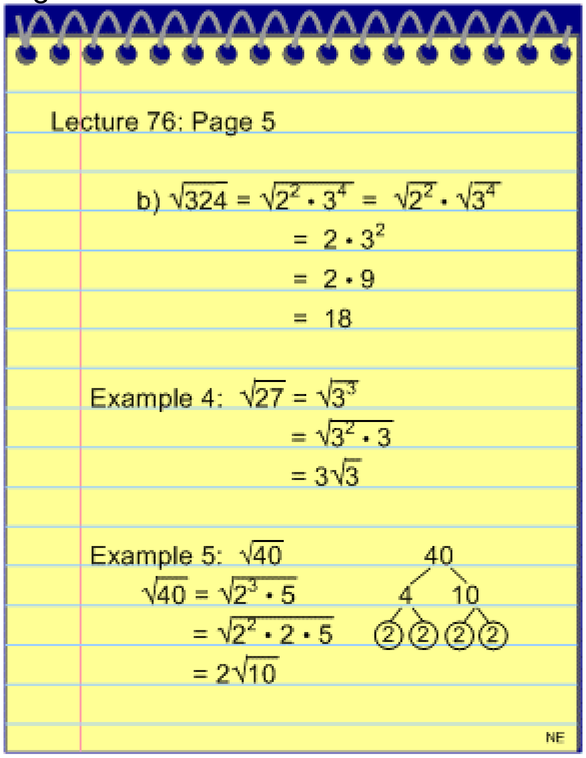
$= 18$

Review:

NE

Lecture 76 Notes, Continued

Alg076-05



Lecture 76: Page 5

b) $\sqrt{324} = \sqrt{2^2 \cdot 3^4} = \sqrt{2^2} \cdot \sqrt{3^4}$

$= 2 \cdot 3^2$

$= 2 \cdot 9$

$= 18$

Example 4: $\sqrt{27} = \sqrt{3^3}$

$= \sqrt{3^2 \cdot 3}$

$= 3\sqrt{3}$

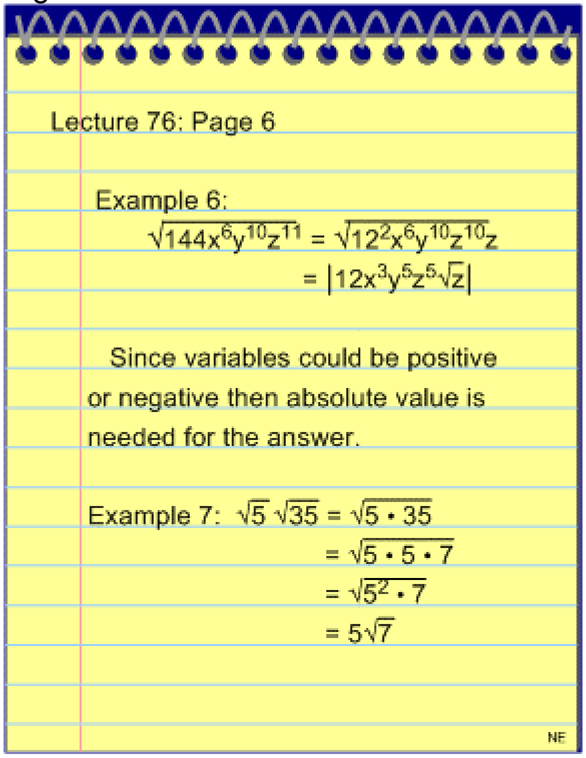
Example 5: $\sqrt{40}$

$\sqrt{40} = \sqrt{2^3 \cdot 5}$

$= \sqrt{2^2 \cdot 2 \cdot 5}$

$= 2\sqrt{10}$

Alg076-06



Lecture 76: Page 6

Example 6:

$\sqrt{144x^6y^{10}z^{11}} = \sqrt{12^2x^6y^{10}z^{10}z}$

$= |12x^3y^5z^5\sqrt{z}|$

Since variables could be positive or negative then absolute value is needed for the answer.

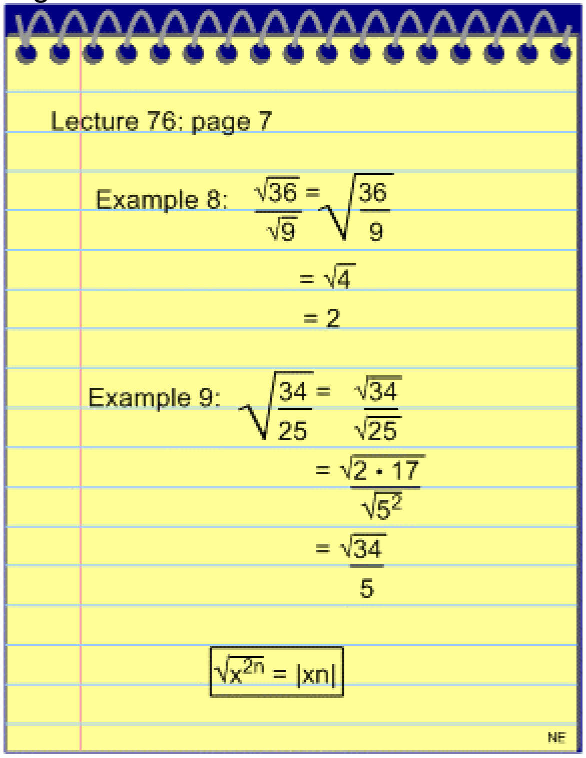
Example 7: $\sqrt{5}\,\sqrt{35} = \sqrt{5 \cdot 35}$

$= \sqrt{5 \cdot 5 \cdot 7}$

$= \sqrt{5^2 \cdot 7}$

$= 5\sqrt{7}$

Alg076-07



Lecture 76: page 7

Example 8: $\dfrac{\sqrt{36}}{\sqrt{9}} = \sqrt{\dfrac{36}{9}}$

$= \sqrt{4}$

$= 2$

Example 9: $\sqrt{\dfrac{34}{25}} = \dfrac{\sqrt{34}}{\sqrt{25}}$

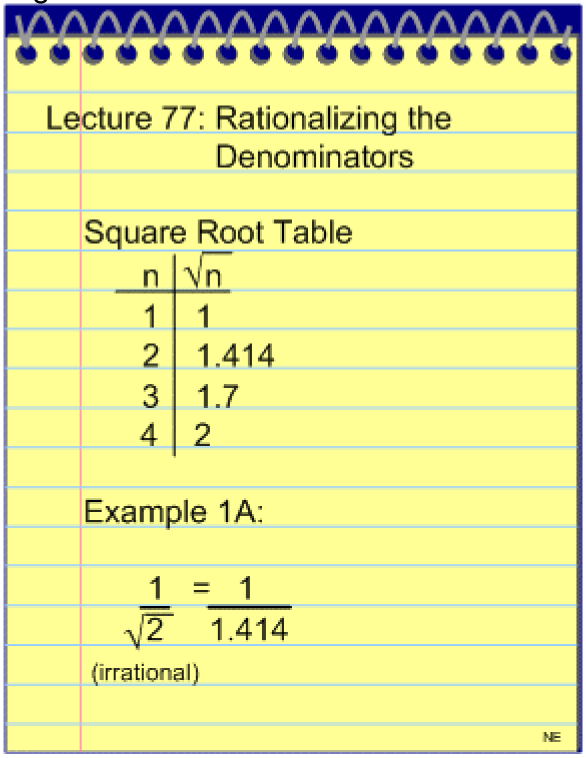
$= \dfrac{\sqrt{2 \cdot 17}}{\sqrt{5^2}}$

$= \dfrac{\sqrt{34}}{5}$

$\boxed{\sqrt{x^{2n}} = |xn|}$

Lecture 77 Notes

Alg077-01



Lecture 77: Rationalizing the Denominators

Square Root Table

n	$\sqrt{n}$
1	1
2	1.414
3	1.7
4	2

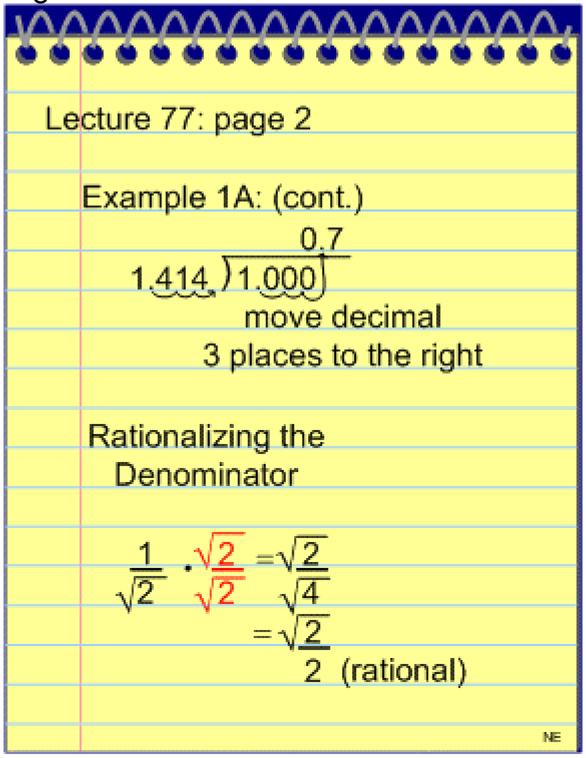
Example 1A:

$$\frac{1}{\sqrt{2}} = \frac{1}{1.414}$$

(irrational)

NE

Alg077-02



Lecture 77: page 2

Example 1A: (cont.)

$$\begin{array}{r} 0.7 \\ 1.414\overline{)1.000} \end{array}$$

move decimal
3 places to the right

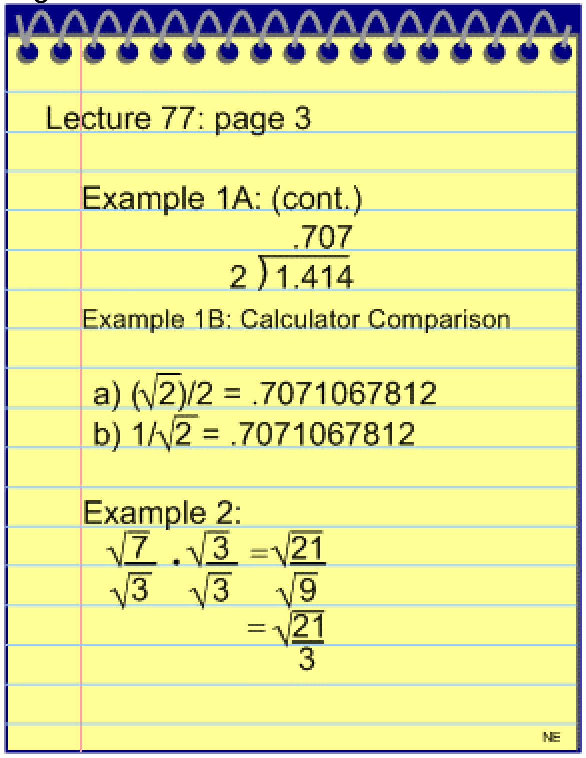
Rationalizing the Denominator

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}}$$

$$= \frac{\sqrt{2}}{2}$$ (rational)

NE

Alg077-03



Lecture 77: page 3

Example 1A: (cont.)

$$\begin{array}{r} .707 \\ 2\overline{)1.414} \end{array}$$

Example 1B: Calculator Comparison

a) $(\sqrt{2})/2 = .7071067812$

b) $1/\sqrt{2} = .7071067812$

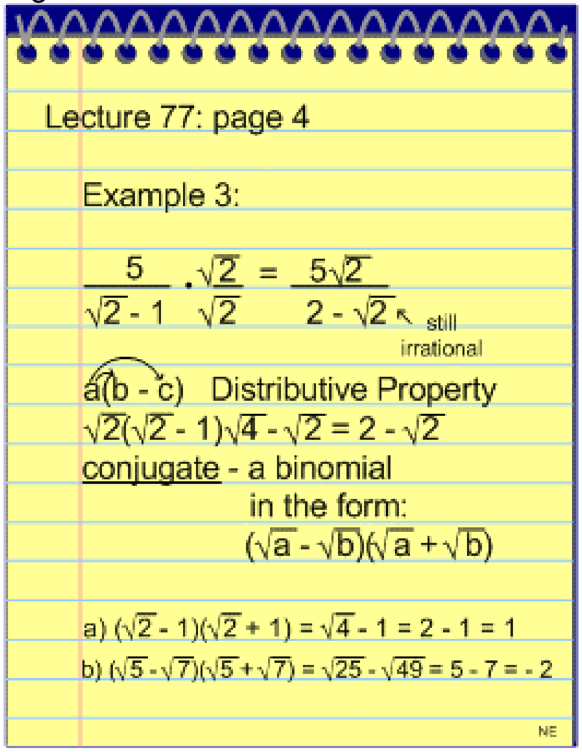
Example 2:

$$\frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{\sqrt{9}}$$

$$= \frac{\sqrt{21}}{3}$$

NE

Alg077-04



Lecture 77: page 4

Example 3:

$$\frac{5}{\sqrt{2}-1} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2-\sqrt{2}}$$ ↖ still irrational

$a(b - c)$ Distributive Property

$\sqrt{2}(\sqrt{2}-1)\sqrt{4}-\sqrt{2} = 2-\sqrt{2}$

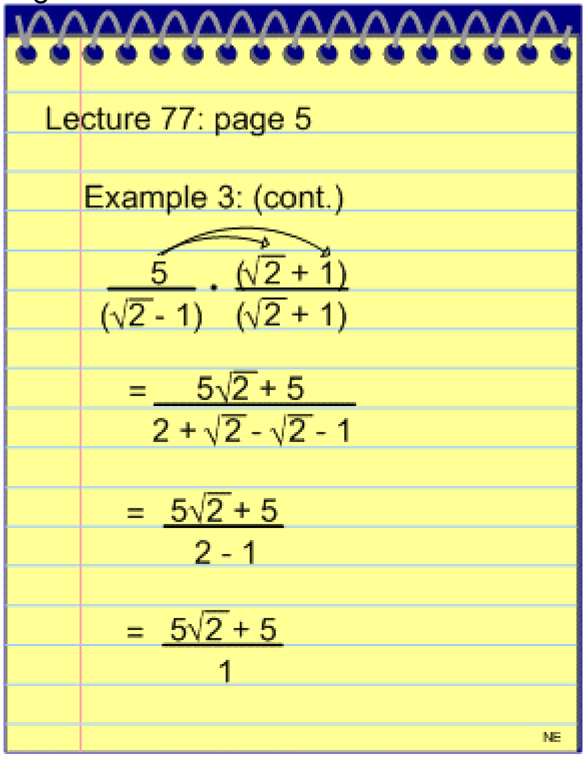
<u>conjugate</u> - a binomial
in the form:
$(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$

a) $(\sqrt{2}-1)(\sqrt{2}+1) = \sqrt{4}-1 = 2-1 = 1$

b) $(\sqrt{5}-\sqrt{7})(\sqrt{5}+\sqrt{7}) = \sqrt{25}-\sqrt{49} = 5-7 = -2$

NE

Alg077-05



Lecture 77: page 5

Example 3: (cont.)

$$\frac{5}{(\sqrt{2}-1)} \cdot \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}$$

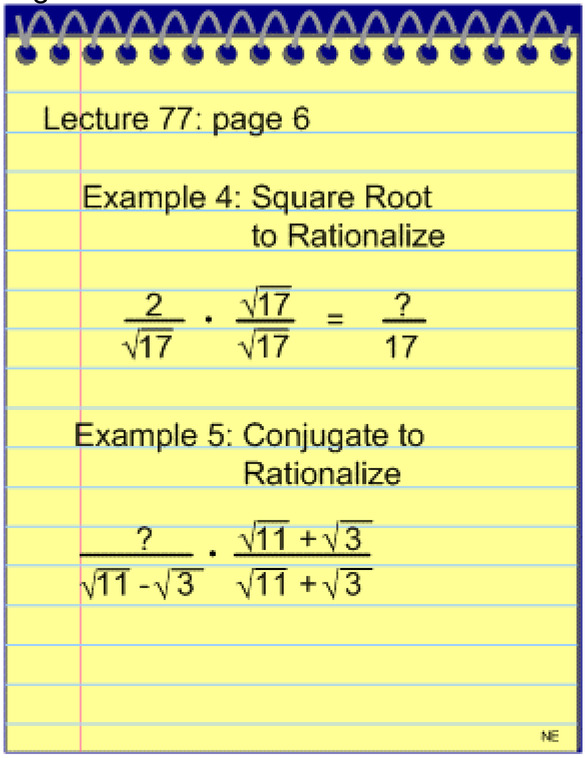
$$= \frac{5\sqrt{2}+5}{2+\sqrt{2}-\sqrt{2}-1}$$

$$= \frac{5\sqrt{2}+5}{2-1}$$

$$= \frac{5\sqrt{2}+5}{1}$$

NE

Alg077-06



Lecture 77: page 6

Example 4: Square Root to Rationalize

$$\frac{2}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = \frac{?}{17}$$

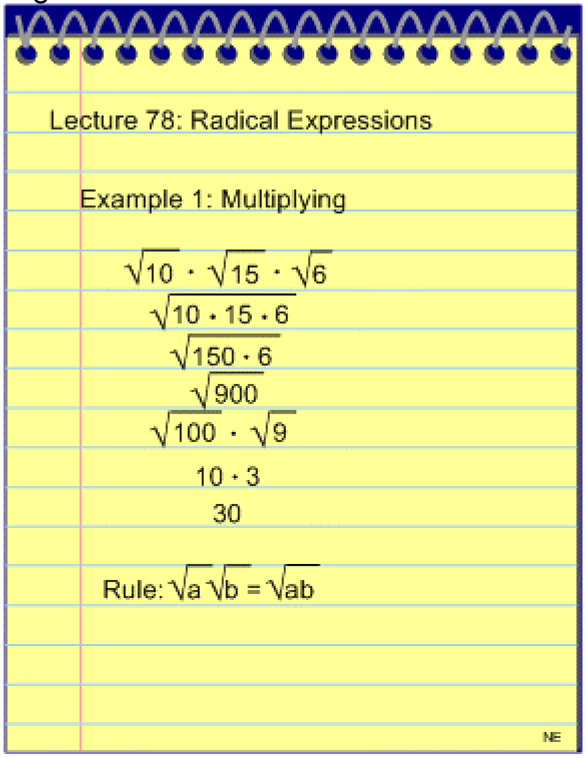
Example 5: Conjugate to Rationalize

$$\frac{?}{\sqrt{11}-\sqrt{3}} \cdot \frac{\sqrt{11}+\sqrt{3}}{\sqrt{11}+\sqrt{3}}$$

NE

Lecture 78 Notes

Alg078-01



Lecture 78: Radical Expressions

Example 1: Multiplying

$\sqrt{10}\cdot\sqrt{15}\cdot\sqrt{6}$

$\sqrt{10\cdot 15\cdot 6}$

$\sqrt{150\cdot 6}$

$\sqrt{900}$

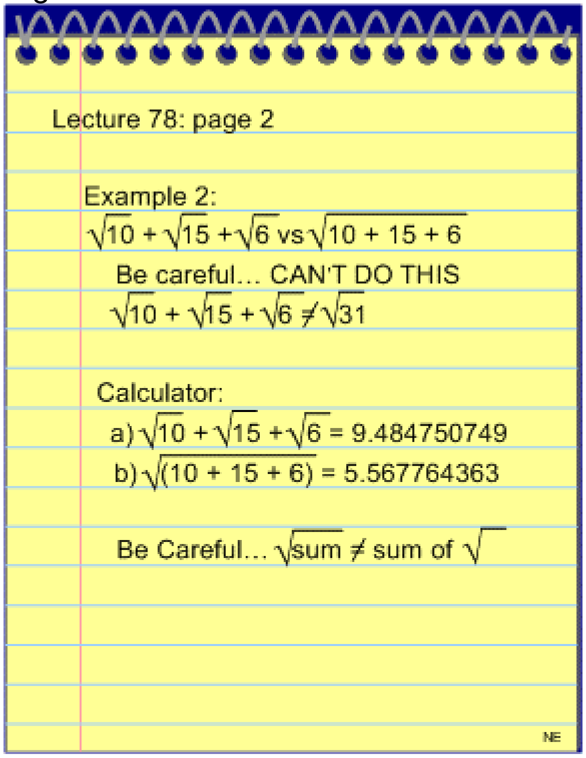
$\sqrt{100}\cdot\sqrt{9}$

$10\cdot 3$

30

Rule: $\sqrt{a}\sqrt{b}=\sqrt{ab}$

Alg078-02



Lecture 78: page 2

Example 2:

$\sqrt{10}+\sqrt{15}+\sqrt{6}$ vs $\sqrt{10+15+6}$

Be careful… CAN'T DO THIS

$\sqrt{10}+\sqrt{15}+\sqrt{6}\neq\sqrt{31}$

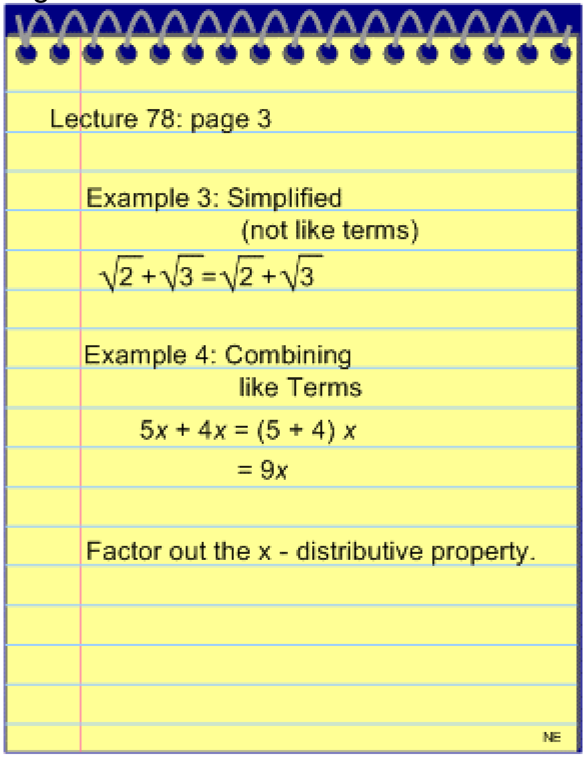
Calculator:

a) $\sqrt{10}+\sqrt{15}+\sqrt{6}=9.484750749$

b) $\sqrt{(10+15+6)}=5.567764363$

Be Careful… $\sqrt{\text{sum}}\neq$ sum of $\sqrt{\ }$

Alg078-03



Lecture 78: page 3

Example 3: Simplified (not like terms)

$\sqrt{2}+\sqrt{3}=\sqrt{2}+\sqrt{3}$

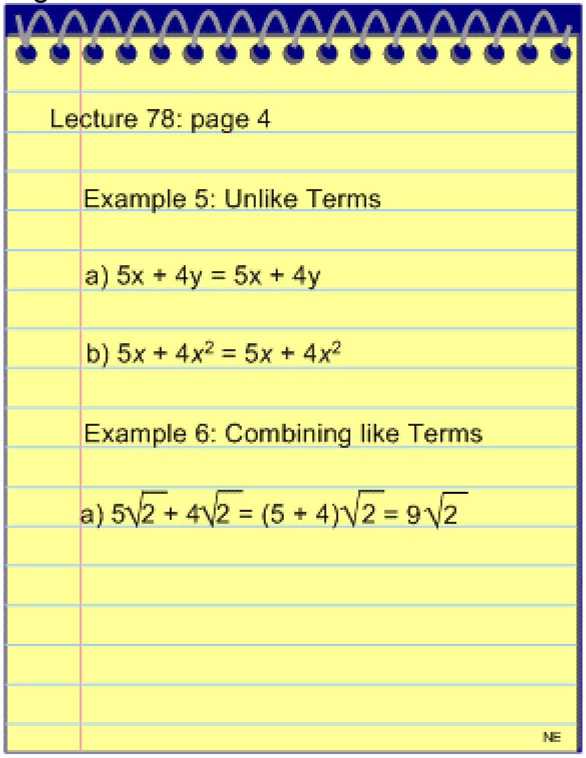
Example 4: Combining like Terms

$5x+4x=(5+4)\,x$

$=9x$

Factor out the x - distributive property.

Alg078-04



Lecture 78: page 4

Example 5: Unlike Terms

a) $5x+4y=5x+4y$

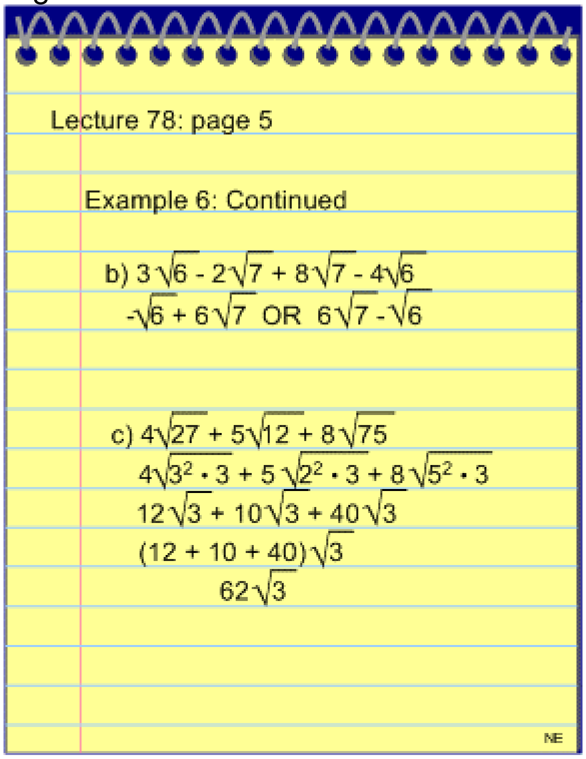
b) $5x+4x^2=5x+4x^2$

Example 6: Combining like Terms

a) $5\sqrt{2}+4\sqrt{2}=(5+4)\sqrt{2}=9\sqrt{2}$

Lecture 78 Notes, Continued

Alg078-05



Lecture 78: page 5

Example 6: Continued

b) $3\sqrt{6} - 2\sqrt{7} + 8\sqrt{7} - 4\sqrt{6}$

$-\sqrt{6} + 6\sqrt{7}$ OR $6\sqrt{7} - \sqrt{6}$

c) $4\sqrt{27} + 5\sqrt{12} + 8\sqrt{75}$

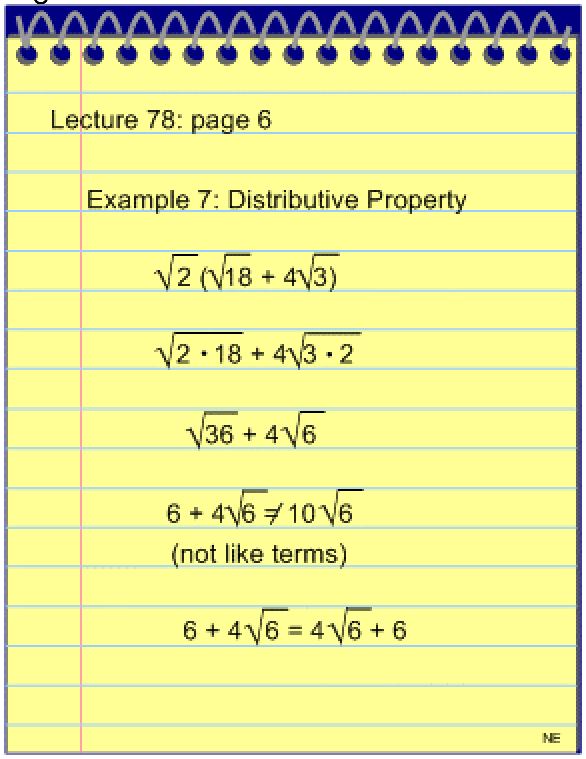
$4\sqrt{3^2 \cdot 3} + 5\sqrt{2^2 \cdot 3} + 8\sqrt{5^2 \cdot 3}$

$12\sqrt{3} + 10\sqrt{3} + 40\sqrt{3}$

$(12 + 10 + 40)\sqrt{3}$

$62\sqrt{3}$

Alg078-06



Lecture 78: page 6

Example 7: Distributive Property

$\sqrt{2}\,(\sqrt{18} + 4\sqrt{3})$

$\sqrt{2 \cdot 18} + 4\sqrt{3 \cdot 2}$

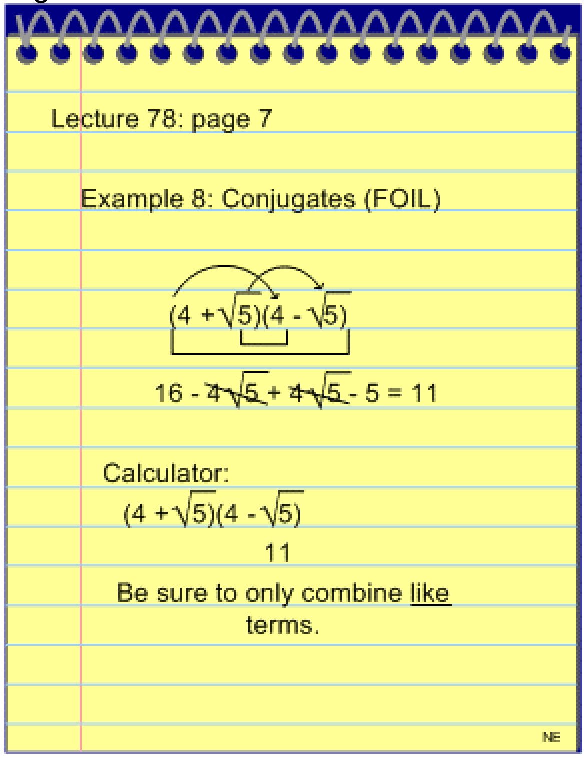
$\sqrt{36} + 4\sqrt{6}$

$6 + 4\sqrt{6} \neq 10\sqrt{6}$

(not like terms)

$6 + 4\sqrt{6} = 4\sqrt{6} + 6$

Alg078-07



Lecture 78: page 7

Example 8: Conjugates (FOIL)

$(4 + \sqrt{5})(4 - \sqrt{5})$

$16 - \cancel{4\sqrt{5}} + \cancel{4\sqrt{5}} - 5 = 11$

Calculator:

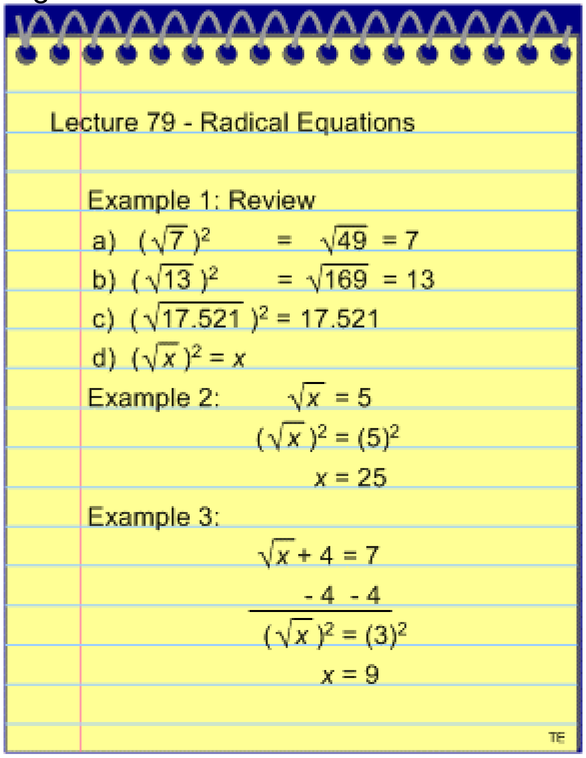
$(4 + \sqrt{5})(4 - \sqrt{5})$

11

Be sure to only combine <u>like</u> terms.

Lecture 79 Notes

Alg079-01



Lecture 79 - Radical Equations

Example 1: Review

a) $(\sqrt{7})^2 = \sqrt{49} = 7$

b) $(\sqrt{13})^2 = \sqrt{169} = 13$

c) $(\sqrt{17.521})^2 = 17.521$

d) $(\sqrt{x})^2 = x$

Example 2: $\sqrt{x} = 5$

$(\sqrt{x})^2 = (5)^2$

$x = 25$

Example 3:

$\sqrt{x} + 4 = 7$

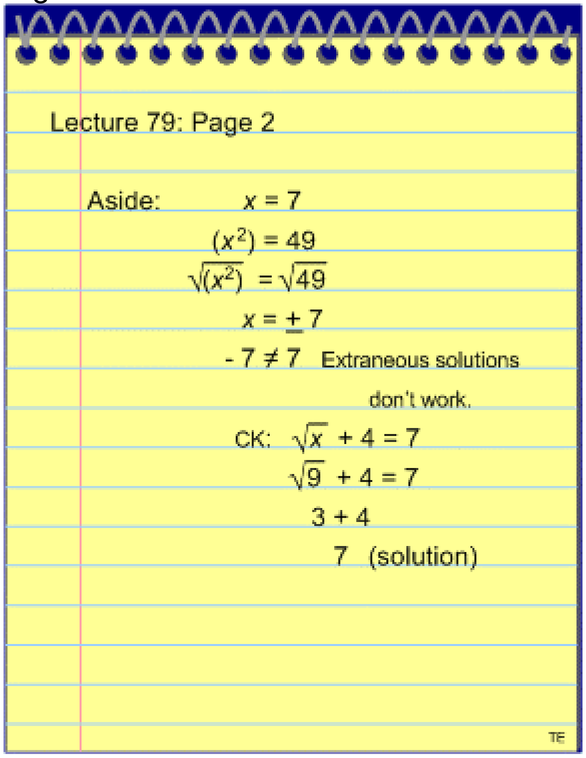
$-4 \quad -4$

$(\sqrt{x})^2 = (3)^2$

$x = 9$

TE

Alg079-02



Lecture 79: Page 2

Aside: $x = 7$

$(x^2) = 49$

$\sqrt{(x^2)} = \sqrt{49}$

$x = \pm 7$

$-7 \neq 7$ Extraneous solutions don't work.

CK: $\sqrt{x} + 4 = 7$

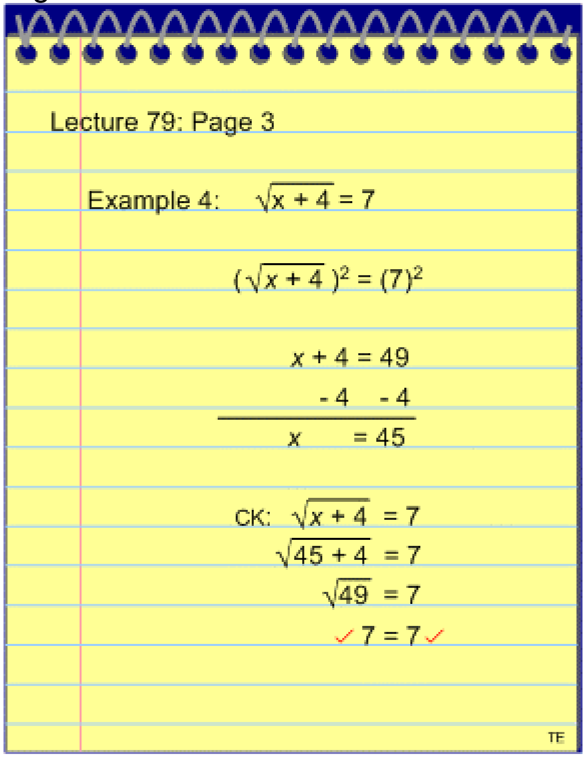
$\sqrt{9} + 4 = 7$

$3 + 4$

7 (solution)

TE

Alg079-03



Lecture 79: Page 3

Example 4: $\sqrt{x + 4} = 7$

$(\sqrt{x + 4})^2 = (7)^2$

$x + 4 = 49$

$-4 \quad -4$

$x = 45$

CK: $\sqrt{x + 4} = 7$

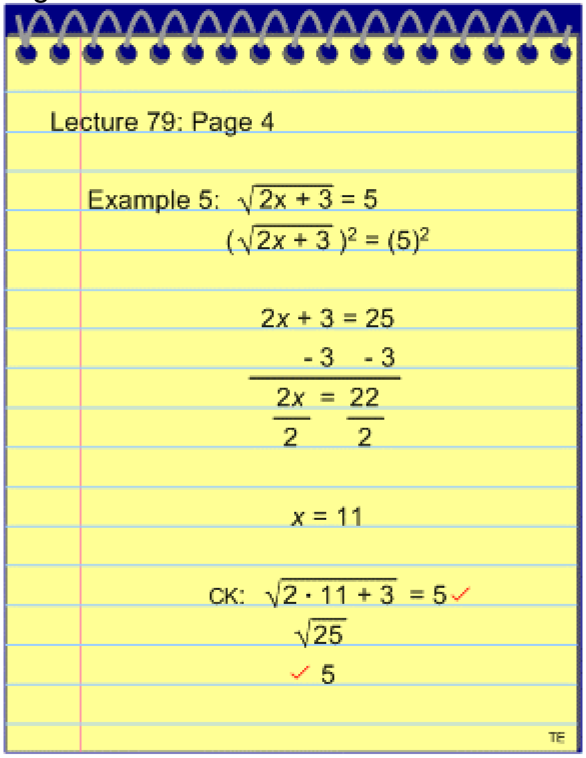
$\sqrt{45 + 4} = 7$

$\sqrt{49} = 7$

✓ $7 = 7$ ✓

TE

Alg079-04



Lecture 79: Page 4

Example 5: $\sqrt{2x + 3} = 5$

$(\sqrt{2x + 3})^2 = (5)^2$

$2x + 3 = 25$

$-3 \quad -3$

$\frac{2x}{2} = \frac{22}{2}$

$x = 11$

CK: $\sqrt{2 \cdot 11 + 3} = 5$ ✓

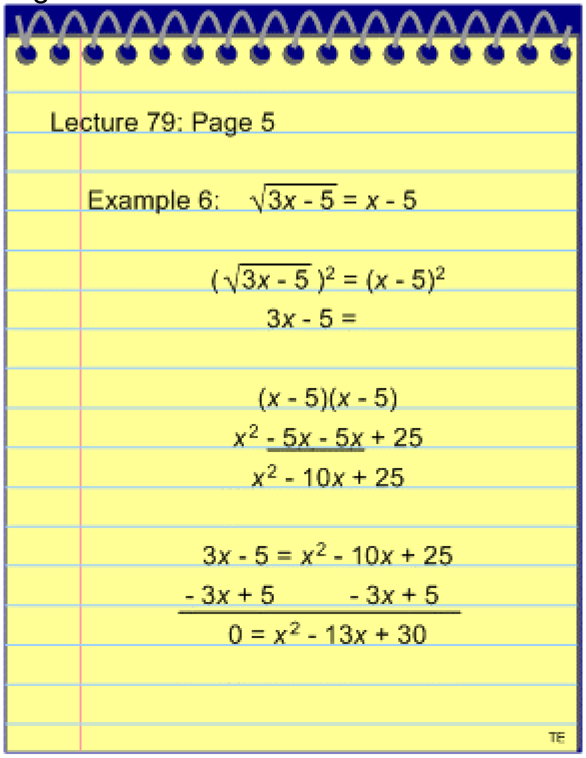
$\sqrt{25}$

✓ 5

TE

Lecture 79 Notes, Continued

Alg079-05



Lecture 79: Page 5

Example 6: $\sqrt{3x - 5} = x - 5$

$(\sqrt{3x - 5})^2 = (x - 5)^2$

$3x - 5 =$

$(x - 5)(x - 5)$

$x^2 \underline{- 5x - 5x} + 25$

$x^2 - 10x + 25$

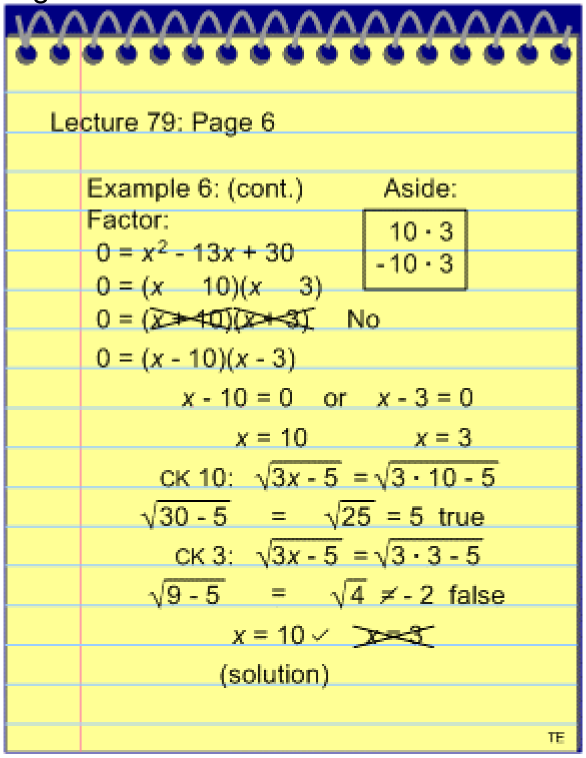
$3x - 5 = x^2 - 10x + 25$

$\underline{- 3x + 5 \qquad - 3x + 5}$

$0 = x^2 - 13x + 30$

TE

Alg079-06



Lecture 79: Page 6

Example 6: (cont.)

Factor:

$0 = x^2 - 13x + 30$

$0 = (x \quad 10)(x \quad 3)$

$0 = \cancel{(x + 10)}\cancel{(x + 3)}$ No

$0 = (x - 10)(x - 3)$

Aside:

$\boxed{\begin{matrix} 10 \cdot 3 \\ -10 \cdot 3 \end{matrix}}$

$x - 10 = 0$ or $x - 3 = 0$

$x = 10 \qquad x = 3$

CK 10: $\sqrt{3x - 5} = \sqrt{3 \cdot 10 - 5}$

$\sqrt{30 - 5} = \sqrt{25} = 5$ true

CK 3: $\sqrt{3x - 5} = \sqrt{3 \cdot 3 - 5}$

$\sqrt{9 - 5} = \sqrt{4} \neq -2$ false

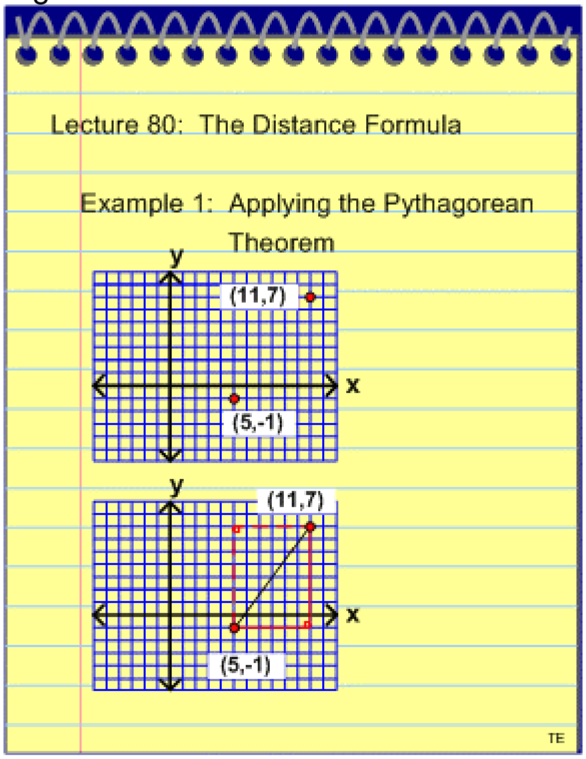
$x = 10$ ✓ $\cancel{x = 3}$

(solution)

TE

Lecture 80 Notes

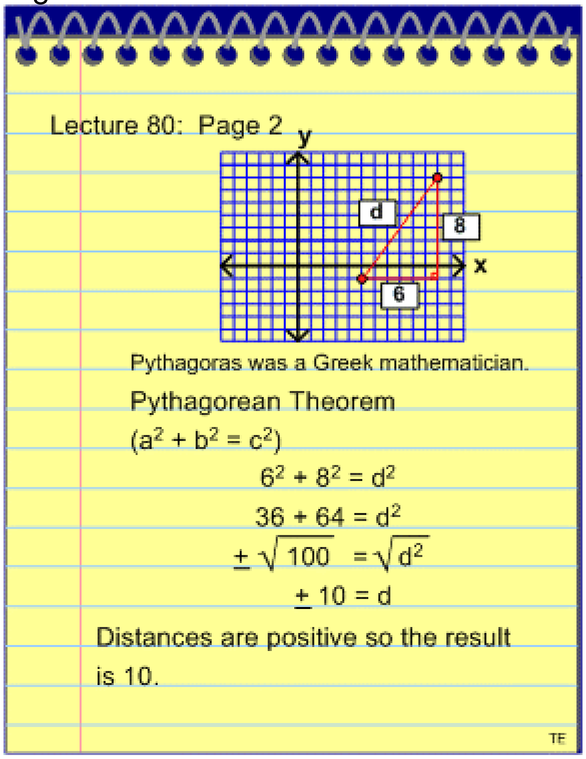
Alg080-01



Lecture 80: The Distance Formula

Example 1: Applying the Pythagorean Theorem

Alg080-02



Lecture 80: Page 2

Pythagoras was a Greek mathematician.

Pythagorean Theorem

$(a^2 + b^2 = c^2)$

$6^2 + 8^2 = d^2$

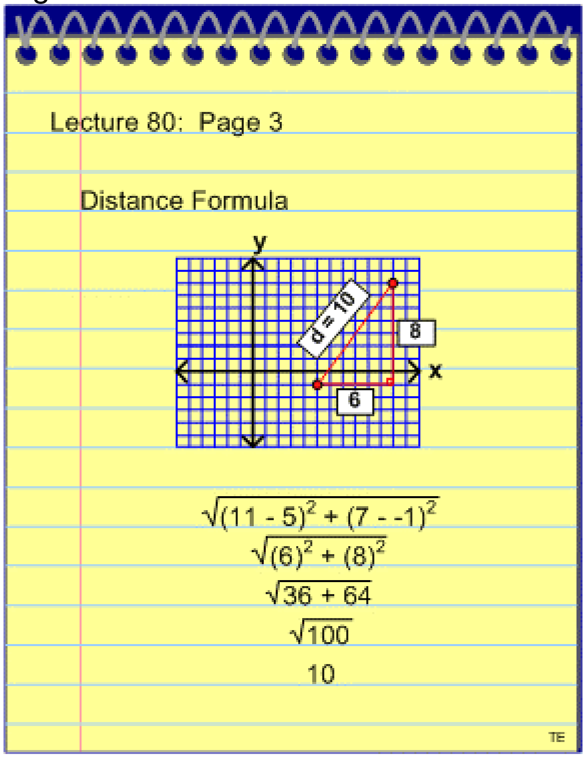
$36 + 64 = d^2$

$\pm\sqrt{100} = \sqrt{d^2}$

$\pm 10 = d$

Distances are positive so the result is 10.

Alg080-03



Lecture 80: Page 3

Distance Formula

$\sqrt{(11 - 5)^2 + (7 - -1)^2}$

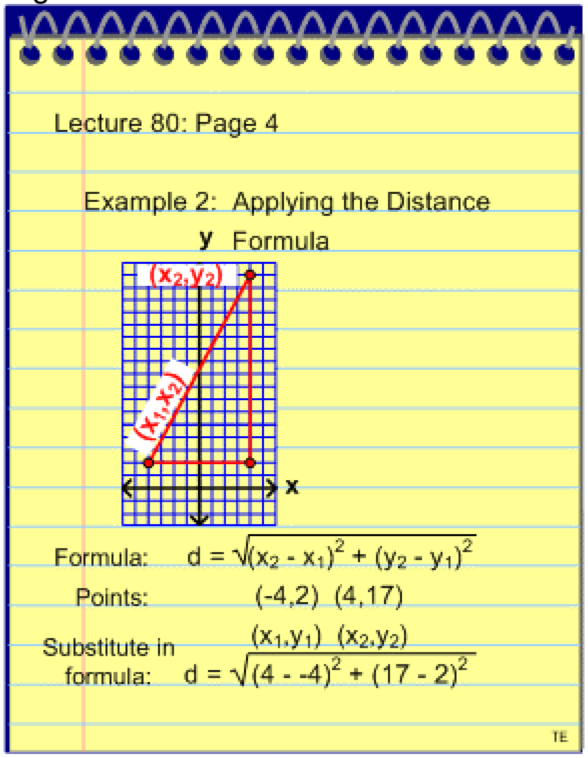
$\sqrt{(6)^2 + (8)^2}$

$\sqrt{36 + 64}$

$\sqrt{100}$

10

Alg080-04



Lecture 80: Page 4

Example 2: Applying the Distance Formula

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

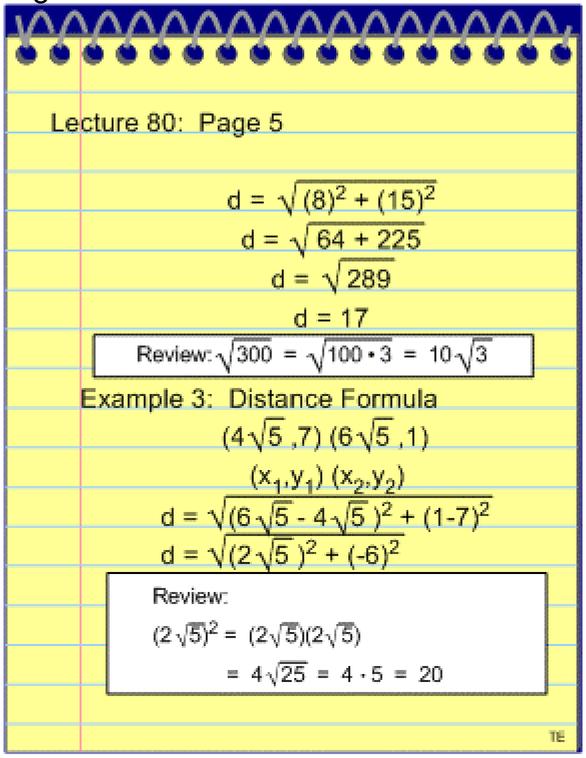
Points: (-4,2) (4,17)

(x_1,y_1) (x_2,y_2)

Substitute in formula: $d = \sqrt{(4 - -4)^2 + (17 - 2)^2}$

Lecture 80 Notes, Continued

Alg080-05



Lecture 80: Page 5

$$d = \sqrt{(8)^2 + (15)^2}$$
$$d = \sqrt{64 + 225}$$
$$d = \sqrt{289}$$
$$d = 17$$

Review: $\sqrt{300} = \sqrt{100 \cdot 3} = 10\sqrt{3}$

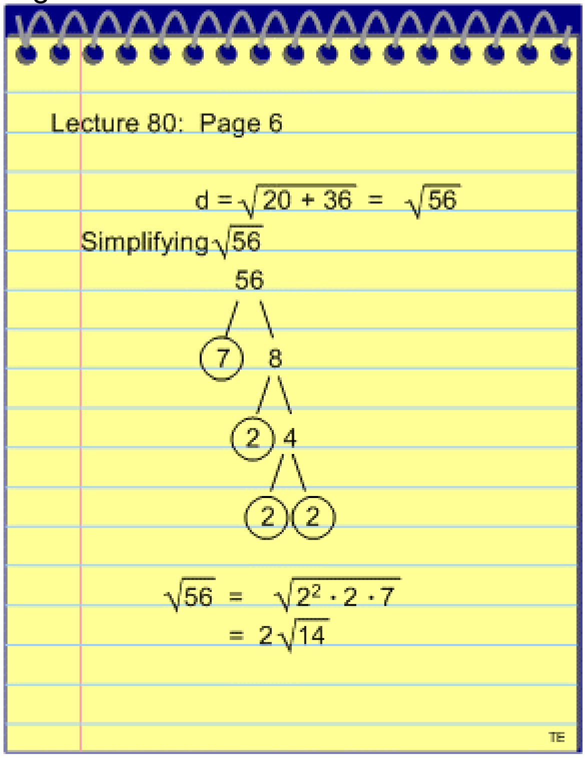
Example 3: Distance Formula

$$(4\sqrt{5},7)\ (6\sqrt{5},1)$$
$$(x_1,y_1)\ (x_2,y_2)$$
$$d = \sqrt{(6\sqrt{5} - 4\sqrt{5})^2 + (1-7)^2}$$
$$d = \sqrt{(2\sqrt{5})^2 + (-6)^2}$$

Review:

$$(2\sqrt{5})^2 = (2\sqrt{5})(2\sqrt{5})$$
$$= 4\sqrt{25} = 4 \cdot 5 = 20$$

Alg080-06



Lecture 80: Page 6

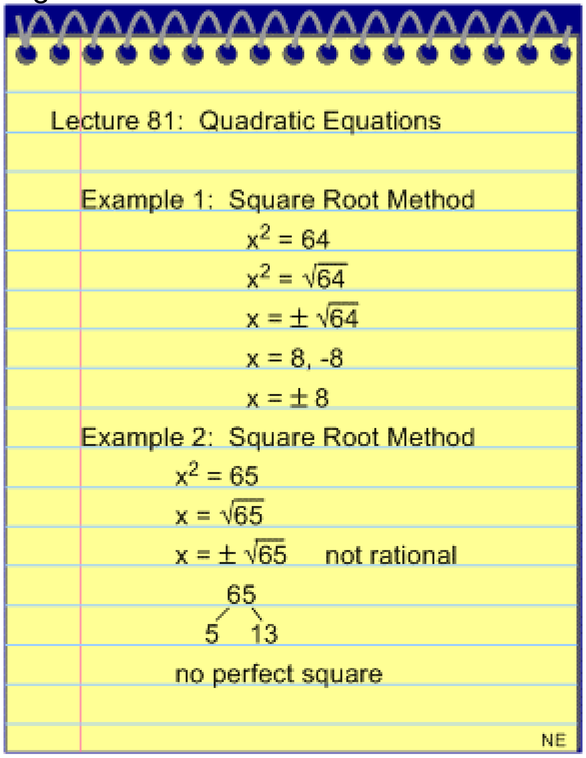
$$d = \sqrt{20 + 36} = \sqrt{56}$$

Simplifying $\sqrt{56}$

$$\sqrt{56} = \sqrt{2^2 \cdot 2 \cdot 7}$$
$$= 2\sqrt{14}$$

Lecture 81 Notes

Alg081-01



Lecture 81: Quadratic Equations

Example 1: Square Root Method

$x^2 = 64$

$x^2 = \sqrt{64}$

$x = \pm\sqrt{64}$

$x = 8, -8$

$x = \pm 8$

Example 2: Square Root Method

$x^2 = 65$

$x = \sqrt{65}$

$x = \pm\sqrt{65}$ not rational

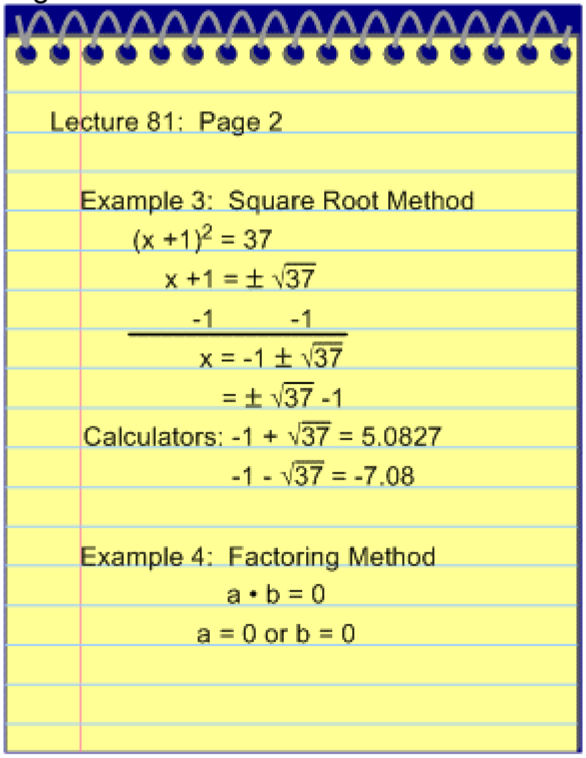
65

5 13

no perfect square

NE

Alg081-02



Lecture 81: Page 2

Example 3: Square Root Method

$(x + 1)^2 = 37$

$x + 1 = \pm\sqrt{37}$

$\underline{\quad -1 \qquad -1 \quad}$

$x = -1 \pm \sqrt{37}$

$= \pm\sqrt{37} - 1$

Calculators: $-1 + \sqrt{37} = 5.0827$

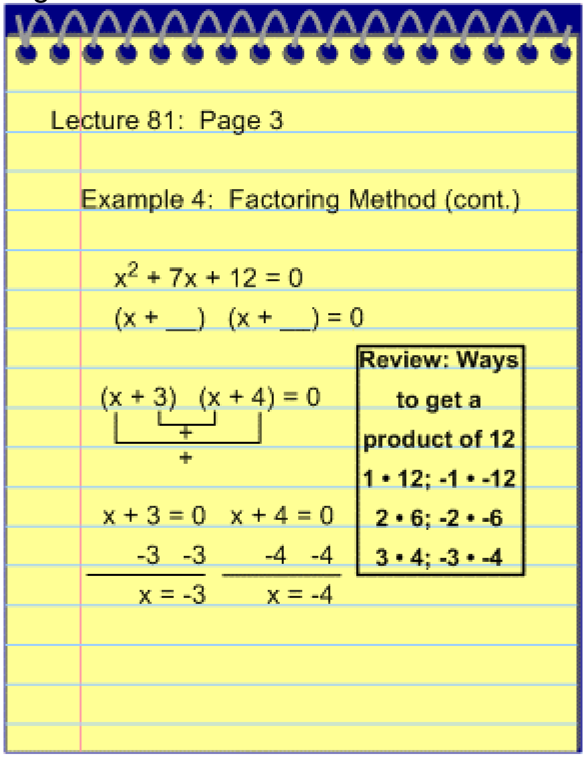
$-1 - \sqrt{37} = -7.08$

Example 4: Factoring Method

$a \cdot b = 0$

$a = 0$ or $b = 0$

Alg081-03



Lecture 81: Page 3

Example 4: Factoring Method (cont.)

$x^2 + 7x + 12 = 0$

$(x + \_\_)\ (x + \_\_) = 0$

$(x + 3)\ (x + 4) = 0$

+

+

$x + 3 = 0$ $x + 4 = 0$

$\underline{\quad -3 \quad -3}$ $\underline{\quad -4 \quad -4}$

$x = -3$ $x = -4$

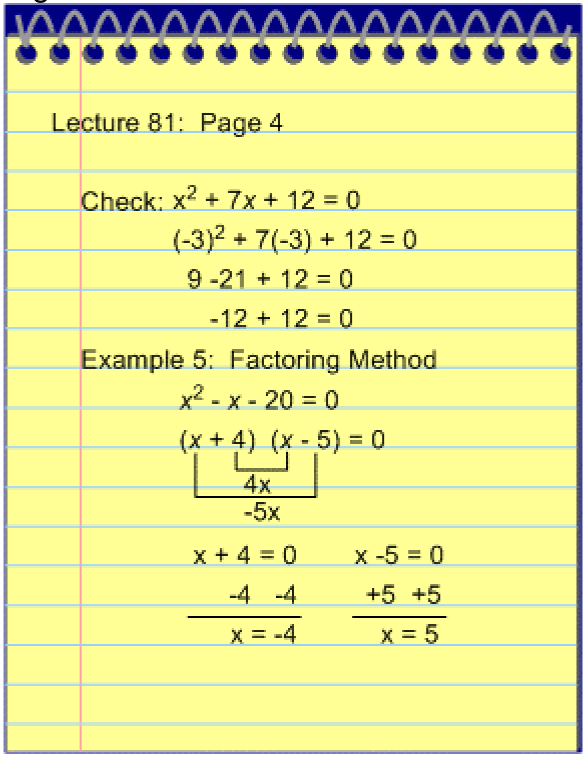
Review: Ways to get a product of 12

1 • 12; -1 • -12

2 • 6; -2 • -6

3 • 4; -3 • -4

Alg081-04



Lecture 81: Page 4

Check: $x^2 + 7x + 12 = 0$

$(-3)^2 + 7(-3) + 12 = 0$

$9 - 21 + 12 = 0$

$-12 + 12 = 0$

Example 5: Factoring Method

$x^2 - x - 20 = 0$

$(x + 4)\ (x - 5) = 0$

4x

-5x

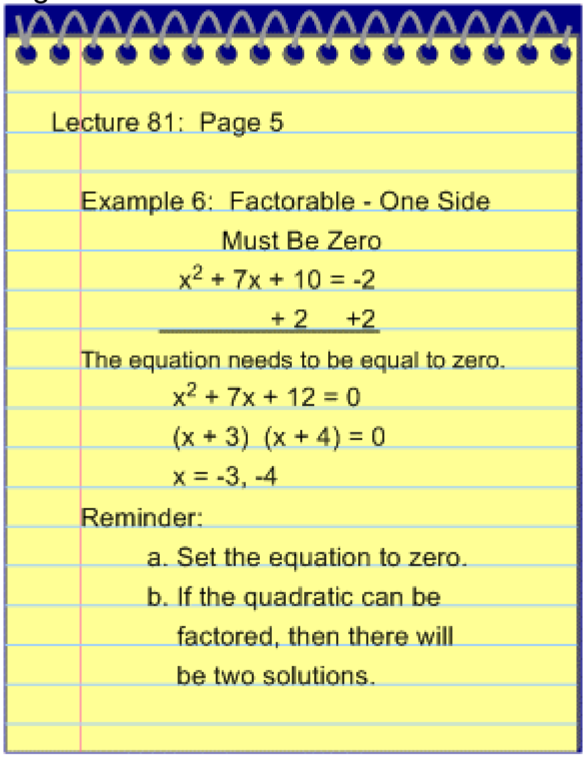
$x + 4 = 0$ $x - 5 = 0$

$\underline{\quad -4 \quad -4}$ $\underline{\quad +5 \quad +5}$

$x = -4$ $x = 5$

Lecture 81 Notes, Continued

Alg081-05



Lecture 81: Page 5

Example 6: Factorable - One Side Must Be Zero

$$x^2 + 7x + 10 = -2$$

$$\underline{\qquad\qquad + 2 \quad +2}$$

The equation needs to be equal to zero.

$$x^2 + 7x + 12 = 0$$

$$(x + 3)\ (x + 4) = 0$$

$$x = -3, -4$$

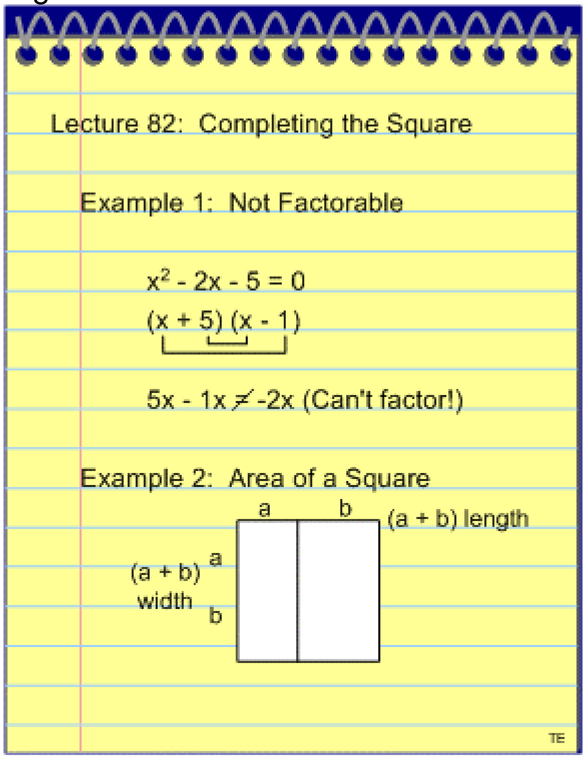
Reminder:

a. Set the equation to zero.

b. If the quadratic can be factored, then there will be two solutions.

Lecture 82 Notes

Alg082-01



Lecture 82: Completing the Square

Example 1: Not Factorable

$x^2 - 2x - 5 = 0$

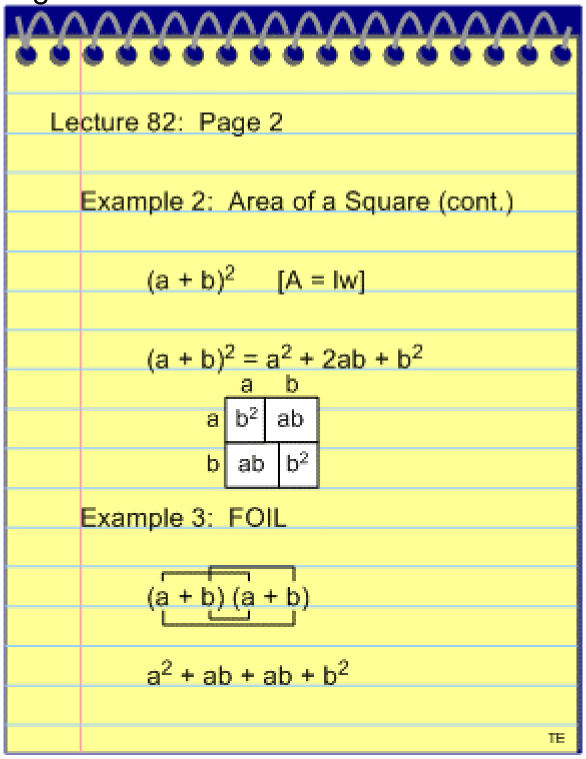
$\underbrace{(x + \underbrace{5)\ (x}_{} - 1)}_{}$

$5x - 1x \neq -2x$ (Can't factor!)

Example 2: Area of a Square

TE

Alg082-02



Lecture 82: Page 2

Example 2: Area of a Square (cont.)

$(a + b)^2$ [A = lw]

$(a + b)^2 = a^2 + 2ab + b^2$

	a	b
a	b^2	ab
b	ab	b^2

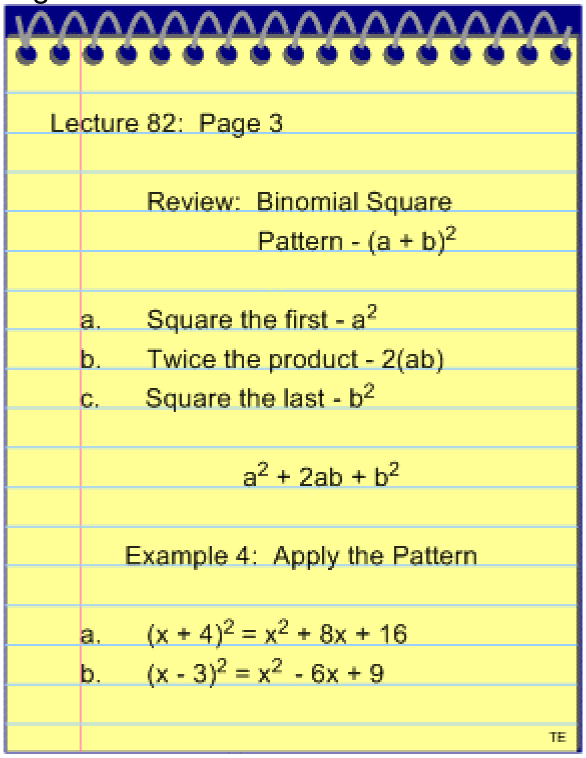
Example 3: FOIL

$(a + b)\ (a + b)$

$a^2 + ab + ab + b^2$

TE

Alg082-03



Lecture 82: Page 3

Review: Binomial Square Pattern - $(a + b)^2$

a. Square the first - a^2
b. Twice the product - 2(ab)
c. Square the last - b^2

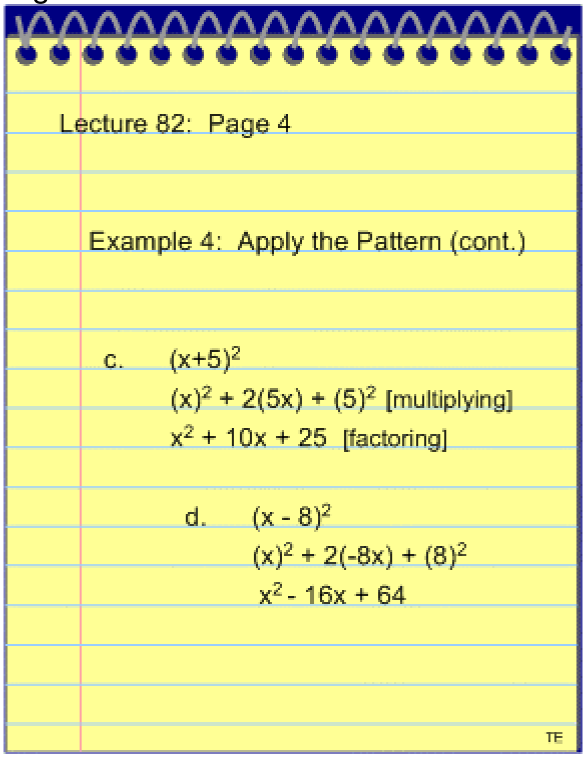
$a^2 + 2ab + b^2$

Example 4: Apply the Pattern

a. $(x + 4)^2 = x^2 + 8x + 16$
b. $(x - 3)^2 = x^2 - 6x + 9$

TE

Alg082-04



Lecture 82: Page 4

Example 4: Apply the Pattern (cont.)

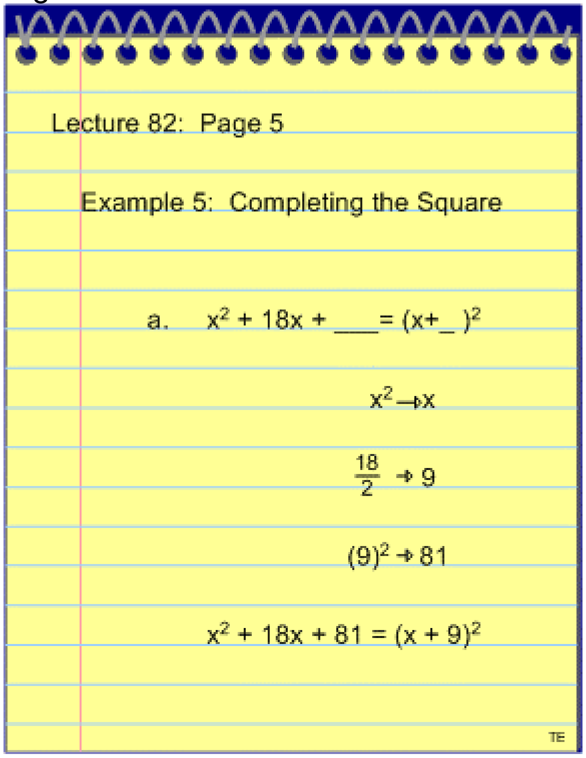
c. $(x+5)^2$
$(x)^2 + 2(5x) + (5)^2$ [multiplying]
$x^2 + 10x + 25$ [factoring]

d. $(x - 8)^2$
$(x)^2 + 2(-8x) + (8)^2$
$x^2 - 16x + 64$

TE

Lecture 82 Notes, Continued

Alg082-05



Lecture 82: Page 5

Example 5: Completing the Square

a. $x^2 + 18x + \_\_\_ = (x + \_)^2$

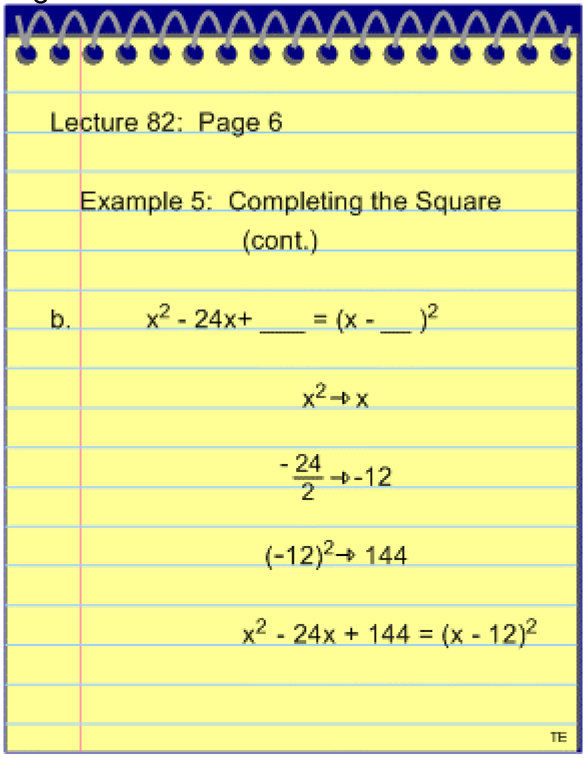
$x^2 \rightarrow x$

$\frac{18}{2} \rightarrow 9$

$(9)^2 \rightarrow 81$

$x^2 + 18x + 81 = (x + 9)^2$

Alg082-06



Lecture 82: Page 6

Example 5: Completing the Square (cont.)

b. $x^2 - 24x + \_\_\_ = (x - \_\_)^2$

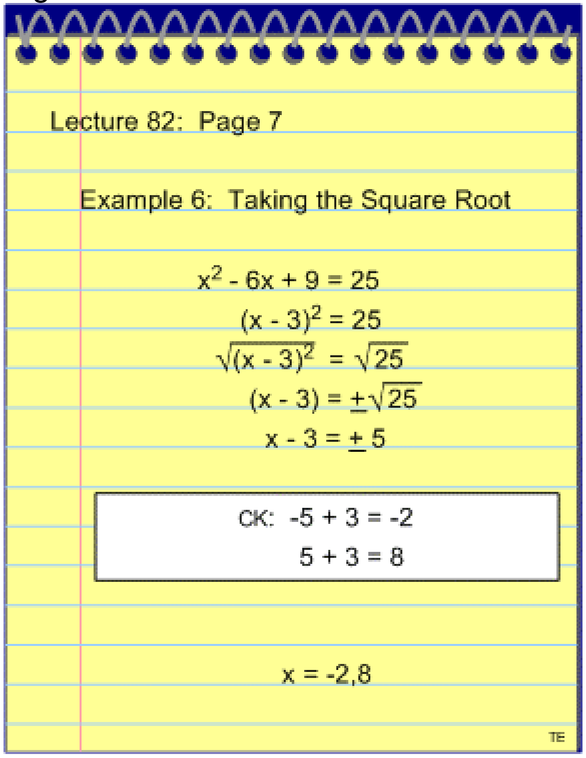
$x^2 \rightarrow x$

$-\frac{24}{2} \rightarrow -12$

$(-12)^2 \rightarrow 144$

$x^2 - 24x + 144 = (x - 12)^2$

Alg082-07



Lecture 82: Page 7

Example 6: Taking the Square Root

$x^2 - 6x + 9 = 25$

$(x - 3)^2 = 25$

$\sqrt{(x - 3)^2} = \sqrt{25}$

$(x - 3) = \pm\sqrt{25}$

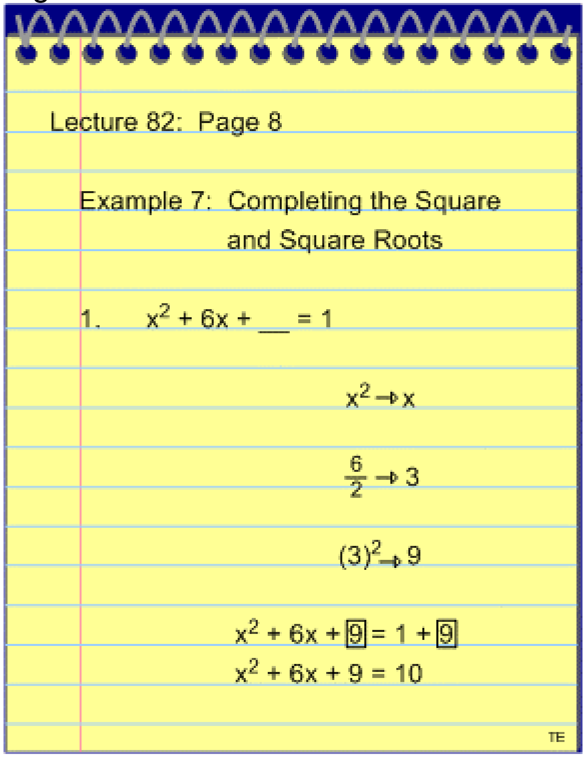
$x - 3 = \pm 5$

CK: $-5 + 3 = -2$

$5 + 3 = 8$

$x = -2, 8$

Alg082-08



Lecture 82: Page 8

Example 7: Completing the Square and Square Roots

1. $x^2 + 6x + \_\_ = 1$

$x^2 \rightarrow x$

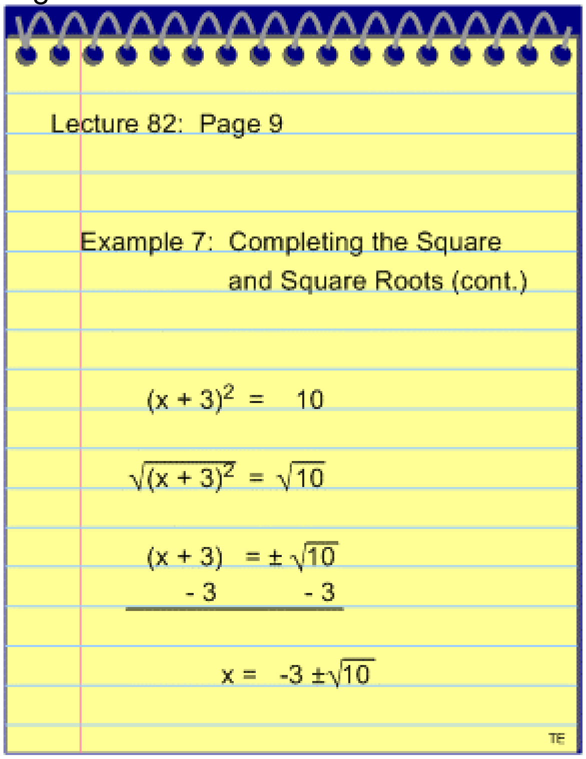
$\frac{6}{2} \rightarrow 3$

$(3)^2 \rightarrow 9$

$x^2 + 6x + \boxed{9} = 1 + \boxed{9}$

$x^2 + 6x + 9 = 10$

Alg082-09



Lecture 82: Page 9

Example 7: Completing the Square and Square Roots (cont.)

$(x + 3)^2 = 10$

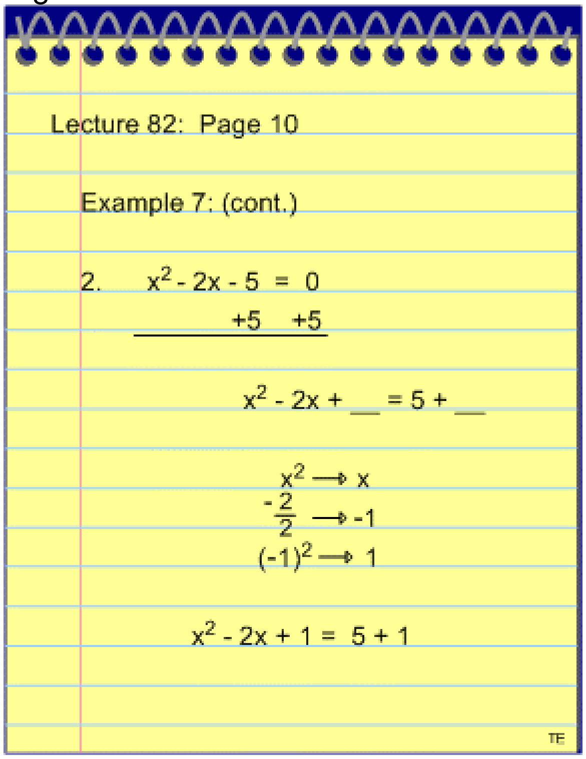
$\sqrt{(x + 3)^2} = \sqrt{10}$

$(x + 3) = \pm\sqrt{10}$

$\underline{\quad - 3 \qquad\quad - 3}$

$x = -3 \pm\sqrt{10}$

Alg082-10



Lecture 82: Page 10

Example 7: (cont.)

2. $x^2 - 2x - 5 = 0$

$\underline{\qquad +5 \quad +5}$

$x^2 - 2x + \_\_ = 5 + \_\_$

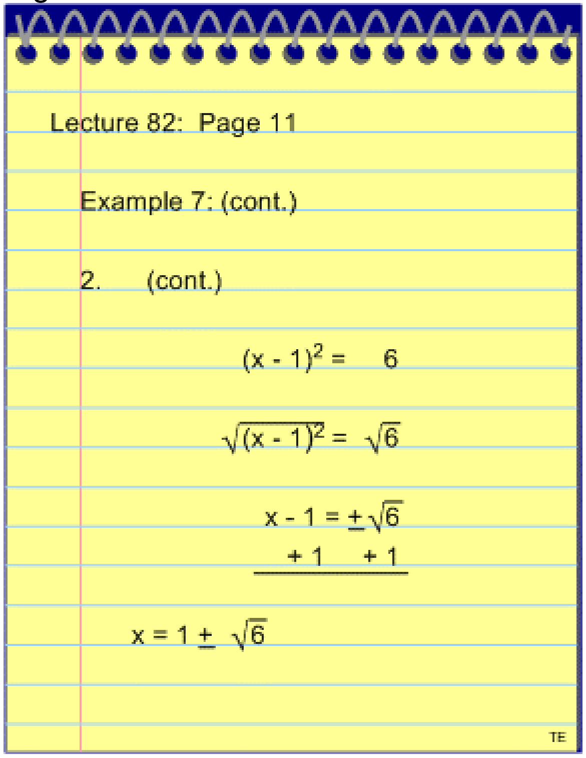
$x^2 \longrightarrow x$

$-\frac{2}{2} \longrightarrow -1$

$(-1)^2 \longrightarrow 1$

$x^2 - 2x + 1 = 5 + 1$

Alg082-11



Lecture 82: Page 11

Example 7: (cont.)

2. (cont.)

$(x - 1)^2 = 6$

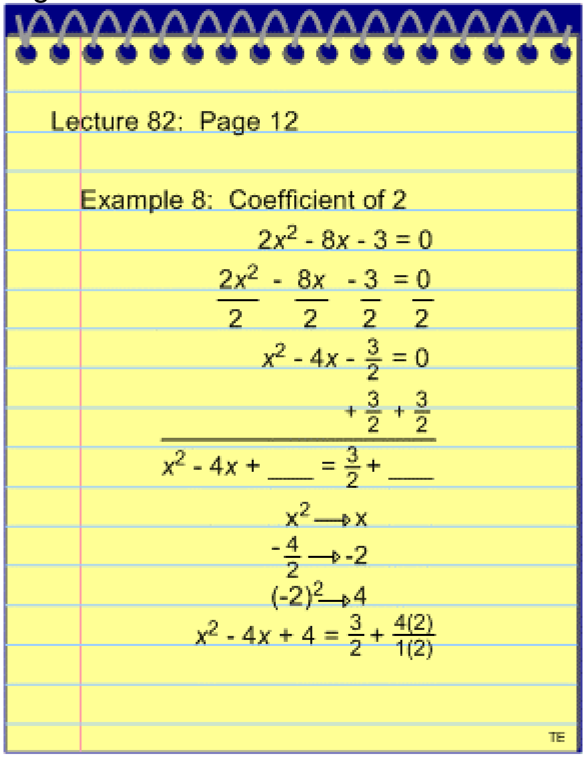
$\sqrt{(x - 1)^2} = \sqrt{6}$

$x - 1 = \pm\sqrt{6}$

$\underline{\quad + 1 \quad + 1}$

$x = 1 \pm \sqrt{6}$

Alg082-12



Lecture 82: Page 12

Example 8: Coefficient of 2

$2x^2 - 8x - 3 = 0$

$\frac{2x^2}{2} - \frac{8x}{2} - \frac{3}{2} = \frac{0}{2}$

$x^2 - 4x - \frac{3}{2} = 0$

$\underline{\qquad\qquad + \frac{3}{2} + \frac{3}{2}}$

$x^2 - 4x + \_\_\_\_ = \frac{3}{2} + \_\_\_\_$

$x^2 \longrightarrow x$

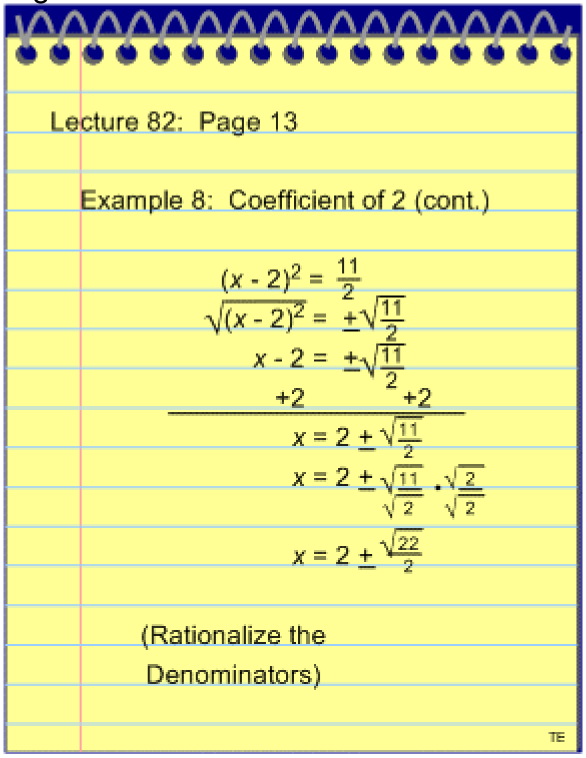
$-\frac{4}{2} \longrightarrow -2$

$(-2)^2 \longrightarrow 4$

$x^2 - 4x + 4 = \frac{3}{2} + \frac{4(2)}{1(2)}$

Lecture 82 Notes, Continued

Alg082-13



Lecture 82: Page 13

Example 8: Coefficient of 2 (cont.)

$$(x - 2)^2 = \frac{11}{2}$$

$$\sqrt{(x - 2)^2} = \pm\sqrt{\frac{11}{2}}$$

$$x - 2 = \pm\sqrt{\frac{11}{2}}$$

$$\underline{\quad +2 \qquad\qquad +2 \quad}$$

$$x = 2 \pm \sqrt{\frac{11}{2}}$$

$$x = 2 \pm \frac{\sqrt{11}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

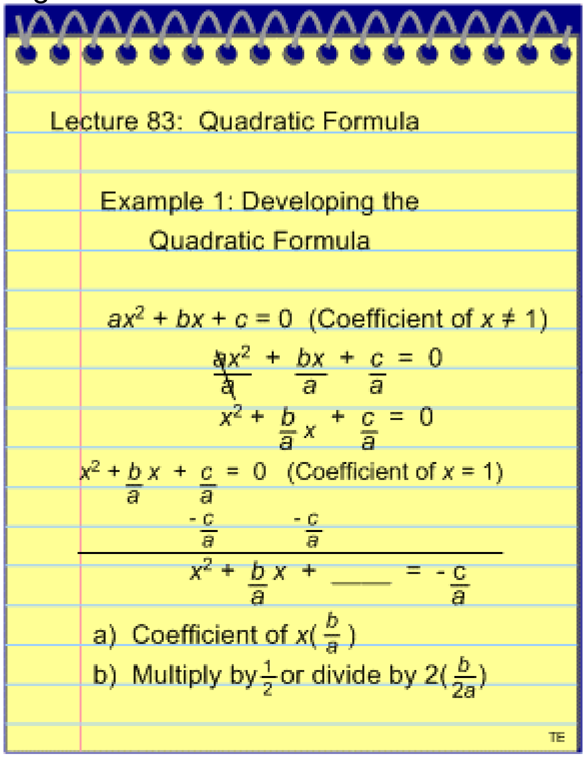
$$x = 2 \pm \frac{\sqrt{22}}{2}$$

(Rationalize the
Denominators)

TE

Lecture 83 Notes

Alg083-01



Lecture 83: Quadratic Formula

Example 1: Developing the Quadratic Formula

$ax^2 + bx + c = 0$ (Coefficient of $x \neq 1$)

$$\frac{\cancel{a}x^2}{\cancel{a}} + \frac{bx}{a} + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \text{(Coefficient of } x = 1\text{)}$$

$$-\frac{c}{a} \qquad -\frac{c}{a}$$

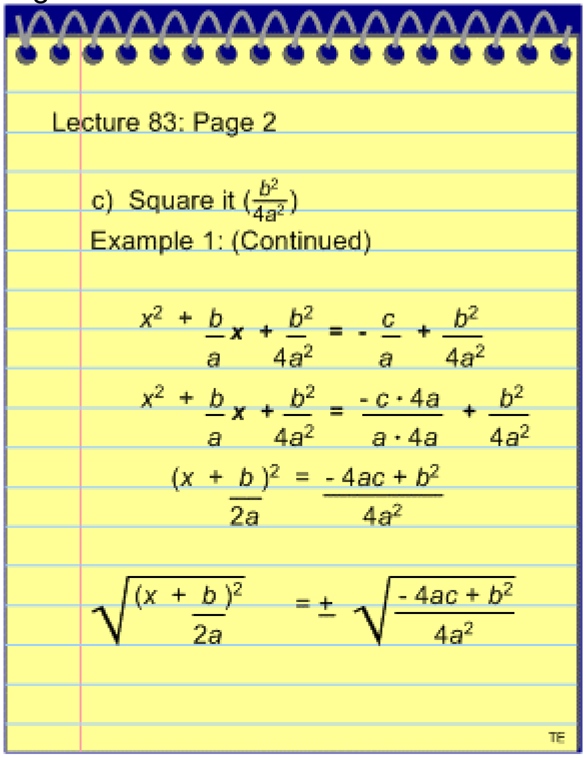
$$x^2 + \frac{b}{a}x + \_\_\_\_ = -\frac{c}{a}$$

a) Coefficient of $x\left(\frac{b}{a}\right)$

b) Multiply by $\frac{1}{2}$ or divide by $2\left(\frac{b}{2a}\right)$

TE

Alg083-02



Lecture 83: Page 2

c) Square it $\left(\frac{b^2}{4a^2}\right)$

Example 1: (Continued)

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

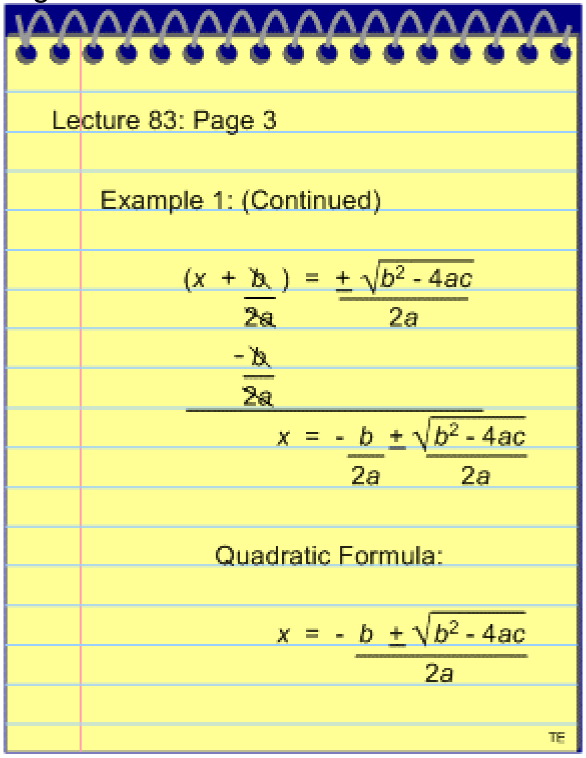
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c \cdot 4a}{a \cdot 4a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$$

TE

Alg083-03



Lecture 83: Page 3

Example 1: (Continued)

$$\left(x + \frac{\cancel{b}}{\cancel{2a}}\right) = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{\cancel{b}}{\cancel{2a}}$$

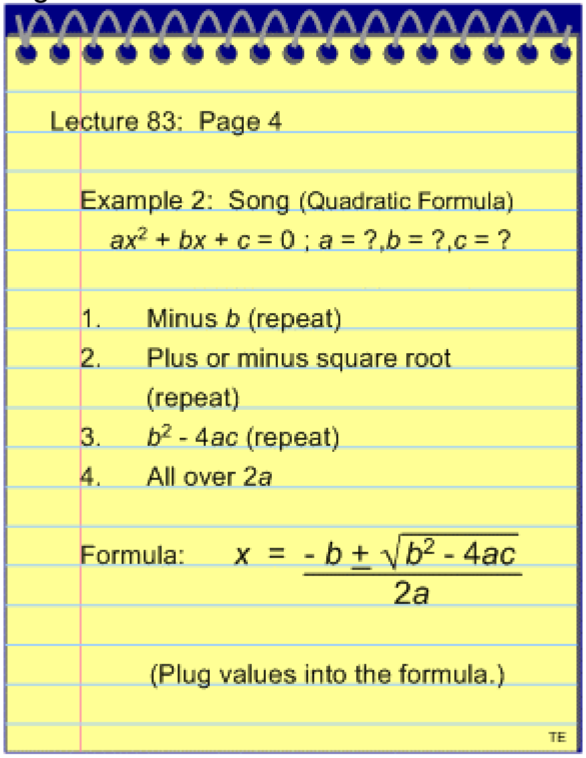
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

TE

Alg083-04



Lecture 83: Page 4

Example 2: Song (Quadratic Formula)

$ax^2 + bx + c = 0$; $a = ?, b = ?, c = ?$

1. Minus b (repeat)
2. Plus or minus square root (repeat)
3. $b^2 - 4ac$ (repeat)
4. All over $2a$

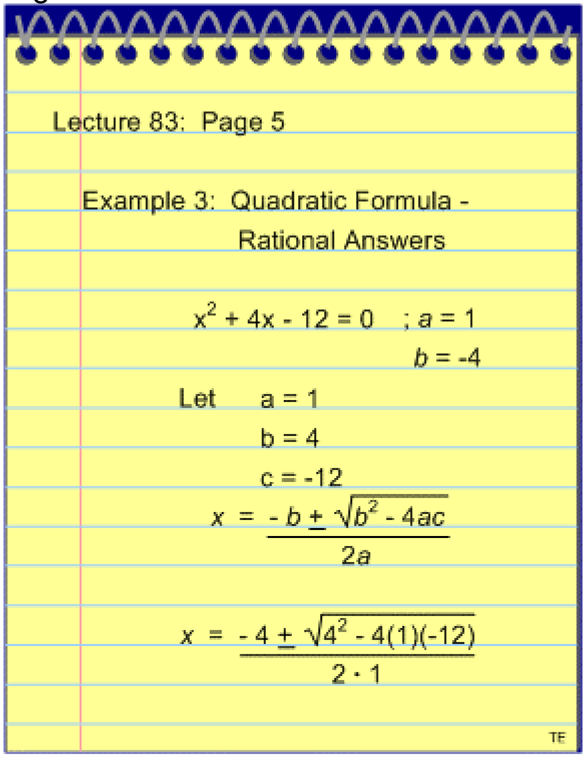
Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(Plug values into the formula.)

TE

Lecture 83 Notes, Continued

Alg083-05



Lecture 83: Page 5

Example 3: Quadratic Formula - Rational Answers

$x^2 + 4x - 12 = 0 \quad ; a = 1$

$b = -4$

Let a = 1

b = 4

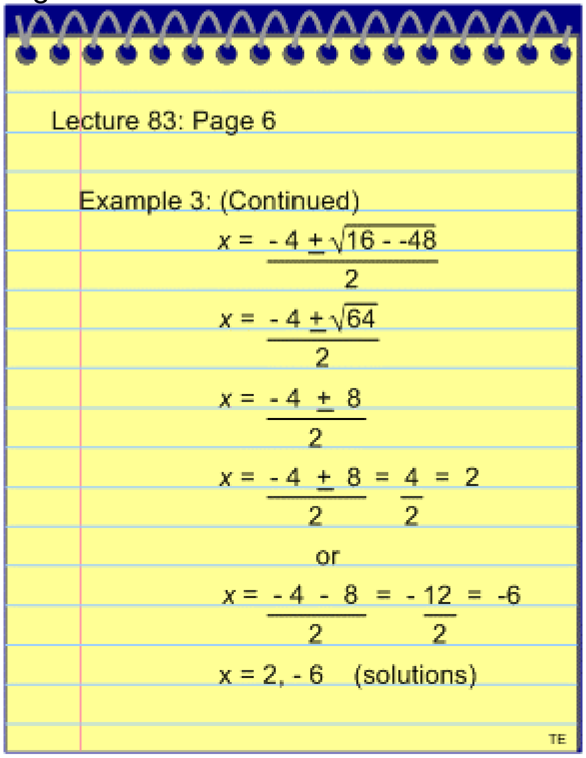
c = -12

$$x = \frac{-b \underline{+} \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \underline{+} \sqrt{4^2 - 4(1)(-12)}}{2 \cdot 1}$$

TE

Alg083-06



Lecture 83: Page 6

Example 3: (Continued)

$$x = \frac{-4 \underline{+} \sqrt{16 - -48}}{2}$$

$$x = \frac{-4 \underline{+} \sqrt{64}}{2}$$

$$x = \frac{-4 \underline{+} 8}{2}$$

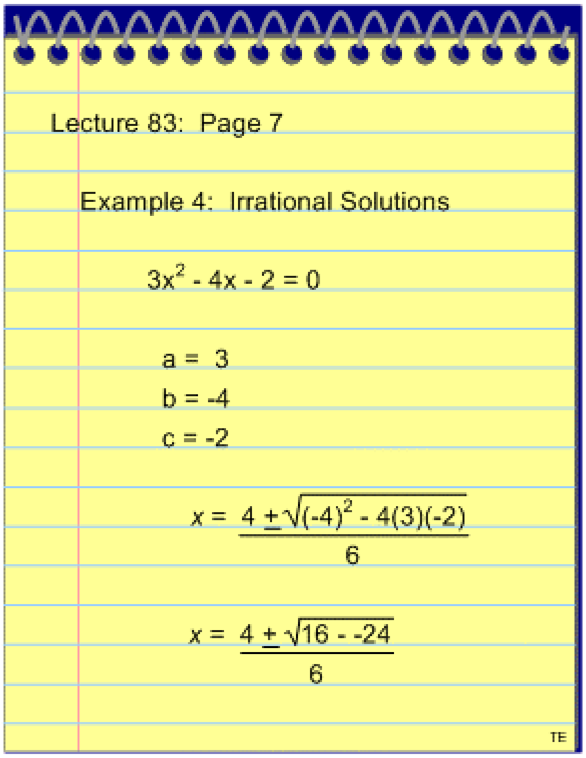
$$x = \frac{-4 + 8}{2} = \frac{4}{2} = 2$$

or

$$x = \frac{-4 - 8}{2} = \frac{-12}{2} = -6$$

x = 2, - 6 (solutions)

TE



Lecture 83: Page 7

Example 4: Irrational Solutions

$3x^2 - 4x - 2 = 0$

a = 3

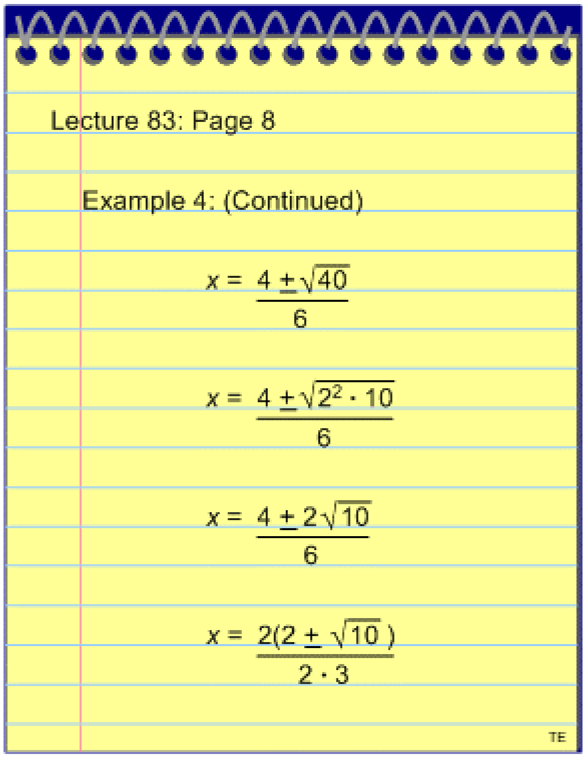
b = -4

c = -2

$$x = \frac{4 \underline{+} \sqrt{(-4)^2 - 4(3)(-2)}}{6}$$

$$x = \frac{4 \underline{+} \sqrt{16 - -24}}{6}$$

TE



Lecture 83: Page 8

Example 4: (Continued)

$$x = \frac{4 \underline{+} \sqrt{40}}{6}$$

$$x = \frac{4 \underline{+} \sqrt{2^2 \cdot 10}}{6}$$

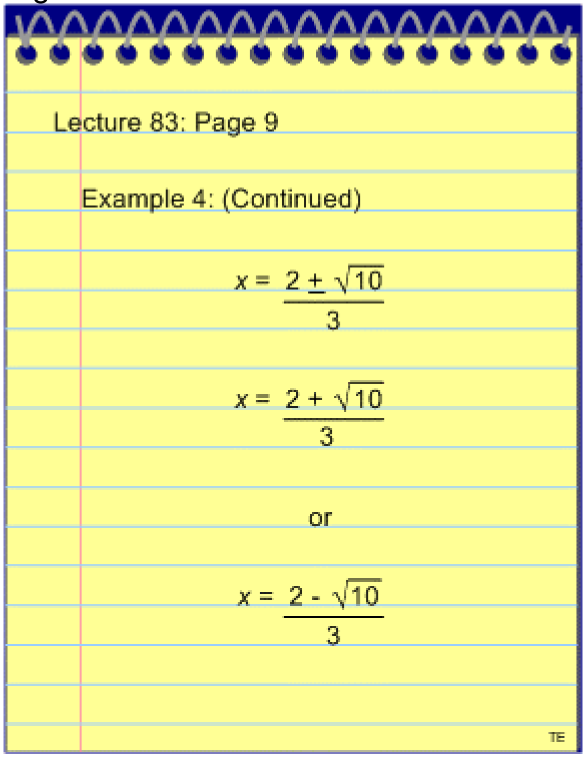
$$x = \frac{4 \underline{+} 2\sqrt{10}}{6}$$

$$x = \frac{2(2 \underline{+} \sqrt{10})}{2 \cdot 3}$$

TE

Lecture 83 Notes, Continued

Alg083-09



Lecture 83: Page 9

Example 4: (Continued)

$$x = \frac{2 \pm \sqrt{10}}{3}$$

$$x = \frac{2 + \sqrt{10}}{3}$$

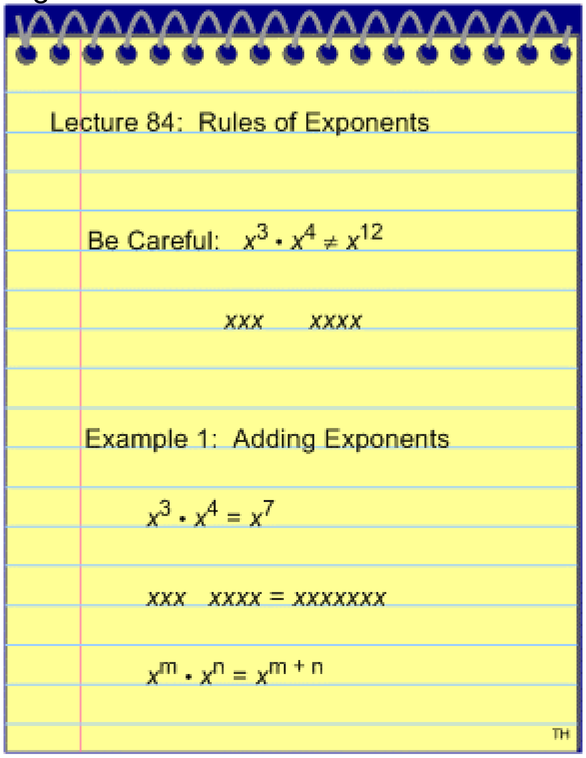
or

$$x = \frac{2 - \sqrt{10}}{3}$$

TE

Lecture 84 Notes

Alg084-01



Lecture 84: Rules of Exponents

Be Careful: $x^3 \cdot x^4 \neq x^{12}$

$xxx \quad xxxx$

Example 1: Adding Exponents

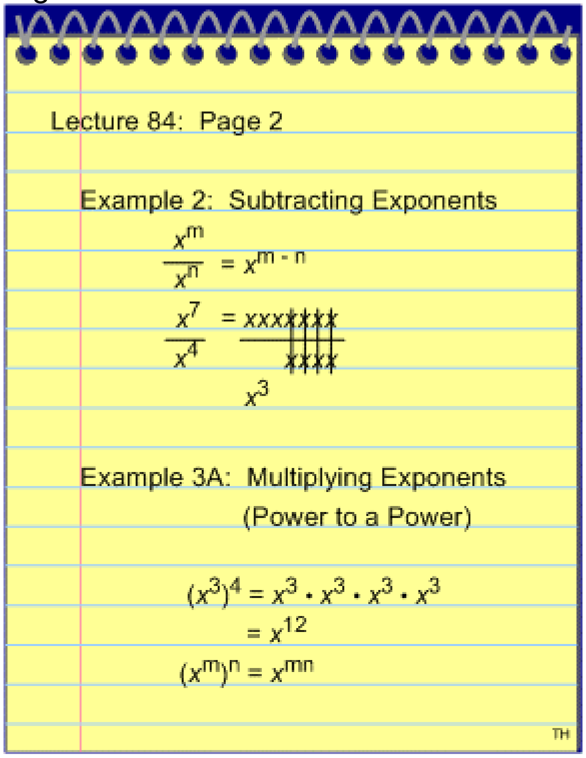
$x^3 \cdot x^4 = x^7$

$xxx \quad xxxx = xxxxxxx$

$x^m \cdot x^n = x^{m+n}$

TH

Alg084-02



Lecture 84: Page 2

Example 2: Subtracting Exponents

$$\frac{x^m}{x^n} = x^{m-n}$$

$$\frac{x^7}{x^4} = \frac{xxx\cancel{xxxx}}{\cancel{xxxx}}$$

$$x^3$$

Example 3A: Multiplying Exponents

(Power to a Power)

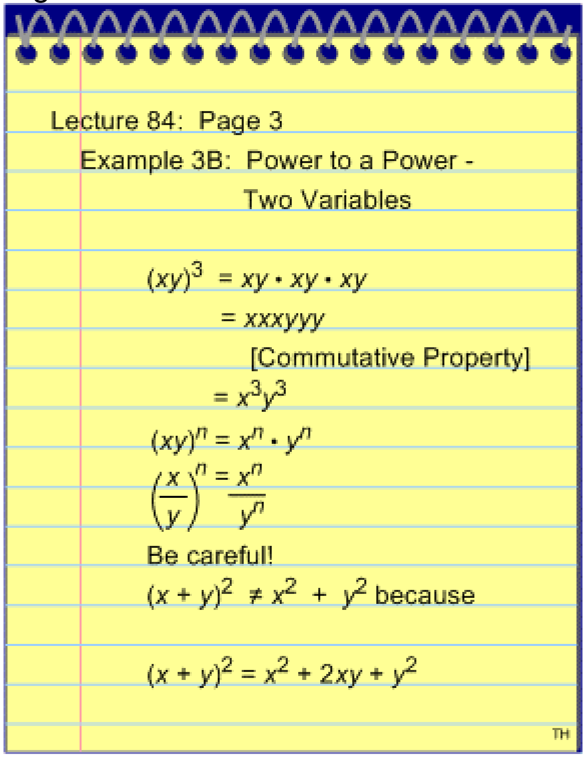
$(x^3)^4 = x^3 \cdot x^3 \cdot x^3 \cdot x^3$

$= x^{12}$

$(x^m)^n = x^{mn}$

TH

Alg084-03



Lecture 84: Page 3

Example 3B: Power to a Power - Two Variables

$(xy)^3 = xy \cdot xy \cdot xy$

$= xxxyyy$

[Commutative Property]

$= x^3y^3$

$(xy)^n = x^n \cdot y^n$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

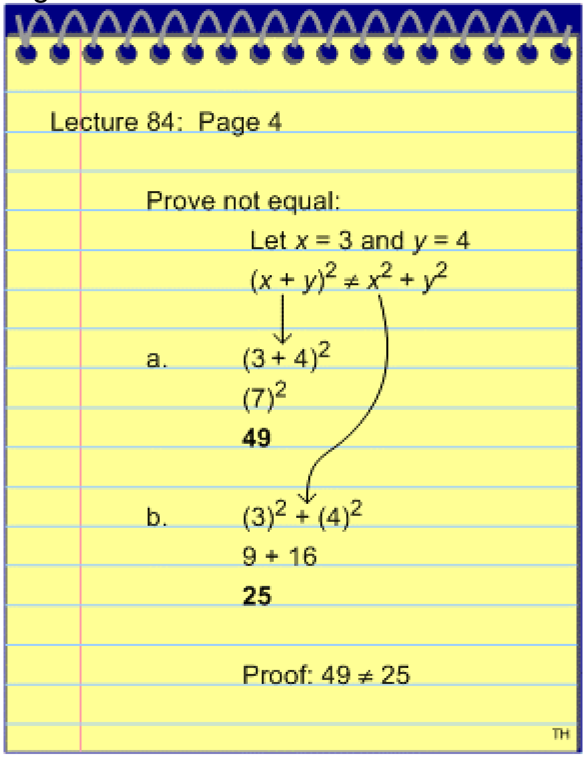
Be careful!

$(x + y)^2 \neq x^2 + y^2$ because

$(x + y)^2 = x^2 + 2xy + y^2$

TH

Alg084-04



Lecture 84: Page 4

Prove not equal:

Let $x = 3$ and $y = 4$

$(x + y)^2 \neq x^2 + y^2$

a. $(3 + 4)^2$

$(7)^2$

49

b. $(3)^2 + (4)^2$

$9 + 16$

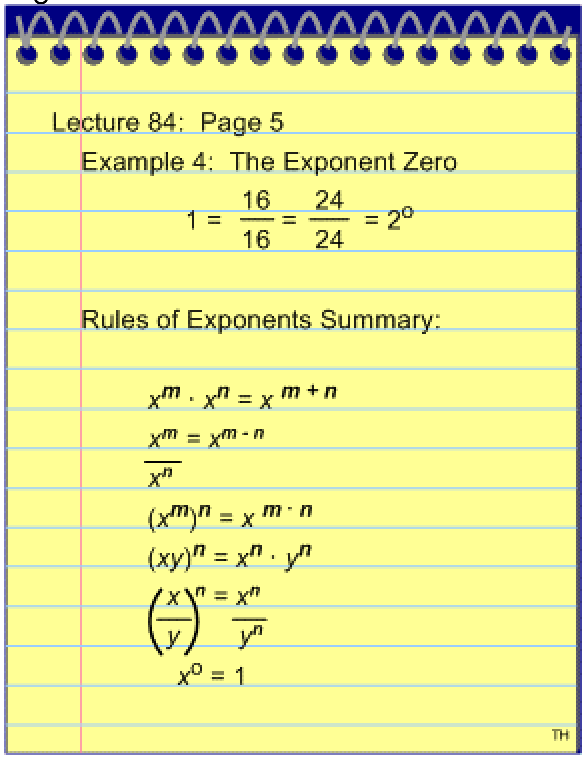
25

Proof: $49 \neq 25$

TH

Lecture 84 Notes, Continued

Alg084-05



Lecture 84: Page 5

Example 4: The Exponent Zero

$$1 = \frac{16}{16} = \frac{24}{24} = 2^0$$

Rules of Exponents Summary:

$$x^m \cdot x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{m \cdot n}$$

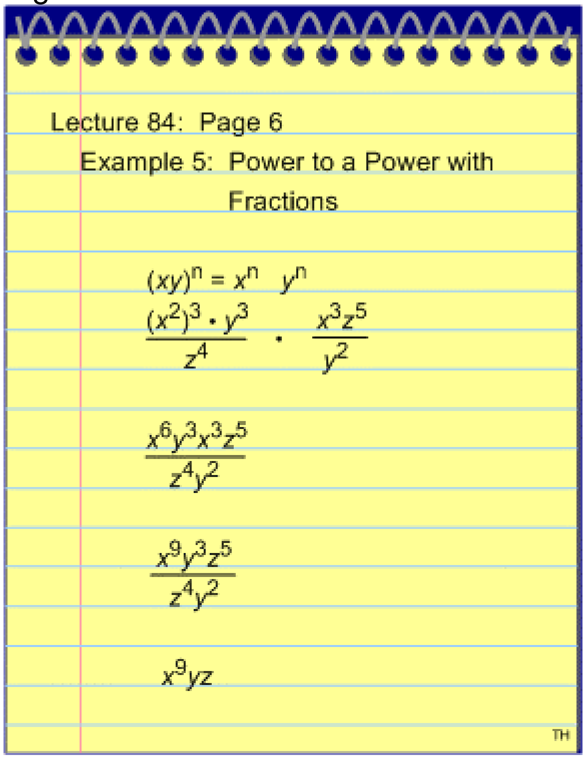
$$(xy)^n = x^n \cdot y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^0 = 1$$

TH

Alg084-06



Lecture 84: Page 6

Example 5: Power to a Power with Fractions

$$(xy)^n = x^n \quad y^n$$

$$\frac{(x^2)^3 \cdot y^3}{z^4} \cdot \frac{x^3z^5}{y^2}$$

$$\frac{x^6y^3x^3z^5}{z^4y^2}$$

$$\frac{x^9y^3z^5}{z^4y^2}$$

$$x^9yz$$

TH